MIT CSAIL

6.869: Advances in Computer Vision

Antonio Torralba, 2013

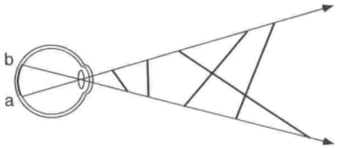
MIT COMPUTER VISION

Lecture 12

Stereo, Shape from X

Depth Perception:

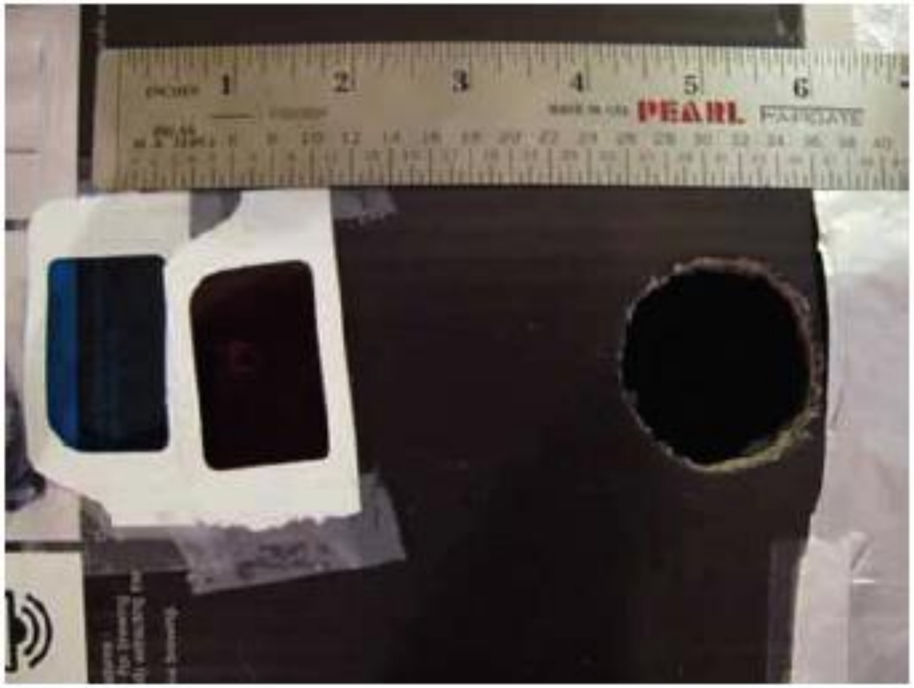
The inverse problem



Monocular cues to depth

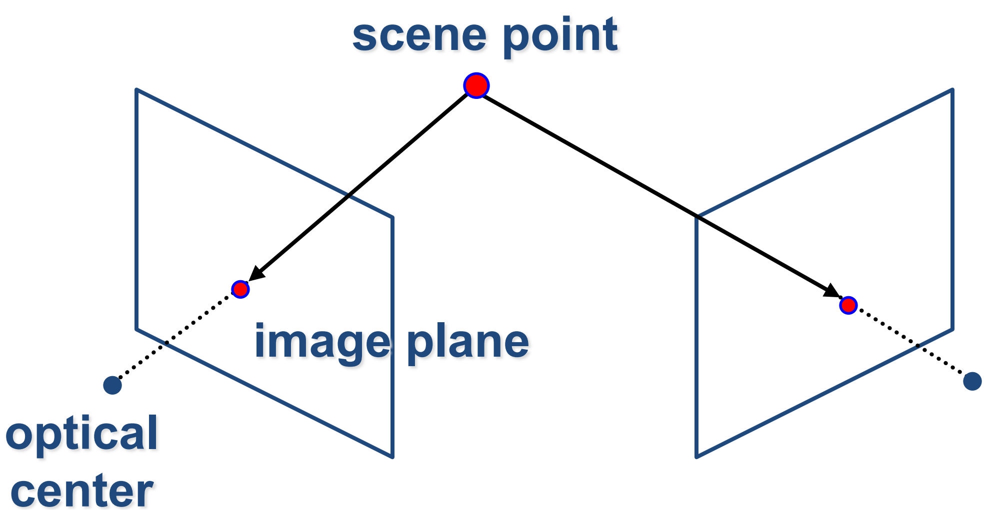
- **Absolute depth cues**: (assuming known camera parameters) these cues provide information about the absolute depth between the observer and elements of the scene
- **Relative depth cues**: provide relative information about depth between elements in the scene (this point is twice as far at that point, …)

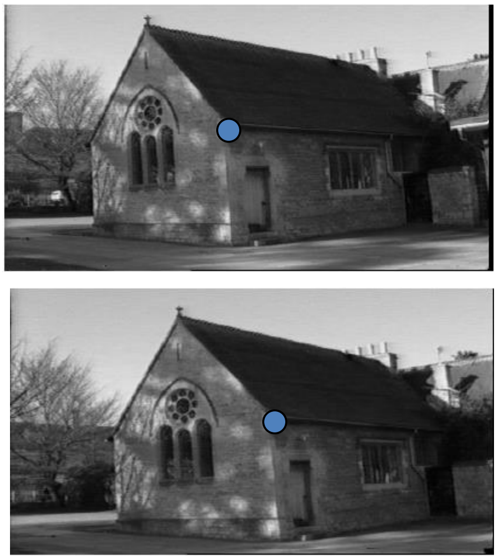
Anaglyph pinhole camera



Estimating depth with stereo

- **Stereo**: shape from disparities between two views
- We'll need to consider:
  - Info on camera pose ("calibration")
  - Image point correspondences

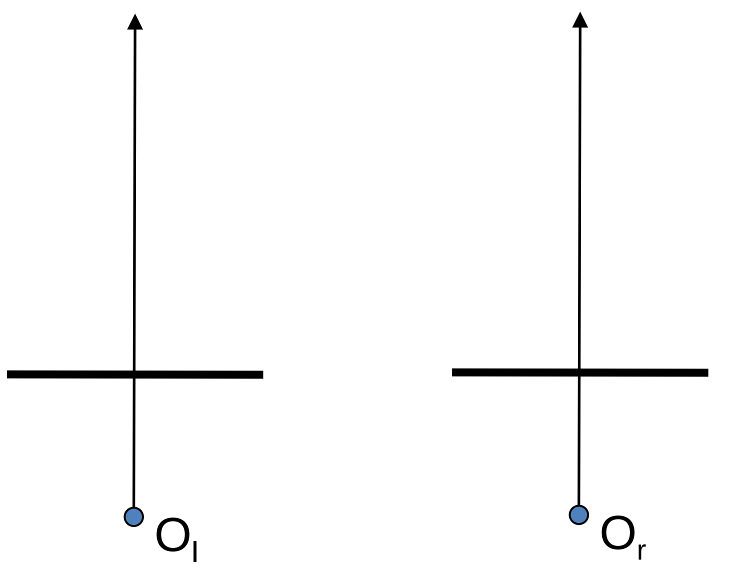


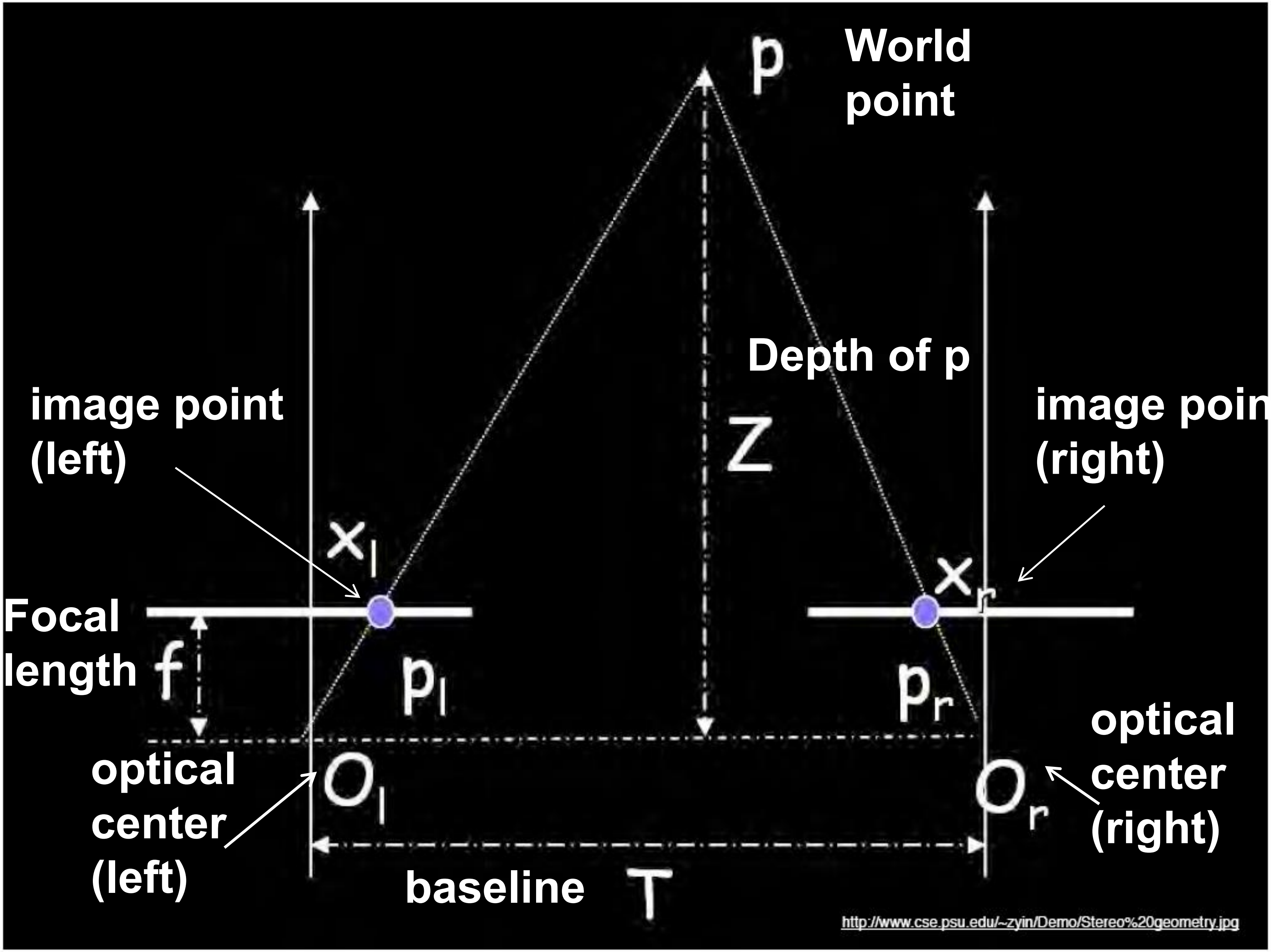


Slide credit: Kristen Grauman

Geometry for a simple stereo system

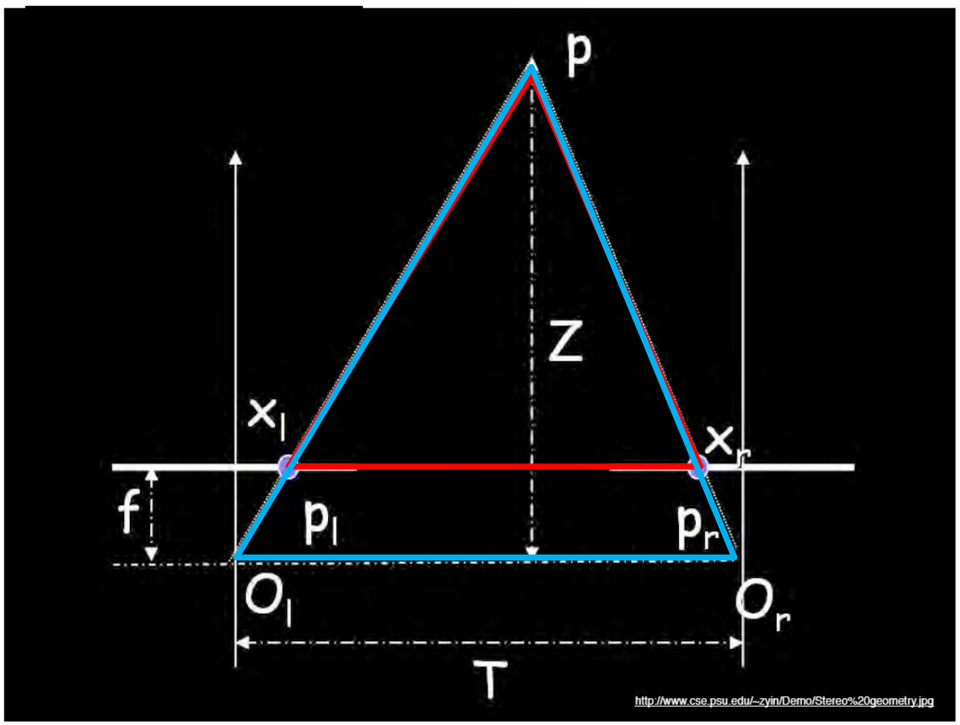
- Assume a simple setting:
  - Two identical cameras
  - parallel optical axes
  - known camera parameters (i.e., calibrated cameras).

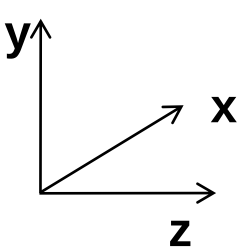



p
World point
Depth of p
Z
image point (left)
image poin (right)
x_l
x_r
Focal length
f
p_l
p_r
optical center (left)
O_l
O_r
optical center (right)
baseline
T
http://www.cse.psu.edu/~zyin/Demo/Stereo%20geometry.jpg

Geometry for a simple stereo system

- Assume parallel optical axes, known camera parameters (i.e., calibrated cameras). We can triangulate via:





Similar triangles (p_l, P, p_r) and (O_l, P, O_r):

$$\frac{T + x_l - x_r}{Z - f} = \frac{T}{Z}$$

$$Z = f \frac{T}{x_r - x_l}$$

disparity

Slide credit: Kristen Grauman

Depth from disparity

image I(x,y)

Disparity map D(x,y)

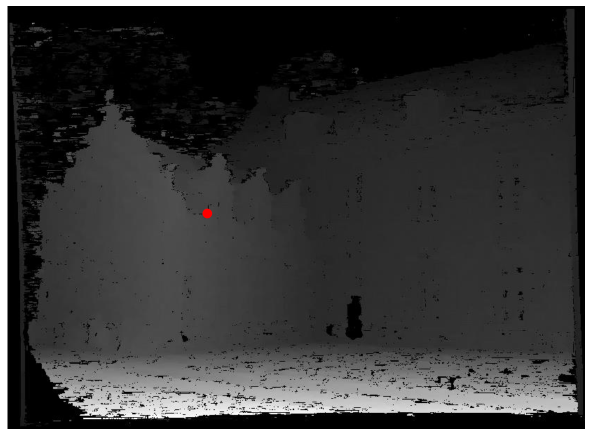


image I´(x´,y´)

$$(x',y') = (x + D(x,y),\ y)$$

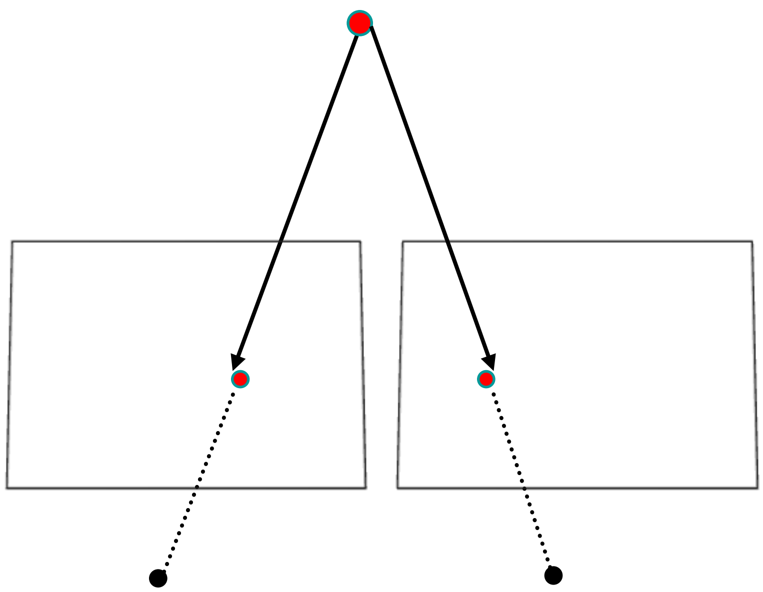
Slide credit: Kristen Grauman

Stereo Topics

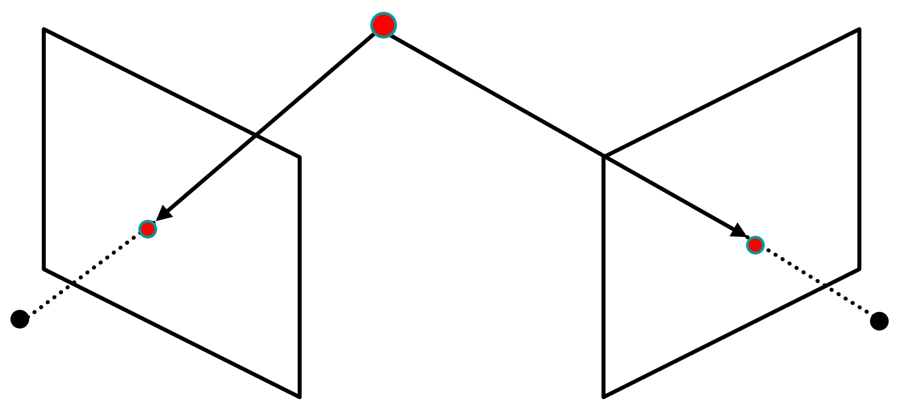
- Special, simple system, main idea
- **More general camera conditions, epipolar constraints**
  - **epipolar geometry**
  - epipolar algebra
- Image rectification
- Stereo matching (likelihood term)
- Stereo regularization (prior term)
- Inference
  - dynamic programming
  - graph cuts
- Structured light

General case, with calibrated cameras

- The two cameras need not have parallel optical axes.

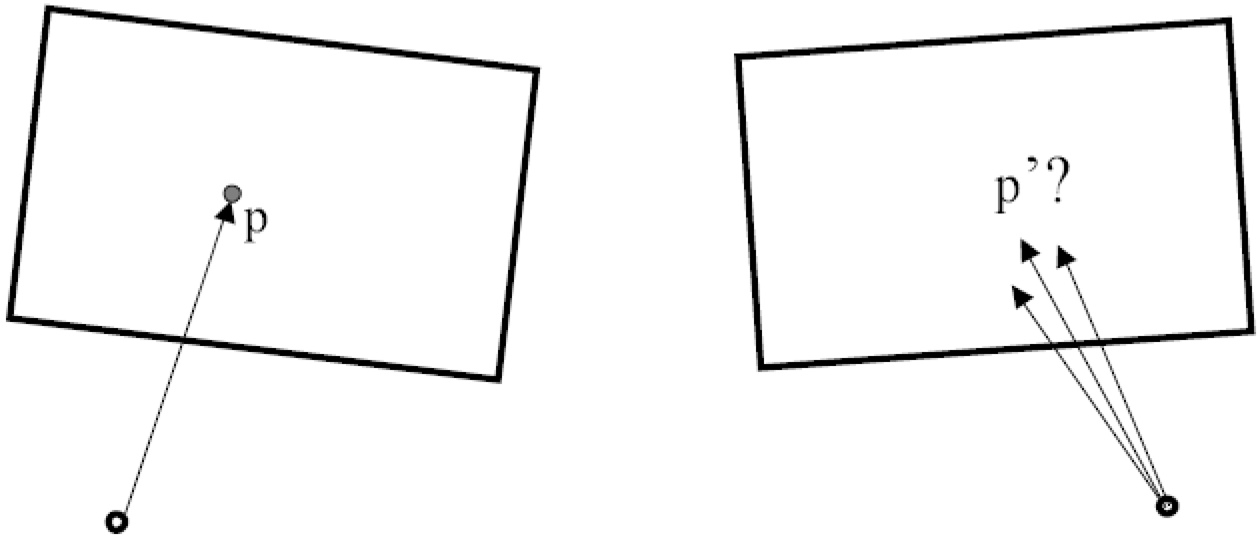


Vs.



Slide credit: Kristen Grauman

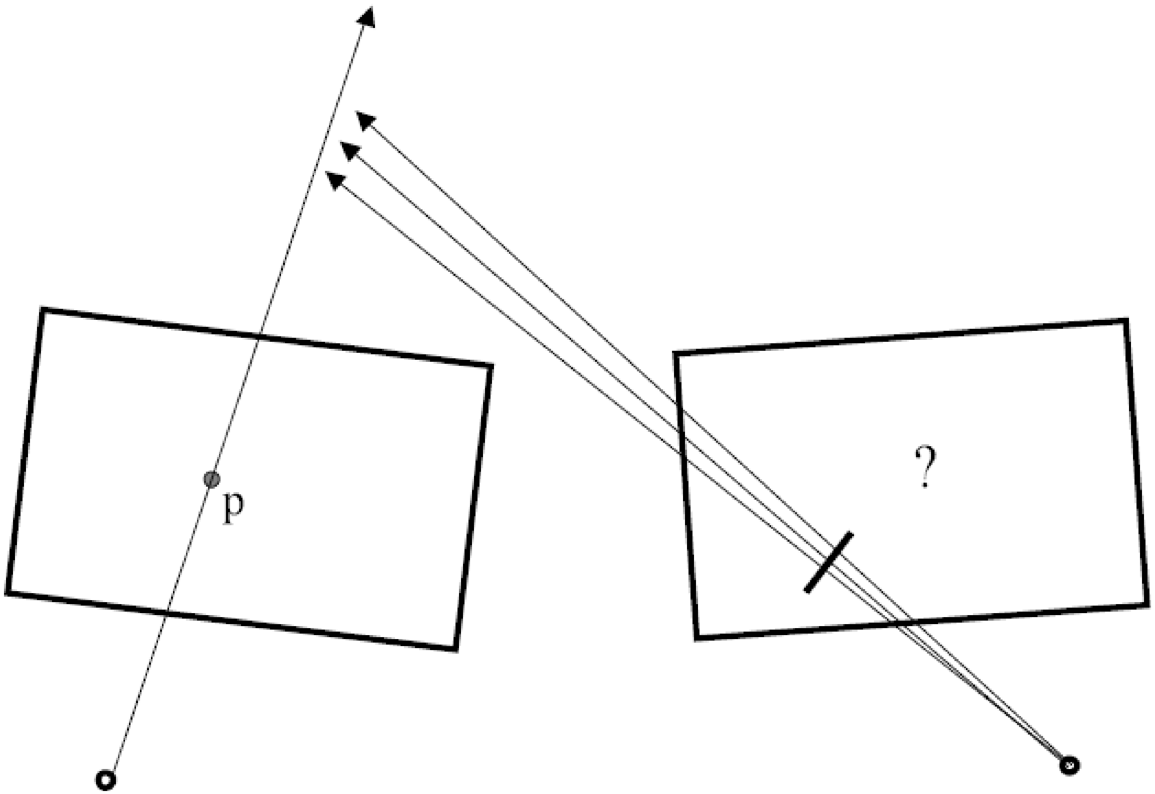
Stereo correspondence constraints



- **Given p in left image, where can corresponding point p' be?**

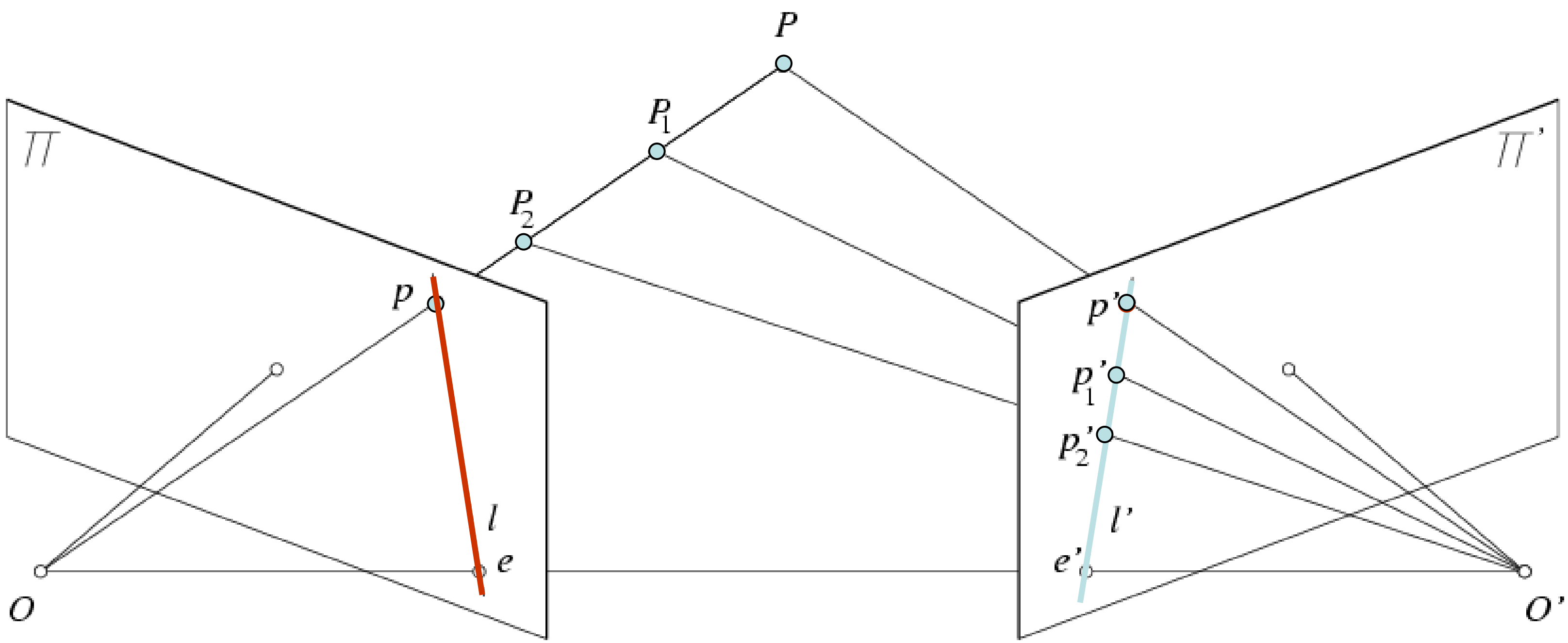
Slide credit: Kristen Grauman

Stereo correspondence constraints



Slide credit: Kristen Grauman

Epipolar constraint



Geometry of two views constrains where the corresponding pixel for some image point in the first view must occur in the second view:

•It must be on the line carved out by a plane connecting the world point and optical centers.

Why is this useful?

Slide credit: Kristen Grauman

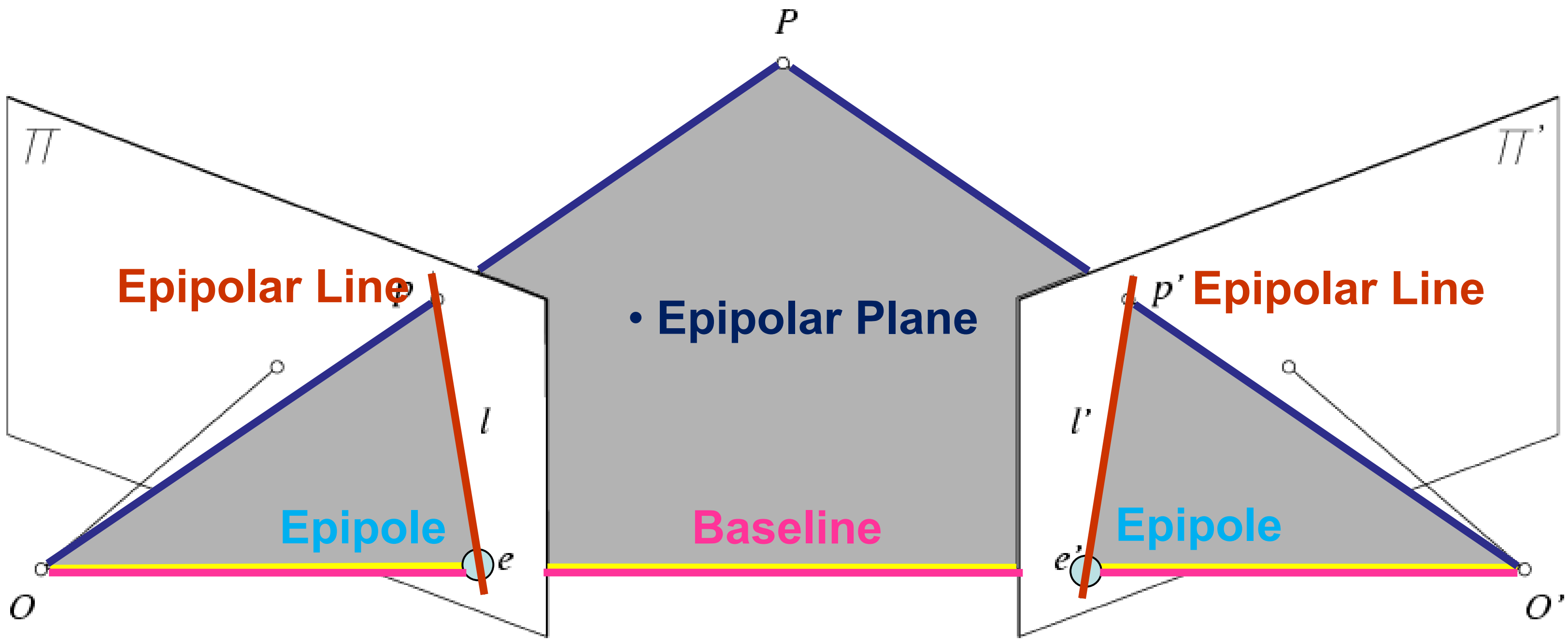
Epipolar constraint

This is useful because it reduces the correspondence problem to a 1D search along an epipolar line.

Image from Andrew Zisserman

Slide credit: Kristen Grauman

Epipolar geometry

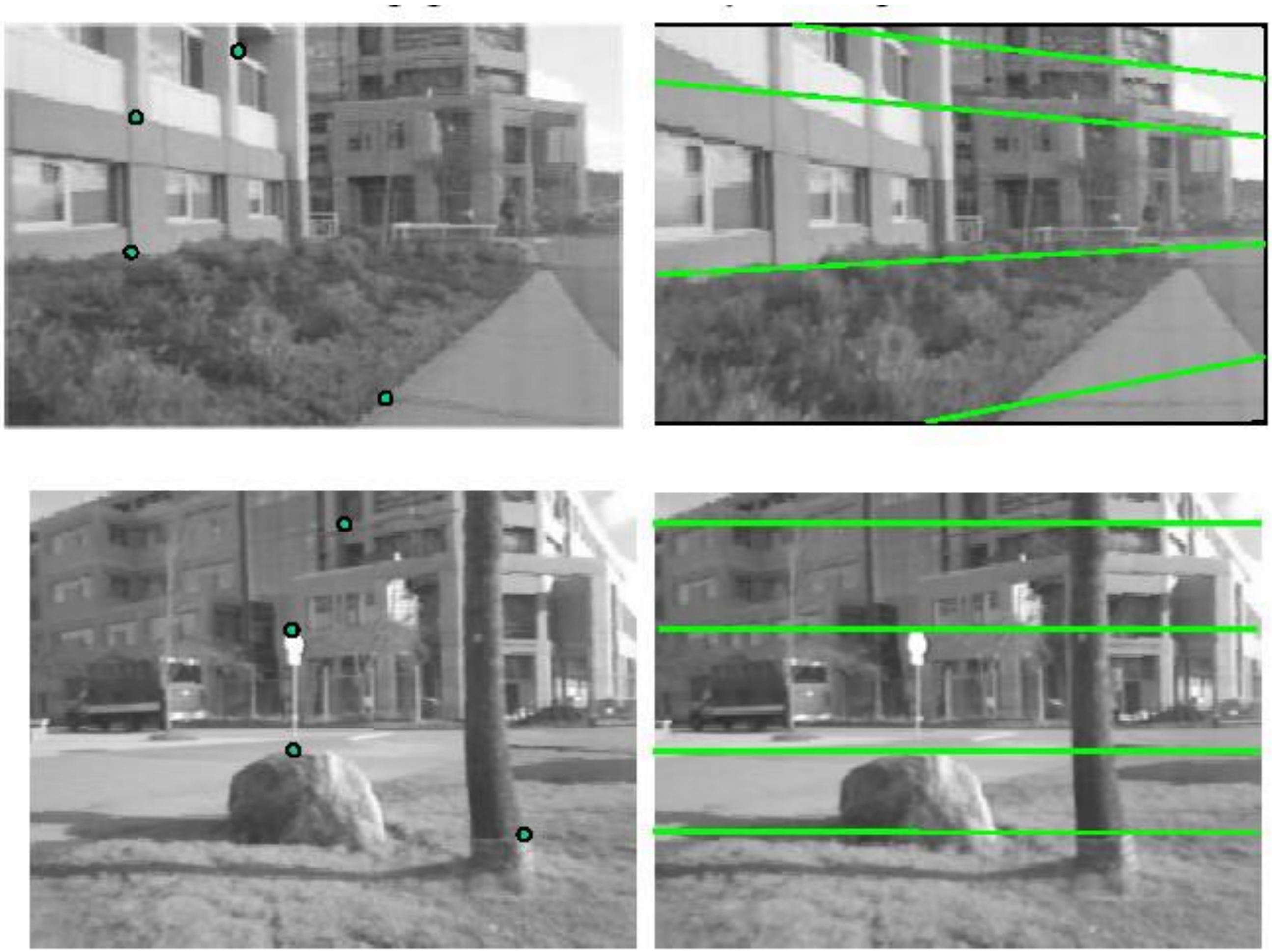


- **Baseline**: line joining the camera centers
- **Epipole**: point of intersection of baseline with the image plane
- **Epipolar plane**: plane containing baseline and world point
- **Epipolar line**: intersection of epipolar plane with the image plane

- All epipolar lines intersect at the epipole
- An epipolar plane intersects the left and right image planes in epipolar lines

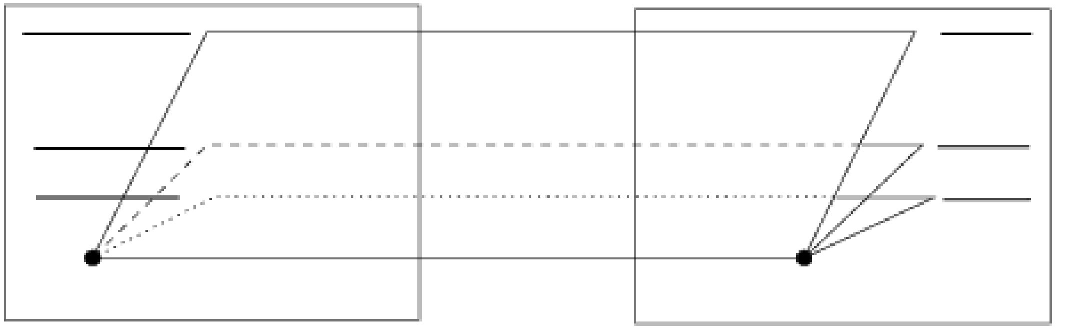
http://www.ai.sri.com/~luong/research/Meta3DViewer/EpipolarGeo.html Slide credit: Kristen Grauman

Example



Slide credit: Kristen Grauman

Example: parallel cameras



Where are the epipoles?

Figure from Hartley & Zisserman

Slide credit: Kristen Grauman

Example: converging cameras

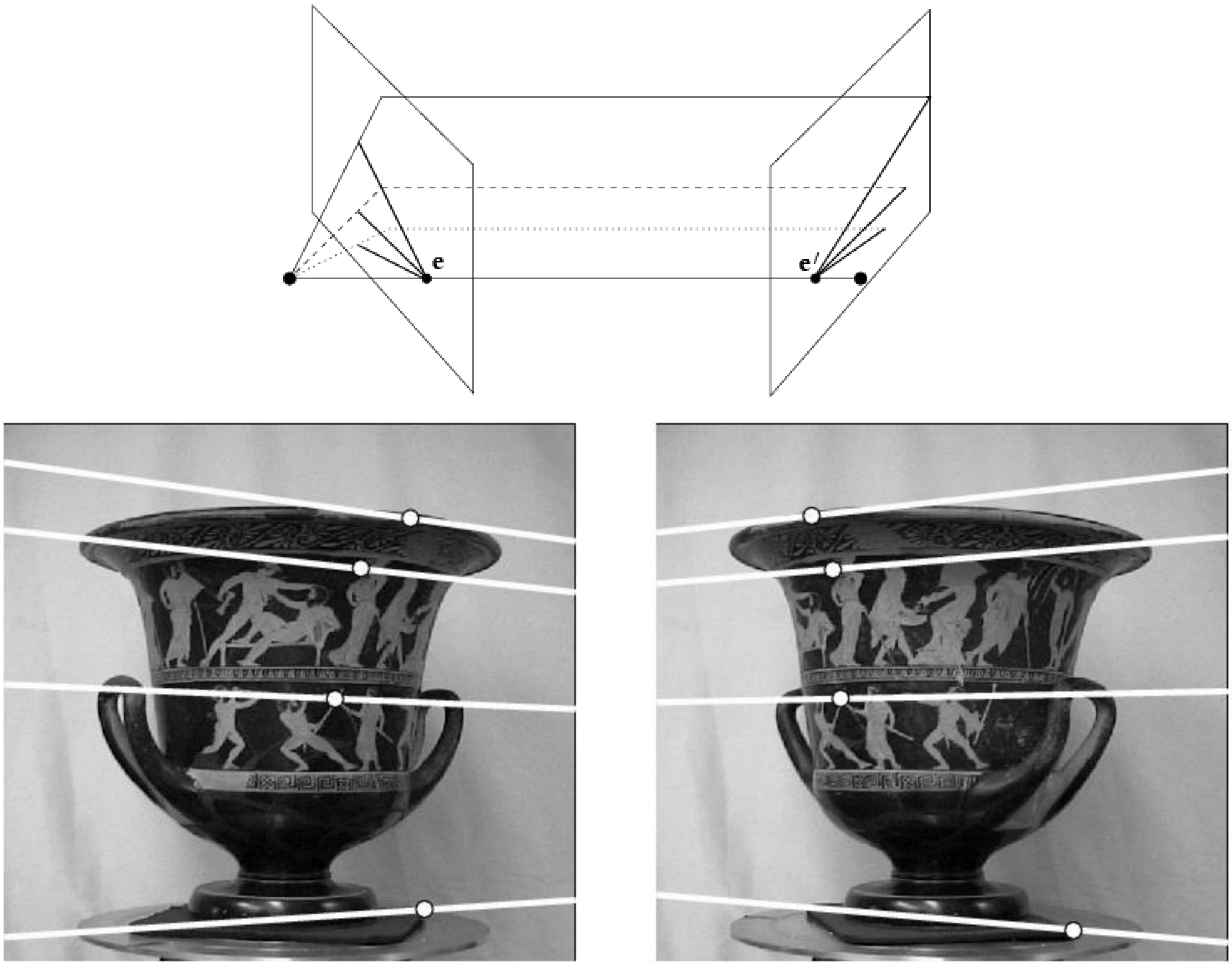


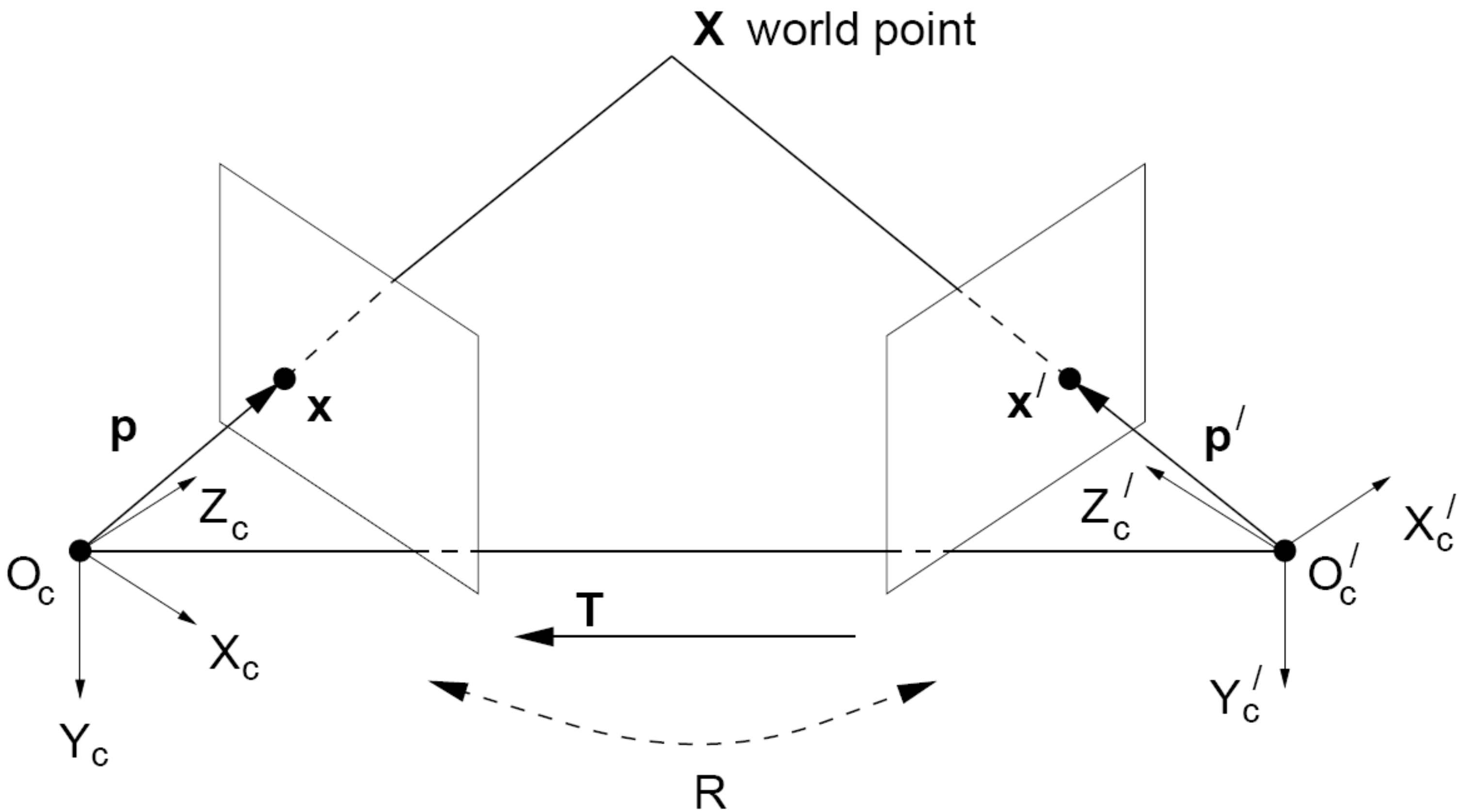
Figure from Hartley & Zisserman

Slide credit: Kristen Grauman

- So far, we have the explanation in terms of geometry.
- Now, how to express the epipolar constraints algebraically?

Slide credit: Kristen Grauman

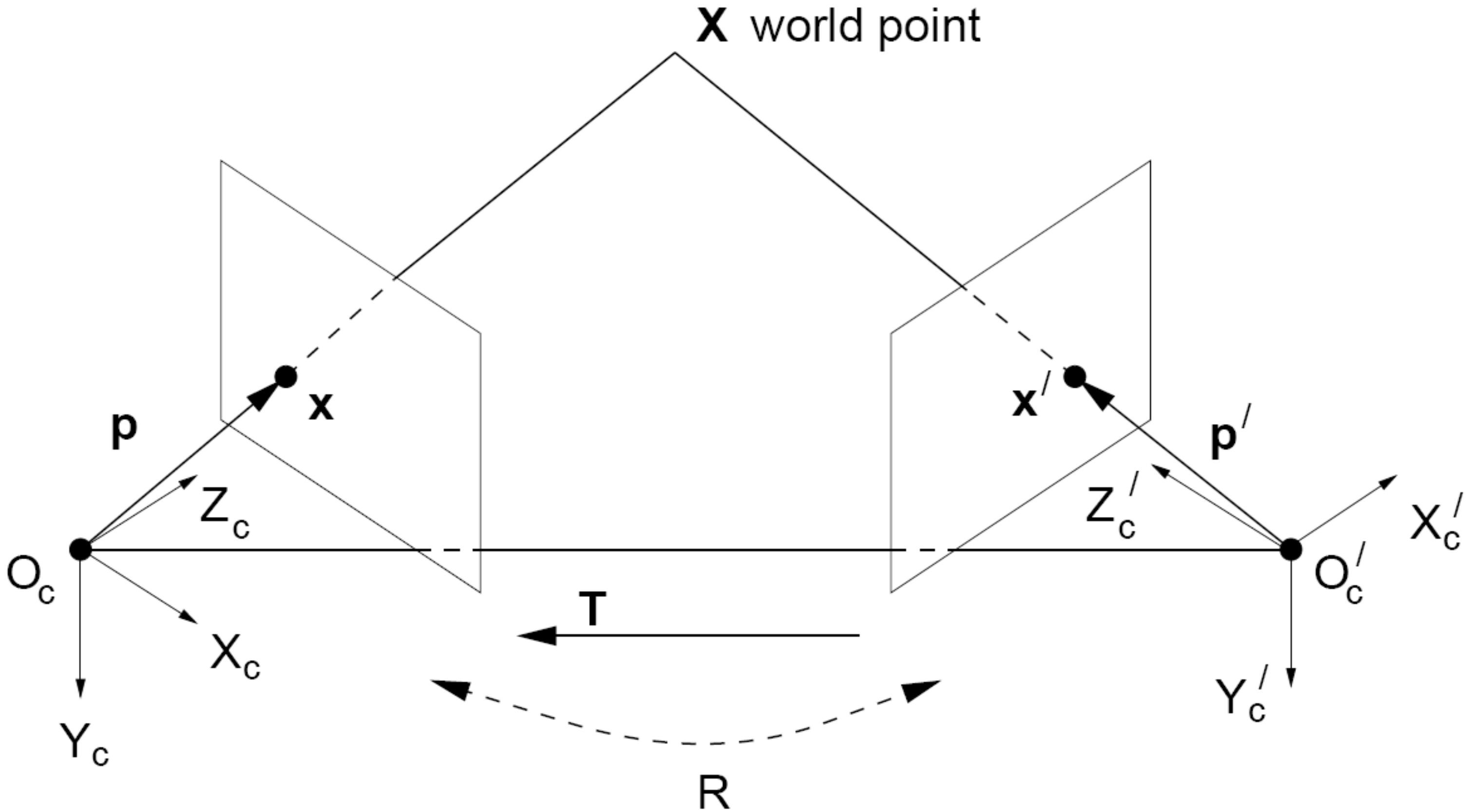
Stereo geometry, with calibrated cameras



Main idea

Slide credit: Kristen Grauman

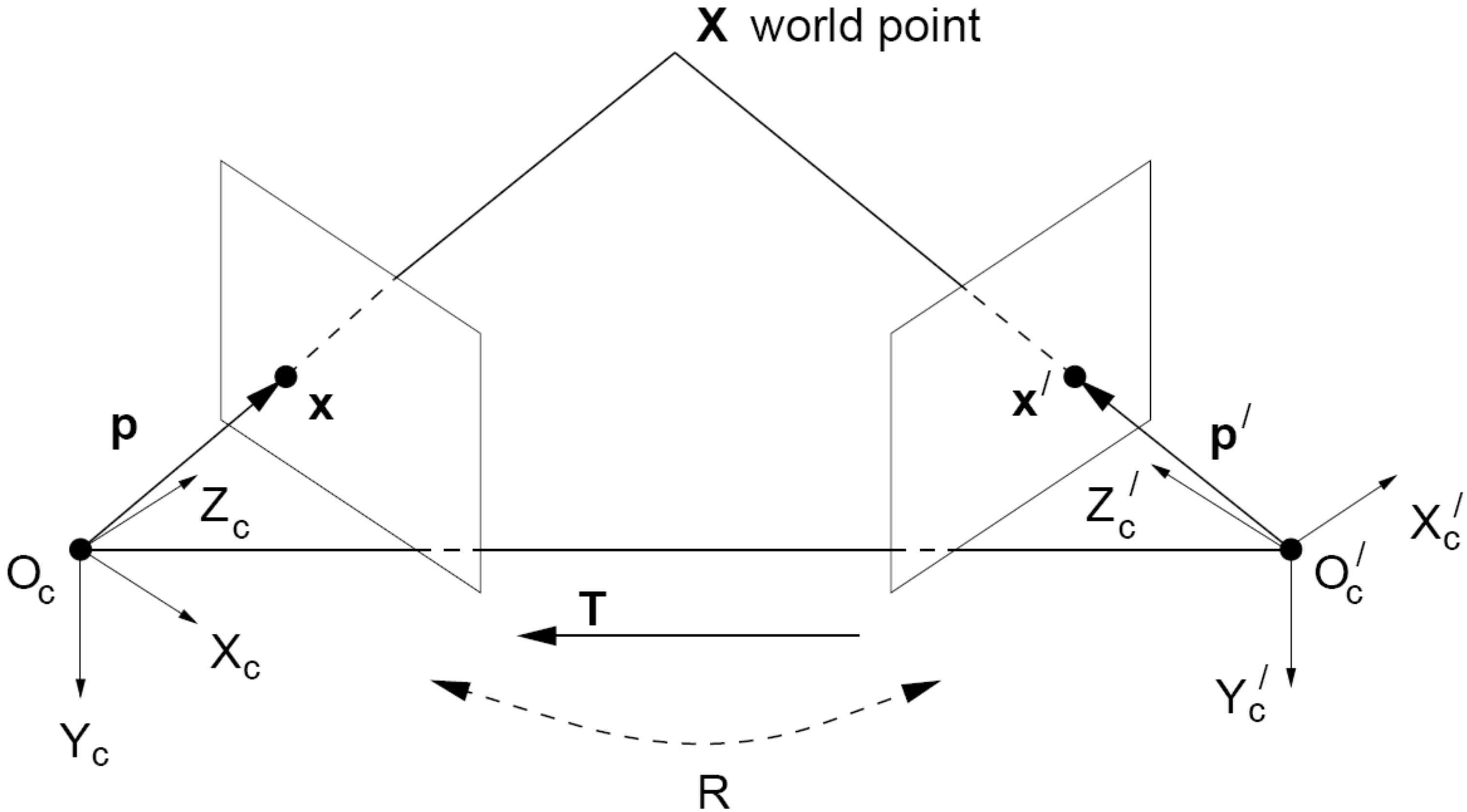
Stereo geometry, with calibrated cameras



If the stereo rig is calibrated, we know :
how to rotate and translate camera reference frame 1 to get to camera reference frame 2.
Rotation: 3 x 3 matrix R; translation: 3 vector T.

Slide credit: Kristen Grauman

Stereo geometry, with calibrated cameras

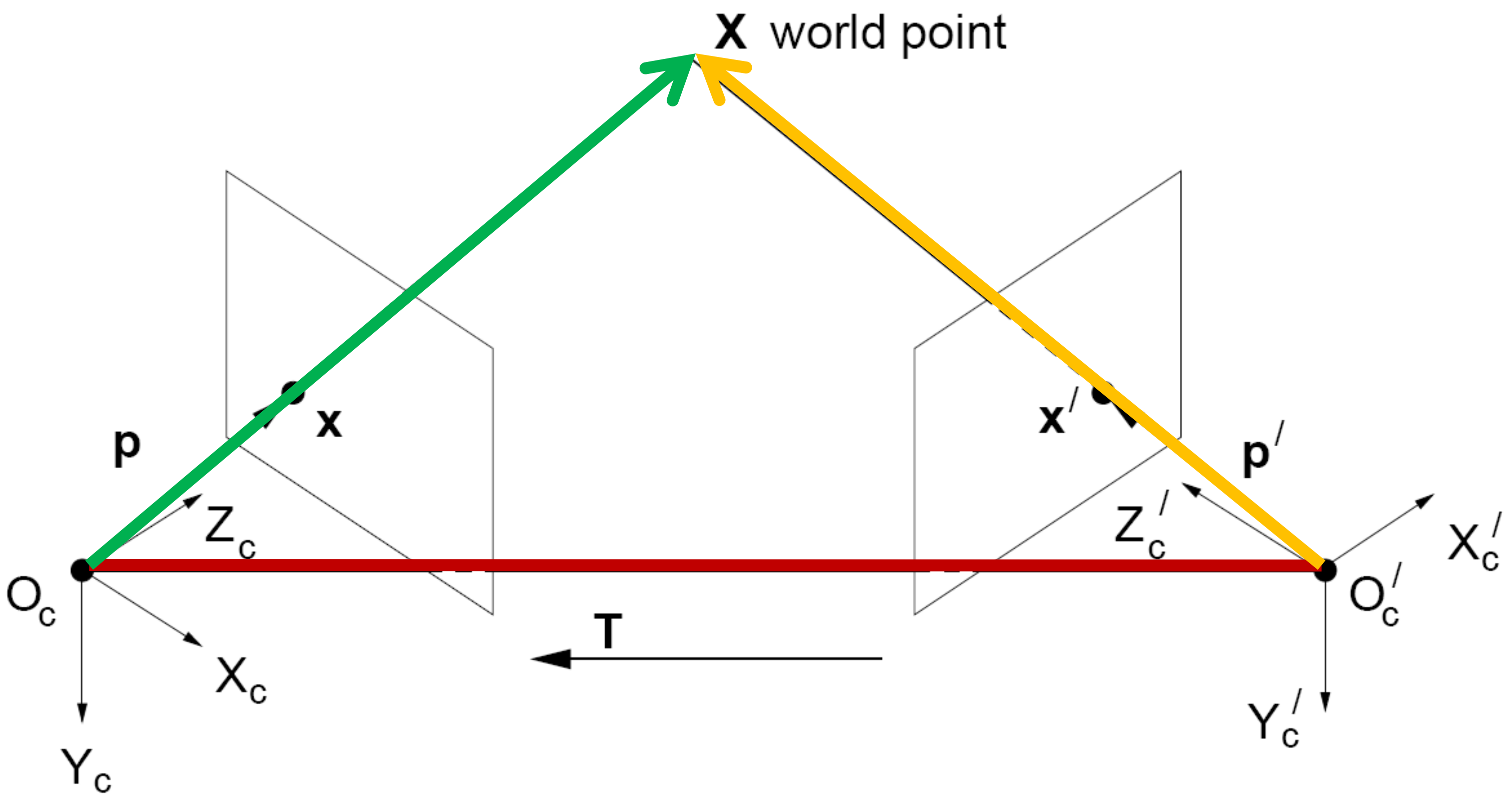


If the stereo rig is calibrated, we know :
how to rotate and translate camera reference frame 1 to get to camera reference frame 2.

$$X'_c = RX_c + T'$$

Slide credit: Kristen Grauman

From geometry to algebra



$$\boxed{\mathbf{X}'} = \mathbf{R}\boxed{\mathbf{X}} + \boxed{\mathbf{T}}$$

$$\underbrace{\mathbf{T}\times\mathbf{X}'}_{\text{Normal to the plane}} = \mathbf{T}\times\mathbf{R}\mathbf{X}$$

$$\mathbf{X}'\cdot\left(\mathbf{T}\times\mathbf{X}'\right) = \mathbf{X}'\cdot\left(\mathbf{T}\times\mathbf{R}\mathbf{X}\right) = 0$$

Slide credit: Kristen Grauman

Aside: cross product

$$\vec{a} \times \vec{b} = \vec{c}$$

$$\vec{a} \cdot \vec{c} = 0$$
$$\vec{b} \cdot \vec{c} = 0$$

Vector cross product takes two vectors and returns a third vector that's perpendicular to both inputs.

So here, c is perpendicular to both a and b, which means the dot product = 0.

Slide credit: Kristen Grauman

Another aside: Matrix form of cross product

$$\vec{a} \times \vec{b} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \vec{c}$$

$$\vec{a} \cdot \vec{c} = 0$$

$$\vec{b} \cdot \vec{c} = 0$$

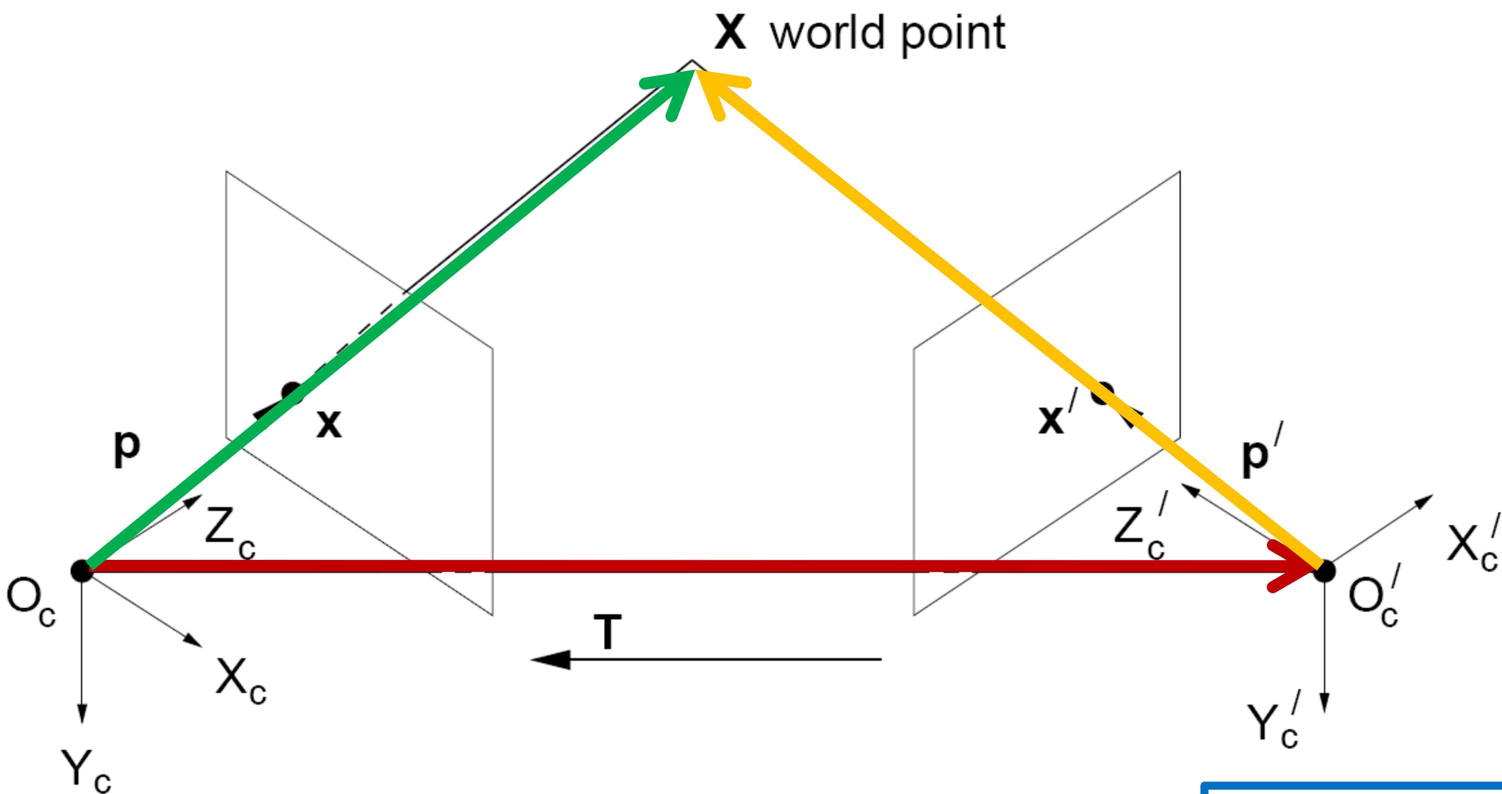
Can be expressed as a matrix multiplication.

$$[a_x] = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

$$\boxed{\vec{a} \times \vec{b} = [a_x]\vec{b}}$$

Slide credit: Kristen Grauman

From geometry to algebra



$$\mathbf{X'} = \mathbf{R}\mathbf{X} + \mathbf{T}$$

$$\underbrace{\mathbf{T} \times \mathbf{X'}}_{\text{Normal to the plane}} = \mathbf{T} \times \mathbf{R}\mathbf{X} + \mathbf{T} \times \mathbf{T}$$

$$= \mathbf{T} \times \mathbf{R}\mathbf{X}$$

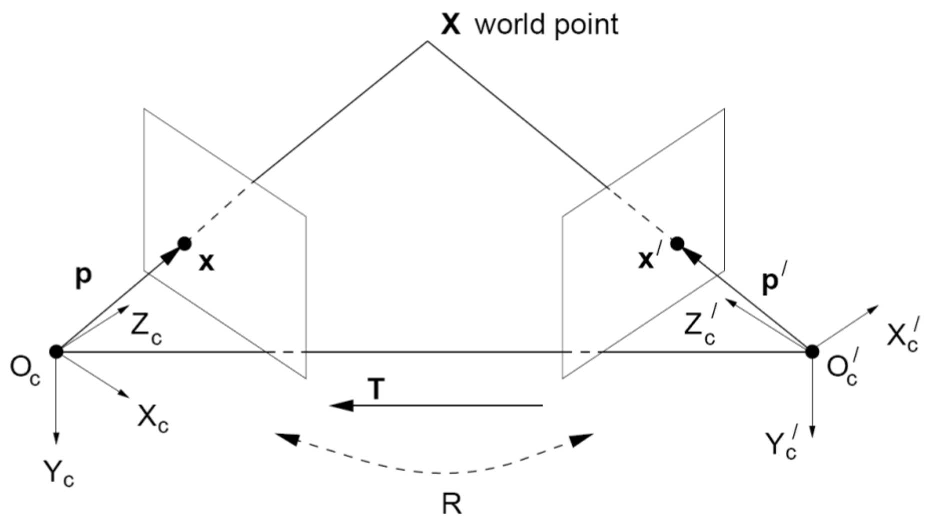
$$\mathbf{X'} \cdot (\mathbf{T} \times \mathbf{X'}) \boxed{= \mathbf{X'} \cdot (\mathbf{T} \times \mathbf{R}\mathbf{X}) \\ = 0}$$

Slide credit: Kristen Grauman

Essential matrix

$$\boxed{\mathbf{X}' \cdot \left(\mathbf{T} \times \mathbf{R}\mathbf{X}\right) = 0}$$

$$\mathbf{X}' \cdot \left(\mathbf{T}_x \ \mathbf{R}\mathbf{X}\right) = 0$$



Let $\mathbf{E} = \mathbf{T}_x\mathbf{R}$

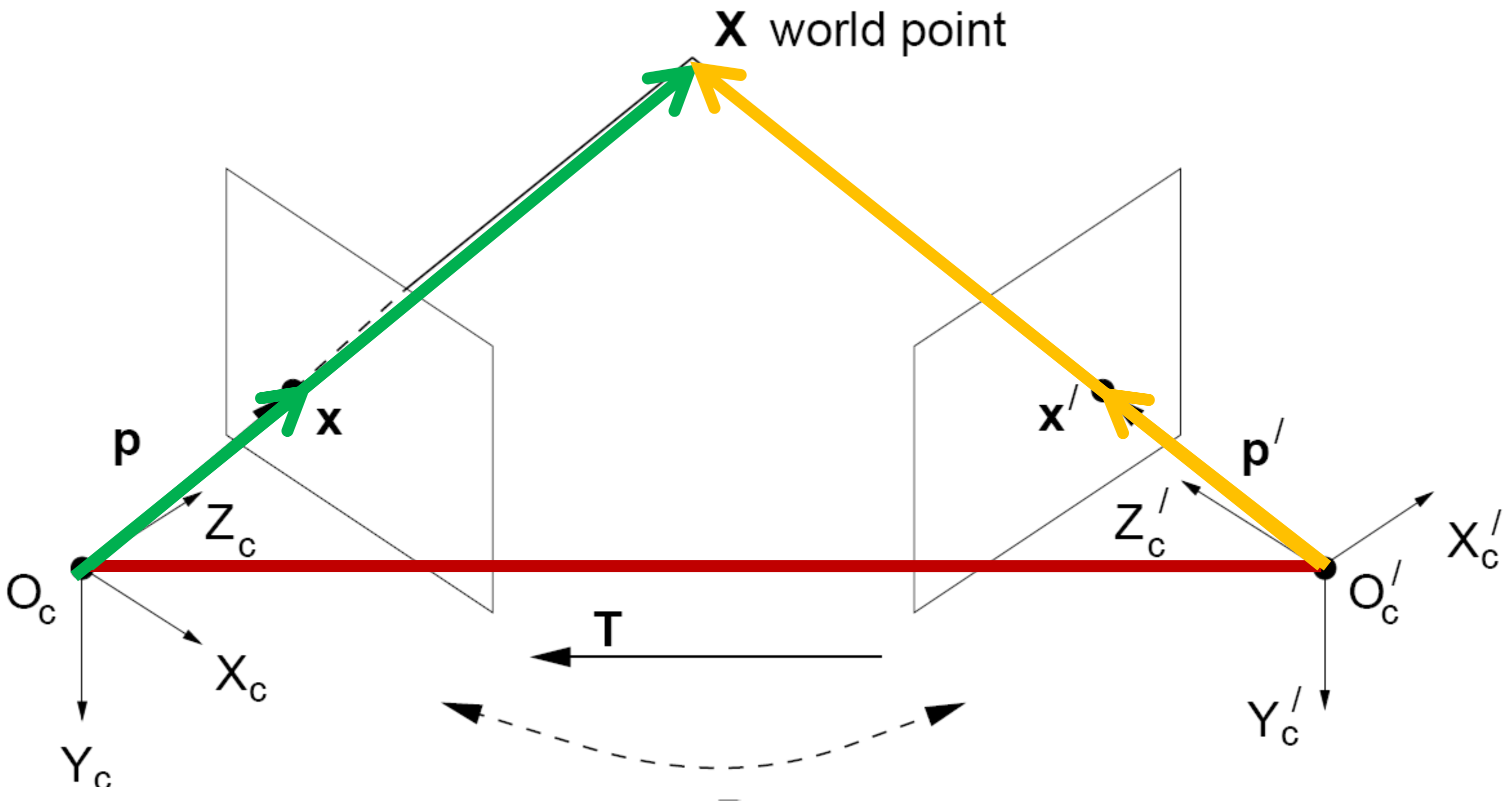
$$\mathbf{X}'^T\mathbf{E}\mathbf{X} = 0$$

E is called the essential matrix, and it relates corresponding image points between both cameras, given the rotation and translation.

If we observe a point in one image, its position in other image is constrained to lie on line defined by above.

Note: these points are in camera coordinate systems.

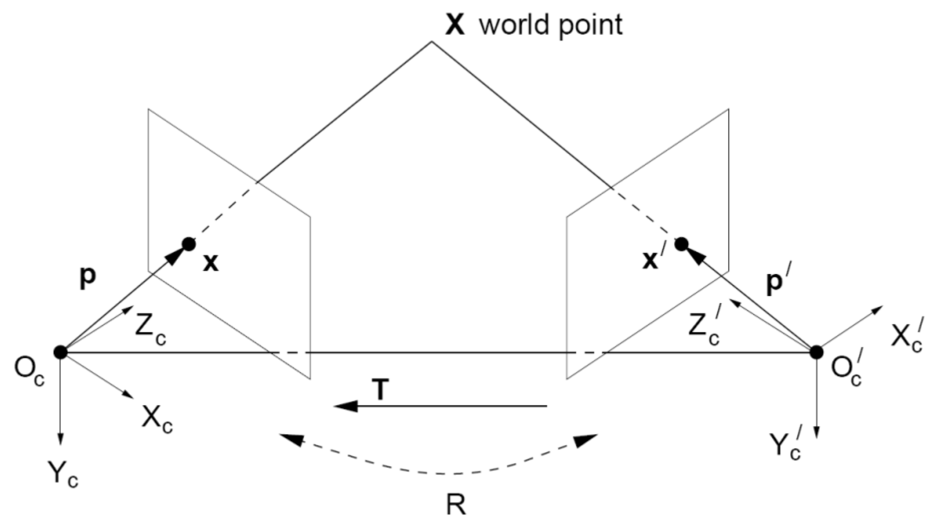
x and x’ are scaled versions of X and X’



$$X' \cdot (T' \times RX) = 0$$

$$X' \cdot (T'_x RX) = 0$$

Let $E = T'_x R$



$$\mathbf{X}'^T \mathbf{E} \mathbf{X} = 0$$

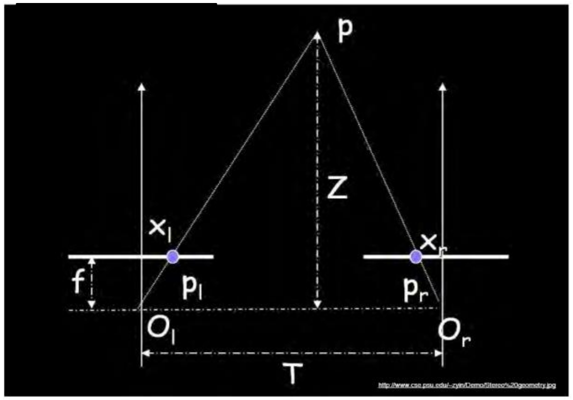
$$x'^T E x = 0$$ pts x and x' in the image planes are scaled versions of X and X'.

E is called the essential matrix, and it relates corresponding image points between both cameras, given the rotation and translation.

If we observe a point in one image, its position in the other image is constrained to lie on line defined by above.

Note: these points are in camera coordinate systems.

Essential matrix example: parallel cameras



$$\mathbf{R} =$$

$$\mathbf{T} =$$

$$\mathbf{E} = [\mathbf{T}_\mathbf{x}]\mathbf{R} =$$

$$\mathbf{p} = [x, y, f]$$

$$\mathbf{p}' = [x', y', f]$$

$$\mathbf{p}'^{\mathrm{T}}\mathbf{E}\mathbf{p} = 0$$

For the parallel cameras, image of any point must lie on same horizontal line in each image plane.

Slide credit: Kristen Grauman

image I(x,y)

Disparity map D(x,y)

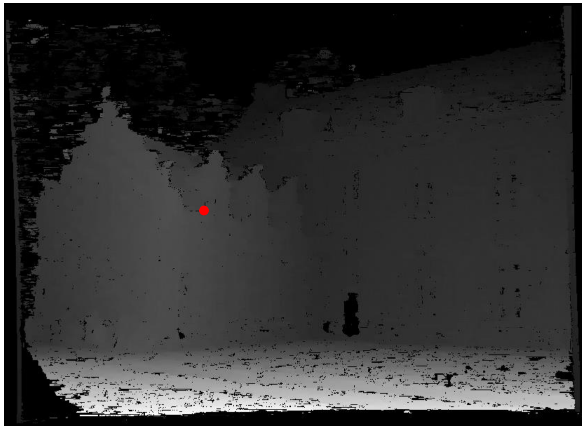


image I´(x´,y´)

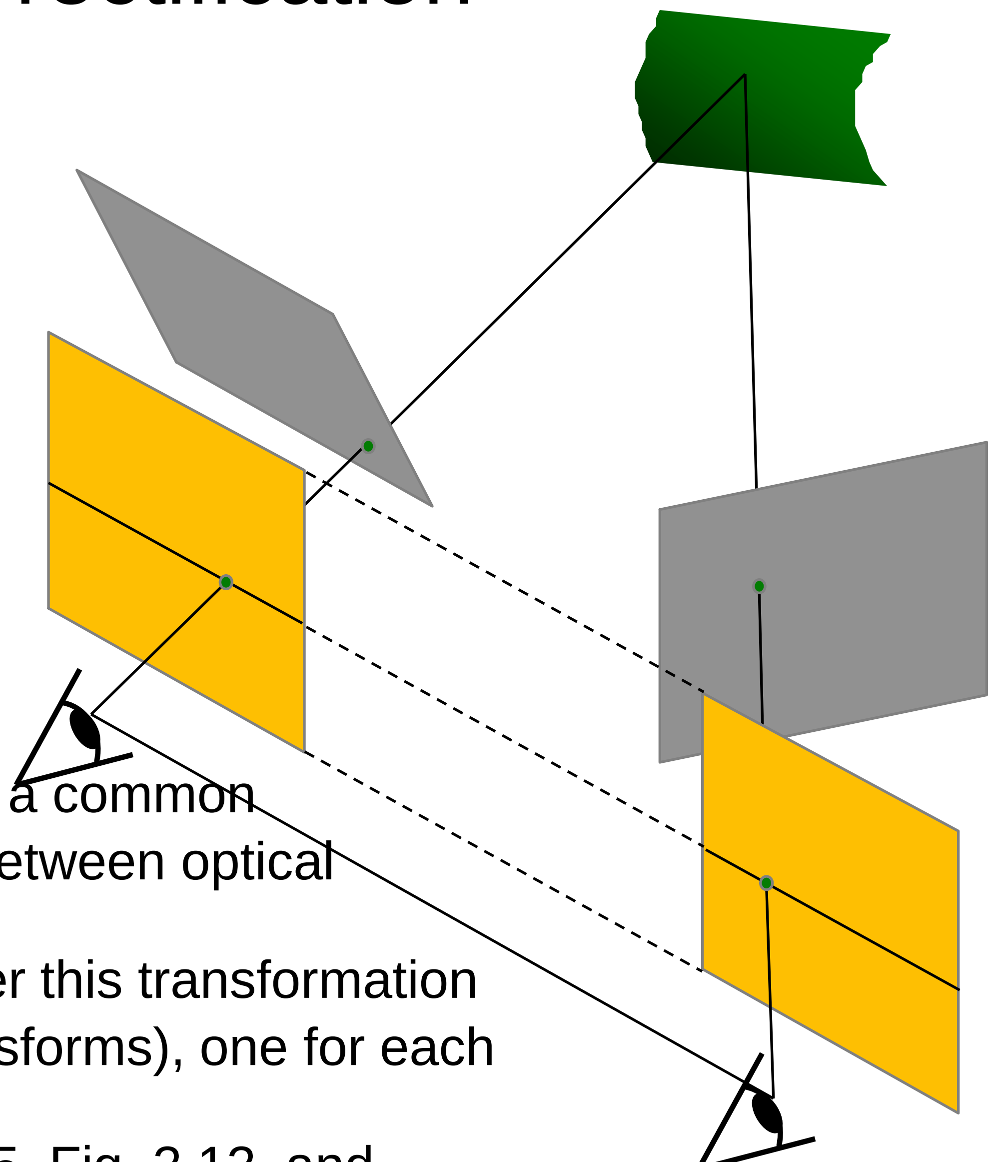
$$(x',y')=(x+D(x,y),y)$$

What about when cameras' optical axes are not parallel?

Slide credit: Kristen Grauman

Stereo image rectification

In practice, it is convenient if image scanlines (rows) are the epipolar lines.

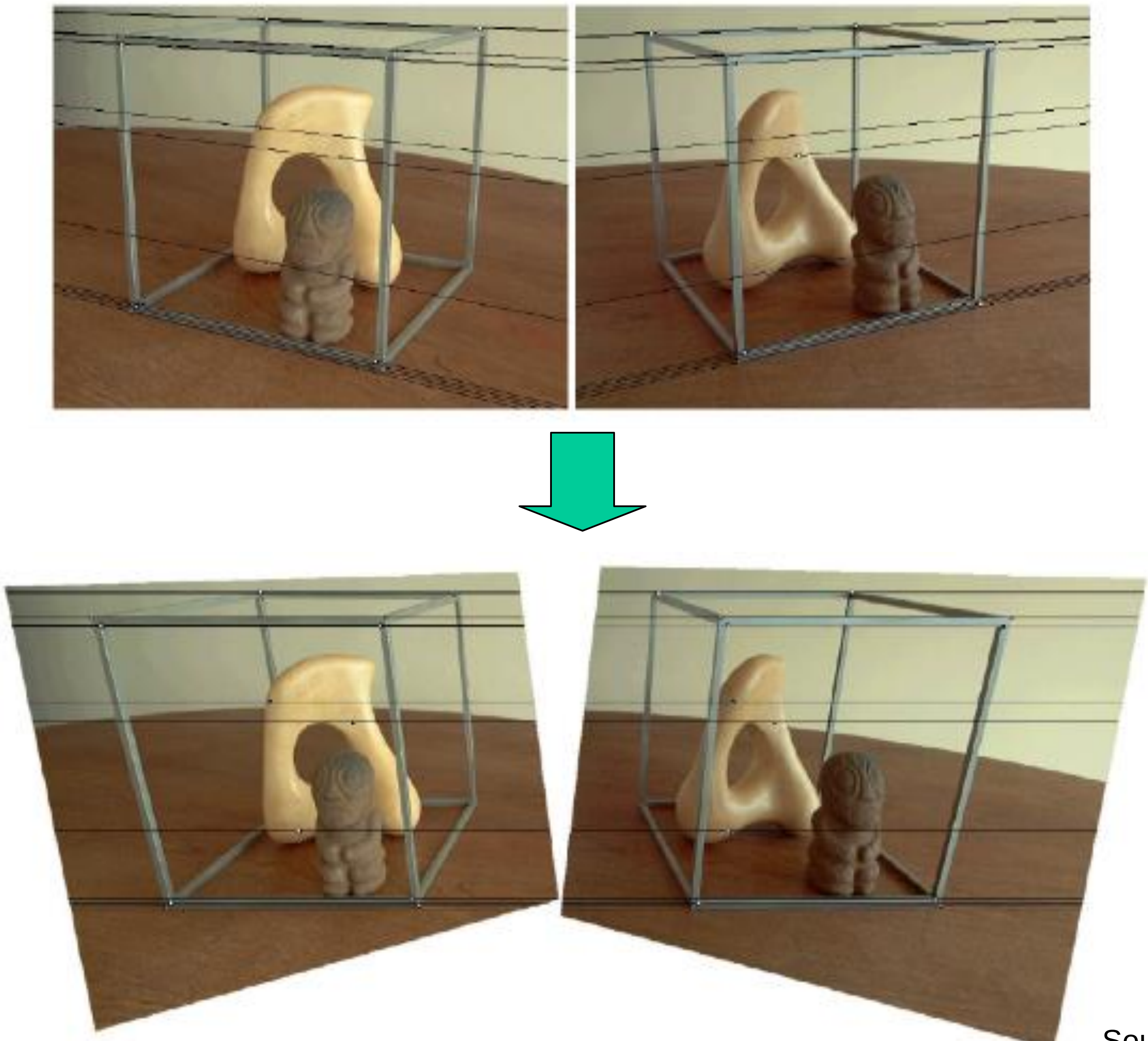


- Reproject image planes onto a common plane parallel to the line between optical centers
- Pixel motion is horizontal after this transformation
- Two homographies (3x3 transforms), one for each input image reprojection
- See Szeliski book, Sect. 2.1.5, Fig. 2.12, and "Mapping from one camera to another" p. 56

Adapted from Li Zhang

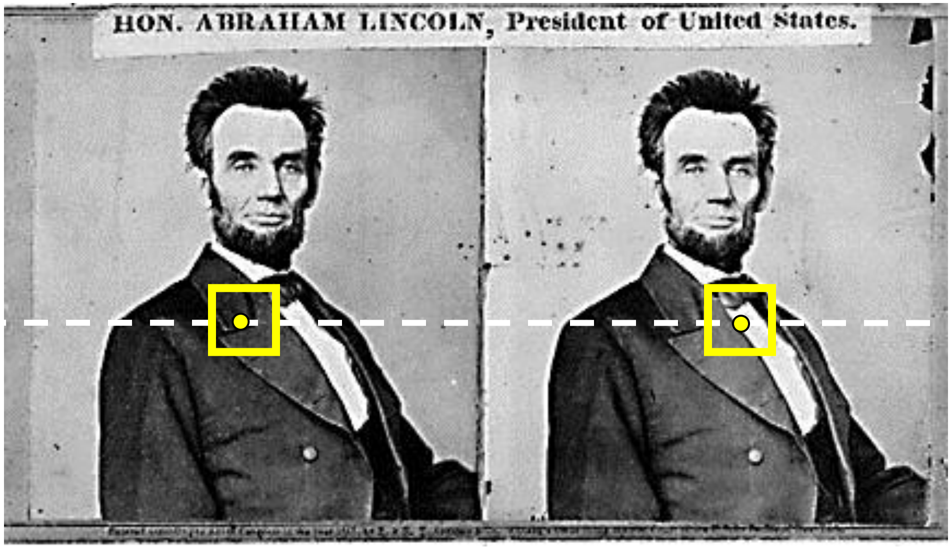
Slide credit: Kristen Grauman

Stereo image rectification: example



Source: Alyosha Efros

Your basic stereo algorithm



For each epipolar line

For each pixel in the left image

- compare with every pixel on same epipolar line in right image
- pick pixel with minimum match cost

Improvement: match ***windows***

Slide credit: Rick Szeliski

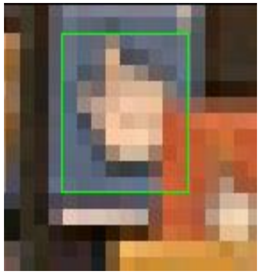
Image block matching

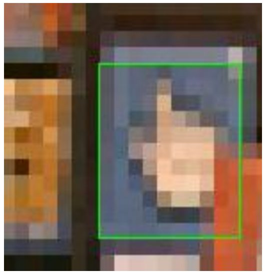
How do we determine correspondences?

- *block matching* or *SSD* (sum squared differences)

$$E(x,y;d) = \sum_{(x',y') \in N(x,y)} [I_L(x'+d,y') - I_R(x',y')]^2$$

d is the *disparity* (horizontal motion)





Slide credit: Rick Szeliski

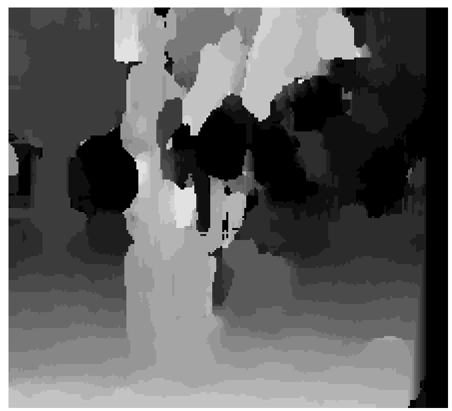
How big should the neighborhood be?

Neighborhood size

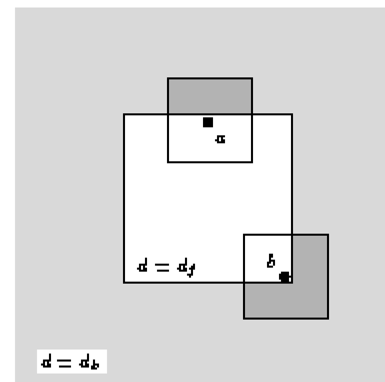
Smaller neighborhood: more details

Larger neighborhood: fewer isolated mistakes

w = 3



w = 20



Slide credit: Rick Szeliski

Matching criteria

Raw pixel values (correlation)

Band-pass filtered images [Jones & Malik 92]

“Corner” like features [Zhang, …]

Edges [many people…]

Gradients [Seitz 89; Scharstein 94]

Rank statistics [Zabih & Woodfill 94]

Slide credit: Rick Szeliski

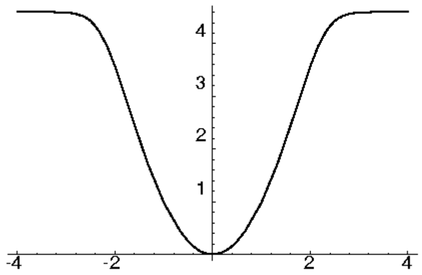
Local evidence framework

For every disparity, compute *raw* matching costs

$$E_0(x, y; d) = \rho(I_L(x' + d, y') - I_R(x', y'))$$

Why use a robust function?

- occlusions, other outliers



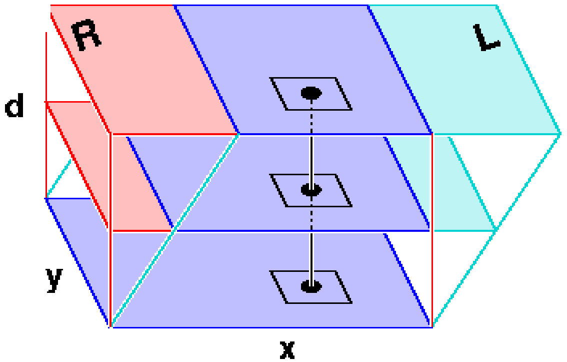
Can also use alternative match criteria

Slide credit: Rick Szeliski

Local evidence framework

Aggregate costs spatially

$$E(x,y;d) = \sum_{(x',y') \in N(x,y)} E_0(x',y',d)$$



Here, we are using a *box filter* (efficient moving average implementation)

Can also use weighted average, [non-linear] diffusion…

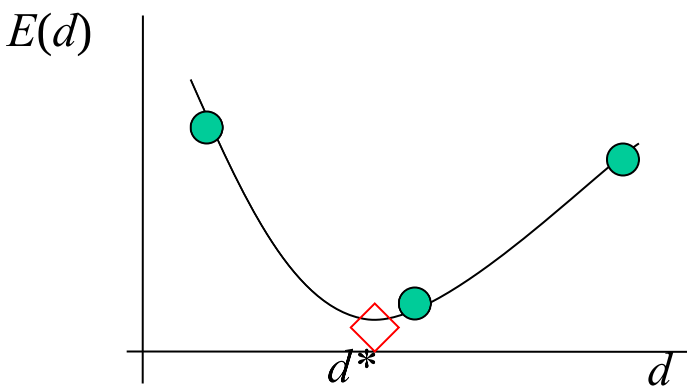
Slide credit: Rick Szeliski

Local evidence framework

Choose winning disparity at each pixel

$$d(x, y) = \arg\min_{d} E(x, y; d)$$

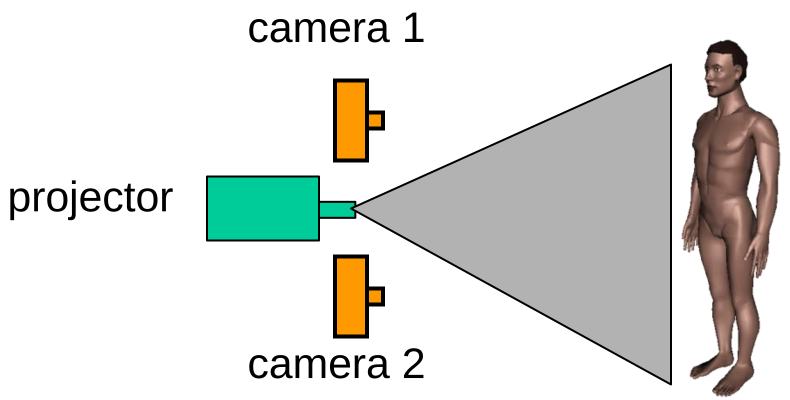
Interpolate to *sub-pixel* accuracy

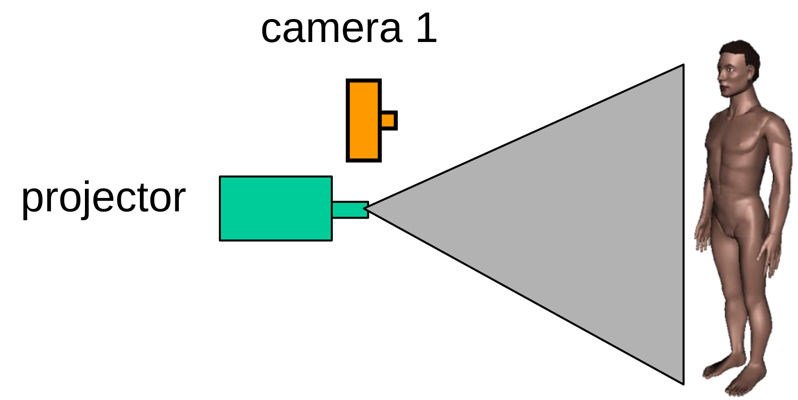


Slide credit: Rick Szeliski

Active stereo with structured light

Li Zhang’s one-shot stereo





Project “structured” light patterns onto the object

- simplifies the correspondence problem

Li Zhang, Brian Curless, and Steven M. Seitz. Rapid Shape Acquisition Using Color Structured Light and Multi-pass Dynamic Programming. In *Proceedings of the 1st International Symposium on 3D Data Processing, Visualization, and Transmission (3DPVT)*, Padova, Italy, June 19-21, 2002, pp. 24-36.

Slide credit: Rick Szeliski

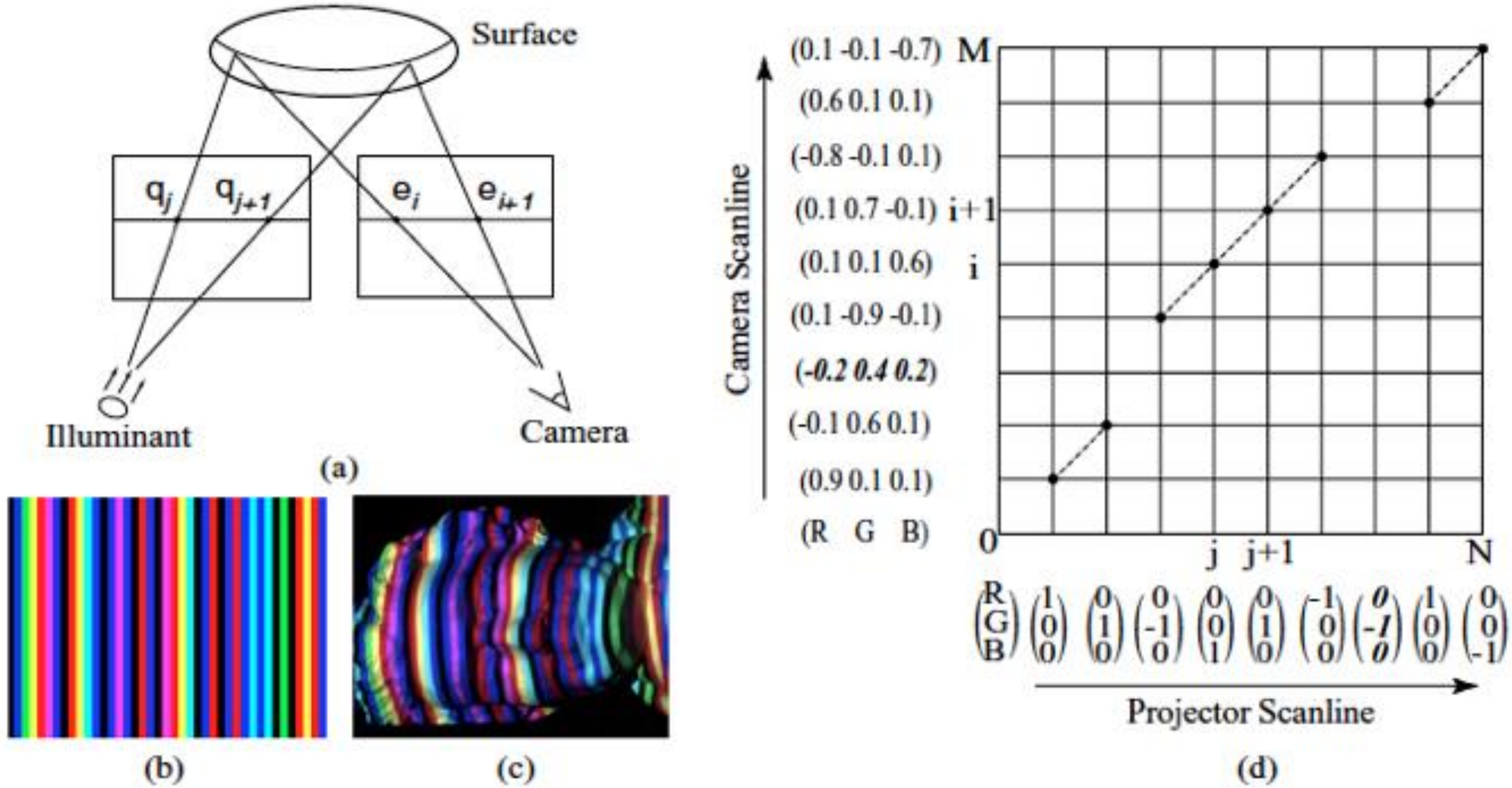
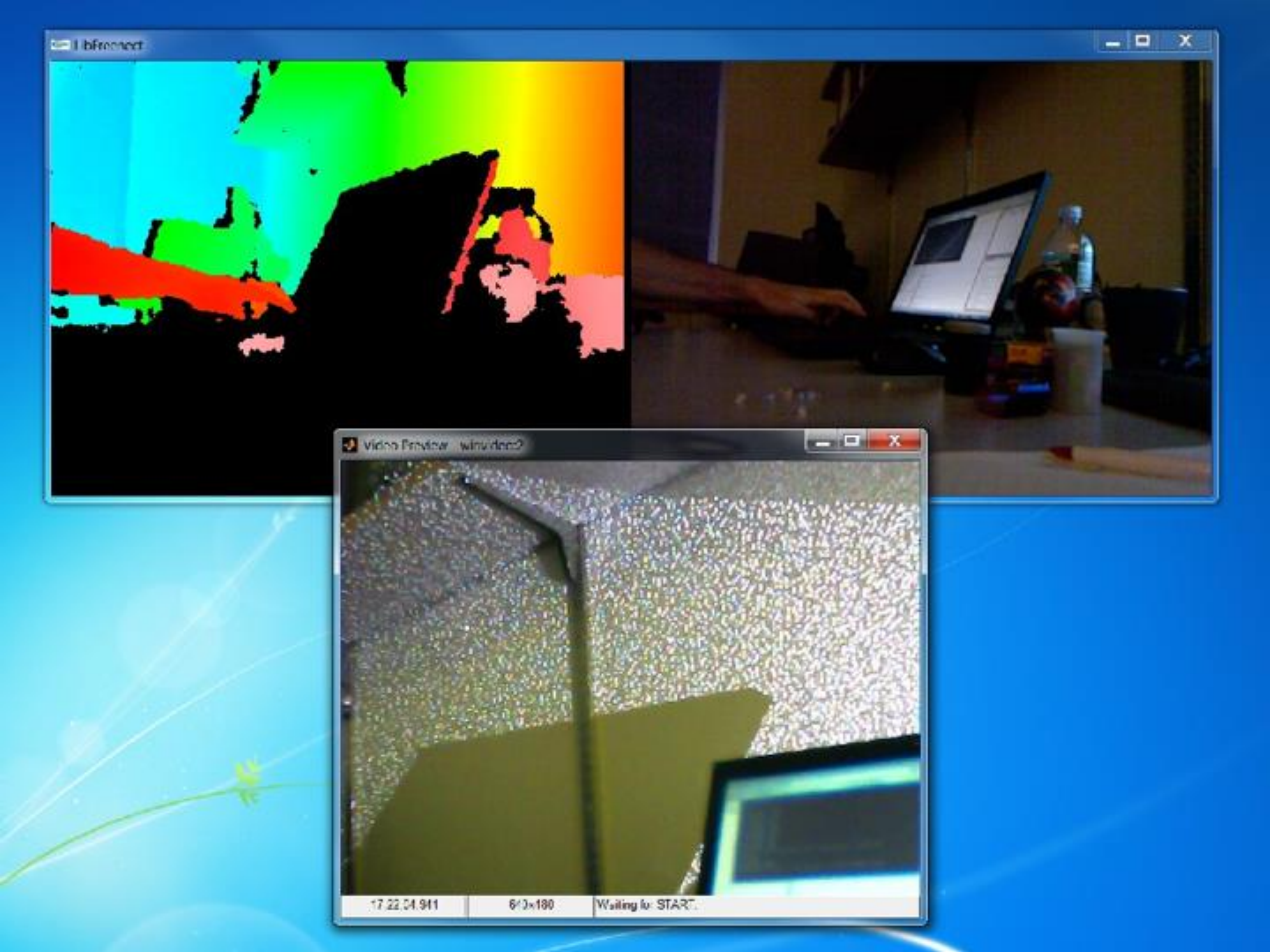
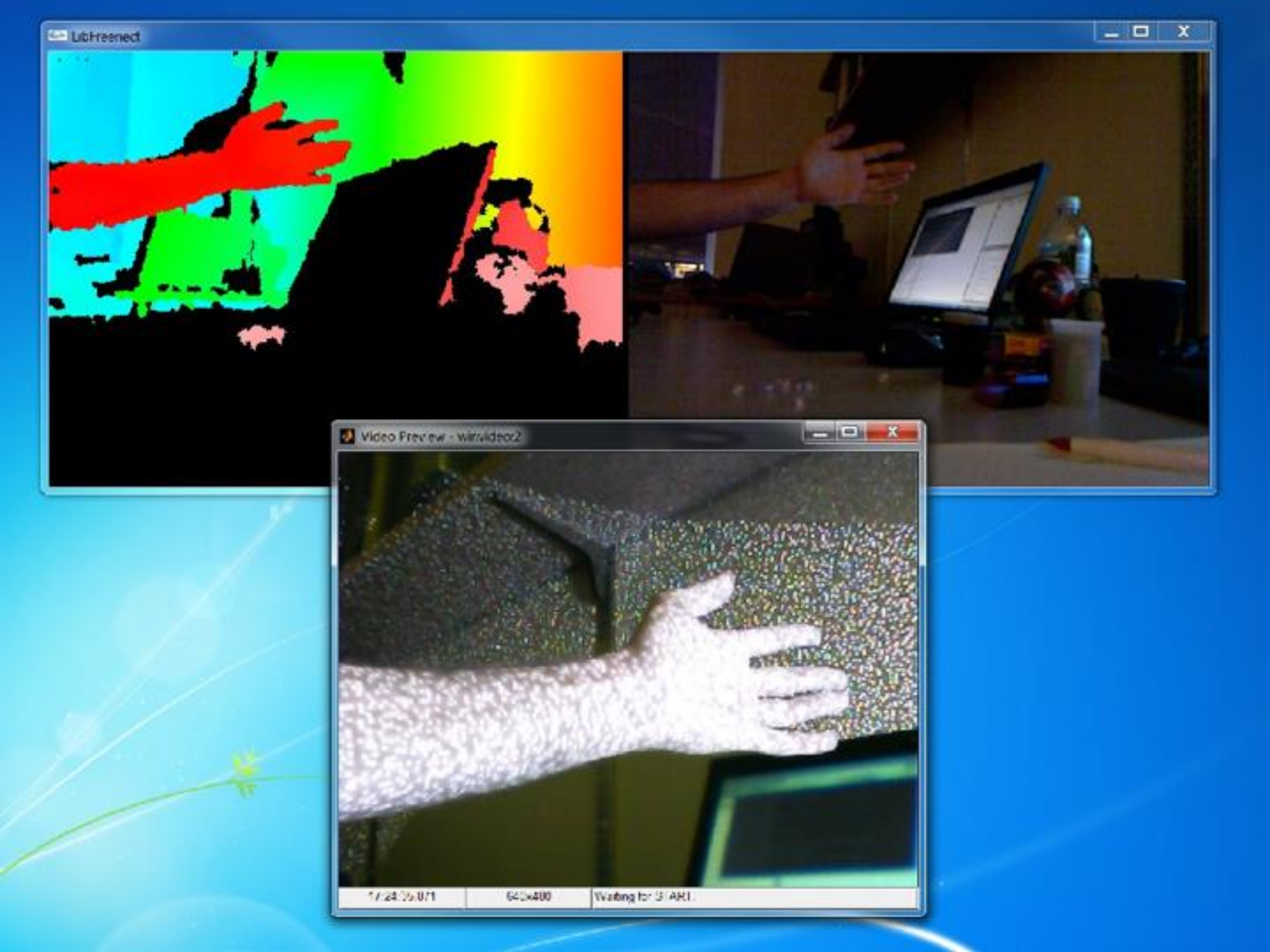


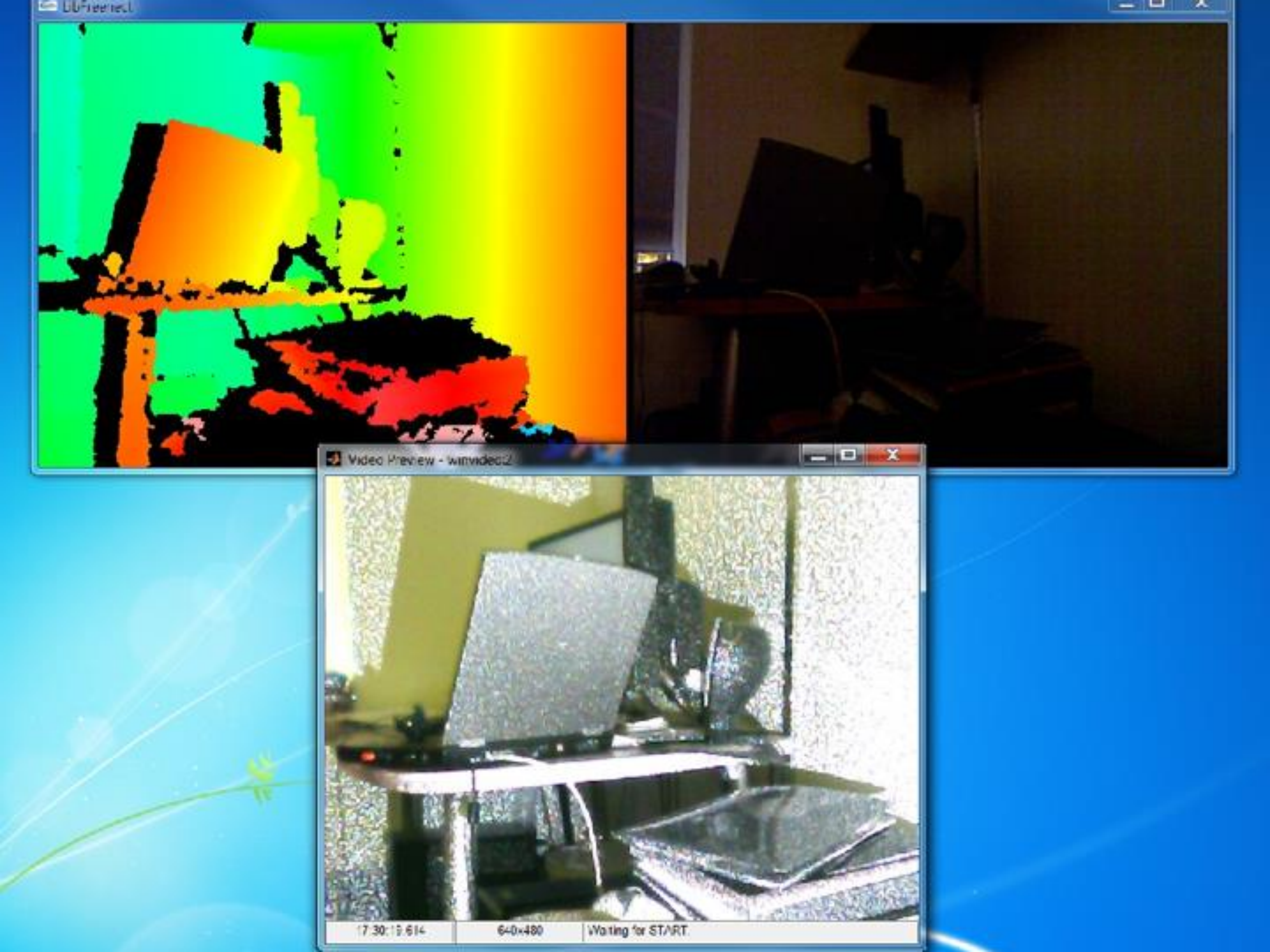
Figure 2. Summary of the one-shot method. (a) In optical triangulation, an illumination pattern is projected onto an object and the reflected light is captured by a camera. The 3D point is reconstructed from the relative displacement of a point in the pattern and image. If the image planes are rectified as shown, the displacement is purely horizontal (one-dimensional). (b) An example of the projected stripe pattern and (c) an image captured by the camera. (d) The grid used for multi-hypothesis code matching. The horizontal axis represents the projected color transition sequence and the vertical axis represents the detected edge sequence, both taken for one projector and rectified camera scanline pair. A match represents a path from left to right in the grid. Each vertex (j, i) has a score, measuring the consistency of the correspondence between e_i, the color gradient vectors shown by the vertical axis, and q_j, the color transition vectors shown below the horizontal axis. The score for the entire match is the summation of scores along its path. We use dynamic programming to find the optimal path. In the illustration, the camera edge in bold italics corresponds to a false detection, and the projector edge in bold italics is missed due to, e.g., occlusion.

Li Zhang, Brian Curless, and Steven M. Seitz


Libfreenect
Video Preview - winvideo2
Waiting for START.


LibFreenect
Video Preview - winvideo2


LibFreenect
Video Preview - winvideo2
Waiting for START


Video Preview - winvidect2
640x480
Waiting for START.

Monocular cues to depth

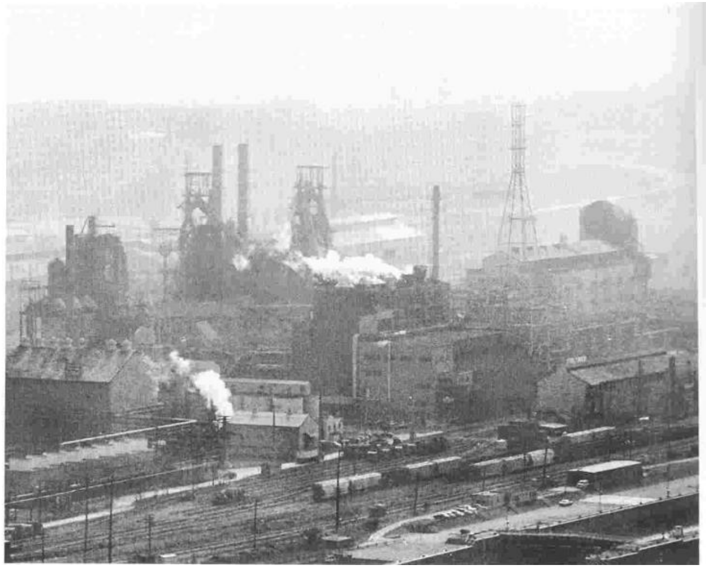
- **Absolute depth cues**: (assuming known camera parameters) these cues provide information about the absolute depth between the observer and elements of the scene
- **Relative depth cues**: provide relative information about depth between elements in the scene (this point is twice as far at that point, …)

Relative depth cues

Simple and powerful cue, but hard to make it work in practice…

Atmospheric perspective

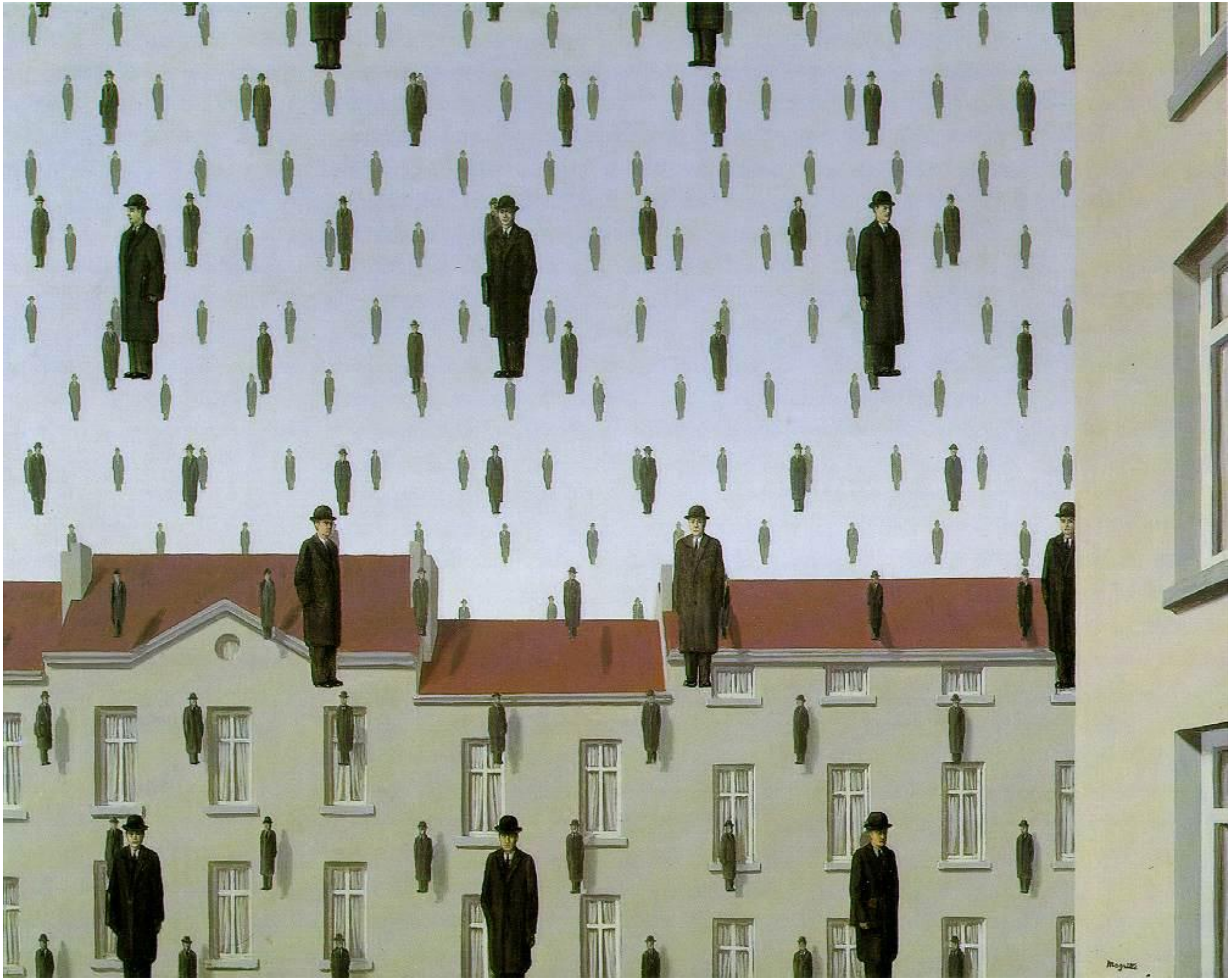
- Based on the effect of air on the color and visual acuity of objects at various distances from the observer.
- Consequences:
  - Distant objects appear bluer
  - Distant objects have lower contrast.



Atmospheric perspective

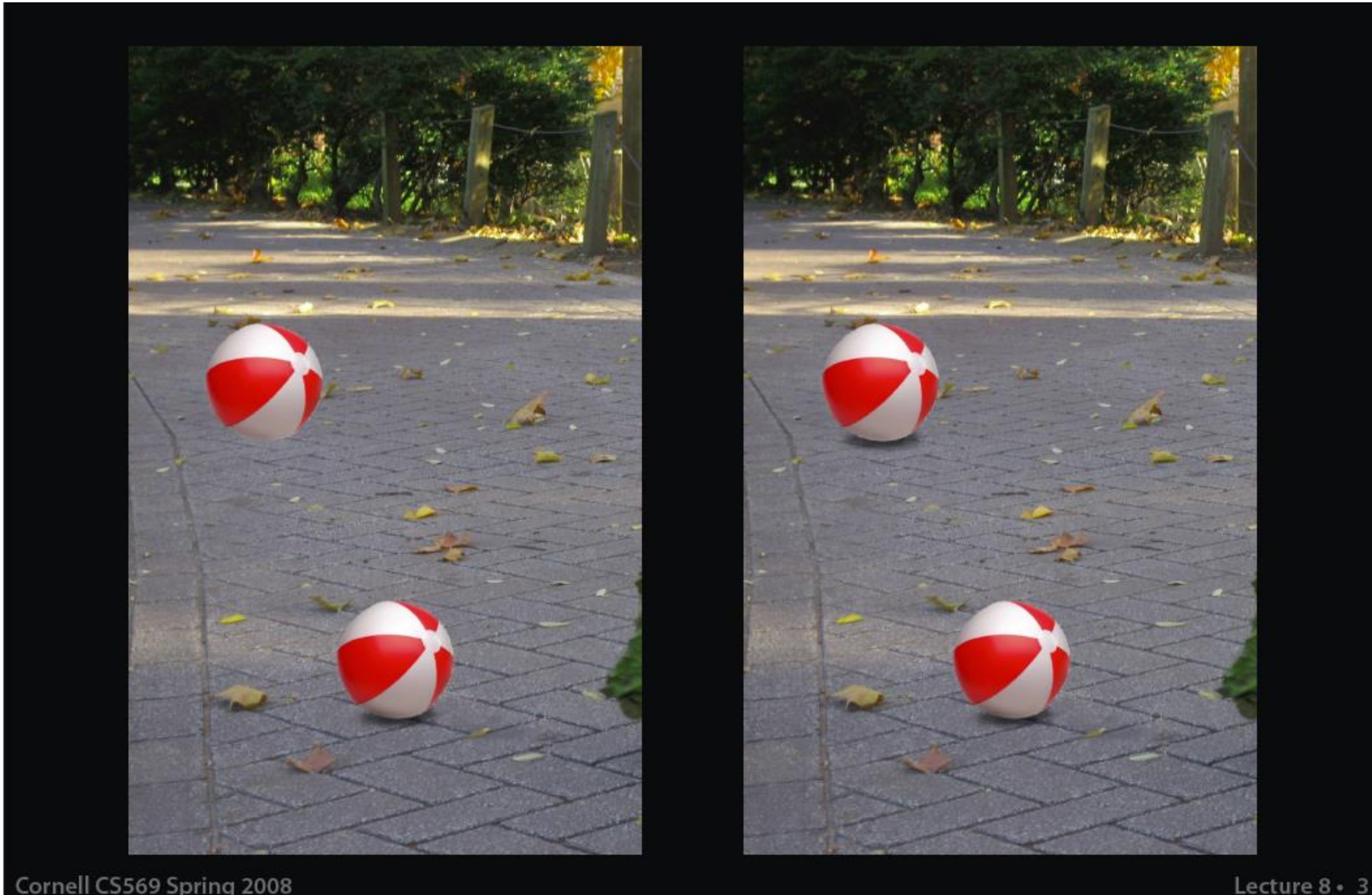
http://encarta.msn.com/medias_761571997/Perception_(psychology).html

Claude Lorrain (artist)
French, 1600 - 1682
Landscape with Ruins, Pastoral Figures, and Trees, 1643/1655



[*Golconde* Rene Magritte]

Shadows



Slide by Steve Marschner

http://www.cs.cornell.edu/courses/cs569/2008sp/schedule.stm

Linear perspective

Linear Perspective

Based on the apparent convergence of parallel lines to common vanishing points with increasing distance from the observer.

(Gibson : “perspective order”)

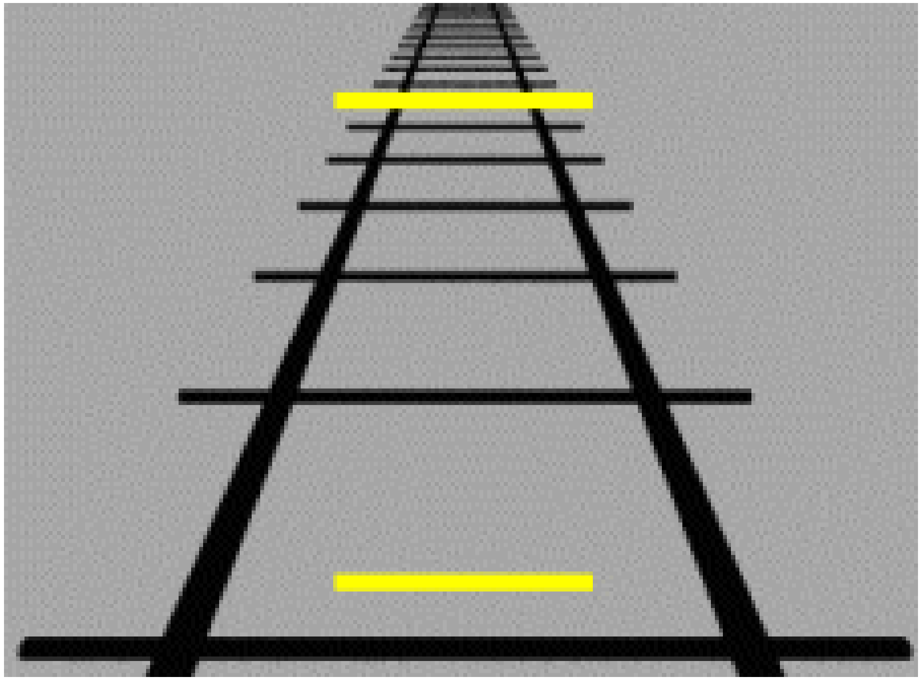
In Gibson’s term, perspective is a characteristic of the visual field rather than the visual world. It approximates how we see (the retinal image) rather than what we see, the objects in the world.

Perspective : a representation that is specific to one individual, in one position in space and one moment in time (a powerful immediacy).

Is perspective a universal fact of the visual retinal image ? Or is perspective something that is learned ?

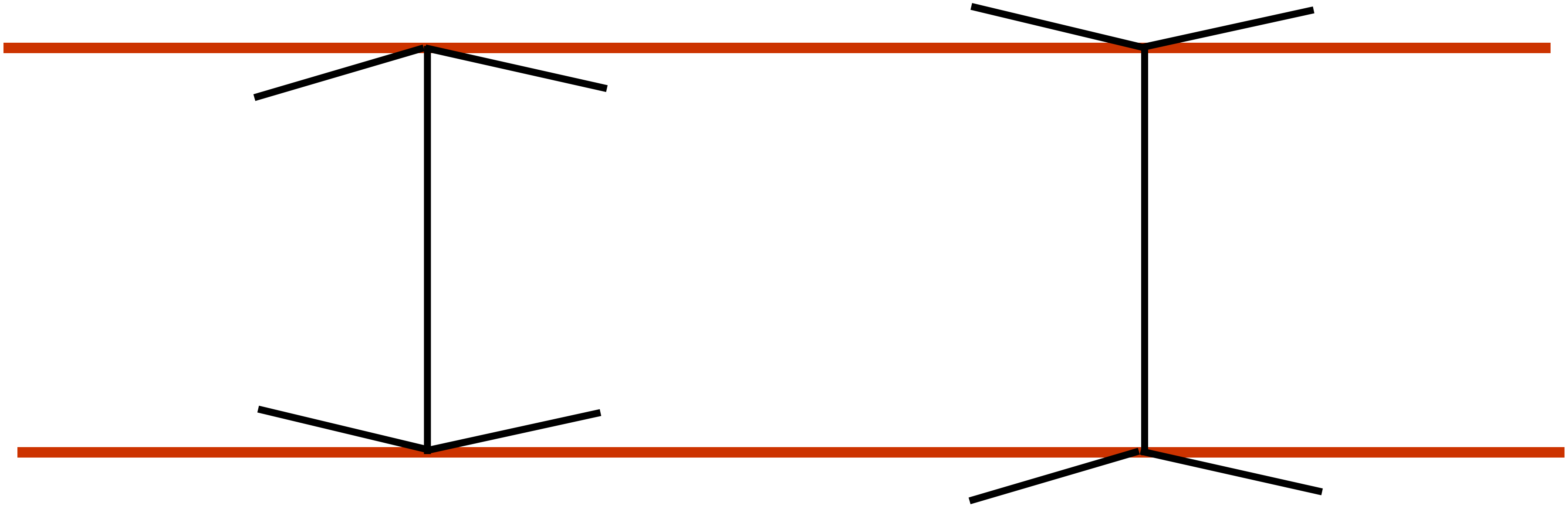
Simple and powerful cue, and easy to make it work in practice…

Linear Perspective



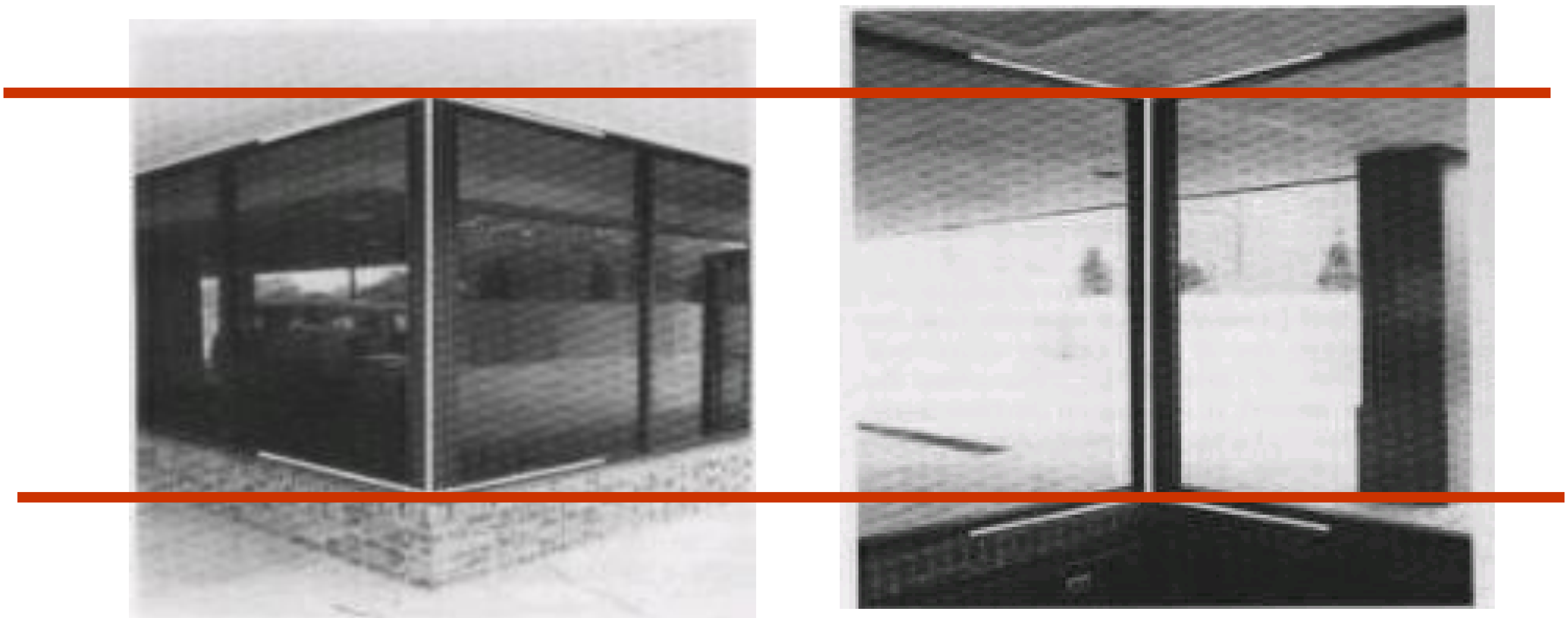
Ponzo’s illusion

Linear Perspective



Muller-Lyer
1889

Linear Perspective

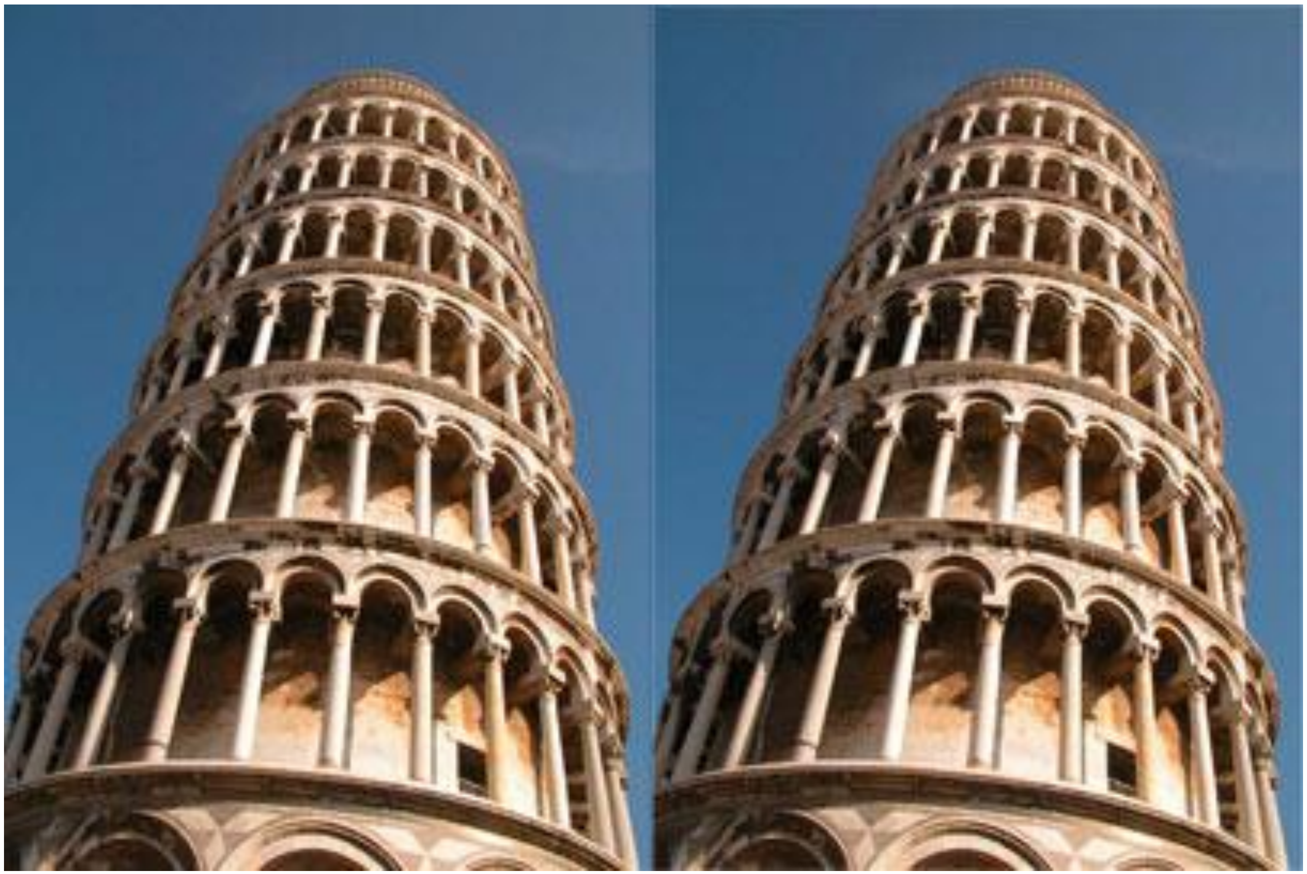


Muller-Lyer
1889

Linear Perspective

Muller-Lyer
1889

The two Towers of Pisa



Frederick Kingdom, Ali Yoonessi and Elena Gheorghiu of McGill Vision Research unit.

The strength of linear perspective

3D percept is driven by the scene, which imposes its ruling to the objects

Manhattan assumption

Application of the statistics of edges: Manhattan World

Many scenes of man-made environments are laid out on a 3-D "Manhattan" grid.

This 3-D structure imposes statistical regularities on the edges, and hence the image gradients, in the image.

These regularities allow us to infer the viewer orientation relative to the Manhattan grid and to detect targets *unaligned* to the grid.

J. Coughlan and A.L. Yuille. "Manhattan World: Orientation and Outlier Detection by Bayesian Inference." Neural Computation. May 2003.

Slide by James Coughlan

Bayesian Model of Manhattan World

Evidence for line edges -- x, y, z or random lines -- provided by the image gradient. Prior on occurrence of these edges.

Image gradient magnitude provides evidence for presence or absence of edges, using P_{on} and P_{off} distributions.

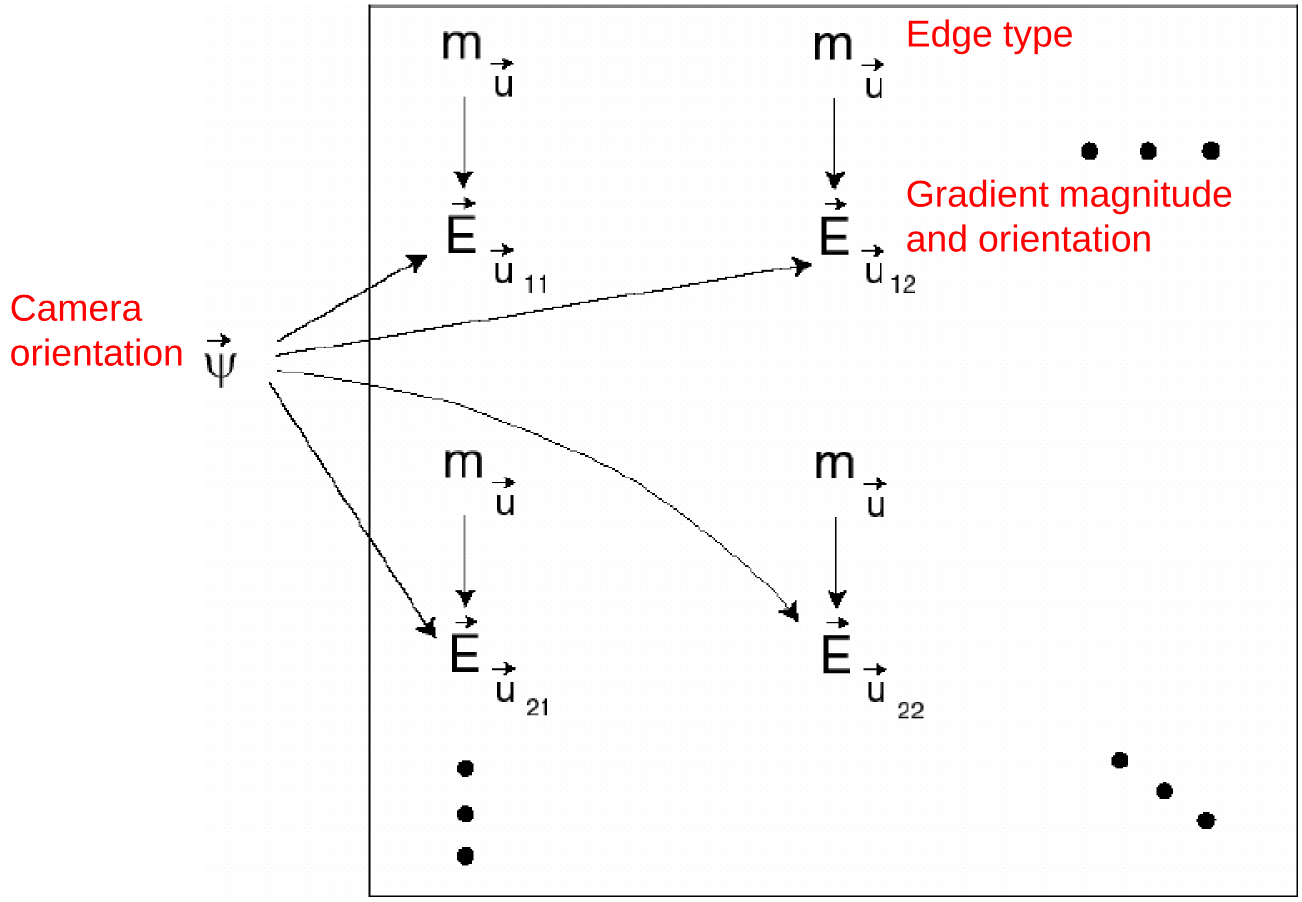
Image gradient direction provides information about edge *orientations*.

Hidden assignment variables: at each pixel, is there an x, y, z or random line, or no edge at all?

If we knew this assignment at each pixel, and the camera orientation Ψ, we could predict likely values of image gradient magnitude and direction, $\vec{E}_{\vec{u}} = (E_{\vec{u}}, \phi_{\vec{u}})$

Slide by James Coughlan

Evidence over all pixels: Bayes net of full Bayesian model

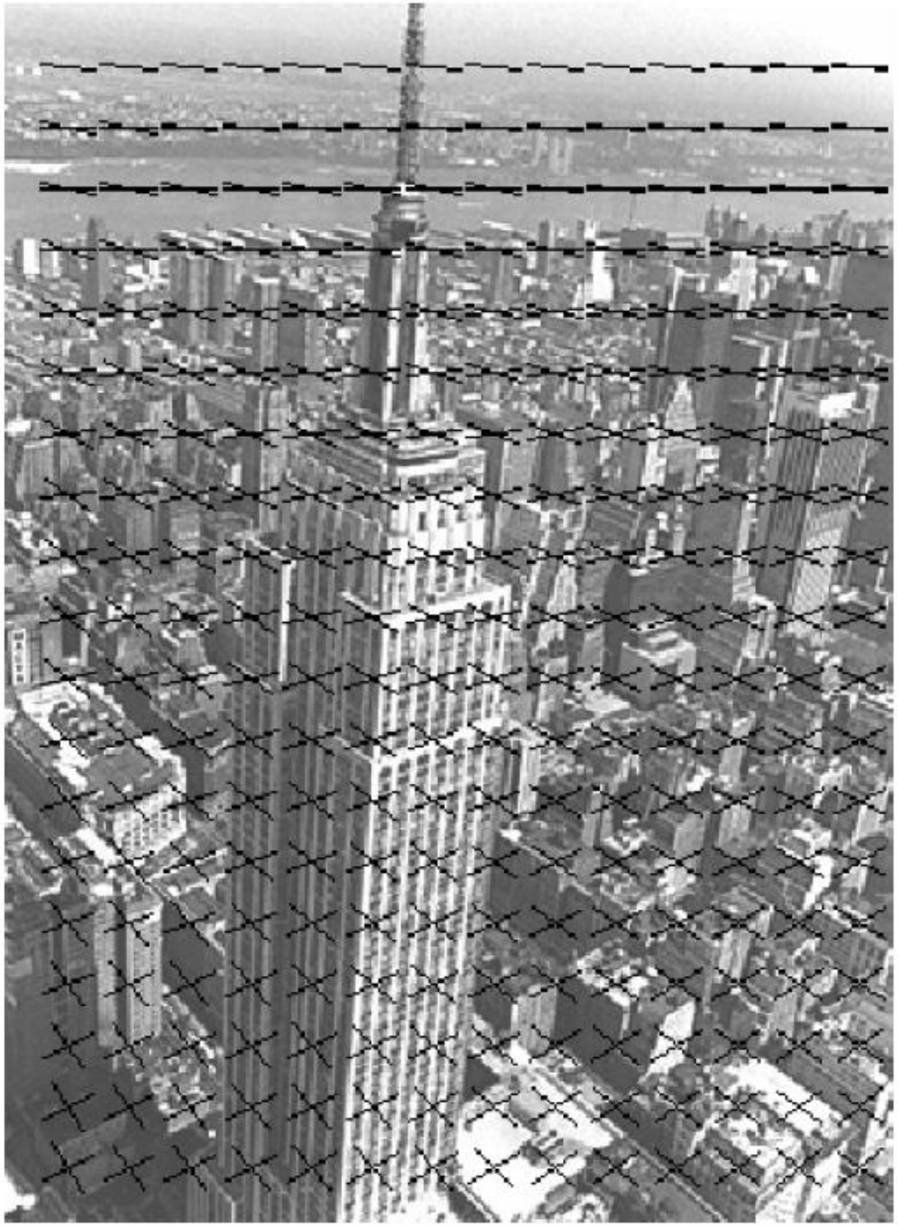


Box represents entire image, with an image gradient vector and assignment variable at each pixel location $\vec{u}_{ij}$

Structure of net graphically illustrates assumption of conditional independence across pixels.

Slide by James Coughlan

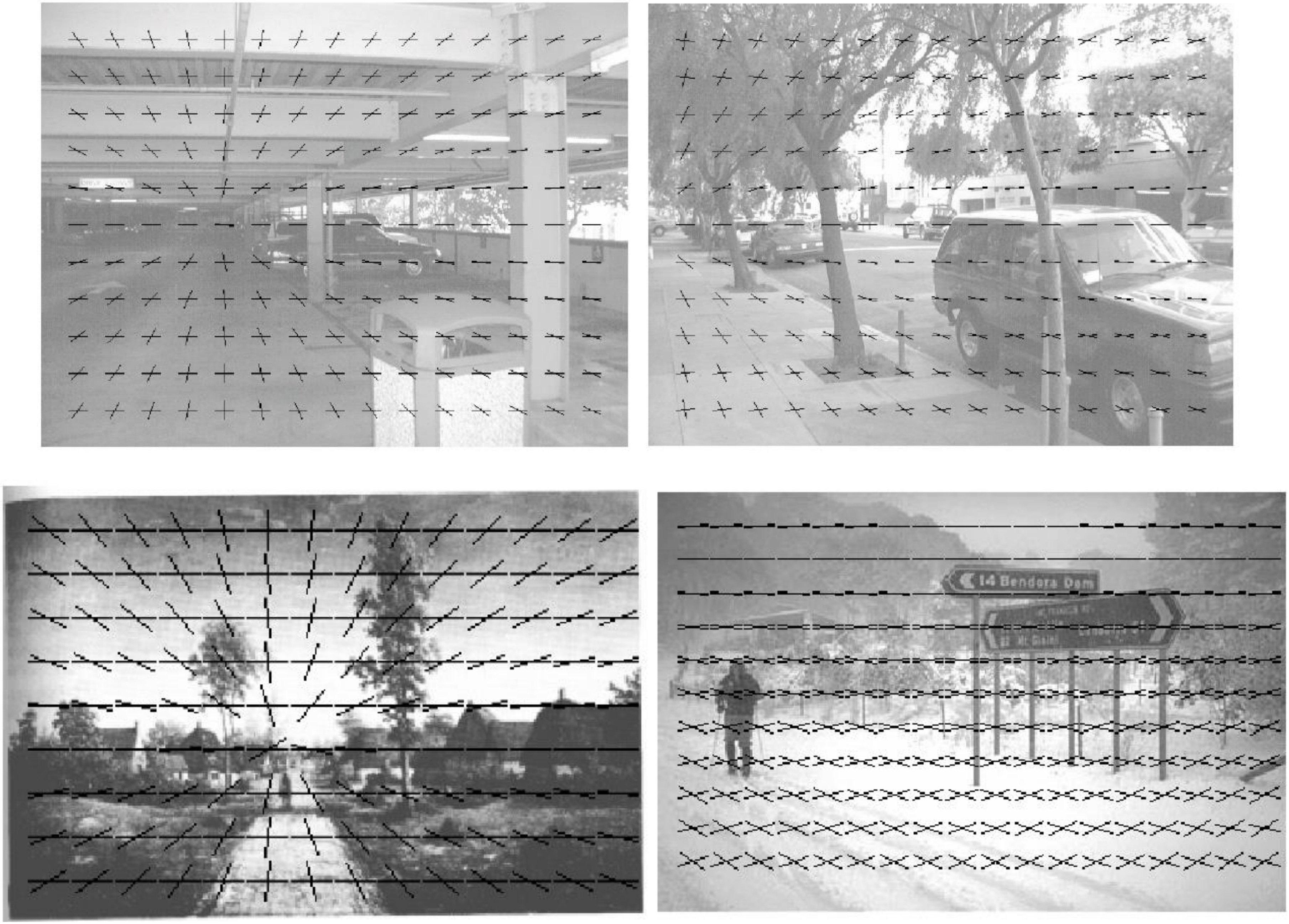
Experimental Results



Estimate of *most probable camera orientation given image*, rendered in terms of the corresponding orientations of x and y lines (drawn in black).

Note how the x lines align with the sides of buildings that are visible and facing left. The y lines align with the other visible sides facing right.

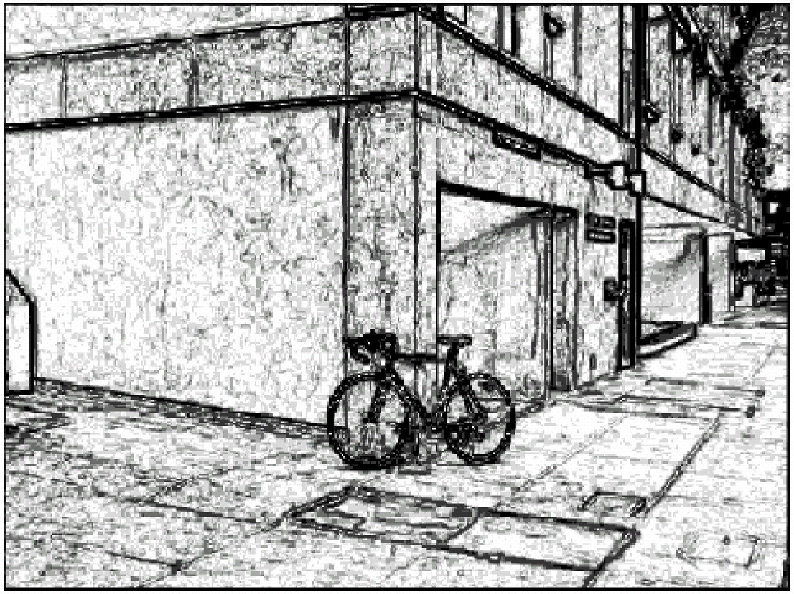
Slide by James Coughlan


14 Bendora Dam

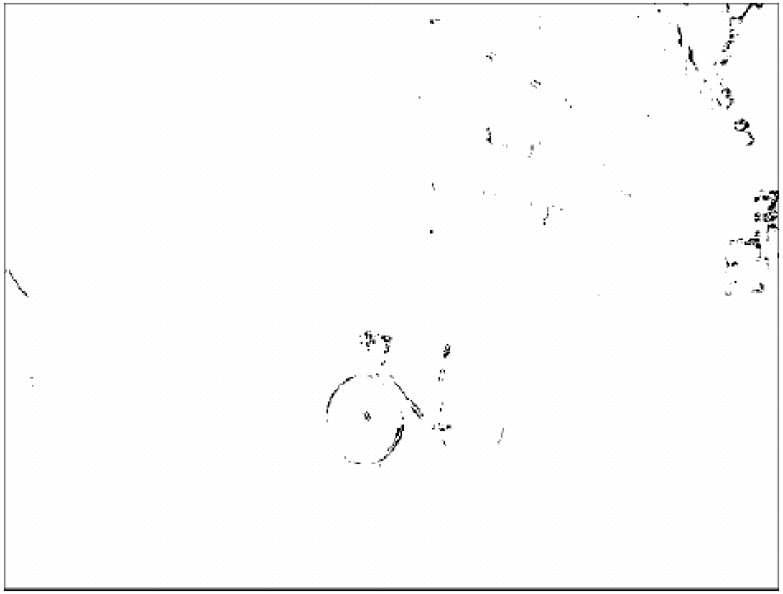
Outlier detection

Slide by James Coughlan

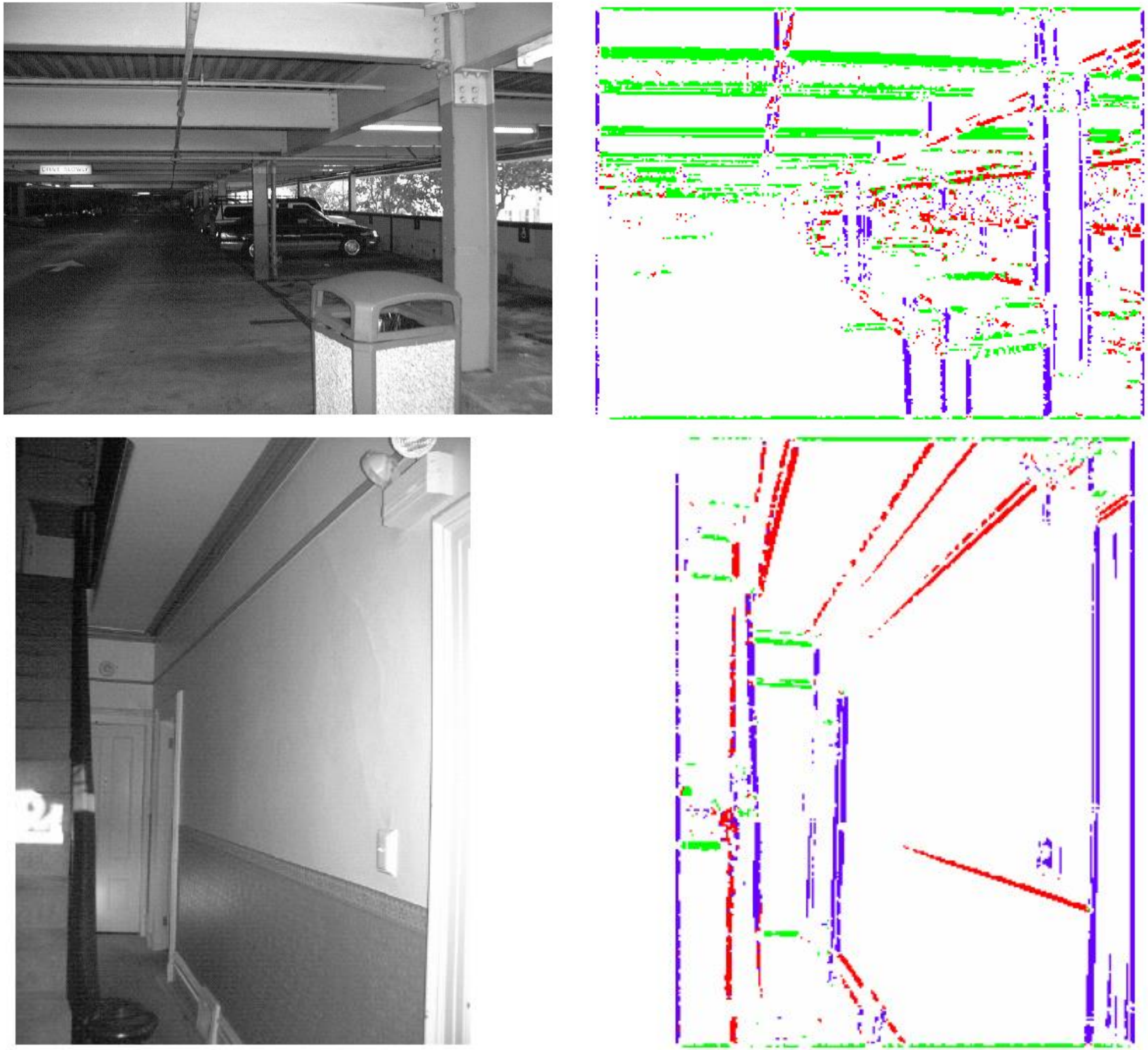
Input image:



$\text{Log}(P_{on}/P_{off})$



Outliers detected

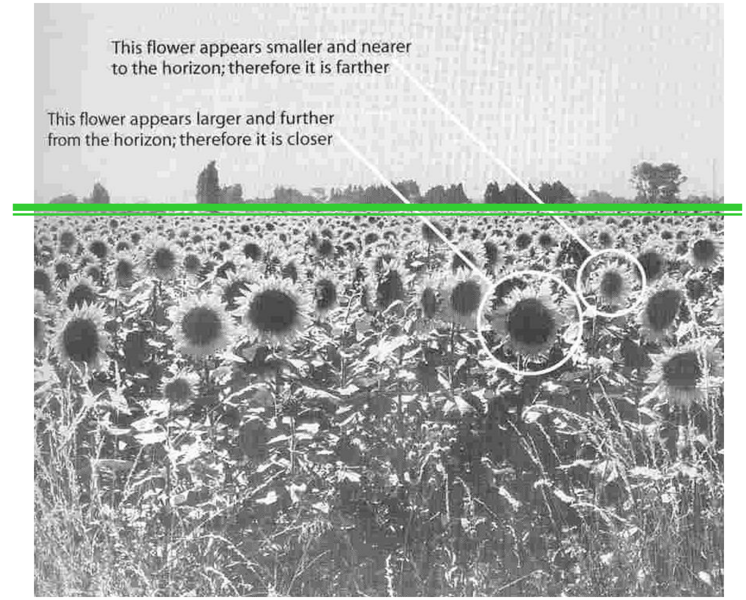


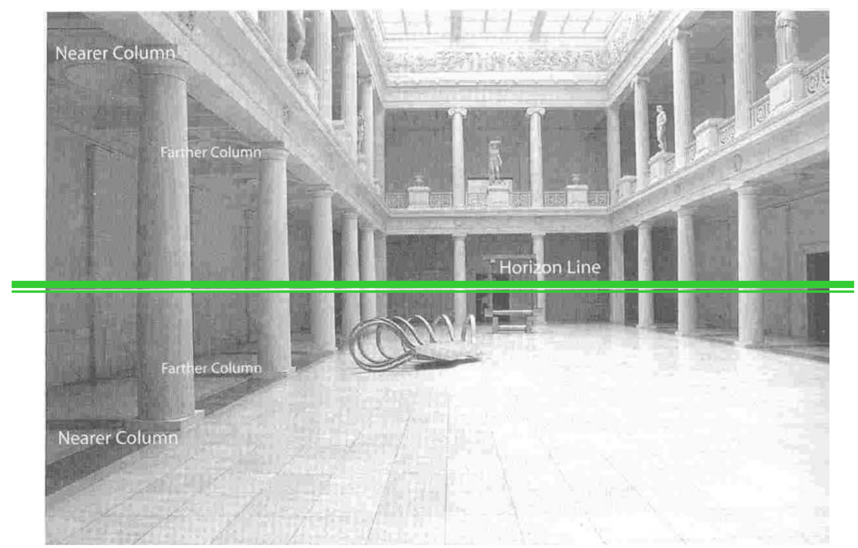
x lines in **red**
y lines in **green**
z lines in **blue**

The importance of the horizon line

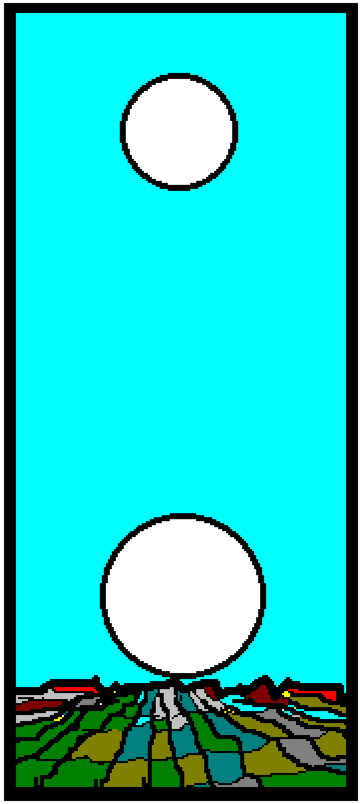
Distance from the horizon line

- Based on the tendency of objects to appear nearer the horizon line with greater distance to the horizon.
- Objects approach the horizon line with greater distance from the viewer. The base of a nearer column will appear lower against its background floor and further from the horizon line. Conversely, the base of a more distant column will appear higher against the same floor, and thus nearer to the horizon line.





Moon illusion

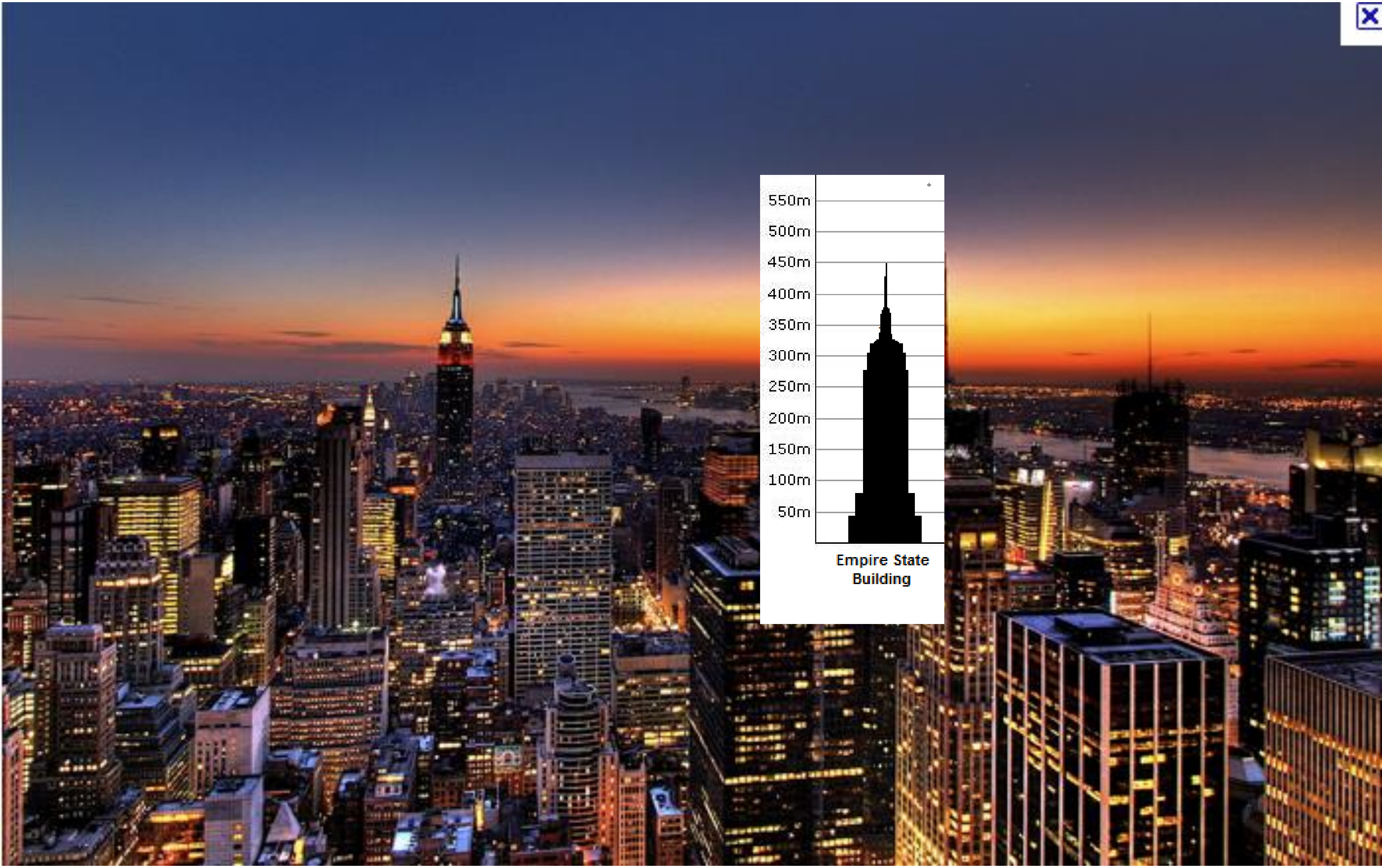


Relative height

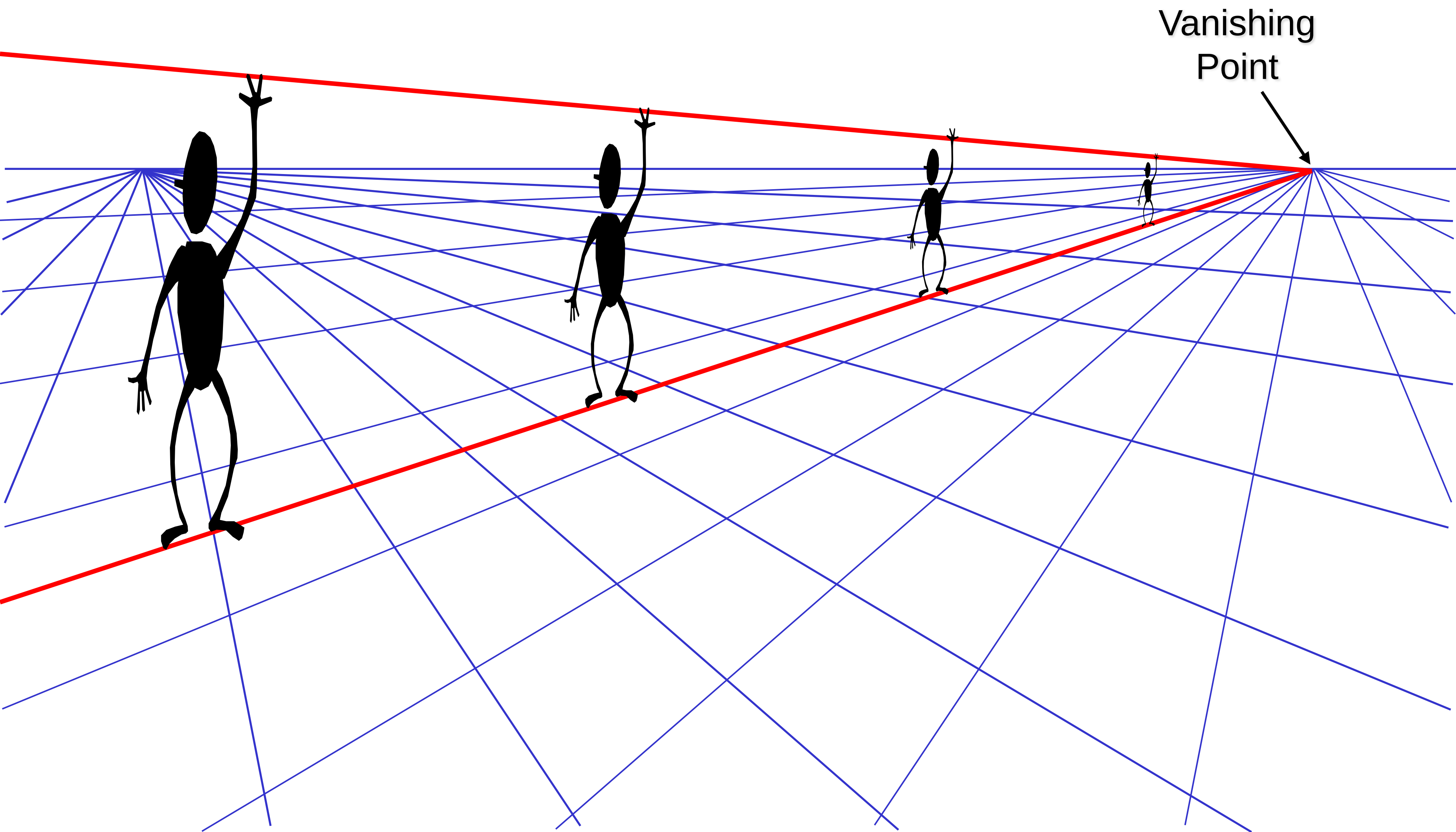
The object closer to the horizon is perceived as farther away, and the object further from the horizon is perceived as closer

If you know camera parameters: height of the camera, then we know real depth

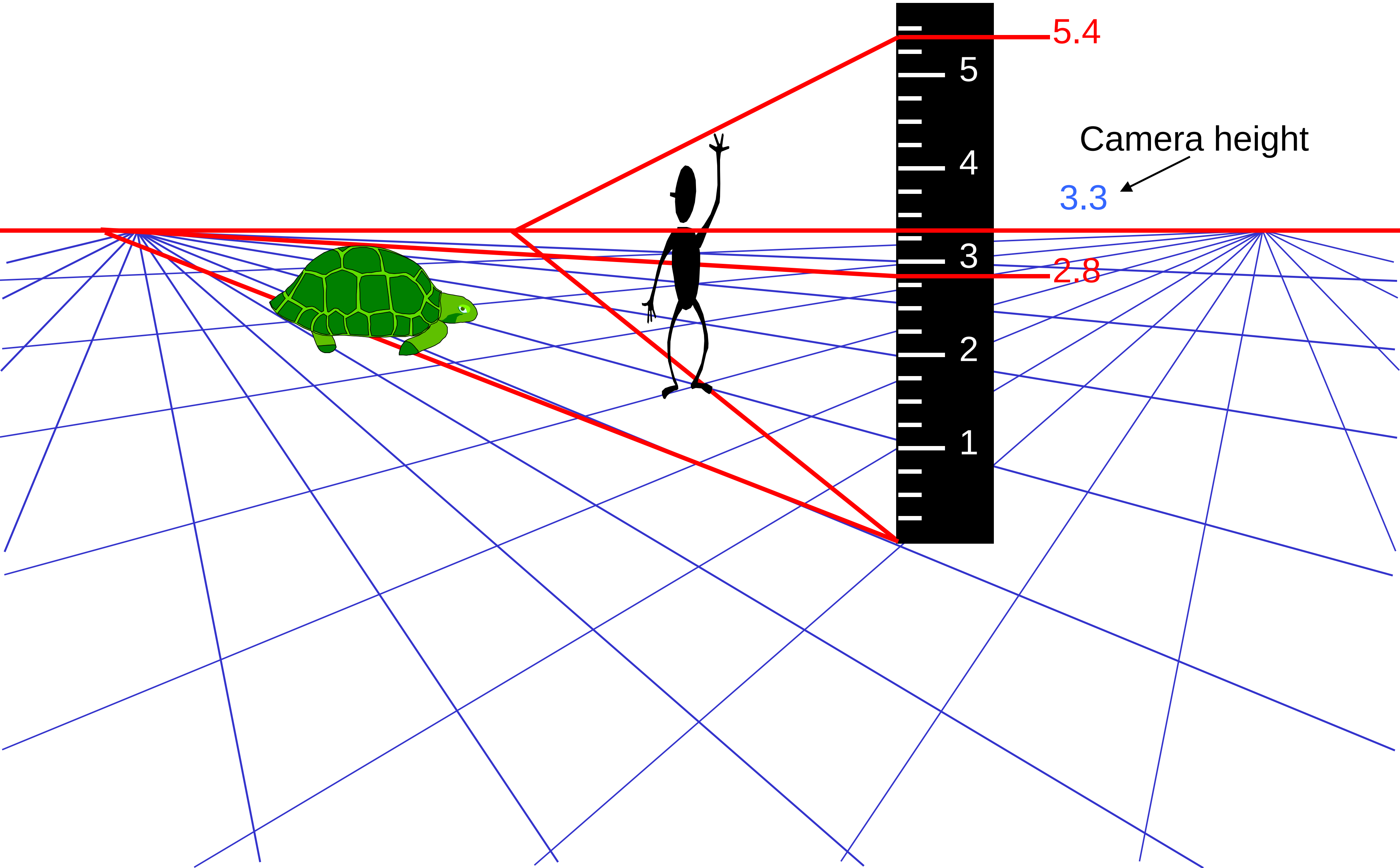
At which elevation has been taken this picture?



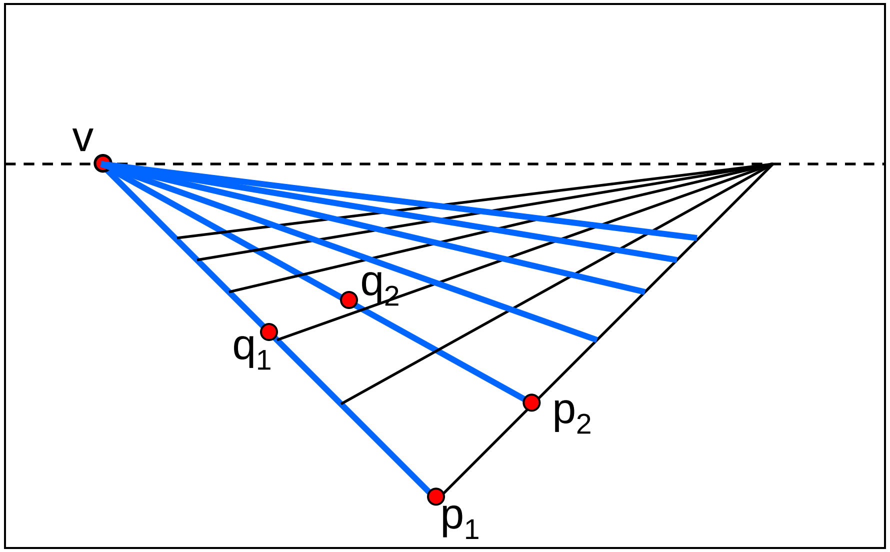
Comparing heights



Measuring height



Computing vanishing points (from lines)



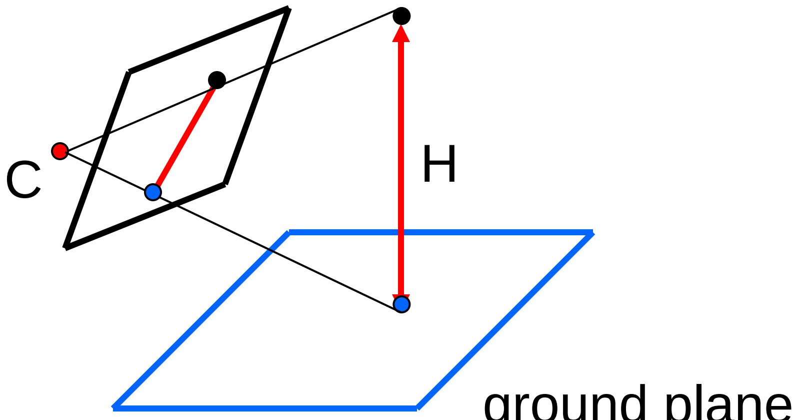
Intersect p_1q_1 with p_2q_2

$$v = (p_1 \times q_1) \times (p_2 \times q_2)$$

Least squares version

- Better to use more than two lines and compute the “closest” point of intersection
- See notes by [Bob Collins]for one good way of doing this:
  - http://www-2.cs.cmu.edu/~ph/869/www/notes/vanishing.txt

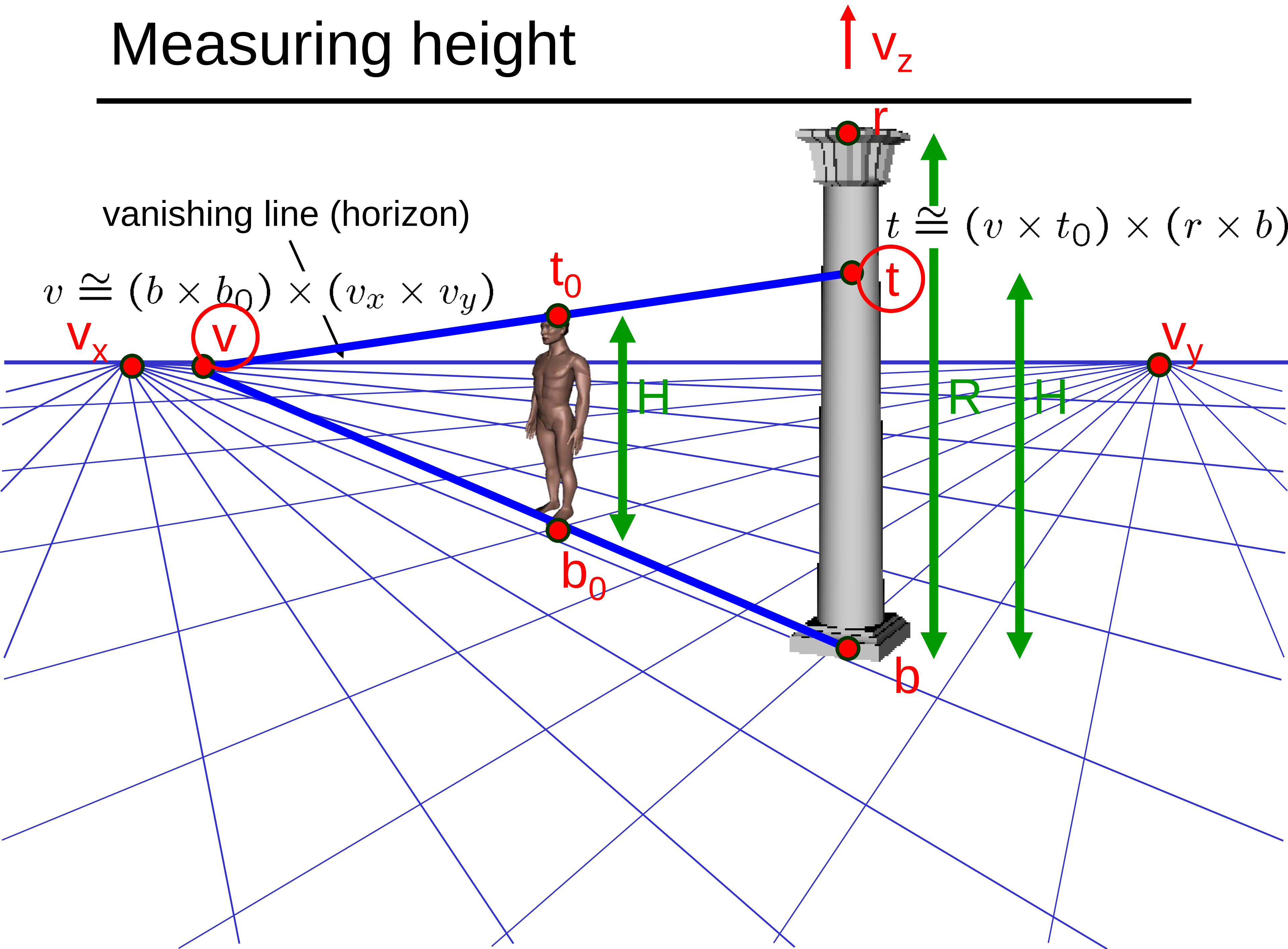
Measuring height without a ruler



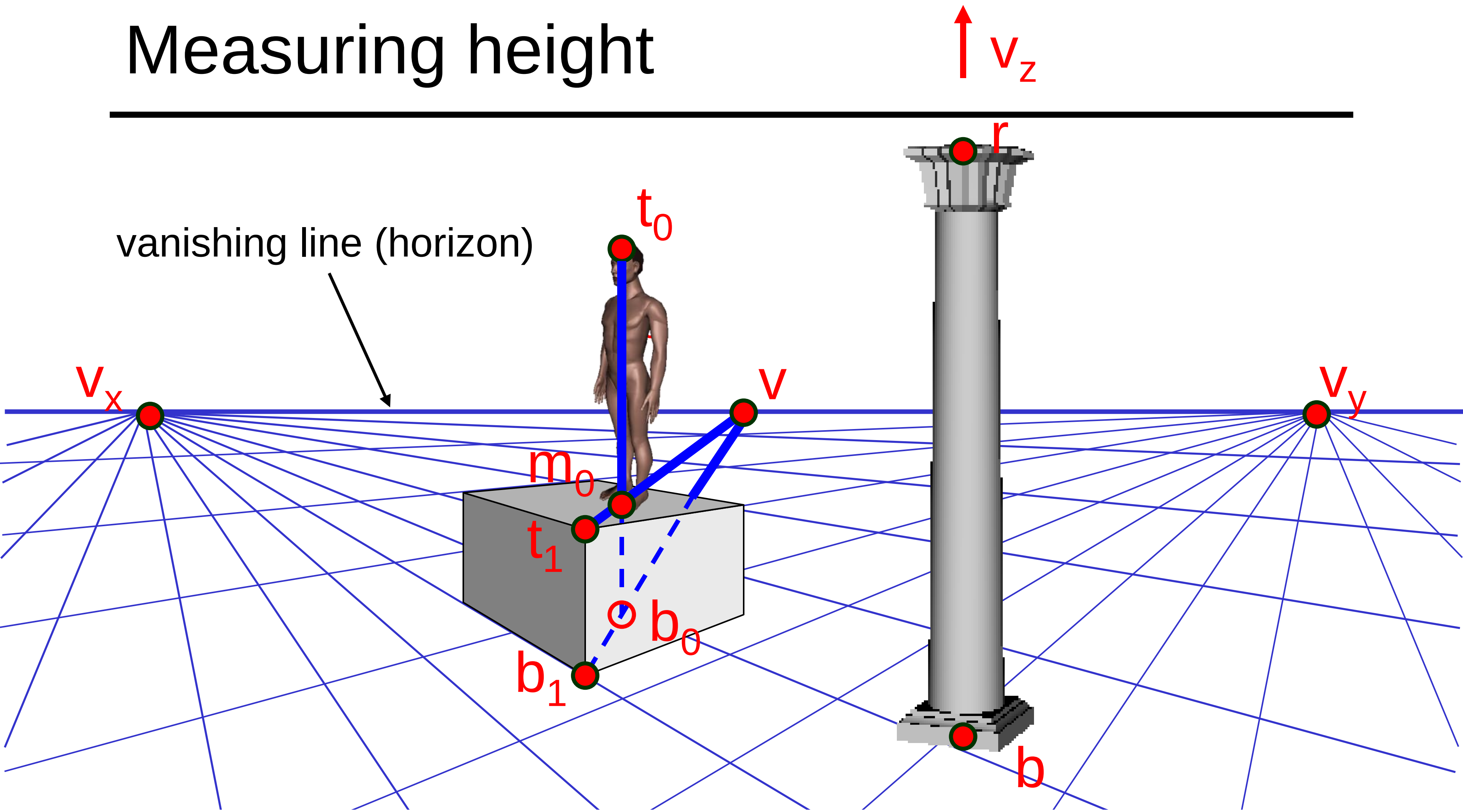
Compute H from image measurements

- Need more than vanishing points to do this

Measuring height



Measuring height



What if the point on the ground plane b_0 is not known?

- Here the guy is standing on the box
- Use one side of the box to help find b_0 as shown above

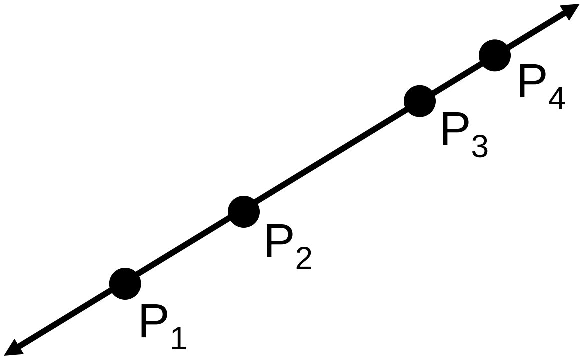
What if v_z is not infinity?

The cross ratio

A Projective Invariant

- Something that does not change under projective transformations (including perspective projection)

The cross-ratio of 4 collinear points



$$\frac{\|\mathbf{P}_3-\mathbf{P}_1\|\,\|\mathbf{P}_4-\mathbf{P}_2\|}{\|\mathbf{P}_3-\mathbf{P}_2\|\,\|\mathbf{P}_4-\mathbf{P}_1\|}$$

$$\mathbf{P}_i=\begin{bmatrix}X_i\\Y_i\\Z_i\\1\end{bmatrix}$$

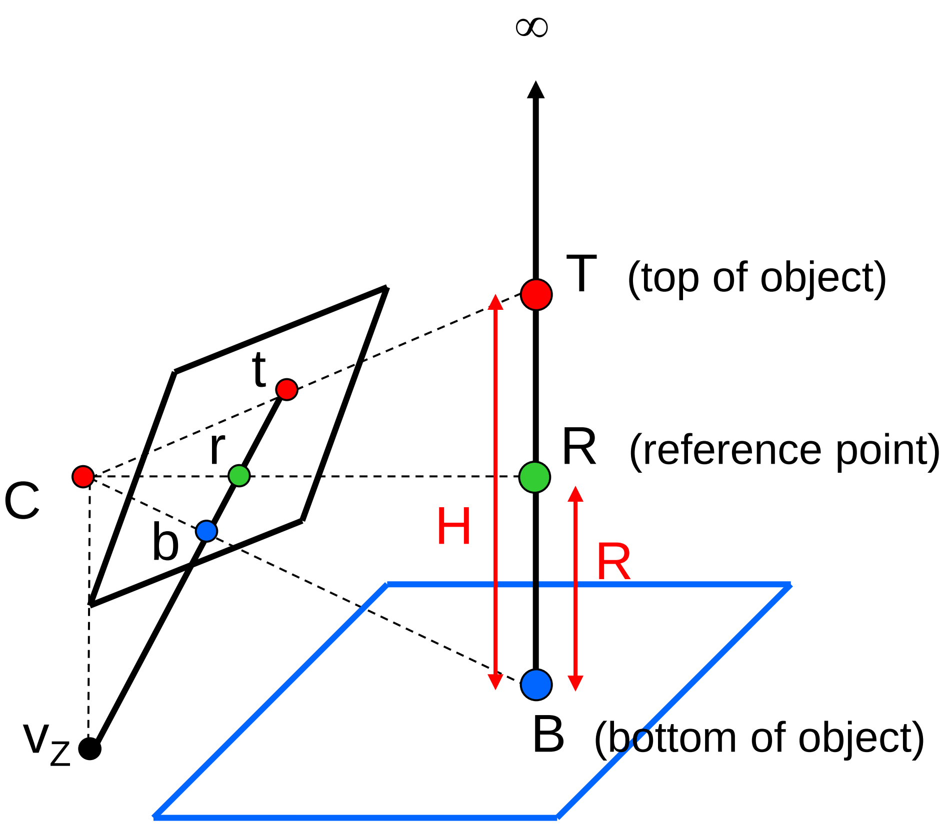
Can permute the point ordering

$$\frac{\|\mathbf{P}_1-\mathbf{P}_3\|\,\|\mathbf{P}_4-\mathbf{P}_2\|}{\|\mathbf{P}_1-\mathbf{P}_2\|\,\|\mathbf{P}_4-\mathbf{P}_3\|}$$

- 4! = 24 different orders (but only 6 distinct values)

This is the fundamental invariant of projective geometry

Measuring height



ground plane

$$\frac{\|\mathbf{T}-\mathbf{B}\|\,\|\infty-\mathbf{R}\|}{\|\mathbf{R}-\mathbf{B}\|\,\|\infty-\mathbf{T}\|} = \frac{H}{R}$$

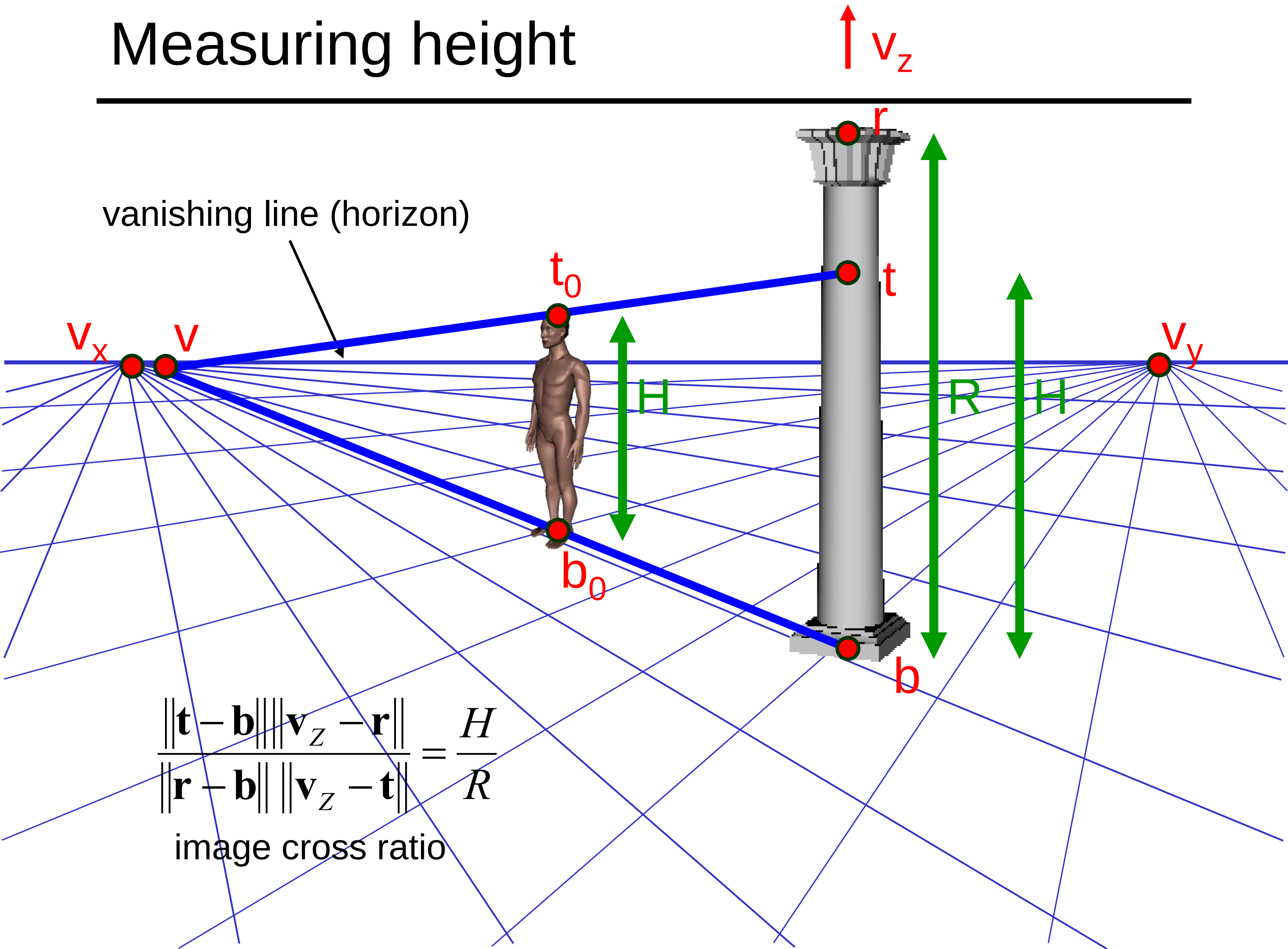
scene cross ratio

$$\frac{\|\mathbf{t}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{t}\|} = \frac{H}{R}$$

image cross ratio

scene points represented as $\mathbf{P} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$ image points as $\mathbf{p} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

Measuring height

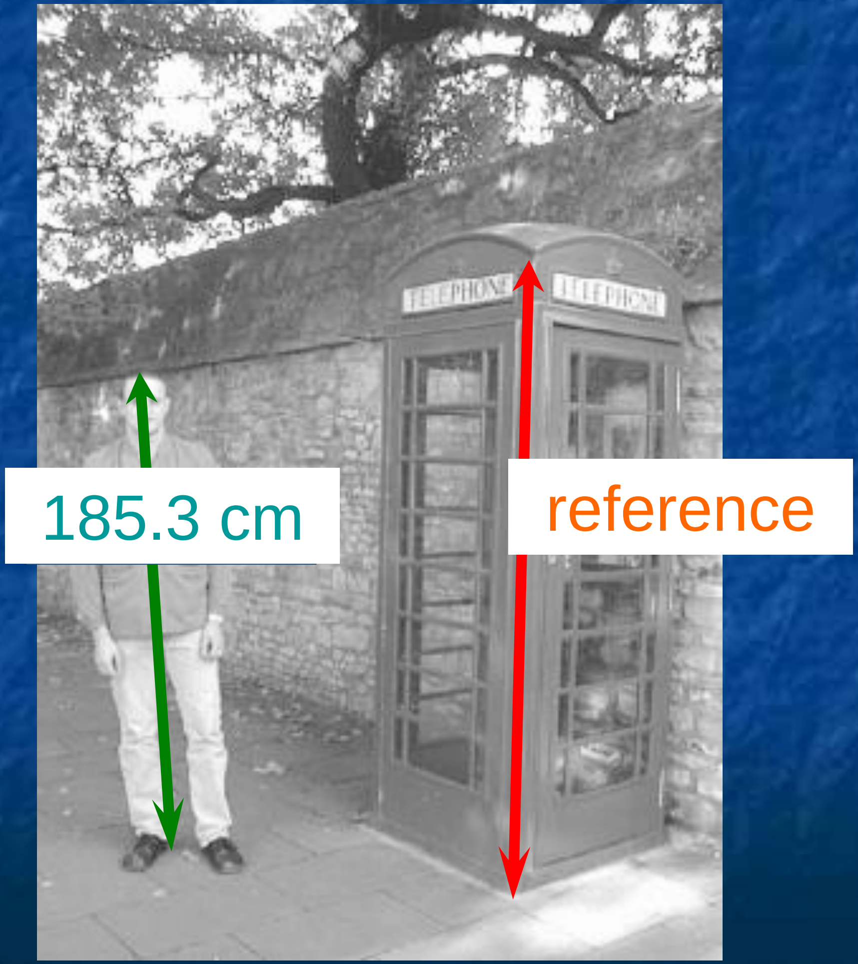


$$\frac{\|\mathbf{t}-\mathbf{b}\|\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\|\,\|\mathbf{v}_Z-\mathbf{t}\|}=\frac{H}{R}$$

image cross ratio

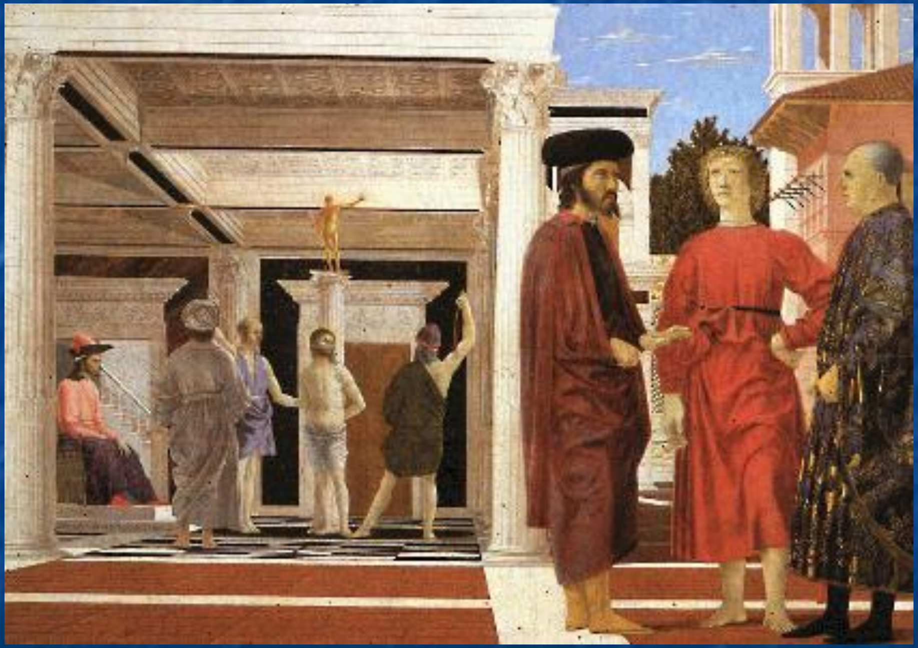
Measuring heights in real photos

Problem: How tall is this person?

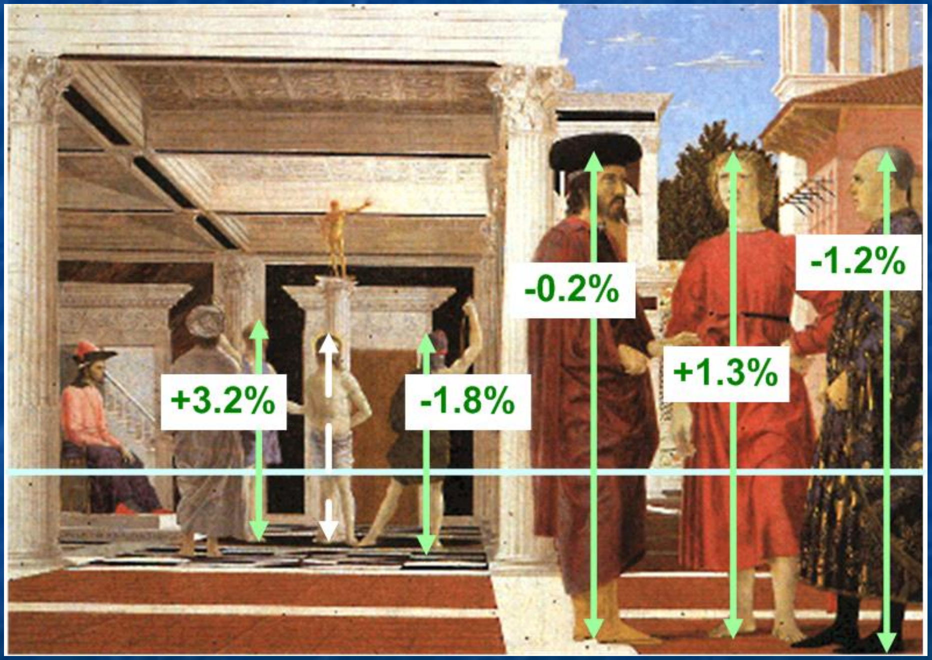


Assessing geometric accuracy

Problem:
Are the heights of the two groups of people consistent with each other?



Piero della Francesca, *Flagellazione di Cristo*, c.1460, Urbino



Measuring relative heights

Single-View Metrology

Complete 3D reconstructions from single views

Texture Gradient

Texture Gradient

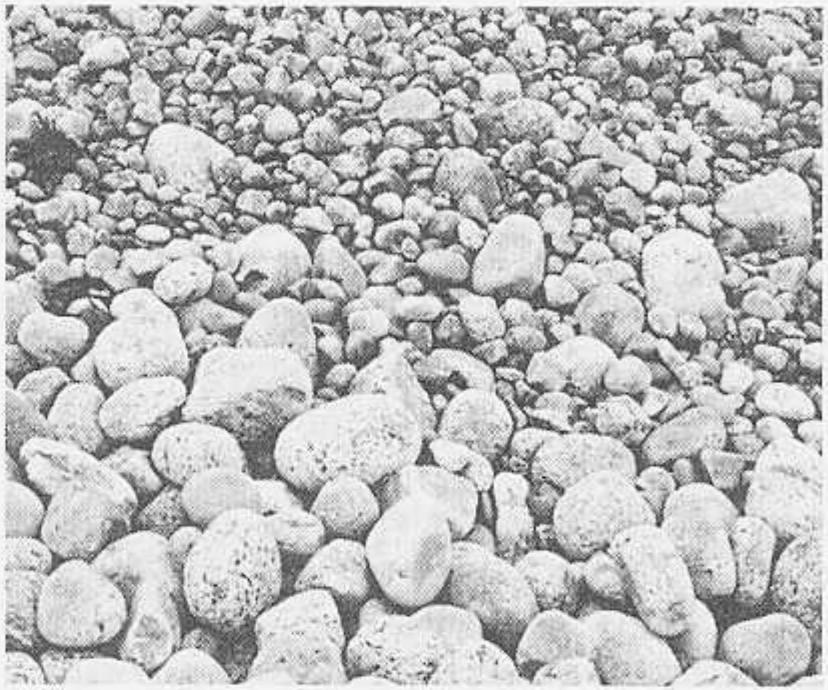


FIGURE 8.27
Texture gradients provide information about depth. (Frank Siteman/Stock, Boston.)

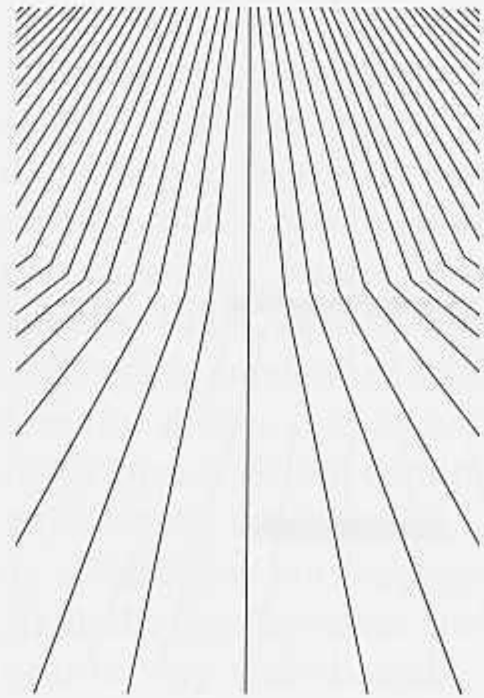
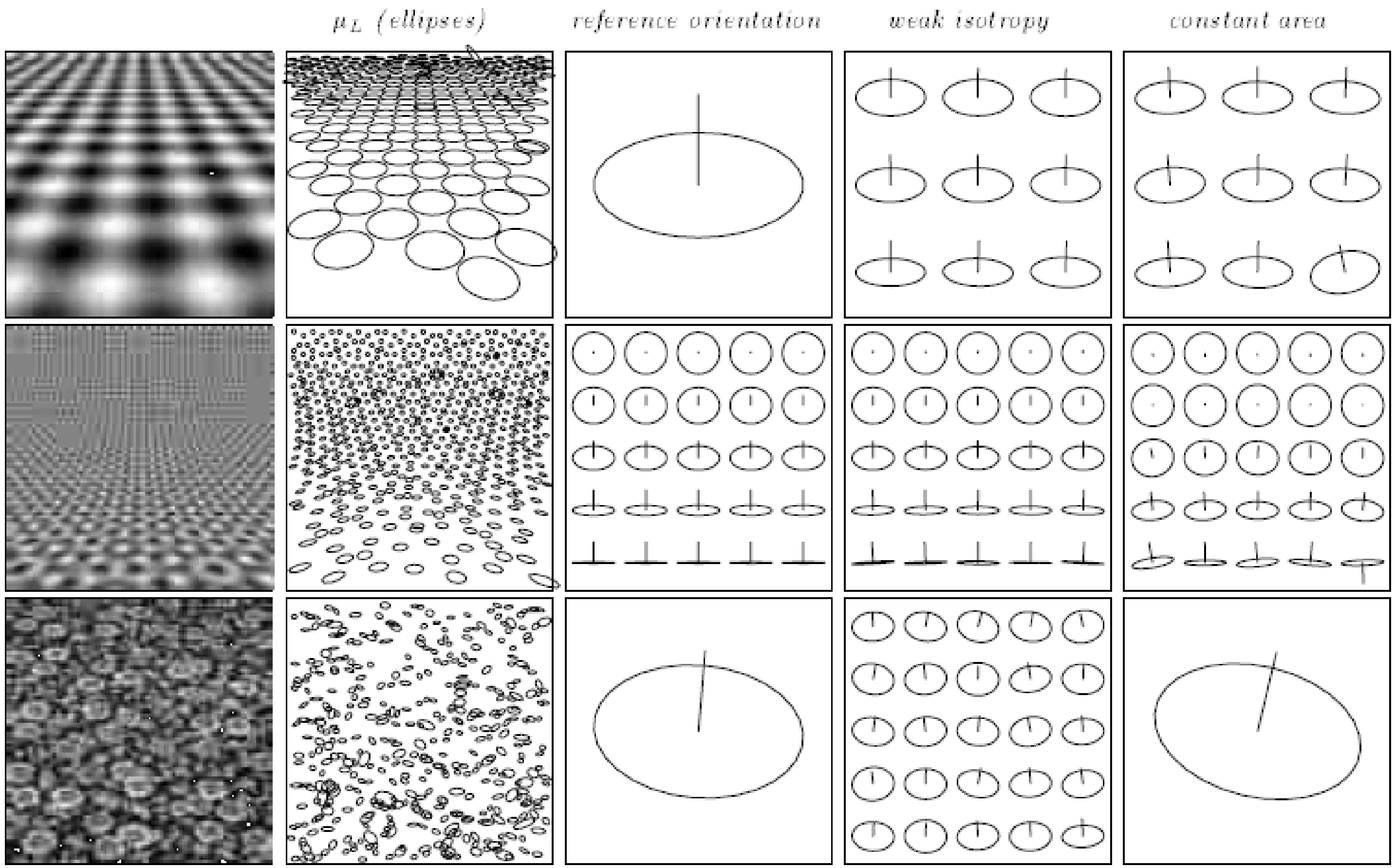


FIGURE 8.28
Texture discontinuity signals the pre
corner.

A Witkin. Recovering Surface Shape and Orientation from Texture (1981)

Texture Gradient



Shape from Texture from a Multi-Scale Perspective. Tony Lindeberg and Jonas Garding. ICCV 93

Texture Gradient

- Filter outputs

- Textons

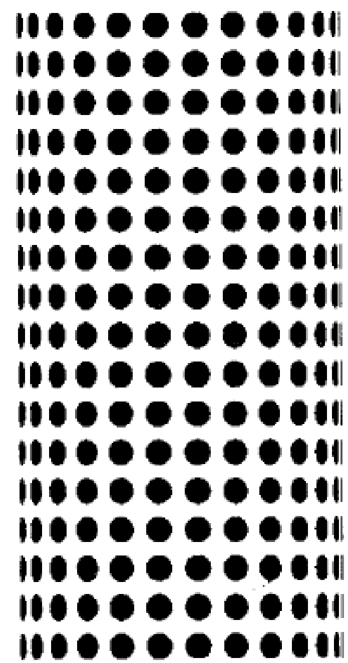
IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE, VOL. 17, NO. 4 APRIL 1995

Shape from Texture Using Local Spectral Moments

Boaz J. Super, *Member, IEEE*, and Alan C. Bovik, *Senior Member, IEEE*

Abstract—We present a non-feature-based solution to the problem of computing the shape of curved surfaces from texture information. First, the use of local spatial-frequency spectra and their moments to describe texture is discussed and motivated. A new, more accurate method for measuring the local spatial-frequency moments of an image texture using Gabor elementary functions and their derivatives is presented. Also described is a technique for separating shading from texture information, which makes the shape-from-texture algorithm robust to the shading effects found in real imagery. Second, a detailed model for the projection of local spectra and spectral moments of any surface reflectance patterns (not just textures) is developed. Third, the conditions under which the projection model can be solved for the orientation of the surface at each point are explored. Unlike earlier non-feature-based, curved surface shape-from-texture approaches, the assumption that the surface texture is isotropic is not required; surface texture homogeneity can be assumed instead. The algorithm's ability to operate on anisotropic and nondeterministic textures, and on both smooth- and rough-textured surfaces, is demonstrated.

Index Items—Shape from texture, shape recovery, surface orientation, moments, wavelet, spatial frequency, Gabor functions, texture, projection.



PLANAR SURFACE ORIENTATION FROM TEXTURE SPATIAL FREQUENCIES

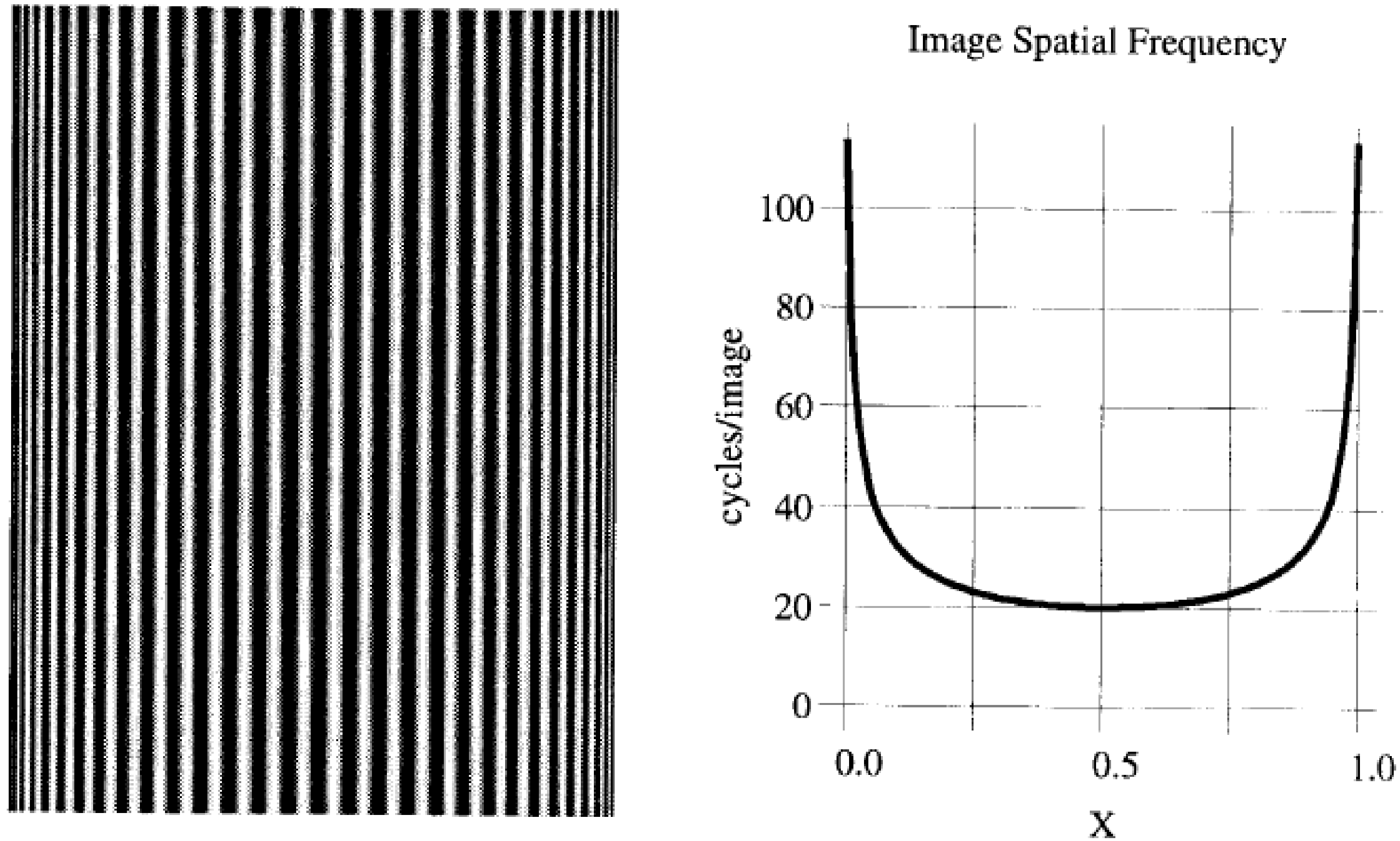
BOAZ J. SUPER *† and ALAN C. BOVIK ‡

Assumptions:

- Smooth closed surface
- Homogeneous texture
- (sometimes, isotropic texture)

Texture description

Use filter outputs to measure local spatial frequency.



(a) (b)

Fig. 2. (a) Cylinder with sinusoidal grating texture. (b) Horizontal component of image spatial frequency on center cross-section of (a).

Texture projection

Assume orthographic projection.

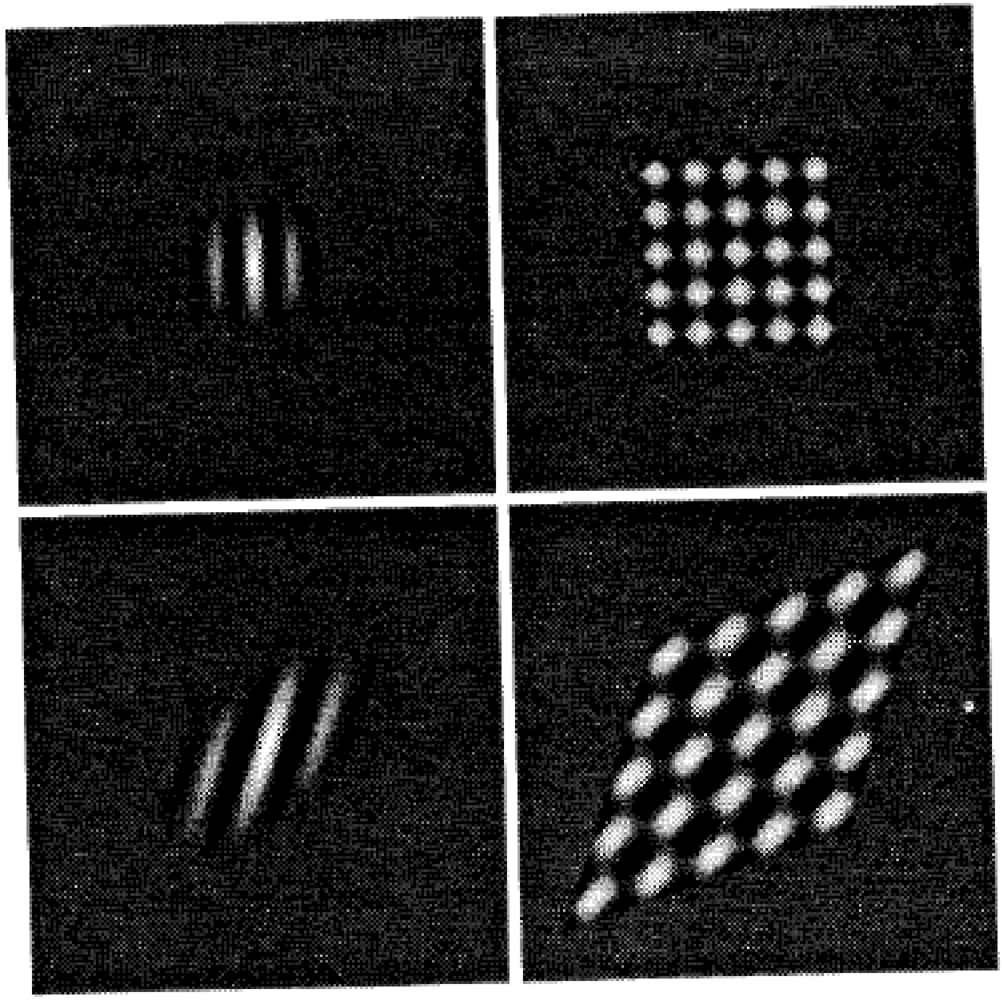
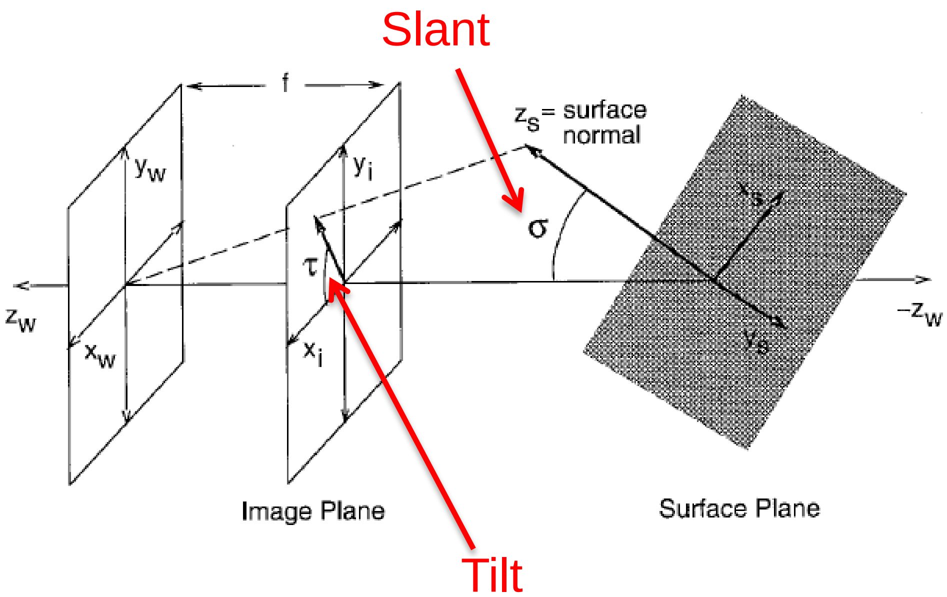
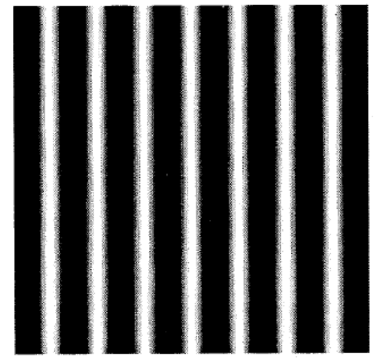


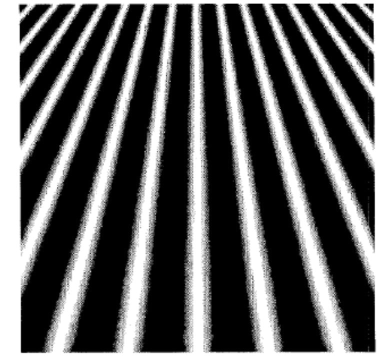
Fig. 5. Top row: real part of Gabor filter with radial frequency of 12 cycles/image, and a texture patch. Bottom row: back-projections of Gabor filter and texture patch onto a plane with orientation $(\sigma, \tau) = (60^\circ, 45^\circ)$.

Slant and tilt

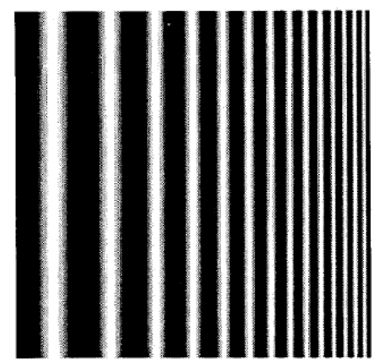




$\sigma = 0°$



$\sigma = 45°$, $\tau = 90°$

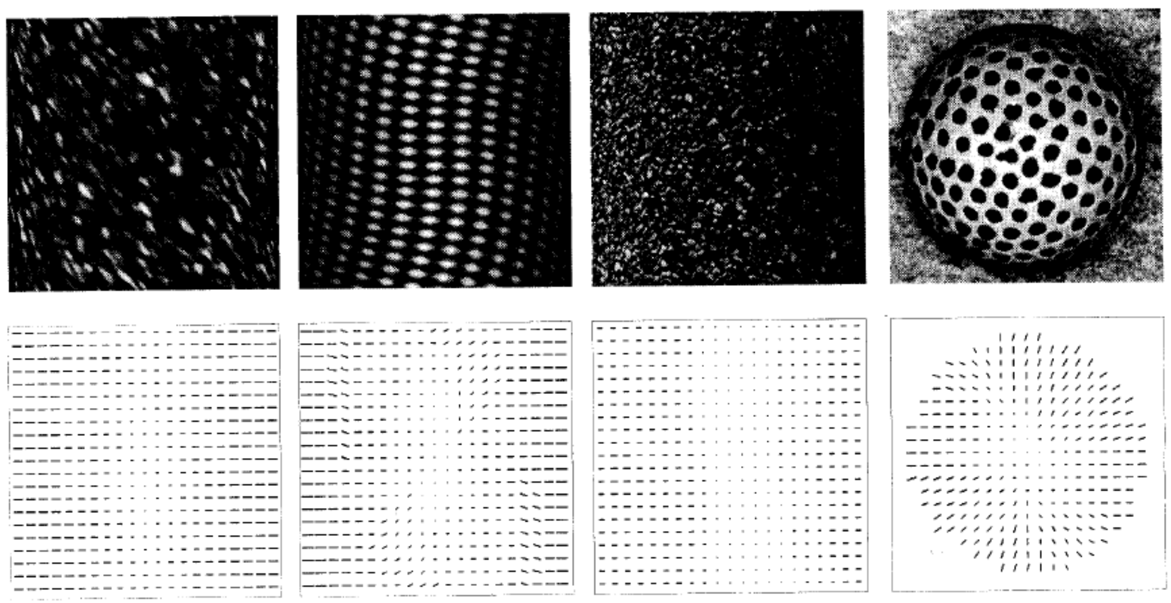


$\sigma = 45°$, $\tau = 0°$

$\sigma = 45°$, $\tau = 45°$

Box 1. Summary of algorithm

1. Convolve the image with Gabor functions and their partial derivatives, and smooth the filter output amplitudes (to reduce noise) by convolving them with a Gaussian.
2. Select the Gabor filter h_k with the largest amplitude output at each point.
3. Compute the (signed) instantaneous frequency $\mathbf{u}_i(\mathbf{x}_i)$ at each point using equation (6).
4. Sample (σ, τ)-space, backprojecting $\mathbf{u}_i(\mathbf{x}_i)$ to compute $\mathbf{u}_s(\mathbf{x}_s)$ using equation (20). Compute the variance $V_{\sigma,\tau}$ of $\mathbf{u}_s(\mathbf{x}_s)$. Coarse-to-fine sampling in multiple stages may be used.
5. Output the values of (σ, τ) for which $V_{\sigma,\tau}$ is a minimum.



(a) (b) (c) (d)

Recovering shape and irradiance maps from rich dense texton fields

Anthony Lobay and D.A. Forsyth

CVPR 04

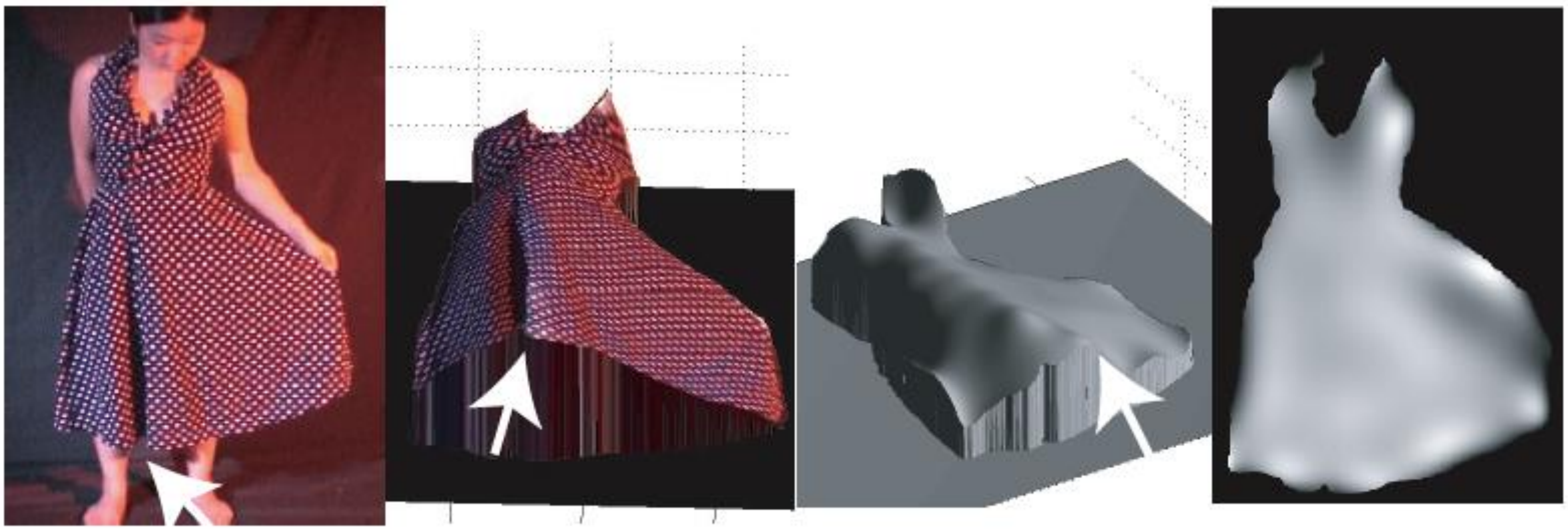


Figure 3: *On the* **left**, *a view of a model in a spotted dress. In the* **center left**, *a textured view of the reconstruction obtained using our method. This reconstruction used 1200 texton instances, in 8 clusters. Note the relatively fine detail that was obtained by the reconstruction, including the two main folds in the skirt (indicated with arrows). Typically, rendering texture on top of the view produces a better looking surface, so we show the surface without texturing on the* **center right**; *arrows indicate the reconstructed folds in the geometry. Notice that the fold in the skirt is well represented. The smoothing term is generally good at resolving normal ambiguities, but patches of surface that are not well connected to the main body can be subjected to a concave-convex ambiguity, as has happened to part of the skirt's bodice here. On the* **right**, *the irradiance map estimated using our method.*

Texture description

Non-occluded textons, and approximated as flat.

The two pieces of the solution

If we knew the transformations

- We can find the textons
- We can find the local intensity contrast

By minimization of:

$$\Sigma_i \; || \; \lambda_i \mathcal{I}_\mu - \mathcal{I}_i \; ||^2$$

Local contrast

Texton

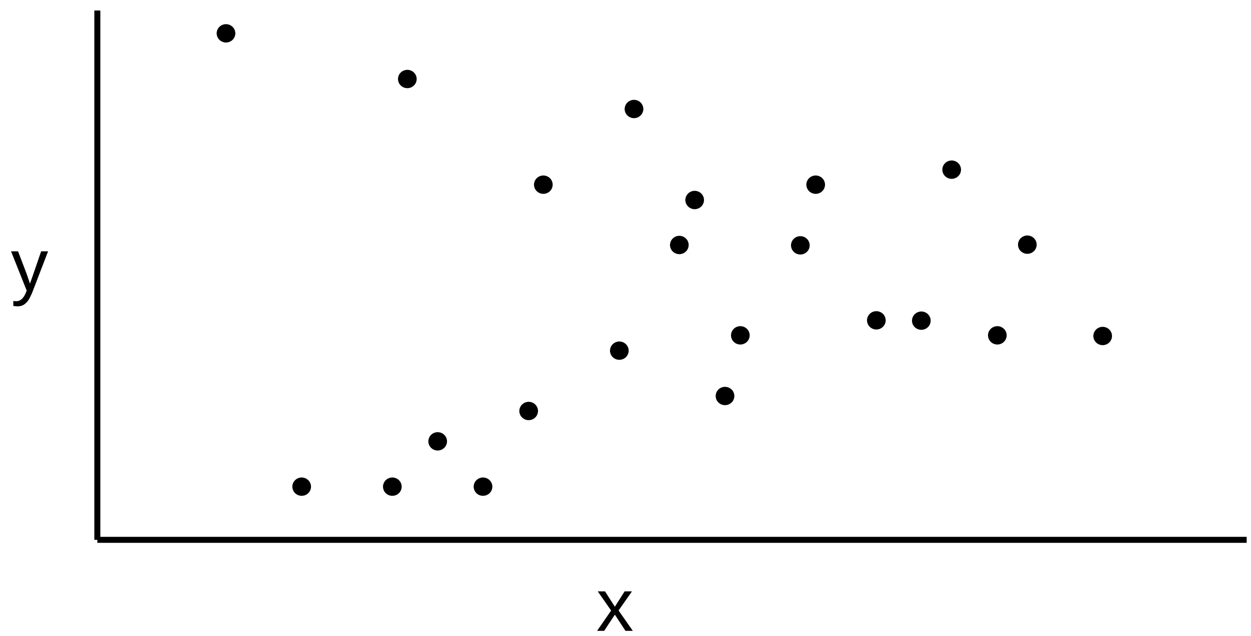
Local texture with inverse transform

If we knew the texton and contrast

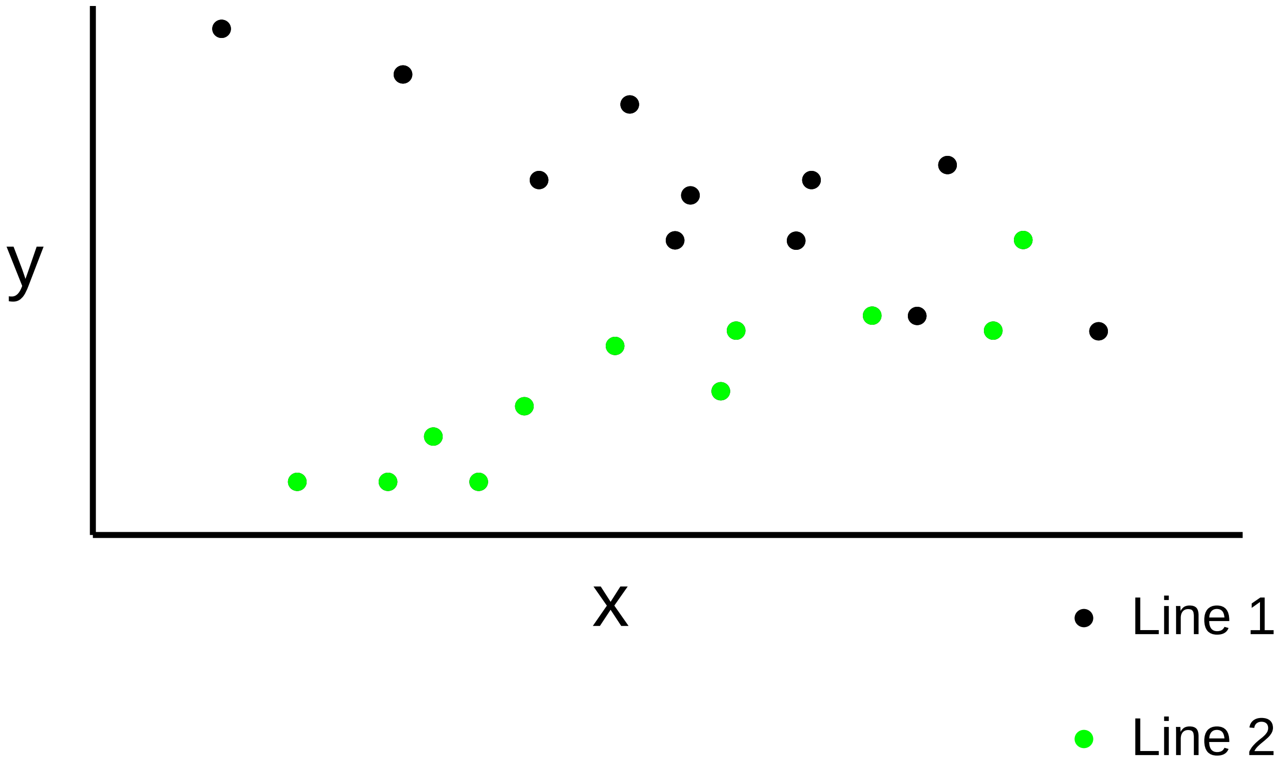
- Recover the transformation by transforming the texton to match each local patch.

Expectation Maximization (EM): a solution to chicken-and-egg problems

Model fitting example
Fitting two lines to observed data



Fitting two lines: on the one hand…



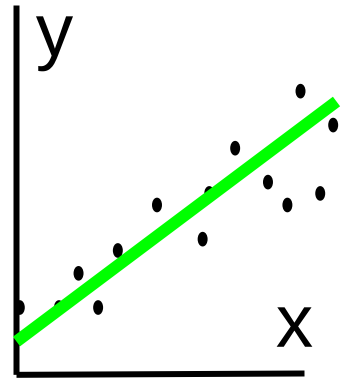
If we knew which points went with which lines, we'd be back at the single line-fitting problem, twice.

Maximum likelihood estimation for the slope of a single line

data: $(X_n, Y_n), n = 1 \ldots, N$

model: $Y = aX + w$

where $w \sim N(\mu = 0, \sigma = 1)$.



Data likelihood for point n:

$$P(X_n, Y_n | a) = c \exp[-(Y_n - aX_n)^2/2]$$

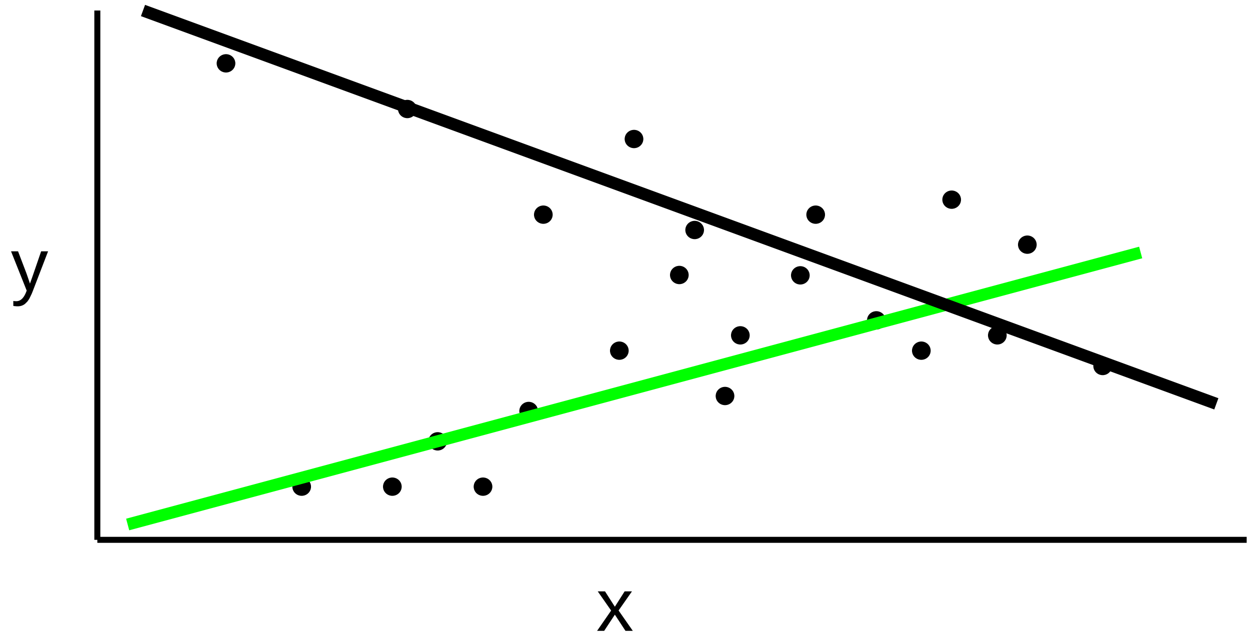
Maximum likelihood estimate:

$$\hat{a} = \arg\max_a p(Y_1, \ldots, Y_n | a) = \arg\max_a \sum_n -d(Y_n; a)^2/2$$

where $d(Y_n; a) = |Y_n - aX_n|$

gives regression formula $\hat{a} = \frac{\sum_n Y_n X_n}{\sum_n X_n^2}$.

Fitting two lines, on the other hand…



We could figure out the probability that any point came from either line if we just knew the two equations for the two lines.

MLE with hidden/latent variables: Expectation Maximisation

General problem:

$$y = (Y_1, \ldots, Y_N); \;\; \theta = (a_1, a_2); \;\; z = (z_1, \ldots, z_N)$$

data parameters hidden variables

For MLE, want to maximise the log likelihood

$$\begin{aligned}\widehat{\theta} &= \text{arg max}_\theta \log p(y|\theta) \\ &= \text{arg max}_\theta \log \textstyle\sum_z p(y, z|\theta)\end{aligned}$$

The sum over z inside the log gives a complicated expression for the ML solution.

Maximizing the log likelihood of the data

if you knew the z_n labels for each sample n:

$$\hat{\theta} = \operatorname{argmax}_{\theta} \sum_{n} \delta(z_n = 1) \log p(y_n \mid z_n = 1, \theta) + \delta(z_n = 2) \log p(y_n \mid z_n = 2, \theta)$$

Maximizing the log likelihood of the data

if you knew the z_n labels for each sample n:

$$\hat{\theta} = \operatorname{argmax}_{\theta} \sum_n \delta(z_n = 1) \log p(y_n \mid z_n = 1, \theta) + \delta(z_n = 2) \log p(y_n \mid z_n = 2, \theta)$$

In the EM algorithm, we replace those known labels with their expectation under the current algorithm parameters. So

$$E[\delta(z_n = i)] = p(z_n = i \mid y, \theta_{old})$$

Call that quantity

$$= \alpha_i(n)$$

$$\propto p(y \mid z_n = i, \theta_{old}) \propto e^{-(y_n - a_i x_n)^2 / 2}$$

Maximizing gives

And then for the estimate of the line parameters, we have

$$\hat{\theta} = \operatorname{argmin}_{\theta} \sum_{n} \alpha_1(n)(y_n - a_1 x_n)^2 + \alpha_2(n)(y_n - a_2 x_n)^2$$

and maximising that gives

$$\hat{a}_i = \frac{\sum_n \alpha_i(n) y_n x_n}{\sum_n \alpha_i(n) x_n^2}$$

EM fitting to two lines

with
$$\alpha_i(n) \propto e^{-(y_n - a_i x_n)^2 / 2}$$

"E-step"

and
$$\alpha_1(n) + \alpha_2(n) = 1$$

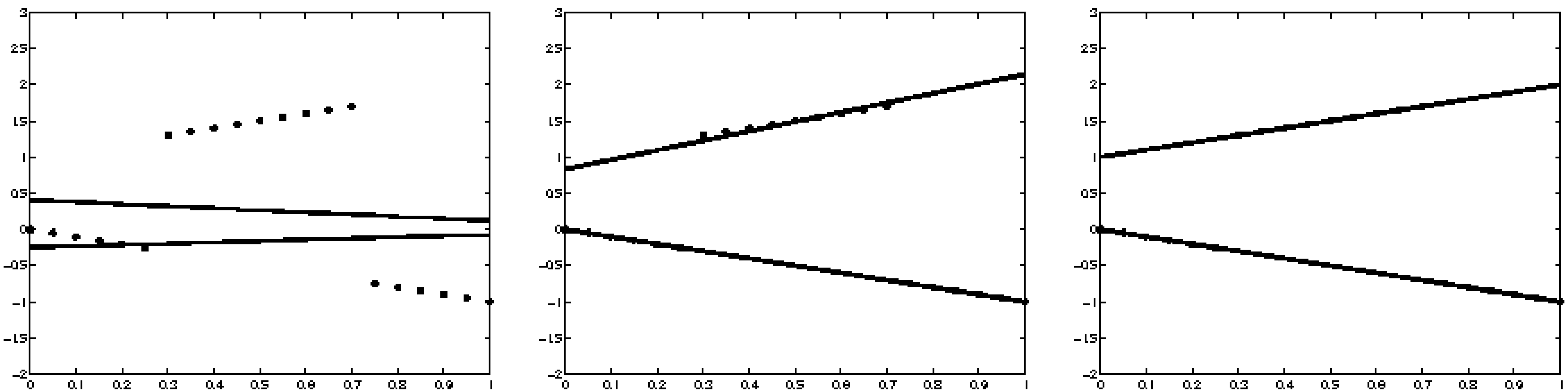
repeat

Regression becomes:
$$\hat{a}_i = \frac{\sum_n \alpha_i(n) y_n x_n}{\sum_n \alpha_i(n) x_n^2}$$

"M-step"

Experiments: EM fitting to two lines

(from a tutorial by Yair Weiss, http://www.cs.huji.ac.il/~yweiss/tutorials.html)



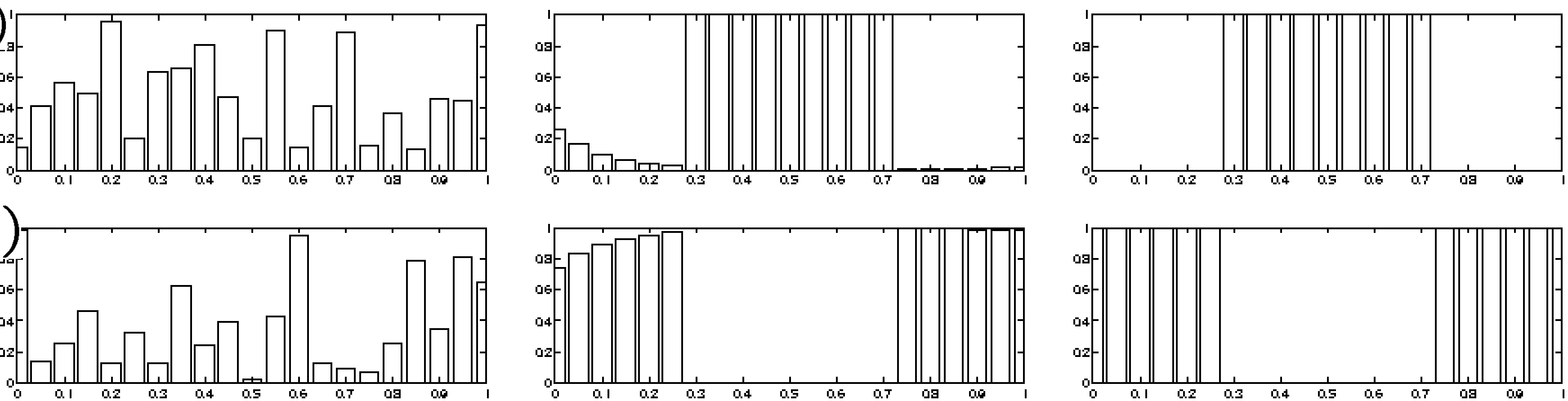
Line weights

$w_1(n)$

line 1

$w_2(n)$

line 2



Iteration 1 2 3

EM

$$\frac{1}{2\sigma_{im}^2}\sum_i \left(\| \lambda_i \mathcal{I}_\mu - \mathcal{T}_i^{-1}\mathcal{I} \|^2 \ \delta_i \right) + \sum_i (1-\delta_i)K + \frac{1}{2\sigma_{light}^2}(\lambda_i - 1)^2 + L$$

Find interest points

EM iterations

1

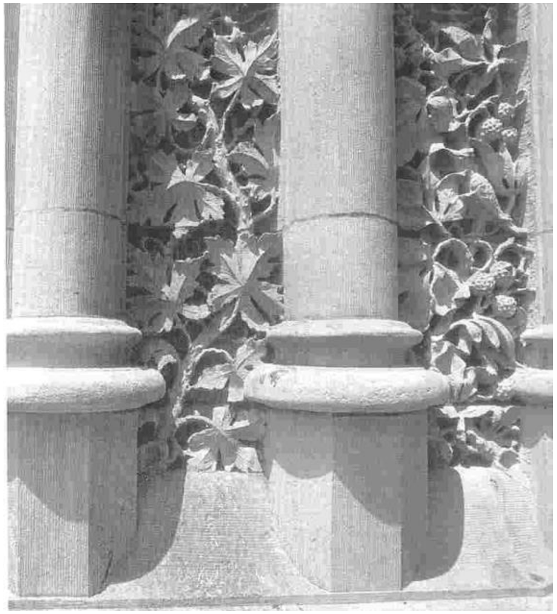
5

10

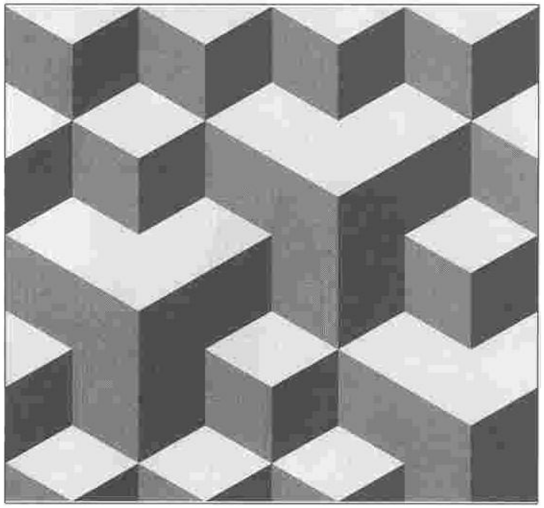
20


300
400
500
100
150
200
250
300
350
400
500
150
200
250
300

Shading

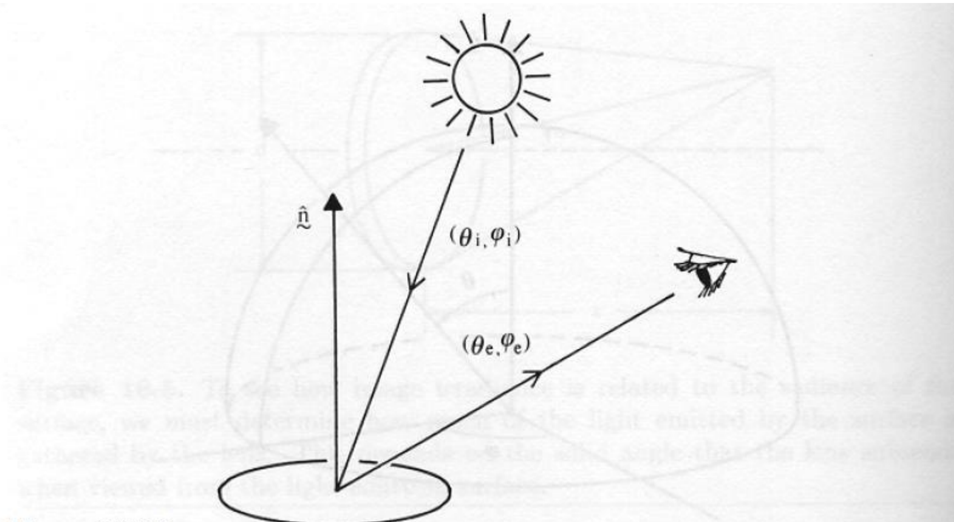


- Based on 3 dimensional modeling of objects in light, shade and shadows.



- Perception of depth through shading alone is always subject to the concave/convex inversion. The pattern shown can be perceived as stairsteps receding towards the top and lighted from above, or as an overhanging structure lighted from below.

Reflectance map



Horn, 1986

Figure 10-7. The bidirectional reflectance distribution function is the ratio of the radiance of the surface patch as viewed from the direction (θ_e, ϕ_e) to the irradiance resulting from illumination from the direction (θ_i, ϕ_i).

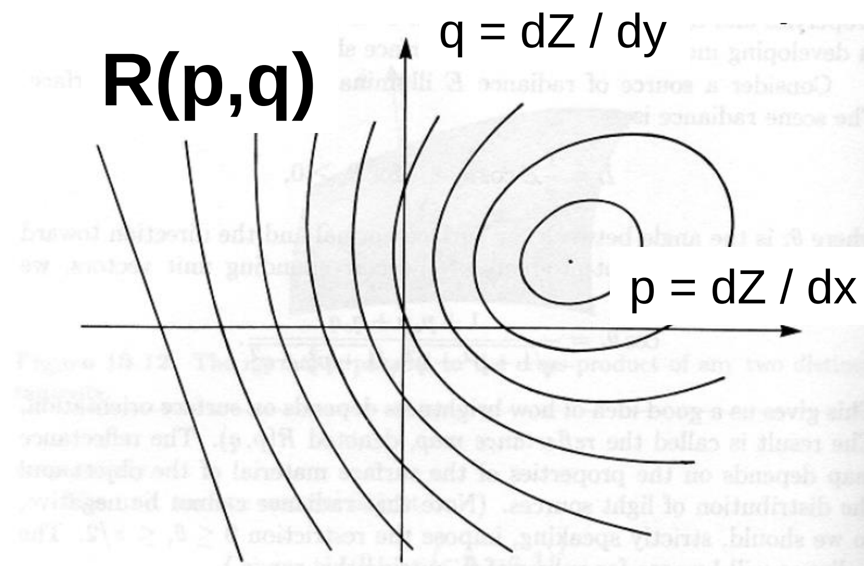


Figure 10-13. The reflectance map is a plot of brightness as a function of surface orientation. Here it is shown as a contour map in gradient space. In the case of a Lambertian surface under point-source illumination, the contours turn out to be nested conic sections. The maximum of $R(p,q)$ occurs at the point $(p,q) = (p_s, q_s)$, found inside the nested conic sections, while $R(p,q) = 0$ all along the line on the left side of the contour map.

$$BRDF = f(\theta_i, \phi_i, \theta_e, \phi_e) = \frac{L(\theta_e, \phi_e)}{E(\theta_i, \phi_i)}$$

Linear shape from shading

Lambertian point source

$$R(p,q) = k\frac{1 + p_s p + q_s q}{\sqrt{1+p^2+q^2}\sqrt{1+p_s^2+q_s^2}}$$

1st order Taylor series about p=q=0

$$\approx k_2 + \left.\frac{\partial R(p,q)}{\partial p}\right|_{p=0,q=0} p + \left.\frac{\partial R(p,q)}{\partial q}\right|_{p=0,q=0} q$$

$$= k_2(1 + p_s p + q_s q)$$

A close form solution can be obtained using the Fourier transform (Pentland 88)

$$\frac{\partial}{\partial x} Z(x,y) \longleftrightarrow F_Z(\omega_1, \omega_2)(-i\omega_1)$$

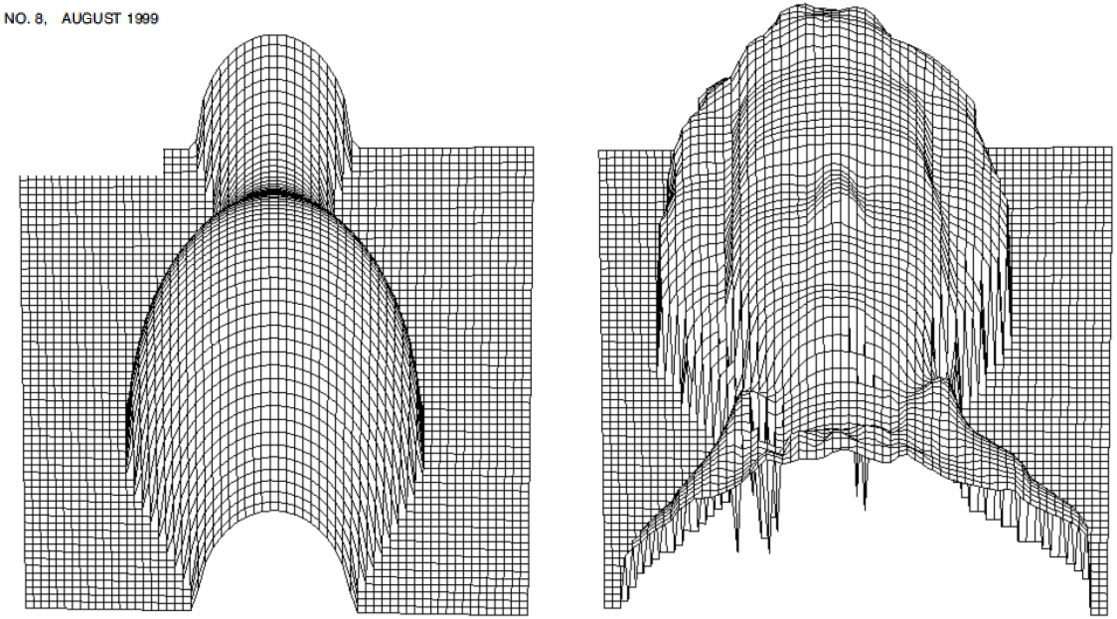
Pentland 88

IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE, VOL. 21, NO. 8, AUGUST 1999

Shape from Shading: A Survey

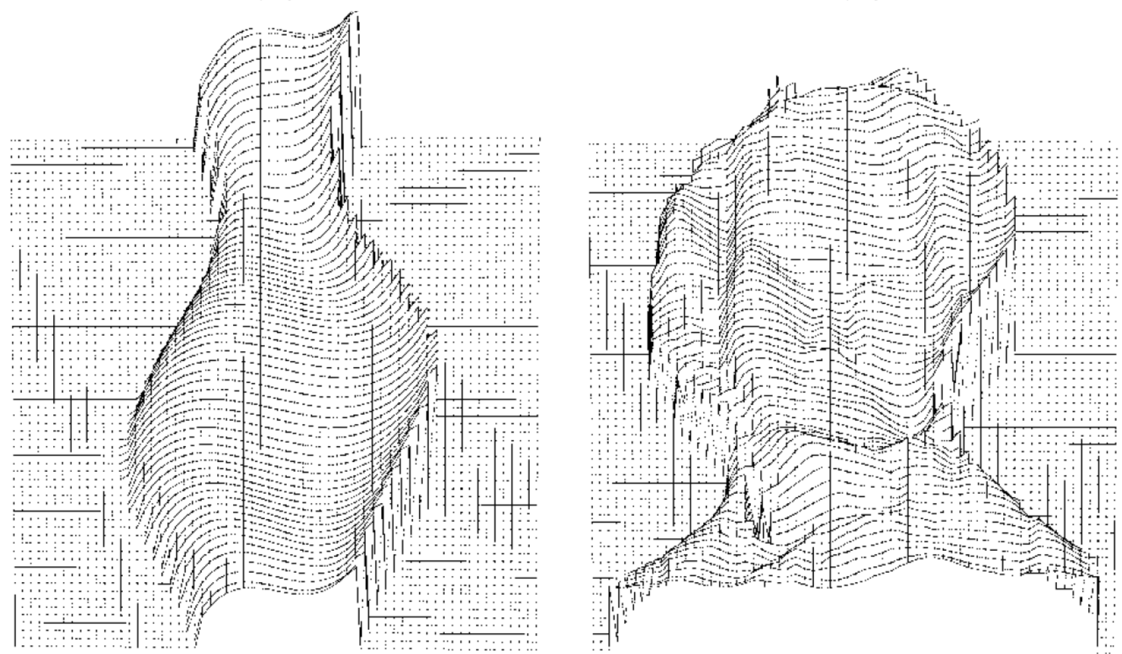
Ruo Zhang, Ping-Sing Tsai, James Edwin Cryer, and Mubarak Shah

Ground truth



(a) (b)

Linear shape from shading



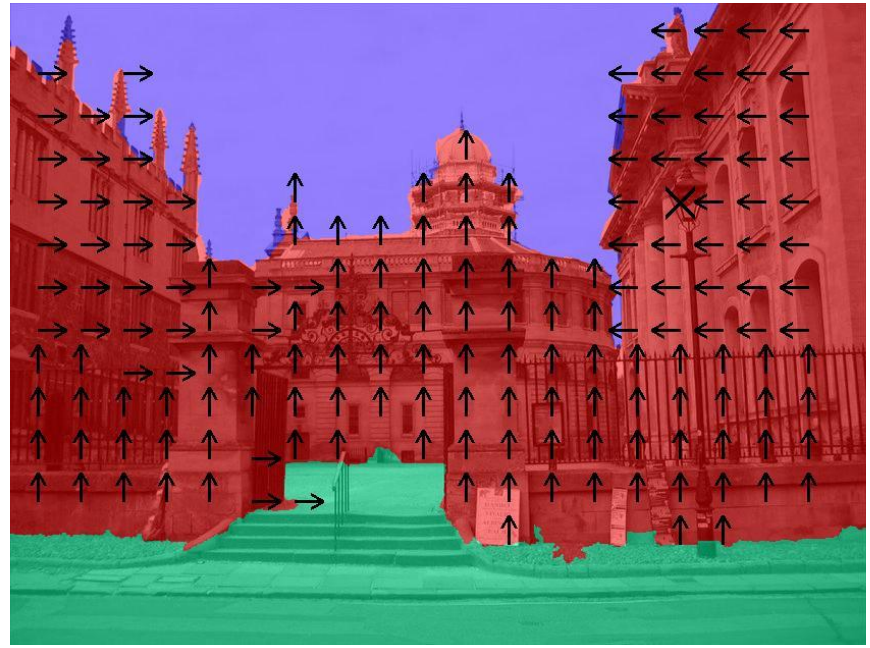
(a) (b)

Learning based methods

- User recognition to learn structure of the world from labeled examples

Slides by Efros

Label Geometric Classes



- **Goal:** learn labeling of image into 7 <u>Geometric Classes</u>:
- **Support (ground)**
- **Vertical**
  - Planar: facing **Left (←)**, **Center (↑)**, **Right (→)**
  - Non-planar: **Solid (X)**, **Porous** or wiry **(O)**
- **Sky**

Slides by Efros

What cues to use?

Vanishing points, lines

Color, texture, image location

Texture gradient

Slides by Efros

Dataset very general













Slides by Efros

The General Case (outdoors)

- Typical outdoor photograph off the Web
  - Got 300 images using Google Image Search keyboards: “outdoor”, “scenery”, “urban”, etc.
- Certainly not random samples from world
  - 100% horizontal horizon
  - 97% pixels belong to 3 classes -- ground, sky, vertical (gravity)
  - Camera axis usually parallel to ground plane
- Still very general dataset!

Slides by Efros

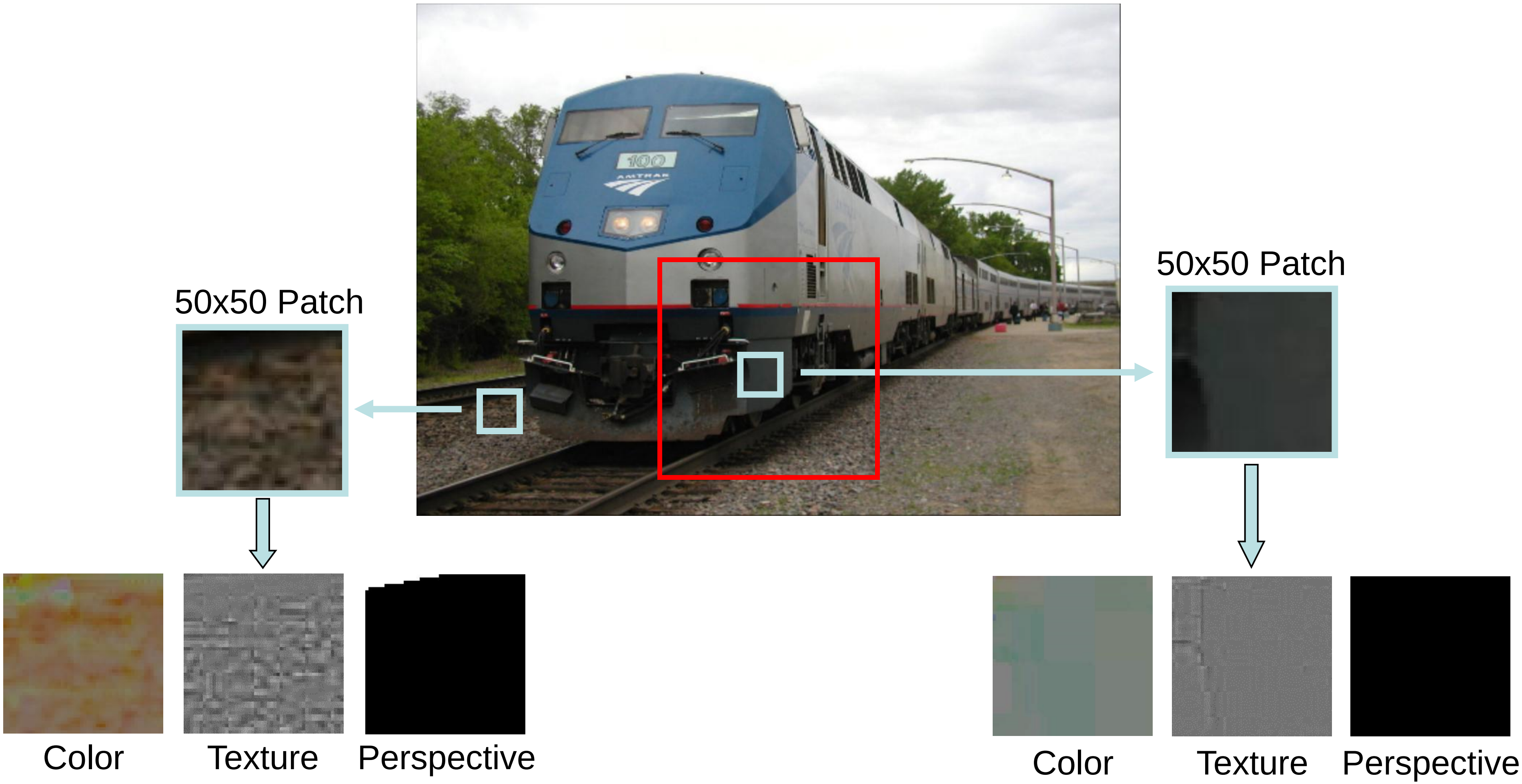
Let's use many <u>weak</u> cues

- Material
- Image Location
- Perspective

SURFACE CUES
Location and Shape
L1. Location: normalized x and y, mean
L2. Location: norm. x and y, 10^{th} and 90^{th} pctl
L3. Location: norm. y wrt estimated horizon, 10^{th}, 90^{th} pctl
L4. Location: whether segment is above, below, or straddles estimated horizon
L5. Shape: number of superpixels in segment
L6. Shape: normalized area in image
Color
C1. RGB values: mean
C2. HSV values: C1 in HSV space
C3. Hue: histogram (5 bins)
C4. Saturation: histogram (3 bins)
Texture
T1. LM filters: mean abs response (15 filters)
T2. LM filters: hist. of maximum responses (15 bins)
Perspective
P1. Long Lines: (num line pixels)/sqrt(area)
P2. Long Lines: % of nearly parallel pairs of lines
P3. Line Intersections: hist. over 8 orientations, entropy
P4. Line Intersections: % right of center
P5. Line Intersections: % above center
P6. Line Intersections: % far from center at 8 orientations
P7. Line Intersections: % very far from center at 8 orientations
P8. Vanishing Points: (num line pixels with vertical VP membership)/sqrt(area)
P9. Vanishing Points: (num line pixels with horizontal VP membership)/sqrt(area)
P10. Vanishing Points: percent of total line pixels with vertical VP membership
P11. Vanishing Points: x-pos of horizontal VP - segment center (0 if none)
P12. Vanishing Points: y-pos of highest/lowest vertical VP wrt segment center
P13. Vanishing Points: segment bounds wrt horizontal VP
P14. Gradient: x, y center of gradient mag. wrt. image center

Slides by Efros

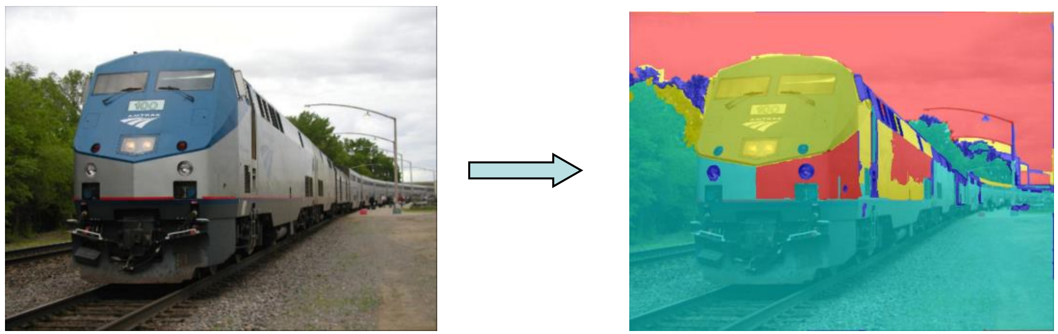
Need Spatial Support



Slides by Efros

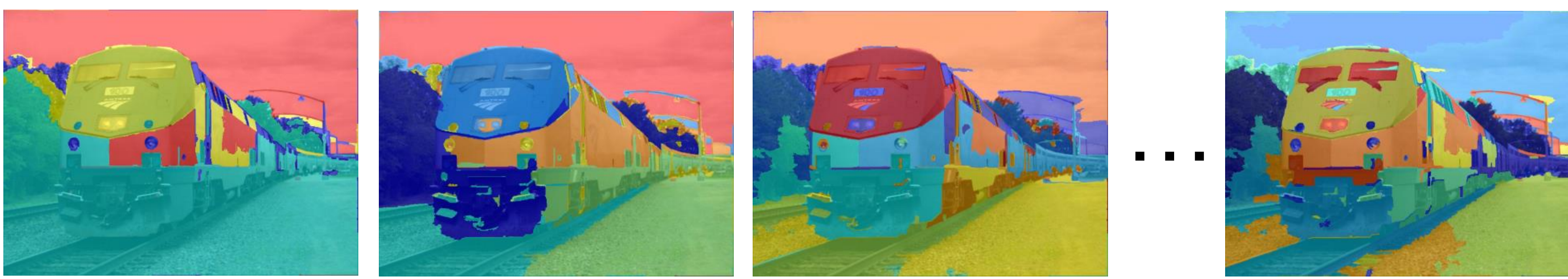
Image Segmentation

- Naïve Idea #1: segment the image



  – Chicken & Egg problem

- Naïve Idea #2: <u>multiple</u> segmentations



  – Decide later which segments are good

Slides by Efros

Estimating surfaces from segments

- We want to know:
  - Is this a good (coherent) segment?

    P(*good segment* | *data*)

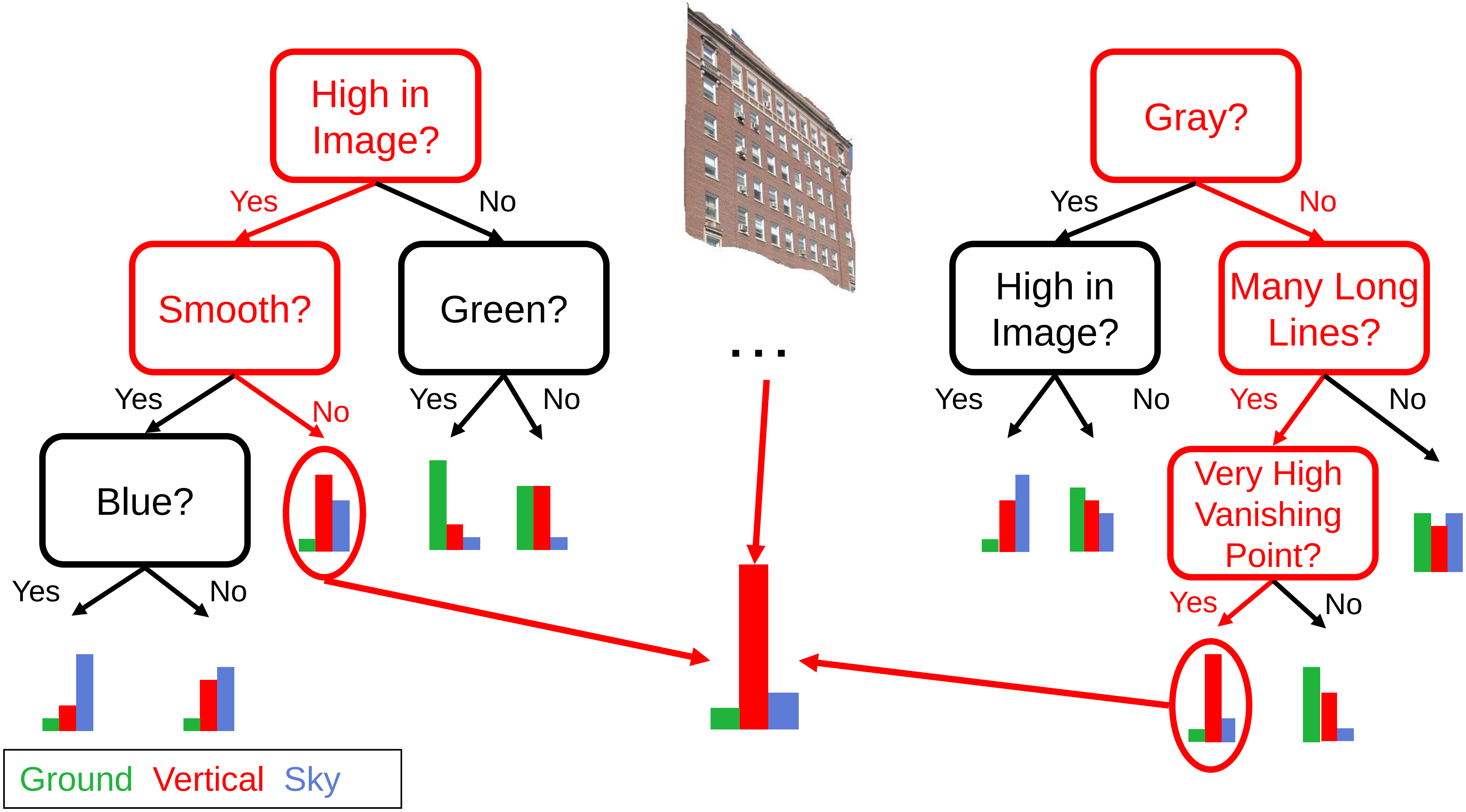
  - If so, what is the surface label?

    P(*label* | *good segment*, *data*)

- *Learn* these likelihoods from training images
  - we use Boosted Decision Trees

Slides by Efros

Boosted Decision Trees



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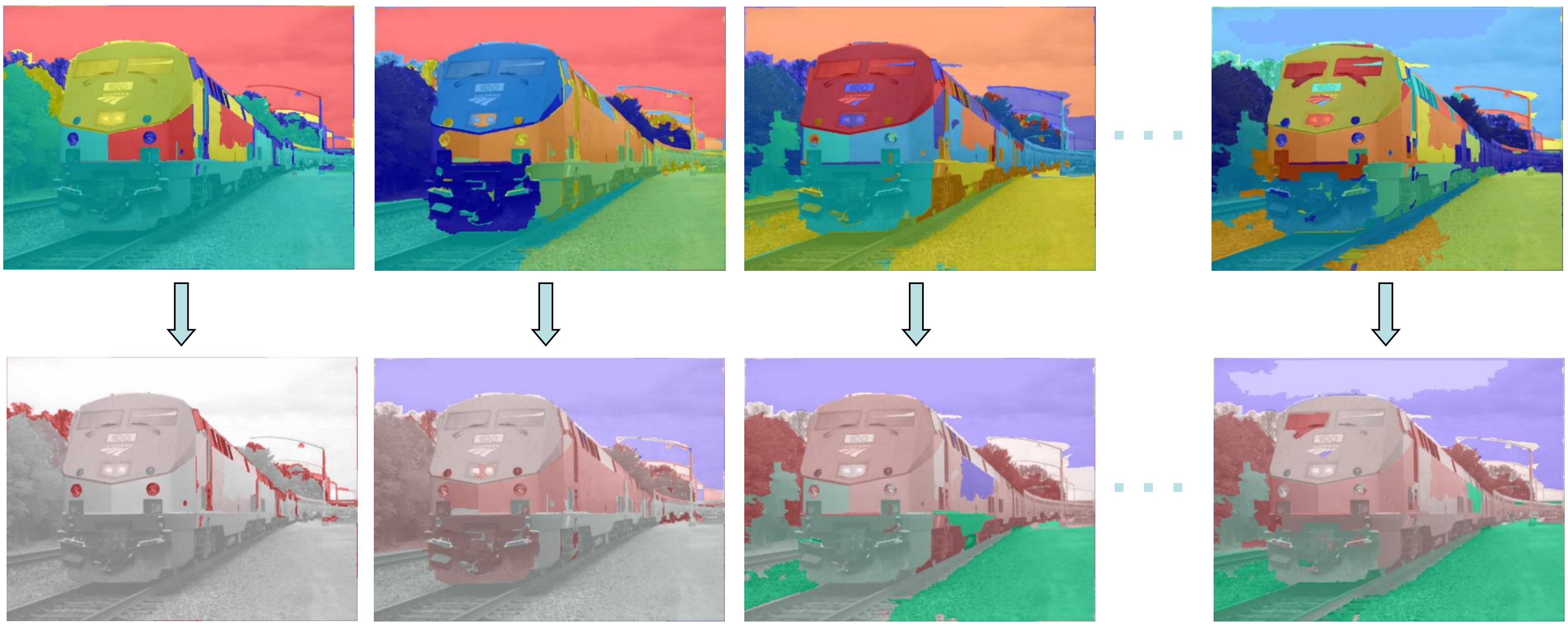
Labeling Segments



For each segment:

- Get P(*good segment* | *data*) P(*label* | *good segment*, *data*)

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Image Labeling

Labeled Segmentations

Labeled Pixels

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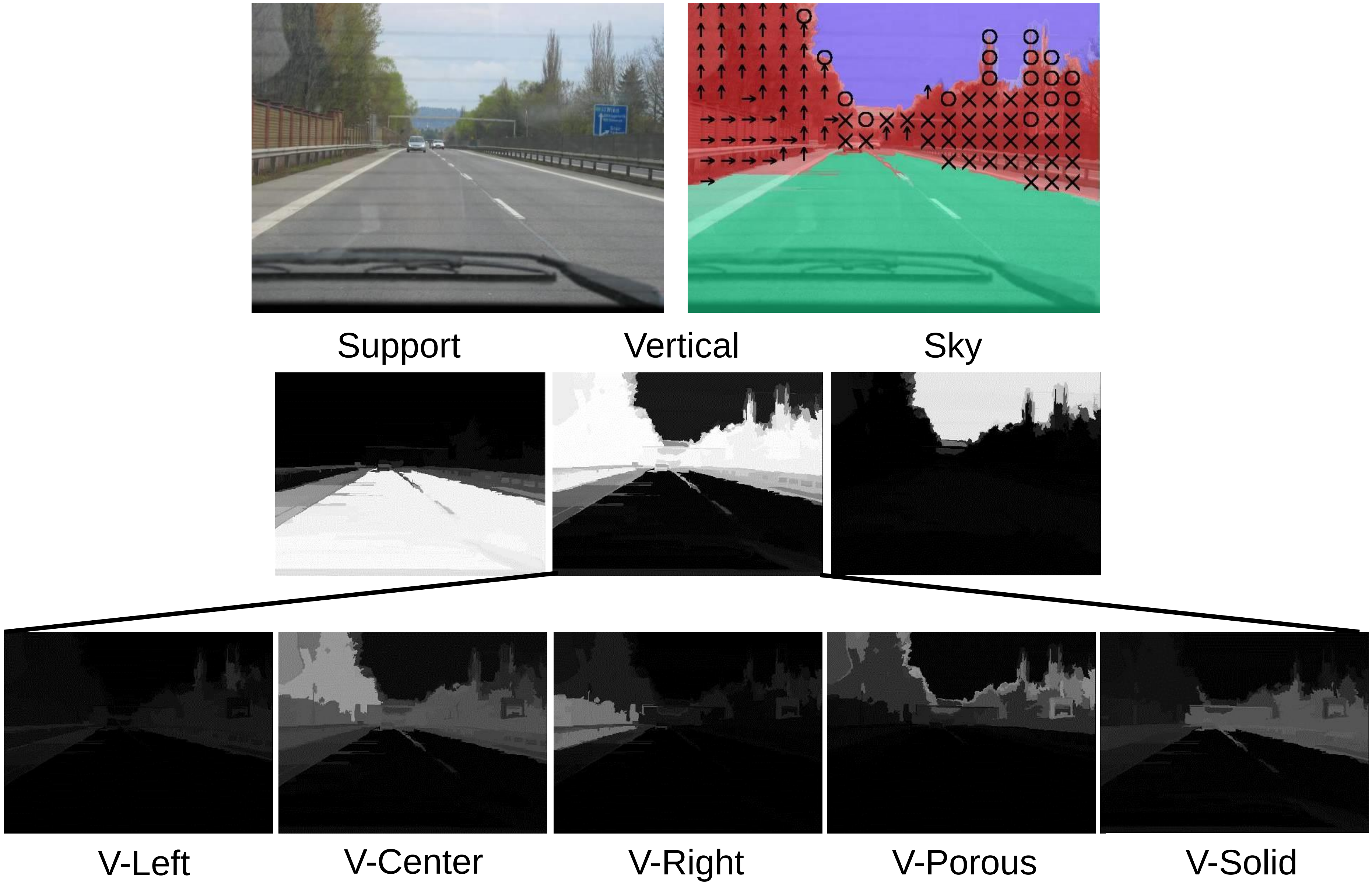
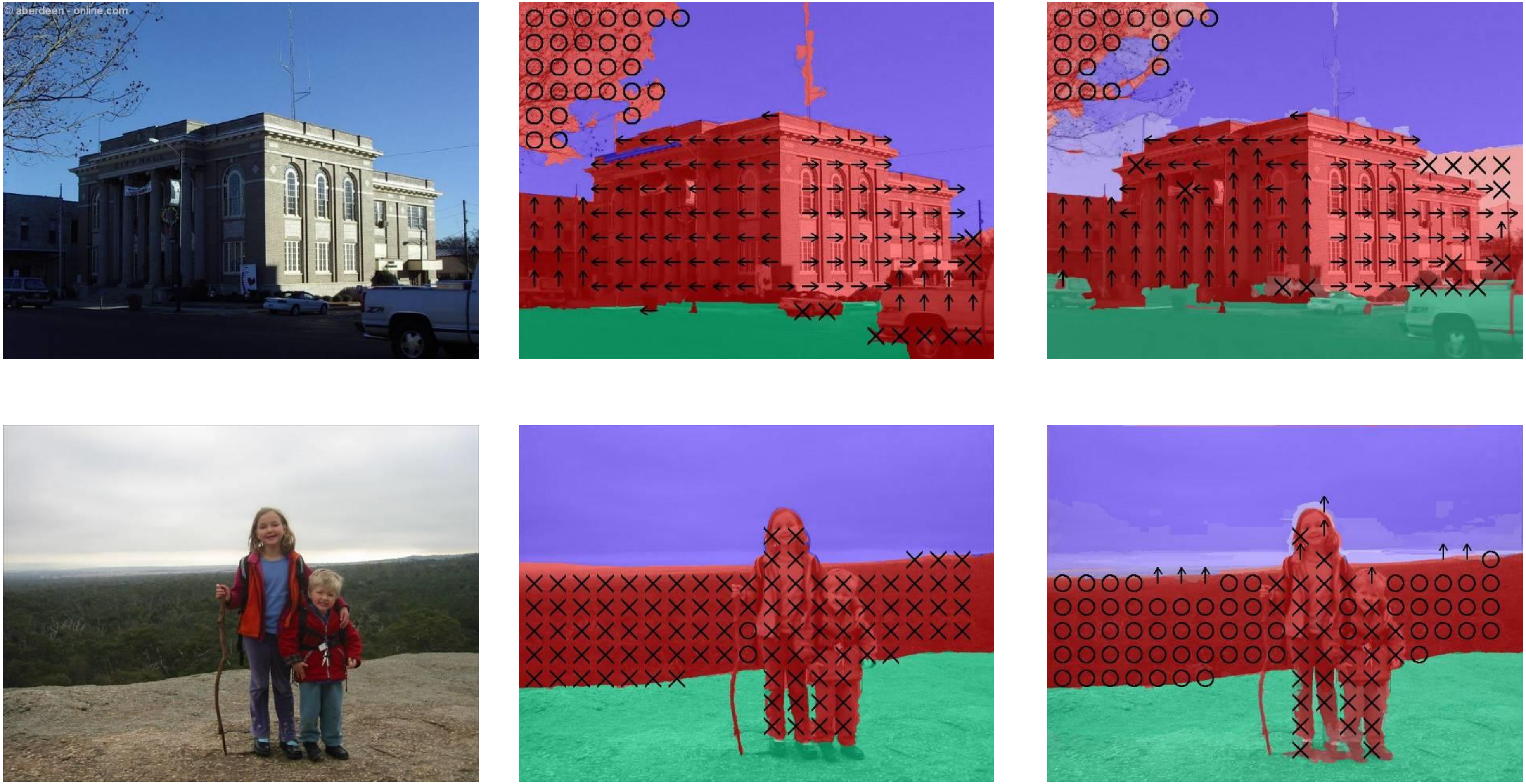
No Hard Decisions



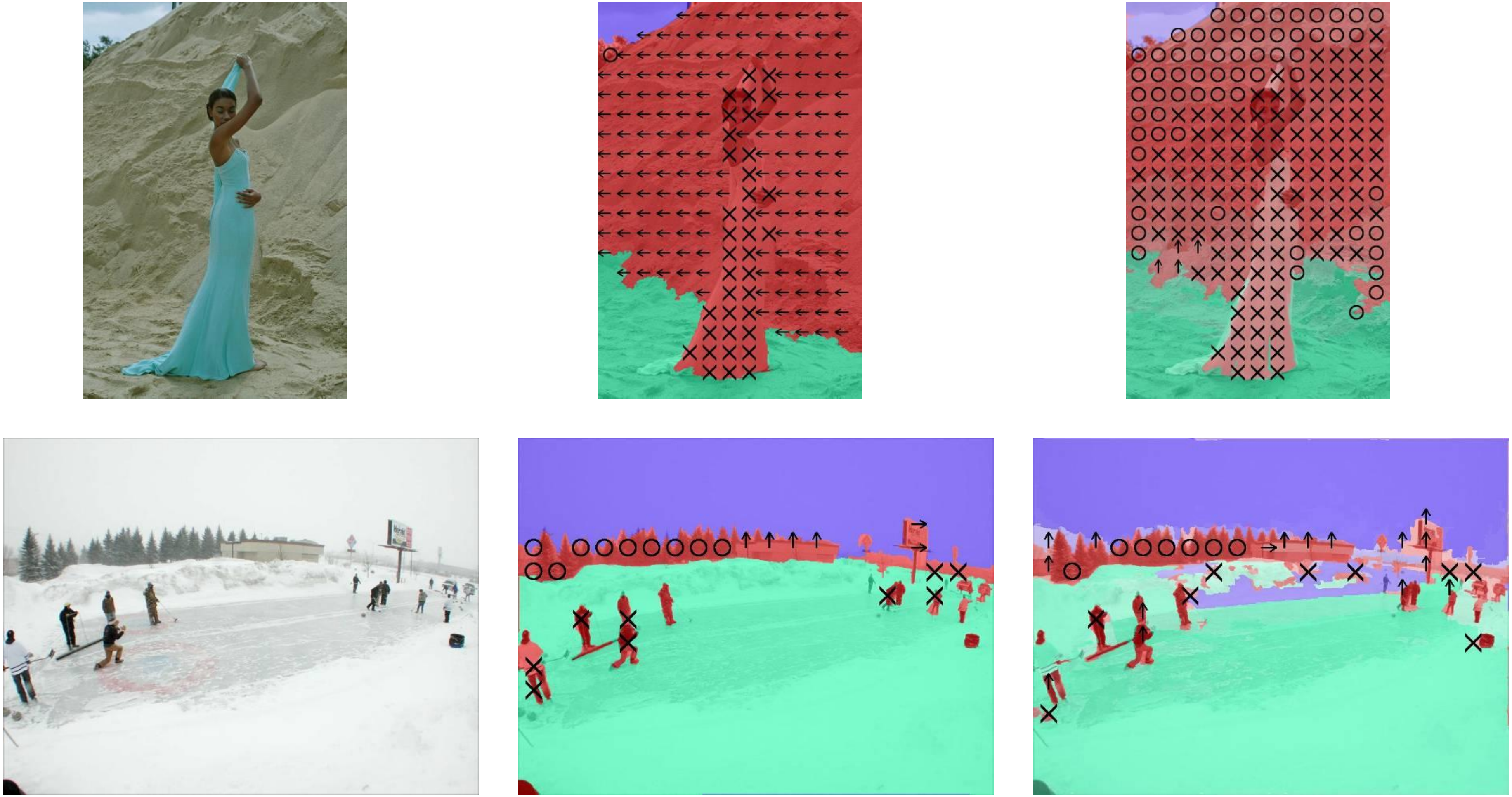
Labeling Results



Input image　　Ground Truth　　Our Result

Slides by Efros

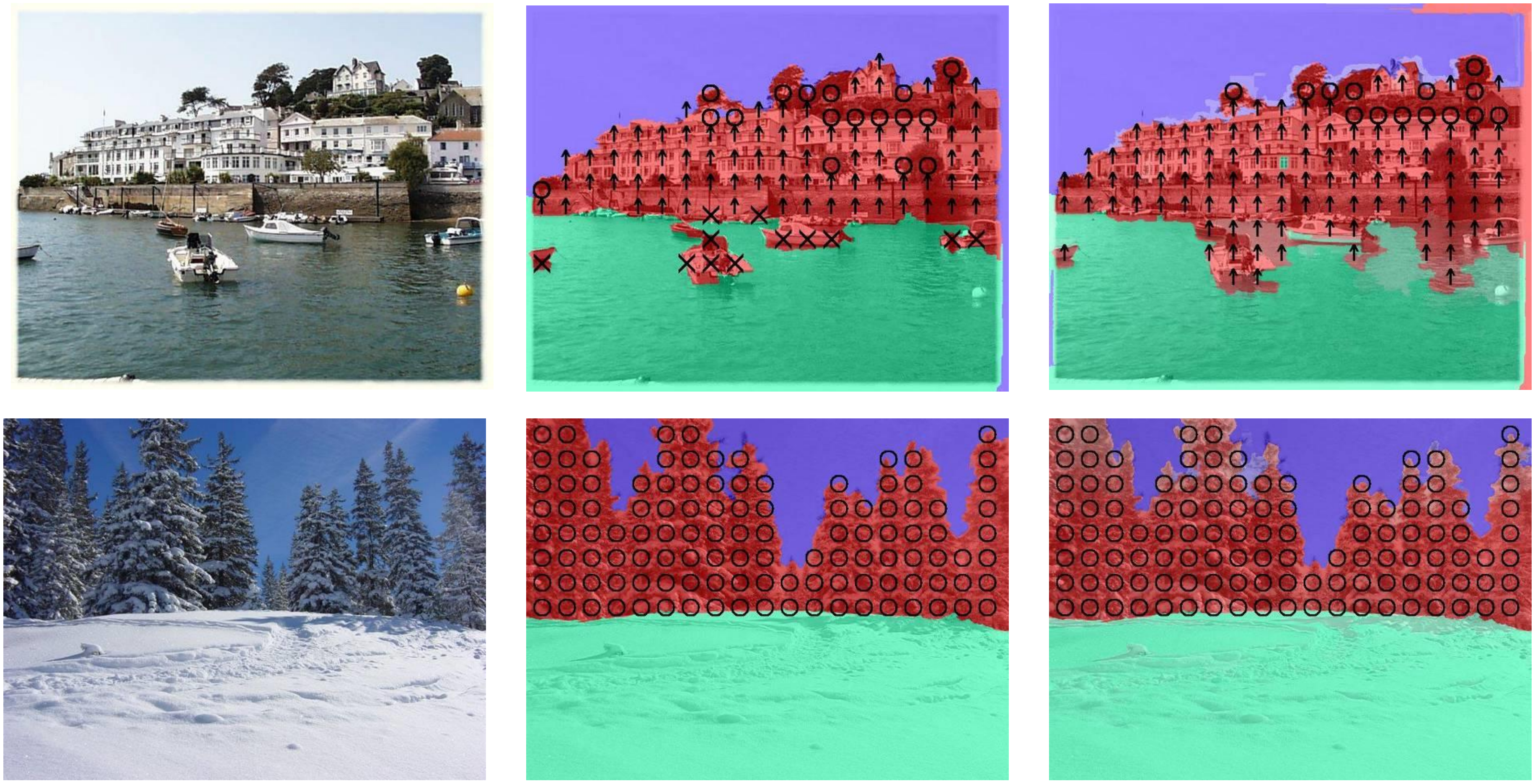
Labeling Results



Input image | Ground Truth | Our Result

Slides by Efros

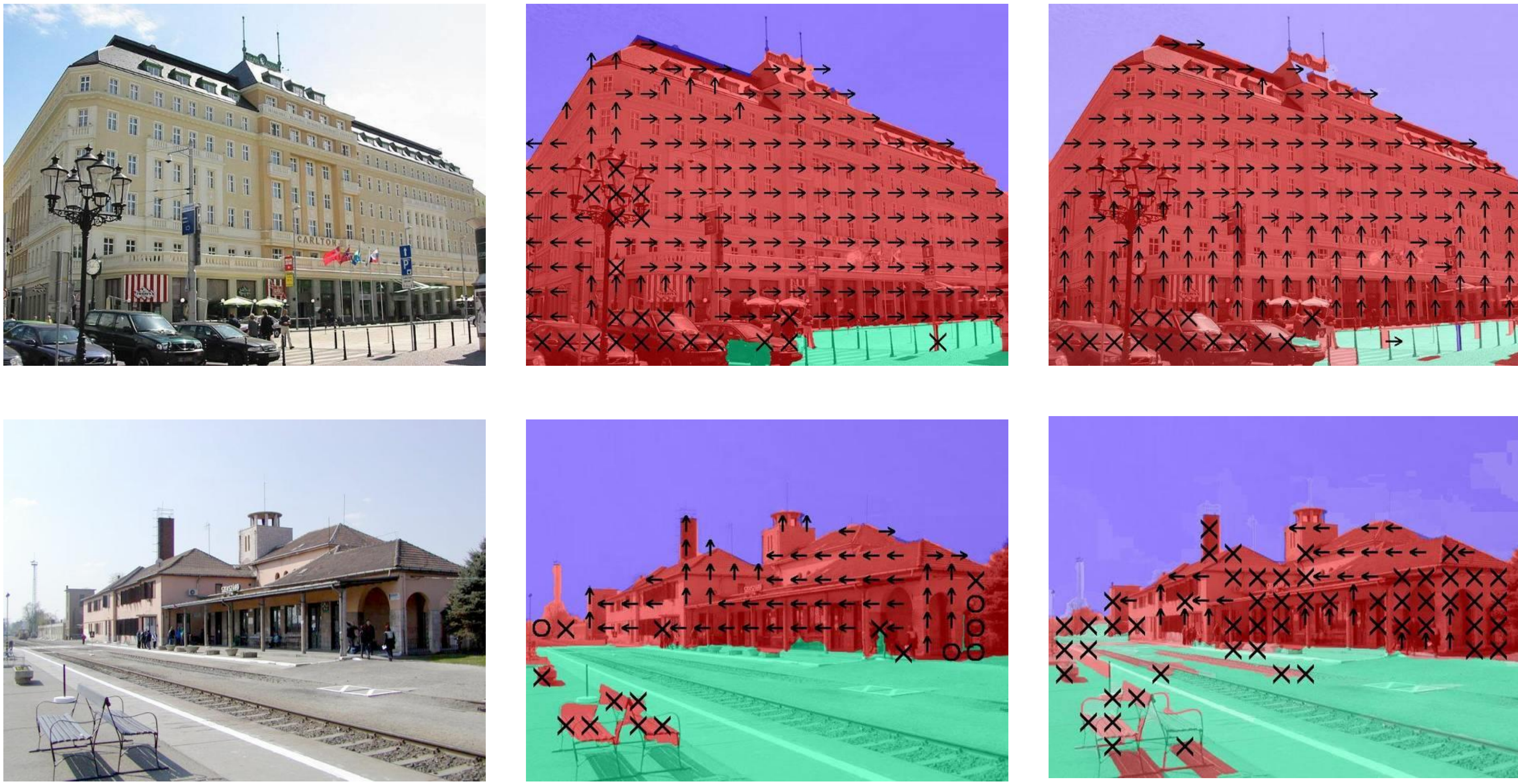
Labeling Results



Input image Ground Truth Our Result

Slides by Efros

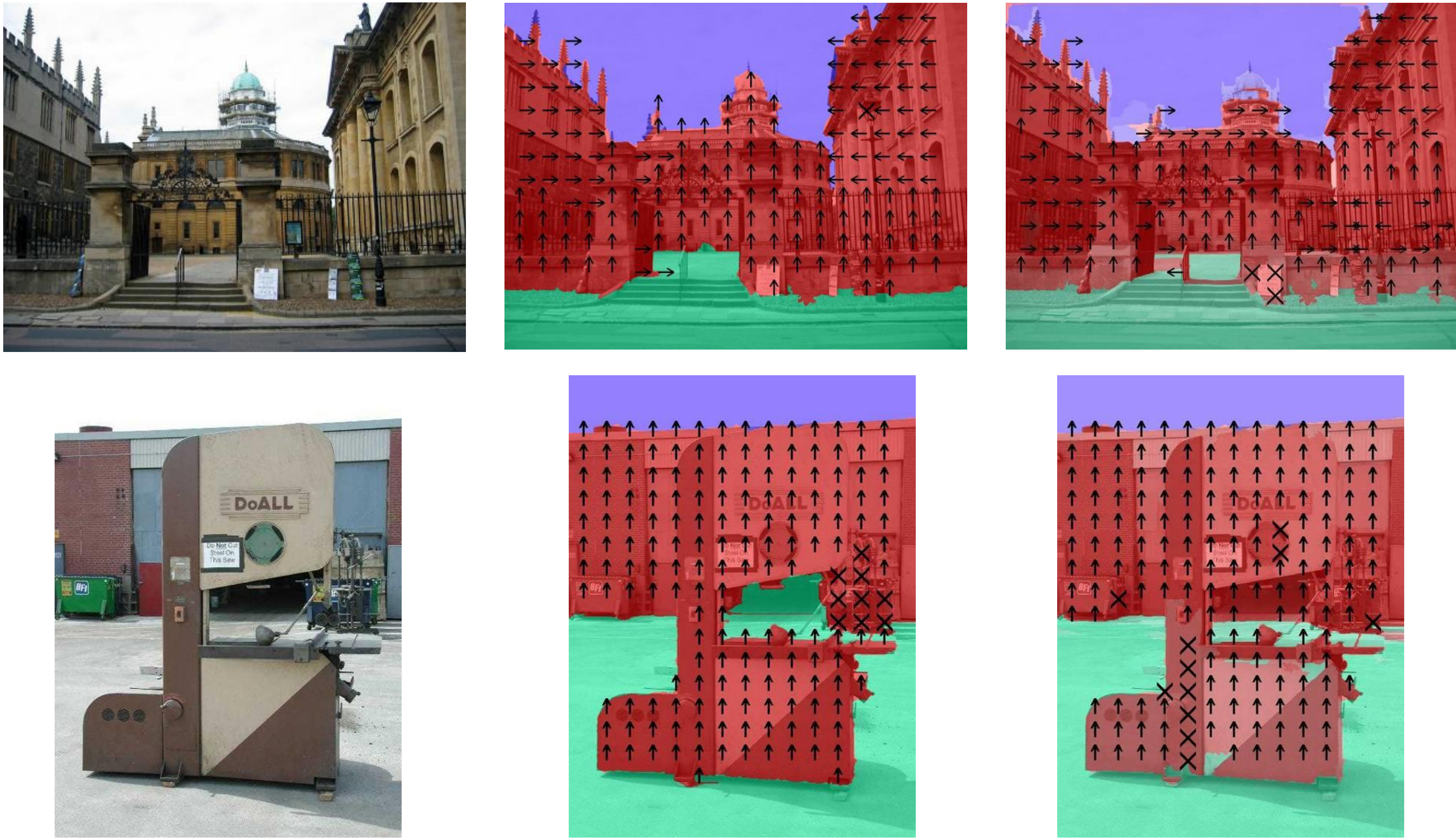
Labeling Results



Input image Ground Truth Our Result

Slides by Efros

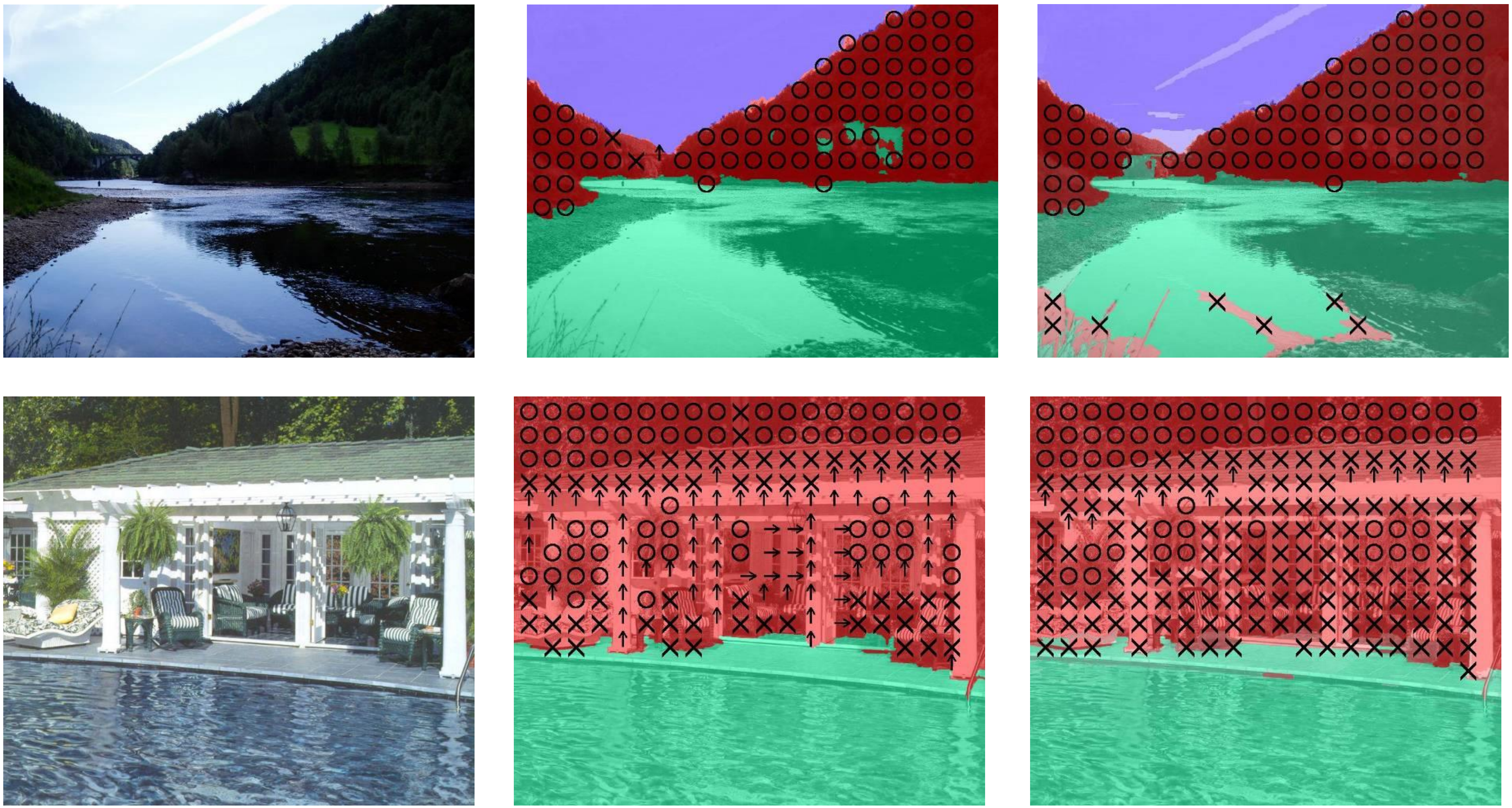
Labeling Results



Input image Ground Truth Our Result

Slides by Efros

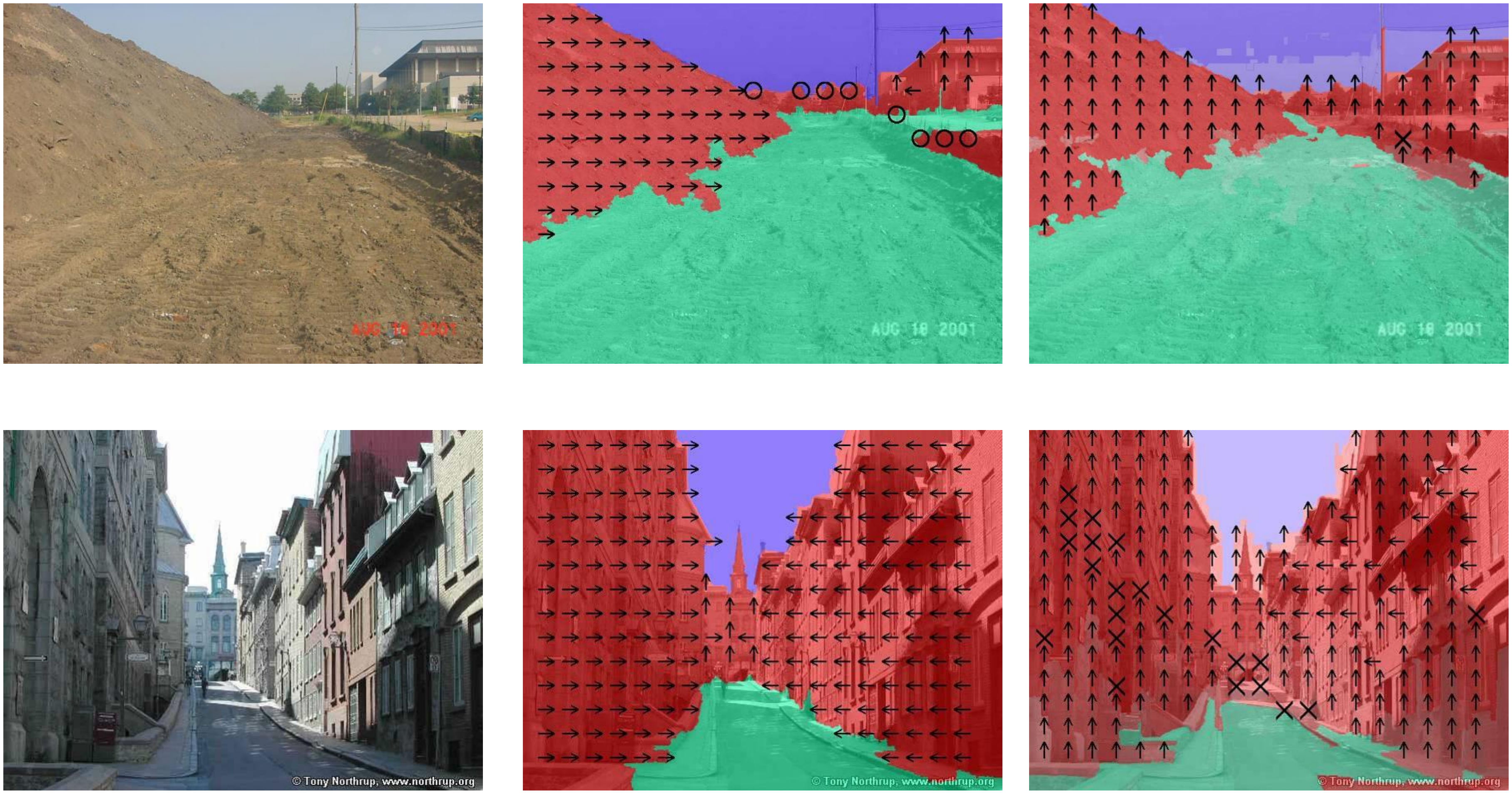
Labeling Results



Input image | Ground Truth | Our Result

Slides by Efros

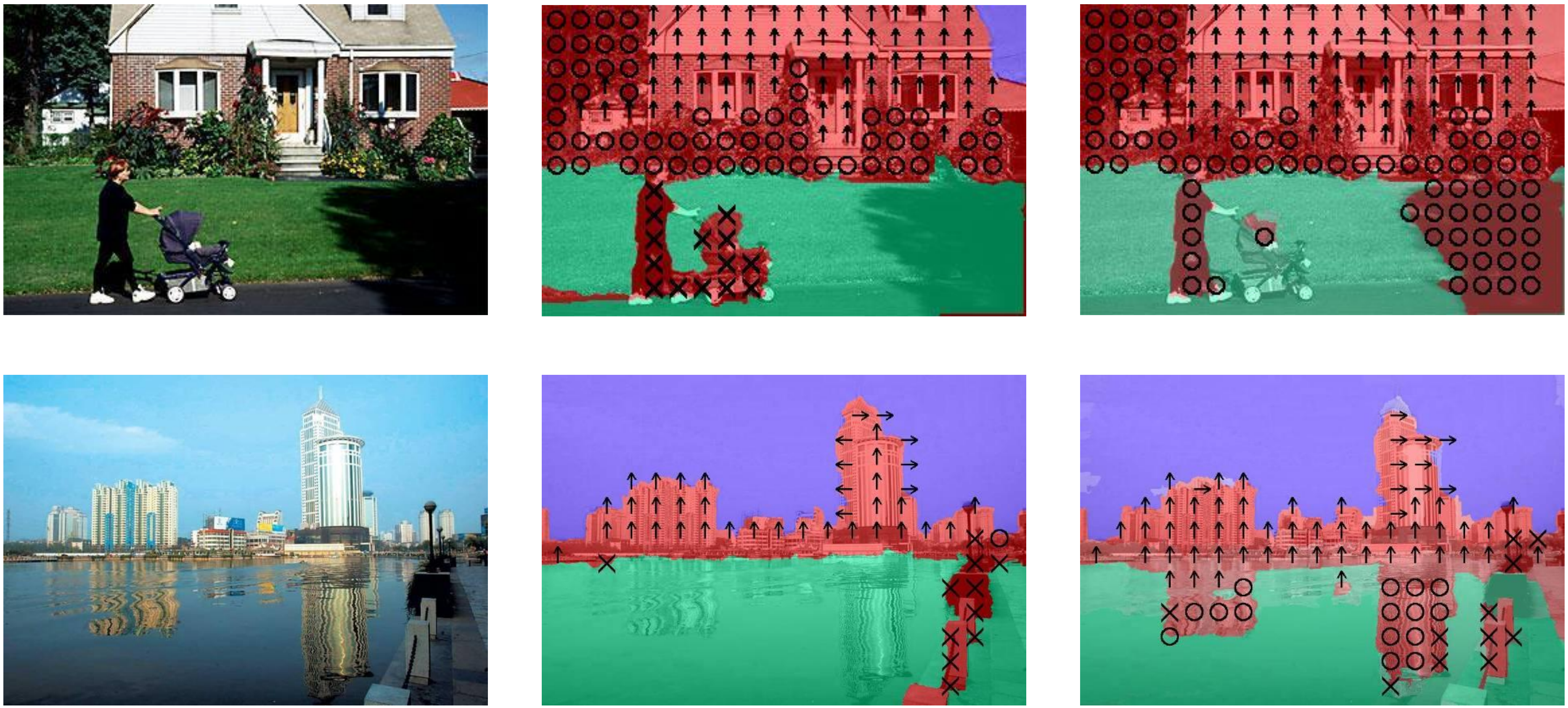
Labeling Results



Input image Ground Truth Our Result

Slides by Efros

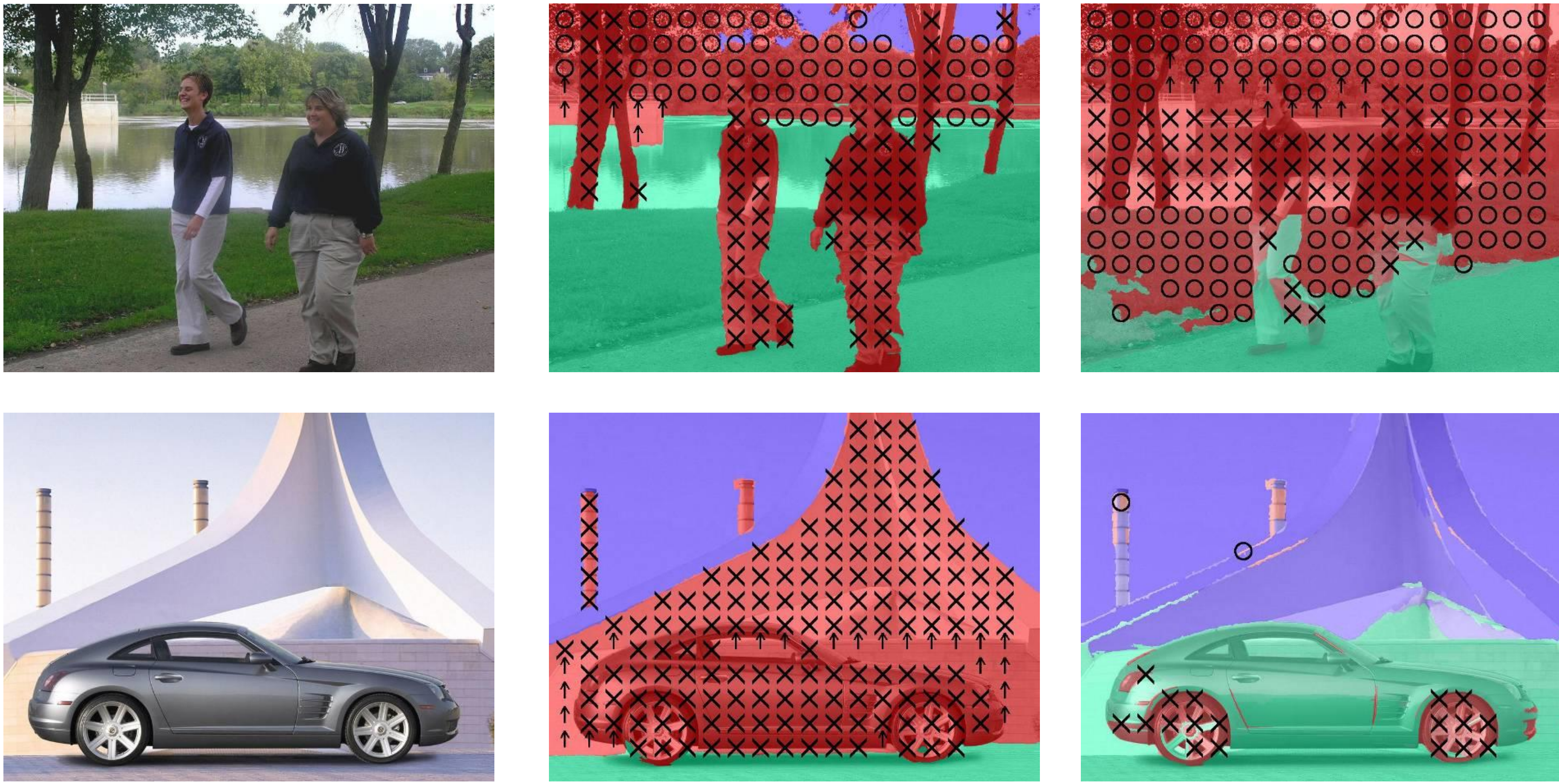
Some Failures



Input image Ground Truth Our Result

Slides by Efros

Catastrophic Failures



Input image Ground Truth Our Result

Slides by Efros

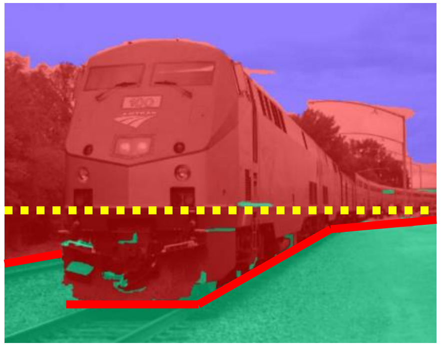
Automatic Photo Popup

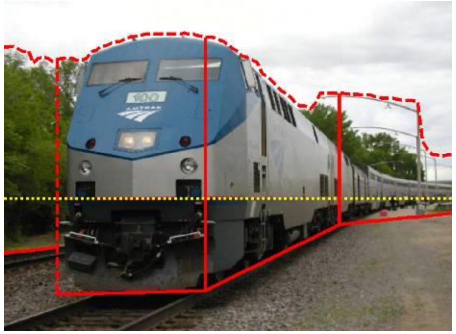
Labeled Image

Fit Ground-Vertical Boundary with Line Segments

Form Segments into Polylines

Cut and Fold





Final Pop-up Model

[Hoiem Efros Hebert 2005]