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CALCULATION OF PRESSURE DISTRIBUTION ON AIRSHIP HULLS

By Theodor Von Karman

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CALCULATION OF PRESSURE DISTRIBUTION ON AIRSHIP HULLS.*

By Theodor Von Karman.

Introduction

These calculations, made at the request of the Zeppelin Airship Company of Friederichshafen, Germany, were based on the shape of the ZR III, with the following simplifications:

Cars, fins, and rudders removed;

All cross sections replaced by equivalent circular cross sections;

Under these assumptions the pressure distribution was calculated for the following cases:

Symmetrical case, or flow parallel to the axis;

Unsymmetrical case, or flow at an angle to the axis.

In both cases the simple potential flow first forms the basis for the determination of the pressure distribution. Case a then yields no drag, while Case b yields a turning moment which tends to bring the hull crosswise to the air stream, but no perpendicular force. For determining the latter, which considerably modifies the pressure distribution, especially at the stern, it was assumed that the hull is followed by a vortex trail in somewhat the same manner as an airplane wing. A simple

*"Berechnung der Druckverteilung an Luftschiffkörpern." From Abhandlungen aus dem Aerodynamischen Institut an der Technischen Hochschule Aachen, 1927, No. 6, pp. 3-17.

assumption regarding the distribution of the vortices leads, in fact, to results which agree very well with the measured ones. The calculations are no more difficult than the static calculation of a statically indeterminate system.

Symmetrical Case

It is assumed that the flow is produced by superposing a flow arising from a system of sources and sinks on the parallel flow of velocity U. The system consists of line sources and sinks of differing productiveness, in which the yield per unit length is kept systematically constant over 10 m (32.8 ft.) lengths. A preliminary survey showed that the flow at the bow is practically independent of the sinks and, conversely, the flow at the stern is independent of the forward sources, so that the calculation can be made separately for the bow and the stern, i.e., for a so-called half-hull, with very close approximation.

A symmetrical flow with respect to the axis can be represented either by the potential function or by the stream function. We first introduce the coordinates of the cylinder:

x, in the direction of the axis of symmetry;

r, as the perpendicular distance from the x axis;

φ, as the angle of orientation of the meridian plane, calculated from the vertical section of the body of revolution.

Then the velocity components in these three directions are:

$$u_x = \frac{\partial \Phi}{\partial x}, \quad u_r = \frac{\partial \Phi}{\partial r}, \quad u_\varphi = 0$$

in which Φ is the potential function. After introducing the stream function ψ, the same velocity components read

$$u_x = \frac{1}{r}\frac{\partial\psi}{\partial r}, \quad u_r = -\frac{1}{r}\frac{\partial\psi}{\partial x}, \quad u_\varphi = 0.$$

If the cylinder coordinates are replaced by spatial polar coordinates,

ρ, the length of the radius vector,

ϑ, the angle between the radius vector and the axis of symmetry,

φ, the potential function,

then the formulas for the velocity components, in the direction of the radius vector and perpendicular to it, read

$$u_\rho = \frac{\partial\Phi}{\partial\rho} = \frac{1}{\rho^2 \sin\vartheta}\frac{\partial\psi}{\partial\vartheta}$$

$$u_\vartheta = \frac{1}{\rho}\frac{\partial\Phi}{\partial\vartheta} = -\frac{1}{\rho \sin\vartheta}\frac{\partial\psi}{\partial\rho}.$$

The two functions Φ and ψ, for a simple source with yield Q, read

Q = strength of sink
– quantity of fluid/unit of time

$$\Phi = -\frac{Q}{4\pi\rho}$$

$$\psi = -\frac{Q}{4\pi}(1 + \cos\vartheta).$$

For the following applications, we will calculate the function of flow and the velocity components for a line source of length a and yield q per unit length.

q - stream function in polar co-ordinates

Stream function.- The contribution of an element $d\xi$ to the stream function at the point P is manifestly

$$d\psi = -\frac{q}{4\pi}(1 + \cos\vartheta)\, d\xi$$

and the stream function of the whole line source is

$$\psi = -\frac{q}{4\pi}\int_0^a (1 + \cos\vartheta)\, d\xi.$$

Now, according to Figure 1,

$$d\xi \cos\vartheta = -d\rho.$$

Therefore, if ρ' and ρ'' respectively, represent the distance of the point P from the left and right end of the line source,

$$\psi = -\frac{q}{4\pi}(a + \rho' - \rho'').$$

If the total yield of the line source $Q = qa$ is introduced, the stream function reads

$$\psi = -\frac{Q}{4\pi}\left(1 + \frac{\rho' - \rho''}{a}\right). \tag{1}$$

Velocity components.- The components u_x and u_r, calculated with the aid of the formulas

$$u_x = \frac{1}{r}\frac{\partial\psi}{\partial r} \quad \text{and} \quad u_r = -\frac{1}{r}\frac{\partial\psi}{\partial x}$$

are

$$u_x = \frac{Q}{4\pi a r}\left(\frac{\partial\rho''}{\partial r} - \frac{\partial\rho'}{\partial r}\right)$$

$$u_r = -\frac{Q}{4\pi a r}\left(\frac{\partial\rho''}{\partial x} - \frac{\partial\rho'}{\partial x}\right).$$

Now, in general, $\rho = \sqrt{r^2 + x^2}$, consequently

$$\frac{\partial \rho}{\partial r} = \frac{r}{\rho} = \sin \vartheta,$$

$$\frac{\partial \rho}{\partial x} = \frac{x}{\rho} = \cos \vartheta$$

and the two velocity components become

$$u_x = \frac{Q}{4 \pi a r} (\sin \vartheta'' - \sin \vartheta'),$$

$$u_r = - \frac{Q}{4 \pi a r} (\cos \vartheta'' - \cos \vartheta'). \qquad (2)$$

The General System of Equations

We retain the division of the construction drawing placed at our disposal by the Zeppelin Airship Company and adopt line sources of 10 m (32.8 ft.) so that the transverse frames 120, 130, etc., lie in the middle of the line source. The corresponding designations of the frames are shown in Figure 2. The line sources are numbered according to the frames, so that, for example, the line source which is symmetrically located with respect to frame 120 is called the twelfth line source and its yield is designated by Q_{12}. Then (on designating the distances of the given streamline point from the end points of the i line source by ρ'_i and ρ''_i) the stream function of the system of sources and sinks reads

$$\Sigma \psi_2 = - \frac{1}{4 \pi} \sum_{i=1}^{n} \left(1 + \frac{\rho'_i - \rho''_i}{a}\right) Q_i$$

The stream function of the parallel flow must then be superposed on this streamline function. If the flow is from the left, the stream function reads

$$\psi_o = U \frac{r^2}{2}$$

The stream function of the whole flow then becomes

$$\psi = \frac{Ur^2}{2} - \sum_{i=1}^{n} \frac{Q_i}{4\pi}\left(1 + \frac{\rho'_i - \rho''_i}{a}\right).$$

The lines ψ = constant represent the streamlines, and the line $\psi = 0$ must yield the axis and the envelope curve. Therefore, if we put $\psi = 0$ for just as many points of the envelope curve as there are unknown line sources, we obtain a system of linear equations for the determination of Q_i.

In the following calculations, the unit is $a = 10$ m, and the nondimensional quantities

$$\frac{Q_i}{2\,U\,\pi\,a^2} = z_i$$

are unknowns. Moreover, ρ'_{ik} and ρ''_{ik} denote the length of the radius vectors which lead from the end points of the i line source to a marginal point on the k frame, and the coefficient

$$1 + \frac{\rho'_{ik} - \rho''_{ik}}{a} = c_{ik}$$

will be designated by c_{ik}. Lastly, r_k denotes the radius of the k frame. Then the condition $\psi = 0$, applied to n line sources and n marginal points, yields the equations

$$
\begin{aligned}
c_{11}\, z_1 + c_{21}\, z_2 + \ldots + c_{n1}\, z_n &= \left(\frac{r_1}{a}\right)^2 \\
\vdots \qquad \vdots \qquad\quad \vdots \qquad & \quad \vdots \\
c_{1n}\, z_1 + c_{2n}\, z_2 + \ldots + c_{nn}\, z_n &= \left(\frac{r_n}{a}\right)^2
\end{aligned}
$$

The system of equations has the following important characteristic. The coefficients

$$c_{ik} = 1 + \frac{\rho'_{ik} - \rho''_{ik}}{a},$$

when k differs greatly from i, approach either 2 or 0, according to whether the frame k lies to the right or left of the i line source. For example, it is obvious that, when the frame k lies to the right, $\rho'_{ik} - \rho''_{ik}$ approaches a and, in the opposite case, $-a$. Values differing substantially from zero or two are therefore to be expected only when k and i lie near each other, that is, in the vicinity of the diagonals of the system of equations. The coefficients c_{11}, c_{22}, etc., are all equal to unity. These characteristics make it possible to solve the equations in a relatively simple manner.

Application to the Bow

Figure 2 represents the forward portion of the hull as employed for the calculation. The line sources are so chosen that the frames 180, 170, 160, etc. coincide with the middle of of the former. The line sources are numbered according to the numbers of the frames. Forward of frame 125 the source strength is represented as zero. Therefore, there are six unknown source strengths, which will be so defined that the streamline $\psi = 0$ will pass through six points of the envelope curve. These points are chosen on the frames 180 to 130, whose radii are designated by r_{18} to r_{13}.

The coefficients of the equation system are then combined according to the above-mentioned points, c_{ik} denoting the contribution of the i line source to the k point. For large values of $k - i$ we have

$$c_{ik} = 1 + \frac{\rho'_{ik} - \rho''_{ik}}{a} \cong \begin{cases} 2 - \frac{1}{2}\left(\frac{r_k}{a}\right)^2 \frac{1}{(i-k)^2} & (k \text{ right}), \\ \frac{1}{2}\left(\frac{r_k}{a}\right)^2 \frac{1}{(i-k)^2} & (k \text{ left}). \end{cases}$$

The 36 coefficients are given in the following table. The last column contains the values of $(r_k/a)^2$ which form the right side of the equations.

k	i = 18	17	16	15	14	13	$\left(\frac{r_k}{a}\right)^2$
18	1.000	0.202	0.058	0.024	0.017	0.009	0.478
17	1.695	1.000	0.305	0.110	0.050	0.031	0.986
16	1.860	1.633	1.000	0.367	0.140	0.080	1.335
15	1.924	1.841	1.602	1.000	0.398	0.159	1.610
14	1.950	1.912	1.825	1.590	1.000	0.410	1.770
13	1.965	1.948	1.910	1.825	1.577	1.000	1.851

The simplest way to solve the system of equations is as follows. For the first approximation, take only the coefficients which lie to the left of the step line in the table, while calling the remaining coefficients zero at first. Then the equations can be solved in the reverse direction. For the first approximation we obtain

$$z_{18} = 0.478,$$

$$z_{17} = 0.988 - 1.695\ z_{18}.$$

Then the values of the first approximation for z_i are introduced into the members at the right of the step line and the reverse operation repeated. The following table gives the values thus obtained to the fourth approximation. It is obvious that the method converges very well. The source intensities are represented graphically in Figure 2.

z_i	I.	II.	III.	IV. (approximate)
18	0.478	0.430	0.424	0.424
17	0.176	0.191	0.194	0.193
16	0.177	0.206	0.211	0.212
15	0.082	0.080	0.075	0.073
14	0.049	0.060	0.069	0.071
13	0.005	0.002	0.008	0.008
$\Sigma =$	0.977	0.965	0.965	0.965

The radius of the half-hull at the bow end, if only the line sources are used, is obtained by the summation of all the sources.

$$\left(\frac{r_\infty}{a}\right)^2 = 2\ \Sigma\ (z_i).$$

In the present case $2\ \Sigma\ (z_i) = 1.932$ and $r_\infty = 13.88$ m. The maximum diameter of the model is $r_{max} = 13.76$ m, hence somewhat less than would correspond to the approximation r_∞ would be reached only at infinity.

On the other hand the course of the streamline can be controlled at the nose. A slight discrepancy is obtained for the center of the nose, namely, 186.9 m, instead of the coordinate 187.4 m of the design. The exact course of the streamline is shown by the dash line in Figure 3. It is evident that the discrepancy is practically negligible. If it is desired to correct this slight discrepancy, a supplementary point source can be adopted at about point 185 of the axis, which brings the center of the nose to the right place. It has a vanishing effect on the further course of the streamline, so that the source strengths are not changed. I have refrained, however, from making this correction.

Application to the Stern

The calculation for the stern is made in the same way as for the bow. The length of the line sinks is likewise assumed as $a = 10$ m. They are numbered according to the frames between -10 and 90. Between frames 90 and 135 the strength of the sources and sinks is assumed to be zero. The distribution of the sources and sinks is shown in the lower part of Figure 3.

Calculation of Pressure Distribution on the Airship Hull

According to Bernoulli's equation, the pressure increase, at any point where the flow velocity has the value kU, is

$$p = \frac{\gamma}{2g} (1 - k^2) U^2.$$

The first of the formulas II is used for calculating k. After the introduction of the quantity $z_i = \frac{Q_i}{2 U a^2 \pi}$ the velocity in the x direction reads

$$u_x = U + U \frac{a}{2r_k} \sum_{i=1}^{n} z_i (\sin \vartheta''_i - \sin \vartheta'_i)$$

and the velocity in the r direction is

$$u_r = -U \frac{a}{2r_k} \sum_{i=1}^{n} z_i (\cos \vartheta''_i - \cos \vartheta'_i).$$

We then obtain:

$$k^2 = [1 - \frac{a}{2r_k} \sum_{i=1}^{n} z_i (\sin \vartheta''_i - \sin \vartheta'_i)]^2 + \frac{a^2}{4{r_k}^2} [\Sigma z_i (\cos \vartheta''_i - \cos \vartheta'_i)]^2$$

The pressure distribution thus obtained is plotted in Figure 3. It agrees remarkably well with Klemperer's results, which are plotted in the same figure (Cf. W. Klemperer's Aachen dissertation which is soon to appear). Only at two points are there noteworthy discrepancies:

a) Between frames 180 and 150 there is an actual negative pressure somewhat smaller than that calculated;

b) At the stern the pressure increase is somewhat less back of the -5 frame.

The discrepancy a may be due to the effect of the cars. The second discrepancy is due to the separation of the flow. It produces the so-called "form drag" of the airship hull. Obviously the separation occurs in the immediate vicinity of the tail.

Unsymmetrical Case

Theory of Unsymmetrical Flow

The oblique flow can always be obtained by superposing

a) The case of zero incidence and

b) The case of 90° incidence.

It is only necessary, therefore, after thoroughly investigating case a, to describe the method of calculation for solving case b.

We assume that the airship is subjected to a perpendicular flow with a velocity W and undertake the task of calculating the flow potential for this case. The principal theorem may be stated as follows:

If the x axis of an $x\ y\ z$ system of coordinates is covered with double sources whose axes are oriented in the z direction, then the flow resulting from these double sources, superposed on a parallel flow in the z direction, produces the streamline form about a body of revolution exposed to a flow at right angles to the axis of symmetry.

We first derive the principal formulas for a double source. For this purpose we consider a source and a sink of like yield Q separated by a distance of 2ϵ. In order to fix the ideas in mind, we will locate the source and sink at the points $x = y = 0$ and $z = \pm \epsilon$ (Fig. 4). The potential function of the source reads

$$\Phi_{(+)} = -\frac{Q}{4\pi}\frac{1}{\sqrt{x^2 + y^2 + (z - \epsilon)^2}}$$

and the potential function of the sink reads

$$\Phi_{(-)} = \frac{Q}{4\pi}\frac{1}{\sqrt{x^2 + y^2 + (z + \epsilon)^2}} .$$

If we add both potential functions and develop the sum according to ϵ, we obtain for the resulting flow

$$\Phi = \Phi_{(+)} + \Phi_{(-)} = -\frac{Q}{4\pi}\frac{2\epsilon z}{(x^2 + y^2 + z^2)^{3/2}} + \text{higher terms.}$$

The z axis, on which the source and sink lie, is the axis of the source pair and its moment is $2 Q \epsilon$. If the angle of inclination of any radius vector to the z axis is designated by γ,

$$\cos \gamma = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

and with

$$\rho = \sqrt{x^2 + y^2 + z^2}$$

we obtain

$$\Phi = -\frac{2 Q \epsilon}{4 \pi \rho^2} \cos \gamma + \text{higher terms}$$

If we now let ϵ decrease toward zero and Q increase toward ∞, but so that $2Q\epsilon$ approaches a finite value M, then

$$\Phi = - \frac{M}{4 \pi \rho^2} \cos \gamma,$$

the potential of a double source or doublet. The corresponding flow about the z axis is symmetrical. The fluid flows in the direction of the positive z axis and from the negative z axis toward the double source.

The superposition of the double source on the parallel flow in the z direction produces the flow about a sphere. The velocity component in the direction of the radisu vector ρ is

$$w_\rho = \frac{M}{2 \pi \rho^3} \cos \gamma .$$

The parallel flow in the z direction at the velocity $-W$ obviously produces a velocity increment in the ρ direction amounting to $\overline{w}_\rho = - W \cos \gamma$. If we put

$$w_\rho + \overline{w}_\rho = 0,$$

it is obvious that, for the surface of a sphere with a radius of

$$\rho = \sqrt[3]{\frac{M}{2 \pi W}}$$

the velocity component perpendicular to the surface of the sphere disappears. Therefore the flow resulting from a parallel flow and a double source produces the flow about a sphere.

If the entire x axis is covered with doublets of constant intensity, whereby the moment per unit length is designated by μ, we obtain the flow crosswise to an infinitely long cylinder. We now introduce the polar coordinates ρ, ϑ and φ. ϑ is the angle between the radius vector and the x axis and φ determines the position of the meridian plane passing through the x axis, $\varphi = 0$ corresponds to the xz plane. Then

$$\cos \gamma = \sin \vartheta \cos \varphi$$

and the potential function of the double source is

$$\Phi = -\frac{M}{4\pi} \frac{\sin \vartheta \cos \varphi}{\rho^2}$$

or of the element having the strength $\mu \, d \, \xi$

$$d\Phi = -\frac{\mu d\xi}{4\pi} \frac{\sin \vartheta \cos \varphi}{\rho^2}$$

From Figure 1 we derive the following expression:

$$x - \xi = r \cot \vartheta,$$

whence

$$-d\xi = -\frac{r}{\sin^2 \vartheta} d\vartheta.$$

Further

$$r = \rho \sin \vartheta,$$

so that

$$d\Phi = -\frac{\mu}{4\pi r} \sin \vartheta \, d \, \vartheta \cos \varphi.$$

For future uses we will calculate the potential for a line of doublets of the length a. We obtain

$$\Phi = \frac{\mu}{4\pi\rho} [\cos \vartheta]_0^a \cos \varphi$$

or, if we indicate the angle ϑ corresponding to the left and right ends of the doublet line by ϑ' and ϑ'', respectively,

$$\Phi = \frac{\mu}{4\pi\rho} (\cos \vartheta'' - \cos \vartheta') \cos \varphi. \qquad (3)$$

If we integrate from $\vartheta = 0$ to $\vartheta = \pi$, that is, from $\xi = -\infty$ to $\xi = \infty$, we obtain

$$\Phi = -\frac{\mu}{2\pi r} \cos \varphi.$$

The velocity component in the r direction is

$$w_r = \frac{\mu}{2\pi r^2} \cos \varphi.$$

Since the parallel flow in the same direction yields the component $W \cos \varphi$, it is obvious that the resulting flow represents a flow around a cylinder with a radius

$$r = \sqrt{\frac{\mu}{2\pi W}}.$$

In other words, the perpendicular flow about a cylinder of radius r can be obtained by covering the cylinder axis with a doublet of moment

$$\mu = 2 \pi r^2 W$$

per unit length.*

*The doublet covering can also be effected by letting the two vortex filaments of a rectilinear pair of vortices come together at constant moment.

If the uniform distribution of the doublets along the x axis extends only from $x = 0$ to $x = \infty$, we obtain the flow about a unilateral body of revolution extending to infinity, whose meridian line can be easily calculated.

The potential function of a system of doublets is expressed by

$$\Phi = -\cos\varphi \sum_i \frac{M_i}{4\pi} \frac{\sin \vartheta_i}{\rho_i^{\,2}} = \cos\varphi \, \Phi_o \, (\rho, \vartheta),$$

in which Φ_o represents the potential function for the vertical section $\varphi = 0$ and depends only on ρ and ϑ or x and r.

The velocities w_x and w_r are derived from this function (Fig. 5) with the aid of the formulas

$$w_x = \frac{\partial \Phi}{\partial x} = \frac{\partial \Phi_o}{\partial x} \cos\varphi = w_{xo} \cos\varphi,$$

$$w_r = \frac{\partial \Phi}{\partial r} = \frac{\partial \Phi_o}{\partial r} \cos\varphi = w_{ro} \cos\varphi,$$

so that, if the velocities w_{xo} and w_{ro} in the plane $\varphi = 0$ are known, the corresponding velocities in any meridian plane can be obtained by multiplying by $\cos\varphi$.

The same is true of the components of the flow velocity W in the meridian plane. Its components are

$$w_x = 0, \qquad w_r = W \cos\varphi.$$

If the streamline pattern in the vertical section $\varphi = 0$ is determined, this immediately furnishes the streamline pattern in any meridian plane, and the corresponding velocity components are obtained by multiplying the velocities found for the meridian plane $\varphi = 0$ by $\cos \varphi$.

The task of determining the perpendicular flow about the airship hull is reduced therefore to the solution of the problem in the plane $\varphi = 0$. The distribution of the doublets along the axis of symmetry must be so determined the the prescribed meridian section becomes a streamline in the plane $\varphi = 0$.

The General System of Equations

The arrangement and notation of the construction drawing are retained again and the axis is covered with doublets, forming line sources of constant intensity (constant moment per unit length). The potential function of the i source is

$$\Phi_i = -\frac{\mu_i}{4\pi r}(\cos \vartheta'_i - \cos \vartheta''_i).$$

Computing the velocity moments w_x and w_r, we obtain

$$w_{xi} = \frac{\partial \Phi_i}{\partial x} = \frac{\mu_i}{4\pi r}\left(\sin \vartheta'_i \frac{\partial \vartheta'_i}{\partial x} - \sin \vartheta''_i \frac{\partial \vartheta''_i}{\partial x}\right)$$

$$w_{ri} = \frac{\partial \Phi_i}{\partial r} = \frac{\mu_i}{4\pi r}\left(\sin \vartheta'_i \frac{\partial \vartheta'_i}{\partial r} - \sin \vartheta''_i \frac{\partial \vartheta''_i}{dr}\right) +$$

$$+ \frac{\mu_i}{4\pi r^2}(\cos \vartheta'_i - \cos \vartheta''_i).$$

Now

$$\vartheta = \text{arc} \tan \frac{r}{x},$$

whence

$$\frac{\partial \vartheta}{\partial x} = \frac{-\frac{r}{x^2}}{1 + \left(\frac{r}{x}\right)^2},$$

$$\frac{\partial \vartheta}{\partial r} = \frac{\frac{1}{x}}{1 + \left(\frac{r}{x}\right)^2}$$

and with

$$\sin \vartheta = \frac{\frac{r}{x}}{\sqrt{1 + \left(\frac{r}{x}\right)^2}}, \qquad \cos \vartheta = \frac{1}{\sqrt{1 + \left(\frac{r}{x}\right)^2}}$$

$$w_{xi} = \frac{\mu_i}{4\pi r^2} (\sin^3 \vartheta''_i - \sin^3 \vartheta'_i),$$

$$w_{ri} = \frac{\mu_i}{4\pi r^2} (2(\cos \vartheta'_i - \cos \vartheta''_i) - (\cos^3 \vartheta'_i - \cos^3 \vartheta''_i)).$$

We introduce the functions

$$f\left(\frac{r}{x}\right) = \frac{\left(\frac{r}{x}\right)^3}{\left(1 + \left(\frac{r}{x}\right)^2\right)^{3/2}} = \sin^3 \vartheta,$$

$$g\left(\frac{r}{x}\right) = \frac{2\left(\frac{r}{x}\right)^2 + 1}{\left(1 + \left(\frac{r}{x}\right)^2\right)^{3/2}} = 2 \cos \vartheta - \cos^3 \vartheta$$

and designate their values by f_i' or f_i'' and g_r' or g_r'', when we calculate x from the left or right terminal of the i source. The functions f and g are represented in Figure 6.

The velocity components produced by the whole system of double sources are

$$w_x = \frac{1}{4\pi r^2} \sum_{i=1}^{n} \mu_i \, (f_i'' - f_i'),$$

$$w_r = \frac{1}{4\pi r^2} \sum_{i=1}^{n} \mu_i \, (g_i' - g_i'').$$

The condition that the prescribed meridian line shall be a streamline may be expressed as follows. The whole flow is produced by superposing the system of doublets on the parallel flow with the components

$$\overline{w}_x = 0, \qquad \overline{w}_r = -W.$$

Let δ denote the angle of inclination of the meridian line. The following expression must then hold good

$$\tan \delta = \frac{w_r + \overline{w}_r}{w_x} = \frac{w_r - W}{w_x}$$

or

$$w_r - w_x \tan \delta = W.$$

Hence

$$\frac{1}{4\pi r^2} \sum_{i=1}^{n} \mu_i \, [(g_i' - g_i'') + \tan \delta \, (f_i' - f_i'')] = W.$$

Introducing

$$z = \frac{\mu_i}{4\pi a^2 W} \qquad (a = \text{unit length of 10 m}),$$

we obtain

$$\sum_{i=1}^{n} z_i [(g_i' - g_i'') + \tan \delta \, (f_i' - f_i'')] = \frac{r^2}{a^2} \tag{4}$$

Applying this equation to n points on the hull curve (n being the number of line sources), we obtain n equations for determining the n unknown z_i.

Application to the Half Hull
(An approximate method)

Figure 7 is a schematic arrangement for the bow. The notations are the same as in the previous calculation of the symmetrical flow. The coefficients

$$g_{ik}' - g_{ik}'' + \tan \delta_k \, (f_{ik}' - f_{ik}'')$$

are most easily calculated by the use of the curves (Fig. 6). The coefficient scheme is characterized by the fact that only the elements in the vicinity of the diagonals differ much from zero, so that one repetition in the reverse direction generally suffices for the determination of the unknown z's.

Figure 6 contains the calculated z_i values. It is obvious that the course of the z_i values conforms to the increment of the radius. If the i airship frame is imagined replaced by a portion of an infinite cylinder of radius r_i, the intensity of the double source is expressed by

$$\mu_i = 2 \pi r_i^2 W$$

or

$$z_i = \frac{1}{2} \left(\frac{r_i}{a}\right)^2.$$

The table contains, along with the values $\frac{1}{2} \left(\frac{r_i}{a}\right)^2$, the computed approximate values of z_i. It can be shown that the slight discrepancy is entirely negligible in the calculation of the pressure distribution. It may therefore be assumed (as a simplified method for calculating the perpendicular flow), that every circular frame of the airship element of mean radius r_i is covered by a line doublet whose intensity is expressed by

$$\mu_i = 2\pi r_i^2 W.$$

Then the velocity components at any point P_k are expressed by the formulas

$$w_x = \frac{W}{2} \Sigma \left(\frac{r_i}{r_k}\right)^2 (f_{ik}'' - f_{ik}''),$$

$$w_r = \frac{W}{2} \Sigma \left(\frac{r_i}{r_k}\right)^2 (g_{ik}' - g_{ik}'')$$

This method leads to the direct determination of the transverse-force distribution, which is more accurate, for example, than Munk's method, in that it indicates correctly the decrease in the transverse forces at the bow and stern, without any difficult computations.

Determination pf the Transverse-Force Distribution

Following Klemperer's example, we have adopted the transverse-force coefficient (Querkraftbreite) with the following notation:

U, the axial component of the flow;

W, the perpendicular component of the flow;

c_x, c_r, c_φ, the three components of the whole flow at any point;

p_o, the pressure at ∞ distance from the airship. We then have

$$p - p_o = \frac{\gamma}{2g} (U^2 + W^2 - c_x^2 - c_r^2 - c_\varphi^2).$$

Now c_x and c_r each consist of two components, the value of the former being independent of φ, while the latter is proportional to $\cos \varphi$. Hence

$$c_x = u_x + w_x \cos \varphi,$$

$$c_r = u_r + w_r \cos \varphi.$$

The third component c_φ is proportional to $\sin \varphi$. Hence we write

$$c_\varphi = w_\varphi \sin \varphi.$$

Introducing these values, we obtain

$$p - p_o = \frac{\gamma}{2g}[U^2 + W^2 - u_x^2 - u_r^2 - w_x^2 \cos^2\varphi - w_r^2 \cos^2 \varphi - \\ - w_\varphi^2 \sin^2\varphi - 2\,(u_x\,w_x + u_r\,w_r) \cos \varphi] .$$

Only the last term

$$\frac{\gamma}{g}\,(u_x\,w_x + u_r\,w_r) \cos\gamma$$

contributes to the transverse force, because all the other terms assume equal values for φ and $\pi - \varphi$, so that the corresponding compression forces are eliminated.

The resulting transverse force, as related to an annular element of unit width, is

$$\frac{dQ}{dx} = \frac{\gamma}{g}\,\pi\,r\,(u_x\,w_x + u_r\,w_r).$$

(the upward direction being considered positive). Klemperer defines the transverse-force coefficient by

$$\beta = \frac{1}{\frac{\gamma}{2g}\,(U^2 + W^2)}\,\frac{dQ}{dx}$$

so that we obtain

$$\beta = 2 \pi r \frac{U W}{U^2 + W^2} \left(\frac{u_x}{U} \frac{w_x}{W} + \frac{u_r}{U} \frac{w_r}{W}\right)$$

in which u_x and u_r are the components of the velocities corresponding to the axial velocity of flow U; w_x and w_r are the velocity components corresponding to the transverse velocity of flow W. If the resulting velocities in the two cases are designated by $\underline{u}$ and $\underline{w}$,

$$u_x w_x + u_r w_r = \underline{u} \, \underline{w},$$

according to the elements of the vector calculation, and the transverse-force coefficient

$$\beta = 2 \pi r \frac{U W}{U^2 + W^2} \frac{\underline{u}}{U} \frac{\underline{w}}{W}.$$

Lastly we write

$$\kappa = \frac{\underline{u}}{U}, \qquad \lambda = \frac{\underline{w}}{W}, \qquad \frac{W}{U} = \tan \alpha ,$$

so that κ or λ represent the velocities corresponding respectively, to the flow velocities $U = 1$ and $W = 1$, and α designates the angle of incidence. We can then write

$$\beta = \pi r \kappa \lambda \sin 2 \alpha .$$

The values κ and λ are given by the corresponding source systems, so that the calculation can cause no difficulty.

Comparison of Theoretical and Experimental Results

Figures 8 and 9 were plotted for the bow and stern, respectively, for an angle of flow of 8°:

a) Transverse-force coefficient according to Munk's theory;

b) Transverse-force coefficient according to the double-source method;

c) Transverse-force coefficient derived from the pressure distribution according to Klemperer.

It is obvious that the double-source method correctly indicates the increase in the transverse force at the bow, while, according to Munk's theory, the transverse force introduces finite values above zero. Both methods give about the same results for the middle portion of an airship hull, which agree well with the experimental results. The determination of the pressure distribution at the stern necessitated a special expedient.

The positive and negative areas limited by the curves in Figures 8 and 9 give, according to the definition of the transverse-force coefficient, the total transverse force acting on the bow and stern, respectively. According to both theories (Munk's and the source-and-sink method) the positive and negative areas are equal, and the resultant of the calculated forces is a simple couple. According to the tests, however, the downward force at the stern is considerably smaller than the upward force at the bow, so that there is a resultant lift for the whole airship. From the momentum theorem it is obvious that this is possible only when there is a difference between the perpendicular components of the momentum passing, per unit time, through a plane in front of and another plane behind the airship. Air must be accelerated downward continuously, just as in the case of an airplane wing. The airship is accordingly followed by a vortex trail, whose intensity and distribution determine the lift. It is possible, by simple and plausible assumptions corresponding to the well-known downflow condition of the wing theory, to develop simple formulas for the magnitude of the lift, which will agree well with the results of experience. The pressure distribution calculated by this method (which will be explained in a future paper) is represented by the curve d in Figure 9. As seen from the figure, the agreement is good. Moreover, I have tried this method of calculation on published examples (measurements by Fuhrmann and measurements on English airship models) with very satisfactory agreement.

Conclusion

It has been shown that the pressure distribution resulting from the potential theory for bodies of revolution can be approximated in a relatively simple manner by covering the axis of symmetry stepwise with sources or double sources. From the practical success of the method, it should not be concludes that the flow about a body of revolution can, in general, be accurately represented by thus covering the axis of symmetry. This is possible only in the exceptional case when the analytical continuation of the potential function, free from singularities in the space outside the body, can be extended to the axis of symmetry without encountering singular spots. It is true, however, that, even in cases for which this method offers no accurate solution, the potential in the surrounding space can be ascertained to any desired degree of approximation by increasing the fineness of division of the line sources. This may justify the method even from the mathematical standpoint.

Translation by Dwight M. Miner,
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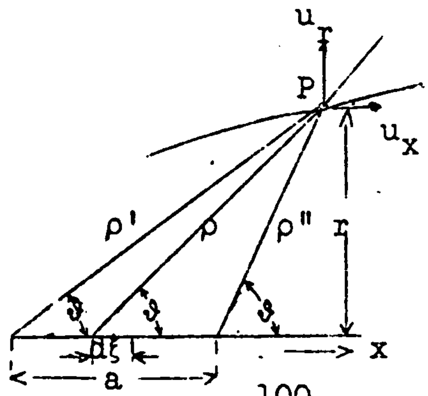
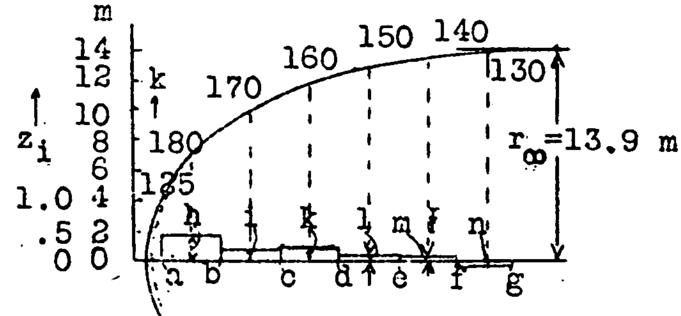


Fig.1



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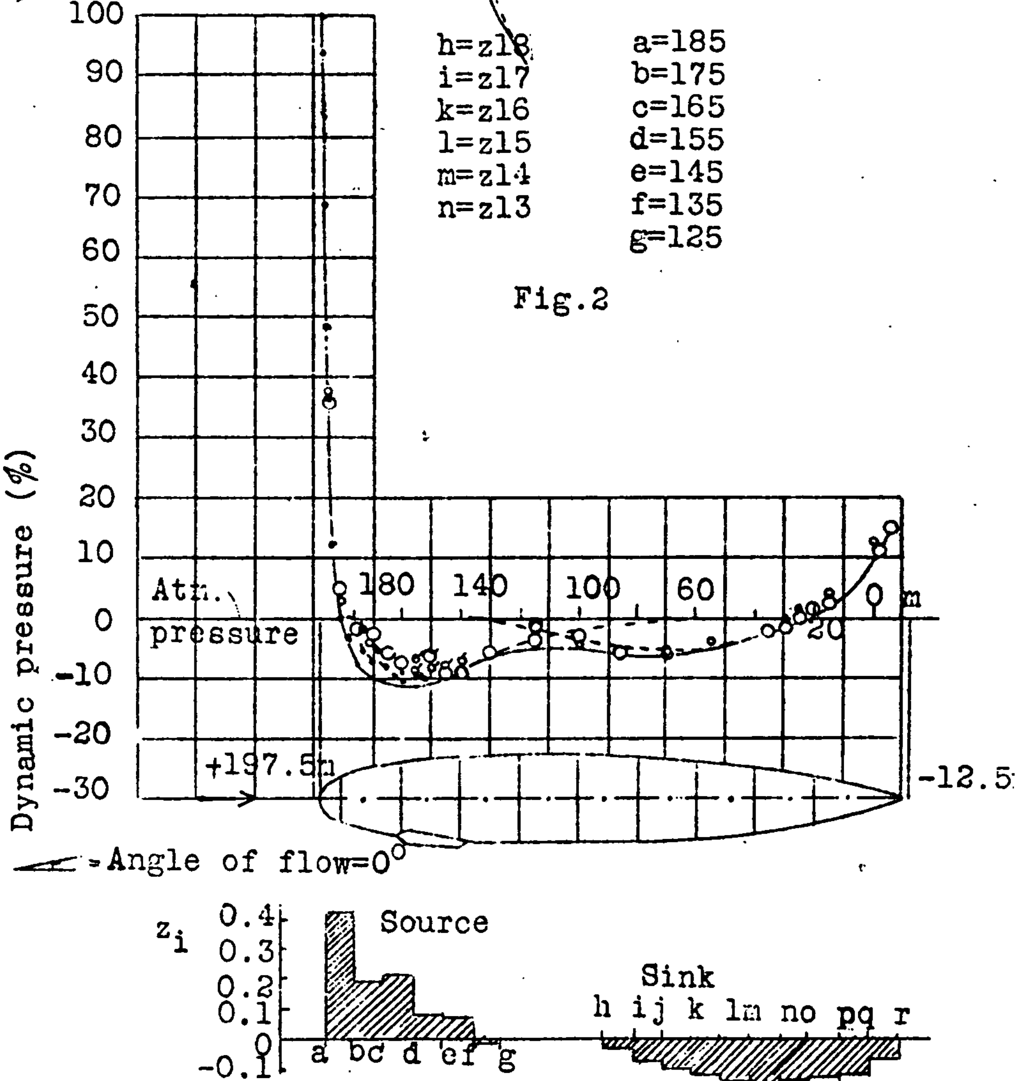
g=125

Fig.2



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k=60 o=20

Fig.3

28

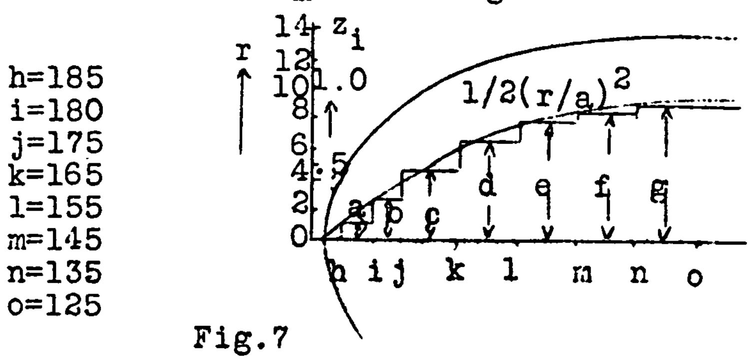

Fig.7

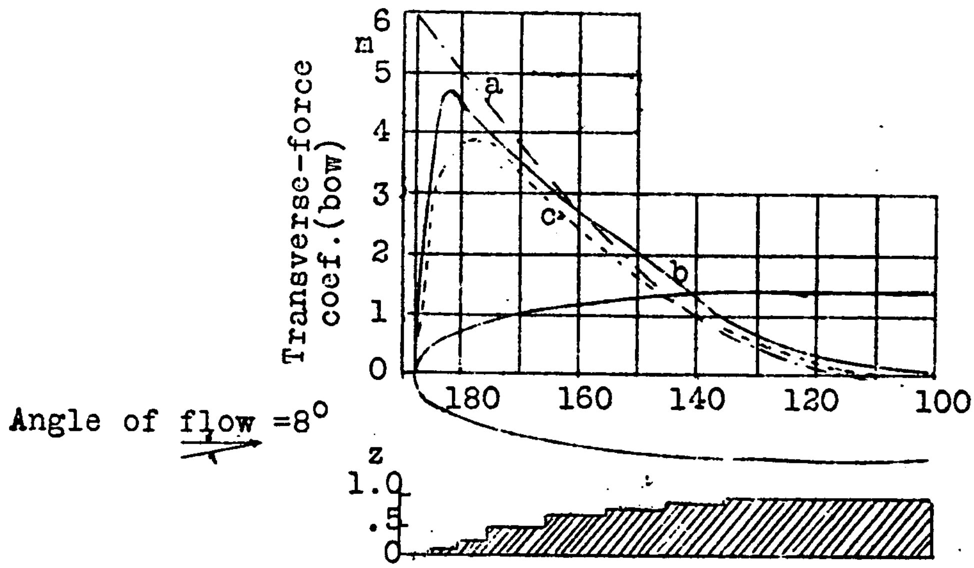

Fig.8

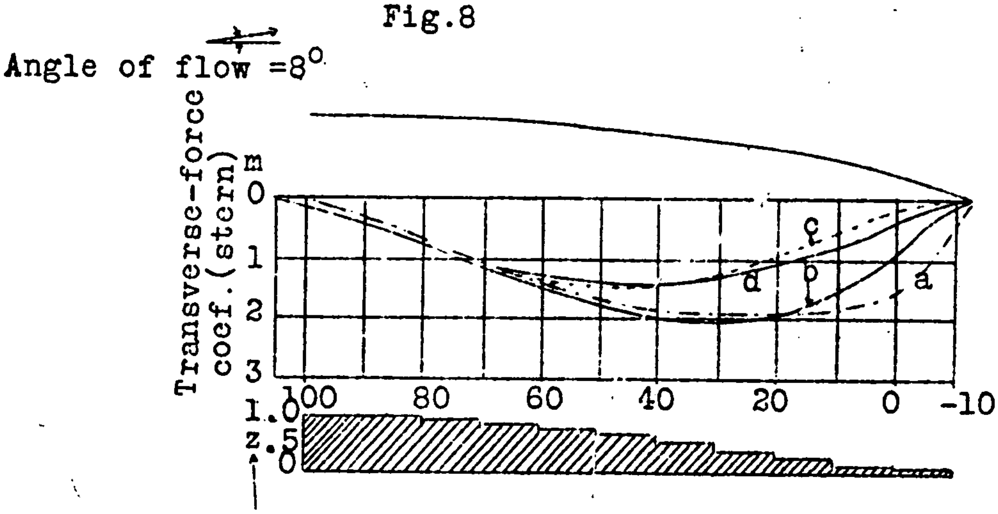

Fig.9

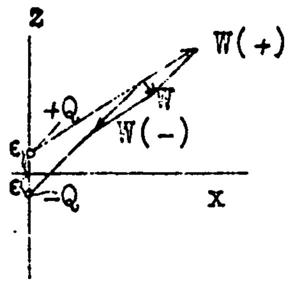

Fig.4

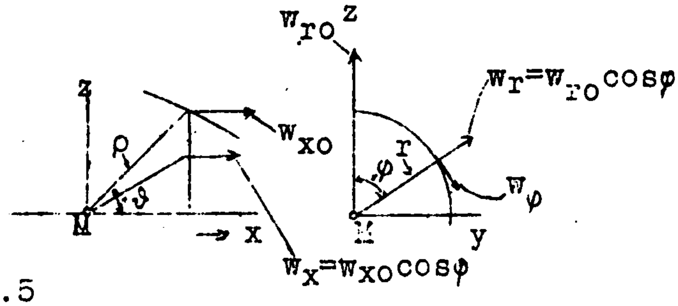

Fig.5

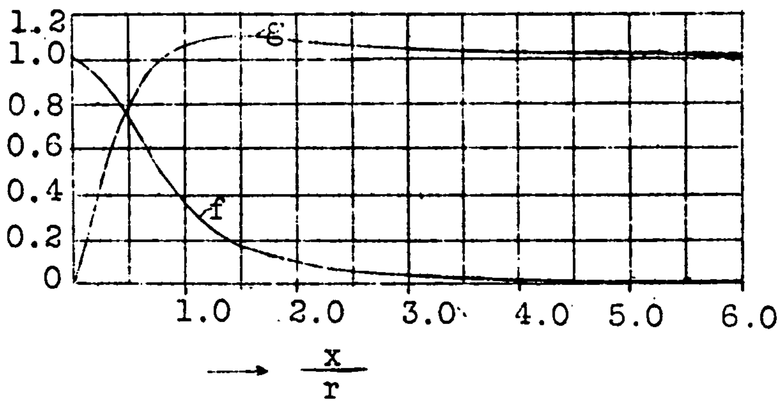

Fig.6