Uniform Panoploid Tetracombs

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A **TETRACOMB** is a four-dimensional *tessellation*. In any tessellation, the *honeycells*, which are the *n*-dimensional polytopes that tessellate the space, must by definition adjoin precisely along their facets, that is, their ($n-1$)-dimensional elements, so that each facet belongs to exactly two honeycells. In the case of tetracombs, the honeycells are four-dimensional polytopes, or *polychora*, and their facets are polyhedra. For a tessellation to be *uniform*, the honeycells must all be uniform polytopes, and the vertices must be transitive on the symmetry group of the tessellation. Loosely speaking, therefore, the vertices must be "surrounded all alike" by the honeycells that meet there.

If a tessellation is such that every point of its space not on a boundary between honeycells lies in the interior of exactly one honeycell, then it is *panoploid*. If one or more points of the space not on a boundary between honeycells lie inside more than one honeycell, the tessellation is *polyploid*. Tessellations may also be constructed that have "holes," that is, regions that lie inside none of the honeycells; such tessellations are called *holeycombs*. It is possible for a polyploid tessellation to also be a holeycomb, but not for a panoploid tessellation, which must fill the entire space exactly once. Polyploid tessellations are also called *starcombs* or *star-tessellations*. Holeycombs usually arise when ($n-1$)-dimensional tessellations are themselves permitted to be honeycells; these take up the otherwise free facets that bound the "holes," so that all the facets continue to belong to two honeycells.

In this essay, as per its title, we are concerned with just the uniform panoploid tetracombs. A natural question is, How many distinct uniform panoploid fillings of four-space exist? By distinct, we mean differing in the shapes or arrangements of their honeycells. Many uniform tessellations, but not all, may be found directly by Wythoff's construction (named after number theorist Willem Abraham Wythoff). Just how to go about doing this was described in some detail in 1930, 1931, 1940, and 1985 by H. S. M. Coxeter, whose graph notation makes the process nearly trivial, once all the Euclidean symmetry groups of *n*-space are enumerated. We assume that the reader is acquainted with Coxeter-Dynkin graphs and their application to uniform tessellations; if not, the reader is referred to chapters in Coxeter's book *Regular Polytopes* for a discussion. What apparently has not yet been carried out is to apply Wythoff's construction in a routine manner to the various four-dimensional space groups to actually compile a table of the different uniform panoploid tetracombs. Here we catalogue and describe them. The names of their honeycells are taken from our website on convex uniform polychora

members.aol.com/Polycell/uniform.html

In particular, we will need the polychora listed in Sections 1–3, 6, and 7 from that site. Those include all the convex uniform polychora that may occur as honeycells in a uniform panoploid tetracomb.

An intuitive starting point for an essay on uniform tetracombs is the unique uniform panoploid filling of one-space with equal line segments connected end to end: the regular **apeirogon**, symbolized by [∞]. From there we proceed to tilings of the plane.

I. Uniform panoploid tilings

A *tiling* is a tessellation of the plane with polygons, that is, a 2-hypercomb or a dicomb. We need to list the uniform panoploid tilings, because a number of uniform panoploid tetracombs are based on them. The first three were known to the Pythogoreans of ancient Greece, who showed that they were the only ways to fill a plane with identical regular polygons edge-to-edge with no overlapping. Johannes Kepler described the other eight in 1619. Thus the regular tilings [1] through [3] are often called the *Pythagorean tilings*, the other eight the *Keplerian tilings*. These patterns were also all undoubtedly known to the artisans of medieval Islam, and are (along with numerous nonuniform tilings) of course quite familiar to present-day makers of wallpaper. The names in parentheses are the tilings' Bowers names; the numbers in brackets, parentheses, and so forth are the tilings' typographically compressed Coxeter-Dynkin graphs, not necessarily unique:

[1] **Square tiling** (*Squat*): squares, four at each corner:[4,4]:(001), [4,4]:010, [4,4]:101, [∞][∞]

[2] **Triangular tiling** (*Trat*): triangles, six at each corner: [3,6]:100, P_3:<001>

[3] **Hexagonal tiling** (*Hexat*): hexagons, three at each corner: [3,6]:001, [3,6]:110, P_3:111

[4] **Elongated triangular tiling** (*Etrat*): triangles and squares, 3.3.3.4.4 at each corner: [3,6]:100e, P_3:<001>e

[5] **Trihexagonal tiling** (*That*): triangles and hexagons, 3.6.3.6 at each corner: [3,6]:010, P_3:<011>

[6] **Truncated square tiling** (*Tosquat*): squares and octagons, 4.8.8 at each corner: [4,4]:(011), [4,4]:111

[7] **Truncated hexagonal tiling** (*Toxat*): triangles and dodecagons, 3.12.12 at each corner: [3,6]:011

[8] **Rhombitrihexagonal tiling** (*Rothat*): triangles, squares, and hexagons, 3.4.6.4 at each corner: [3,6]:101

[9] **Omnitruncated trihexagonal tiling** (*Othat*): squares, hexagons, and dodecagons, 4.6.12 at each corner: [3,6]:111

[10] **Snub square tiling** (*Snasquat*): triangles and squares, 3.3.4.3.4 at each corner: [4,4]:s

[11] **Snub trihexagonal tiling** (*Snathat*): triangles and hexagons, 3.3.3.3.6 at each corner: [3,6]:s, or [3,6]:sd and [3,6]:sl if its chiral versions require distinction

Note that the same arrangement of polygons occurs at *each* corner, and that the polygons are regular, convex, and nonoverlapping. This fulfills the definition of a uniform panoploid tiling. Snathat is the only chiral uniform panoploid tiling and occurs in a left-handed (*laevo*) and a right-handed (*dextro*) form that will not coincide unless one form is turned over in three-space. (The alternated hexagonal tiling, h[6,3]:100, is the same as the trat, so the notation h[6,3] may be substituted for [3,6] anywhere in the text. After a while, there are just too many different symbols and notations for the same figures.)

Elongation is the uniform "stretching" of a tessellation by breaking it apart at parallel hyperplanes of vertices and inserting layers (called *laminae*; singular: *lamina*) of prisms to reconnect the pieces into a new tessellation. In *prismatoelongation*, laminae of two interdigitating sets of triangular-polytopic duoprisms—one set with "points up," the other with "points down," are inserted instead of prisms. This operation begins in the plane with the elongated triangular tiling, in which parallel rows of squares (two-dimensional dyadic prisms) are inserted between the rows of triangles. The same tiling is also the prismatoelongated square tiling, when it is considered to be the square tiling broken apart and parallel rows of triangles (two-dimensional triangular duoprisms) are inserted. Whereas in ordinary elongation consecutive laminae are not displaced laterally relative to each other, in prismatoelongation one lamina is translated laterally half an edge-length relative to its predecessor. These operations extend to uniform tessellations in n dimensions, $n>2$. In such spaces it is further possible to uniformly rotate the laminae of duoprisms relative to one another to obtain various *gyrated*, *bigyrated*, and *multigyrated* tessellations.

The compressed Coxeter-Dynkin graph is formed by the designation of a tessellation's symmetry group followed by a colon and a series of binary digits that correspond to ringed (1) and unringed (0) nodes in the graph. At least one digit must be a 1. In cases where a palindrome within the series yields the same geometric figure, the palindromically equivalent digits are enclosed in parentheses. In cases where a cyclic permutation in a series yields the same figure, the cyclically permutable digits are enclosed in angle brackets. And in cases where any permutation in a series yields the same figure, the permutable digits are enclosed in vertical strokes. Cartesian products of lower-dimensional tessellations are simply strung together. Non-Wythoffian tessellations require special designations, such as "e" for "elongated," "g" for "gyrated," "p" for "prismatoelongated," "s" for "snub," "d" for *dextro*, "l" for *laevo*, and so forth. (We only use the special designations when necessary; there's no point to multiplying Wythoffian designations unnecessarily, as, for example, by using [4,3,4]:(0001)e for [4,3,4]:(0001), the same honeycomb.) By convention, the apeirogon is simply denoted [∞], rather than [∞]:(01) or [∞]:11. The first step in finding uniform tessellations is to write down all possible Coxeter-Dynkin graphs for all possible symmetry groups. Then duplicate tessellations must be found and eliminated, and finally the remaining tessellations must be examined for possible non-Wythoffian derivatives.

The *vertex figure* (or *verf*) at a vertex of an *n*-tessellation is the *n*-dimensional polytope whose vertices are the other endpoints of all the edges that meet at that vertex. The other elements of a verf are the verfs of the various elements that meet at a vertex. If the tessellation is uniform, its verfs are all congruent, so we may refer to *the* verf of a uniform tessellation. If the tessellation is regular, then the verf is regular; otherwise the verf is an irregular polytope. In a uniform tessellation, all the edges have the same length, so the verf is always inscribable in an *n*-hypersphere whose radius is the edge length (3-hypersphere is called a *glome*; by convention, the *n* denotes the dimension of the hypersphere's "hypersurface," which is one less than the dimension of the Euclidean space in which it is embedded; Coxeter, however, conventionally used the term "*n*-sphere," for a hypersphere embedded in *n*-space). Also conventionally, the edge length of a uniform tessellation is taken as 1, which fixes the possible edge lengths of a verf of a uniform panoploid tessellation at $2\cos(\pi/p)$, $p>2$: An edge of length $2\cos(\pi/p)$ is the verf of a regular convex *p*-gon. In the case of Euclidean tessellations, these cosines are always either rational or expressible in radicals, which is how they're presented here. Coxeter sometimes used an edge length of 2 for a uniform polytope or tessellation to avoid awkward fractions in equations. In such instances, he also sometimes fixed a vertex figure's vertices at the midpoints of its parent polytope's edges, thereby keeping the vertex figure inscribed in a hypersphere of unit radius.

For the eleven uniform panoploid tilings listed above, the verfs are:

[1] Square, edge length $\sqrt{2}$
[2] Regular hexagon, edge length 1
[3] Equilateral triangle, edge length $\sqrt{3}$ (regular unit hexagon with three alternate corners removed)
[4] Pentagon, edges of lengths 1, 1, 1, $\sqrt{2}$, $\sqrt{2}$ (regular unit hexagon with three consecutive corners removed, replaced by a square of edge length $\sqrt{2}$ with one corner removed)
[5] Rectangle, edges of lengths 1, $\sqrt{3}$, 1, $\sqrt{3}$ (regular unit hexagon with two opposite corners removed)
[6] Isosceles triangle, edges of lengths $\sqrt{2}$, $\sqrt{(2+\sqrt{2})}$, $\sqrt{(2+\sqrt{2})}$
[7] Isosceles triangle, edges of lengths 1, $\sqrt{(2+\sqrt{3})}$, $\sqrt{(2+\sqrt{3})}$
[8] Trapezoid, edges of lengths 1, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{2}$
[9] Scalene triangle, edges of lengths $\sqrt{2}$, $\sqrt{3}$, $\sqrt{(2+\sqrt{3})}$ (chiral, *dextro* and *laevo* versions occur equally)
[10] Pentagon, edges of lengths 1, 1, $\sqrt{2}$, 1, $\sqrt{2}$
[11] Pentagon, edges of lengths 1, 1, 1, 1, $\sqrt{3}$ (regular unit hexagon with one corner removed)

All these verfs are inscribable in a unit circle (1-hypersphere), of course. It is relatively easy to prove, by combinatorial arguments, that these eleven tilings exhaust the possibilities.

II. Uniform panoploid honeycombs

Surprisingly, the tessellations of three-space corresponding to the uniform panoploid tilings, namely, the uniform panoploid *honeycombs*, were noticed only after the start of the 20th century by several geometers, beginning with Alfredo Andreini in 1905 and including H. S. M. Coxeter and Norman Johnson. Their disparate works were summarized as recently as 1994 by Branko Grünbaum, whose list of 28 honeycombs is considered complete (but not yet proved so). Inasmuch as there are several workers in this field, these honeycombs have acquired several names. The long names in ordinary type (not boldface) following our own name in boldface are Norman Johnson's; the italicized short names in parentheses are the Bowers acronyms. Norman uses the term "cellulation" instead of "honeycomb," reserving the term "honeycomb" for a general tessellation of *n*-space, and the term "tiling" for any panoploid tessellation of *n*-space. Following Norman's name (and perhaps his alternative name) may be one or more older versions of our name for the same honeycomb. Their Coxeter-Dynkin graphs are encapsulated in the numbers immediately after each name, sometimes several different ones, separated by commas, that yield the same honeycomb:

[1] **Cubic honeycomb** (*Chon*)
    [4,3,4]:(0001), [4,3,4]:1001, h[4,3,4]:1000, [4,4]:(001)[∞], [4,4]:010[∞],
        [4,4]:101[∞], [∞][∞][∞]
    cubic cellulation
    runcinated cubic cellulation
    square prismatic honeycomb
    Honeycells: cubes

[2] **Triangular prismatic honeycomb** (*Tiph*)
    [3,6]:100[∞], P_3:<001>[∞]
    triangular prismatic cellulation
    Honeycells: triangular prisms (aligned)

[3] **Hexagonal prismatic honeycomb** (*Hiph*)
    [3,6]:001[∞], [3,6]:110[∞], P_3:111[∞]
    hexagonal prismatic cellulation
    Honeycells: hexagonal prisms

[4] **Elongated triangular prismatic honeycomb** (*Etoph*)
    [3,6]:100[∞]e, P_3:<001>[∞]e, [3,6]:100e[∞], P_3:<001>e[∞]
    elongated antiprismatic prismatic cellulation
    Honeycells: triangular prisms (aligned) and cubes

[5] **Trihexagonal prismatic honeycomb** (*Thiph*)
    [3,6]:010[∞], P_3:<011>[∞]
    trihexagonal prismatic cellulation
    triangular-hexagonal prismatic honeycomb

Honeycells: triangular prisms and hexagonal prisms (four of each per vertex)

[6] **Truncated square prismatic honeycomb** (*Tassiph*)
[4,4]:(011)[∞], [4,4]:111[∞]
tomo-square prismatic cellulation
Honeycells: cubes and octagonal prisms

[7] **Truncated hexagonal prismatic honeycomb** (*Thaph*)
[3,6]:011[∞]
tomo-hexagonal prismatic cellulation
Honeycells: triangular prisms and dodecagonal prisms

[8] **Rhombitrihexagonal prismatic honeycomb** (*Rothaph*)
[3,6]:101[∞]
rhombitrihexagonal prismatic cellulation
rhombitriangular-hexagonal prismatic honeycomb
Honeycells: triangular prisms, cubes, and hexagonal prisms

[9] **Omnitruncated trihexagonal prismatic honeycomb** (*Otathaph*)
[3,6]:111[∞]
tomo-trihexagonal prismatic cellulation
omnitruncated triangular-hexagonal prismatic honeycomb
Honeycells: cubes, hexagonal prisms, and dodecagonal prisms

[10] **Snub square prismatic honeycomb** (*Sassiph*)
[4,4]:s[∞]
simo-square prismatic cellulation
Honeycells: triangular prisms and cubes

[11] **Snub trihexagonal prismatic honeycomb** (*Snathaph*)
[3,6]:s[∞]
simo-trihexagonal prismatic cellulation
snub triangular-hexagonal prismatic honeycomb
Honeycells: triangular prisms and hexagonal prisms (eight of the former and two of the latter per vertex)

[12] **Gyrated triangular prismatic honeycomb** (*Gytoph*)
[3,6]:010[∞]g, P_3:<011>[∞]g
parasquare fastigial cellulation
Honeycells: triangular prisms (gyrated)

[13] **Gyrated elongated triangular prismatic honeycomb** (*Gyetaph*)
[3,6]:010[∞]ge, P_3:<011>[∞]ge
elongated parasquare fastigial cellulation

gyroelongated triangular prismatic honeycomb
Honeycells: triangular prisms (gyrated) and cubes

[14] **Truncated cubic honeycomb** (*Tich*)
[4,3,4]:(0011), h[4,3,4]:1100
truncated cubic cellulation
Honeycells: octahedra and truncated cubes

[15] **Rectified cubic honeycomb** (*Rich*)
[4,3,4]:(0010), h[4,3,4]:0100, h[4,3,4]:0011, P_4:<0101>
rectified cubic cellulation
Honeycells: octahedra and cuboctahedra

[16] **Bitruncated cubic honeycomb** (*Batch*)
[4,3,4]:0110, h[4,3,4]:0111, P_4:1111
bitruncated cubic cellulation
truncated-octahedral honeycomb
Honeycells: truncated octahedra

[17] **Cantellated cubic honeycomb** (*Srich*)
[4,3,4]:(0101), h[4,3,4]:1011
cantellated cubic cellulation
small rhombated cubic honeycomb
Honeycells: cubes, cuboctahedra, and rhombicuboctahedra

[18] **Cantitruncated cubic honeycomb** (*Grich*)
[4,3,4]:(0111), h[4,3,4]:1111
cantitruncated cubic cellulation
great rhombated cubic honeycomb
Honeycells: cubes, truncated octahedra, and truncated cuboctahedra

[19] **Runcitruncated cubic honeycomb** (*Prich*)
[4,3,4]:(1011)
runcitruncated cubic cellulation
prismatorhombated cubic honeycomb
Honeycells: cubes, octagonal prisms, truncated cubes, and rhombicuboctahedra

[20] **Omnitruncated cubic honeycomb** (*Otch*)
[4,3,4]:1111
omnitruncated cubic cellulation
Honeycells: octagonal prisms and truncated cuboctahedra

[21] **Alternated cubic honeycomb** (*Octet*)
h[4,3,4]:00|01|, P_4:<0001>

half cubic cellulation
tetrahedral-octahedral honeycomb
octahedral-tetrahedral honeycomb
Honeycells: tetrahedra and octahedra (aligned)

[22] **Gyrated alternated cubic honeycomb** (*Gytoh*)
h[4,3,4]:00|01|g, P_4:<0001>g
triangular antiprismatic cellulation
gyrated tetrahedral-octahedral honeycomb
Honeycells: tetrahedra and octahedra (gyrated)

[23] **Elongated alternated cubic honeycomb** (*Etoh*)
h[4,3,4]:00|01|e, P_4:<0001>e
elongated triangular gyroprismatic cellulation
elongated tetrahedral-octahedral honeycomb
Honeycells: tetrahedra, octahedra, and triangular prisms (aligned)

[24] **Gyrated elongated alternated cubic honeycomb** (*Gyetoh*)
h[4,3,4]:00|01|ge, P_4:<0001>ge
elongated triangular antiprismatic cellulation
gyroelongated alternated cubic honeycomb
gyroelongated tetrahedral-octahedral honeycomb
Honeycells: tetrahedra, octahedra, and triangular prisms (gyrated)

[25] **Truncated alternated cubic honeycomb** (*Tatoh*)
h[4,3,4]:01|01|, P_4:<0111>
cantic cubic cellulation
truncated tetrahedral-octahedral honeycomb
Honeycells: cuboctahedra, truncated tetrahedra, and truncated octahedra

[26] **Runcinated alternated cubic honeycomb** (*Ratoh*)
h[4,3,4]:10|01|
runcic cubic cellulation
runcinated tetrahedral-octahedral honeycomb
rhombated tetrahedral-octahedral honeycomb
Honeycells: tetrahedra, cubes, and rhombicuboctahedra

[27] **Quarter cubic honeycomb** (*Batatoh*)
P_4:<0011>
quarter cubic cellulation
bitruncated alternated cubic honeycomb
bitruncated tetrahedral-octahedral honeycomb
Honeycells: tetrahedra and truncated tetrahedra

[28] **Runcicantic cubic honeycomb** (*Gratoh*)
h[4,3,4]:11|01|
runicantic cubic cellulation
cantitruncated alternated cubic honeycomb
cantitruncated tetrahedral-octahedral honeycomb
great rhombated tetrahedral-octahedral honeycomb
Honeycells: truncated tetrahedra, truncated cubes, and truncated cuboctahedra

The cubic honeycomb was, of course, known to humanity since the dawn of architecture. The honeycells of the cubic honeycomb are all identical cubes, which, since the cube is a regular polyhedron, makes the cubic a regular panoploid honeycomb—incidentally the *only* regular honeycomb (that is, three-dimensional tessellation; in two dimensions there are the above three regular tilings, and in four dimensions there are three regular tetracombs; higher dimensions than that are all like three-space in having only the hypercubic tessellation as regular). It is rather astonishing, however, that most of other uniform panoploid honeycombs remained uncharacterized for so long, because finding them is a natural extension of the problem of finding the uniform panoploid tilings, and it is not even necessary to work in a space of more than three dimensions. The list of 28 above is considered complete just because it is difficult to imagine a 29th; a formal proof remains elusive.

Of the 28, the first eleven are *prismatic* honeycombs: uniform honeycombs developed from the uniform tilings by simply expanding each into a lamina of polygonal prisms and then stacking the laminae to infinity in three-space. They are the Cartesian products of the eleven plane tilings with the apeirogon. This process, generalized to space of n dimensions, may be referred to as the *prismatization* of an n-tessellation; it results in a prismatic (n+1)-tessellation.

Honeycombs in the list that have the same kinds of honeycells differ in the arrangements of the honeycells at each corner, or, if they have the same arrangement at a corner, they differ in their symmetry groups. The terms *aligned* and *gyrated* distinguish different ways of stacking laminae of the same kinds of polyhedra in a honeycomb. Although the prismatization of a chiral tiling, honeycomb [11] above is not itself chiral, because it may simply be turned upside-down to become its “mirror image.” Alas, there appear to be no chiral uniform panoploid honeycombs. In the plane, tiling [11] is chiral because we must turn it upside-down in three-space, out of its plane, in order to bring it into coincidence with its mirror image. This same motion also turns honeycomb [11] into its “mirror image.”

The verfs of the uniform panoploid honeycombs are generally irregular, and sometimes fairly strange-looking, polyhedra. Nevertheless, they are all vertex-inscribable in a unit sphere. The faces of the verfs are verfs of the honeycells, and they show how the honeycells come together at each vertex of the honeycomb. The verf of any prismatic honeycomb is a bipyramid whose lateral edges have length $\sqrt{2}$ and whose base polygon is the verf of the tiling that was prismatized. In the case of the cubic honeycomb, which is a prismatized square tiling, the bipyramid is a regular octahedron.

[1] Regular octahedron, edges of length √2
[2] Hexagonal bipyramid, equatorial edges of length 1, lateral edges of length √2
[3] Triangular bipyramid, equatorial edges of length √3, lateral edges of length √2
[4] Pentagonal bipyramid, equatorial edges of lengths 1, 1, 1, √2, √2, lateral edges of length √2
[5] Rectangular bipyramid, equatorial edges of lengths 1, √3, 1, √3, lateral edges of length √2
[6] Isosceles-triangular bipyramid, equatorial edges of lengths √2, √(2+√2), √(2+√2), lateral edges of length √2
[7] Isosceles-triangular bipyramid, equatorial edges of lengths 1, √(2+√3), √(2+√3), lateral edges of length √2
[8] Trapezoidal bipyramid, equatorial edges of lengths 1, √2, √3, √2, lateral edges of length √2
[9] Scalene-triangular bipyramid, equatorial edges of lengths √2, √3, √(2+√3), lateral edges of length √2 (chiral, *dextro* and *laevo* versions occur equally)
[10] Pentagonal bipyramid, equatorial edges of lengths 1, 1, √2, 1, √2, lateral edges of length √2
[11] Pentagonal bipyramid, equatorial edges of lengths 1, 1, 1, 1, √3, lateral edges of length √2
[12] Hexagonal bipyramid as for [2], but split apart along a diametral square and rejoined with one half rotated 90°
[13] Same as [4]
[14] Equilateral-triangular pyramid, base edges of length √2, lateral edges of length √(2+√2)
[15] Square prism, base edges of length √2, height √3
[16] Tetragonal disphenoid, two opposite edges of length √3, four lateral edges of length √2
[17] Rectangular wedge, base a rectangle with edges of length 1, √2, 1, √2, lateral faces alternately equilateral triangles with edges of length √2 and trapezoids with edges of lengths 1, √2, √2, √2
[18] Equilateral-triangular pyramid, base edge of length √2, lateral edges of lengths √3, √3, √(2+√2)
[19] Trapezoidal pyramid, base edges of lengths 1, √2, √2, √2, lateral edges of lengths √2, √2, √(2+√2), √(2+√2) ordered so that edges 1, √(2+√2), √(2+√2) form a lateral isosceles triangle
[20] Irregular chiral tetrahedron, three zigzag edges of lengths √(2+√2), √3, √(2+√2), other three zigzag edges all of length √2 (half *dextro* and half *laevo* versions)
[21] Cuboctahedron, edges of length 1
[22] Triangular gyrobicupola, edges of length 1 (cuboctahedron of [21] split along equatorial hexagon and rejoined with one half rotated 60°)
[23] Triangular cupola (half of a cuboctahedron), edges of length 1, joined at base

hexagon to hexagonal pyramid, lateral edges of length $\sqrt{2}$
[24] Same as [23]
[25] Rectangular pyramid, base edges of lengths 1, $\sqrt{2}$, 1, $\sqrt{2}$, lateral edges of lengths $\sqrt{3}$
[26] Frustum of equilateral-triangular pyramid, upper base with edges of length 1, lower base with edges of length $\sqrt{2}$, lateral faces trapezoids with edges of length 1, $\sqrt{2}$, $\sqrt{2}$, $\sqrt{2}$
[27] Elongated equilateral-triangular antiprism, base edges of length 1, lateral edges of length $\sqrt{3}$
[28] Tetrahedron, two opposite edges of lengths 1, $\sqrt{2}$, four lateral edges of lengths $\sqrt{3}$, $\sqrt{3}$, $\sqrt{(2+\sqrt{2})}$, $\sqrt{(2+\sqrt{2})}$, ordered so that its unit edge belongs to two different isosceles triangles

III. Uniform panoploid prismatic and duoprismatic tetracombs
There are 83 known ways to generate distinct uniform panoploid tetracombs using the above 11 tilings and 28 honeycombs. In addition, the four-dimensional analogues of the operations of expansion and contraction (as described by Alicia Boole Stott), in their general form as Wythoff's construction, may be applied to the space groups of the regular tetracombs to generate numerous nonprismatic uniform panoploid tetracombs. And there are also a number of uniform panoploid tetracombs that are not directly Wythoffian or prismatizations of lower-dimensional tessellations. The simplest and most obvious uniform panoploid tetracombs, however, are the prismatizations of the 28 uniform panoploid honeycombs. The first eleven of these are also the Cartesian products of their parent tilings with the square tiling; the Cartesian products commute, so the order of the tilings in their designations doesn't matter. We begin with these:

[1] **Tesseractic tetracomb**
[4,3,3,4]:(00001), [4,3,3,4]:10001, h[4,3,3,4]:10000, [4,3,4]:<0001>[∞], [4,4]:(001)[4,4]:(001), [4,4]:(001)[4,4]:010, [4,4]:(001)[4,4]:101, [4,4]:010[4,4]:010, [4,4]:010[4,4]:101, [4,4]:101[4,4]:101, [4,3,4]:1001[∞], h[4,3,4]:1000[∞], [4,4]:(001)[∞][∞], [4,4]:010[∞][∞], [4,4]:101[∞][∞], [∞][∞][∞][∞]
chon × apeirogon; squat × squat; squat × apeirogon × apeirogon; apeirogon × apeirogon × apeirogon × apeirogon
Honeycells: tesseracts
Verf: regular hexadecachoron, edge length $\sqrt{2}$
This tetracomb is regular, being the four-dimensional analogue of the checkerboard and cubic honeycomb. It has several derivations in addition to being the prismatization of the cubic honeycomb. It is also the Cartesian product of two square tilings, as well as the Cartesian product of a square tiling and two apeirogons, or of four apeirogons. And it is its own expansion (or runcination), like its lower-dimensional analogues.

[2] **Triangular-square duoprismatic tetracomb**
[3,6]:100[4,4]:(001), P_3:<001>[4,4]:(001), [3,6]:100[4,4]:010, P_3:<001>[4,4]:010,

[3,6]:100[4,4]:101, P_3:<001>[4,4]:101, [3,6]:100[∞][∞], P_3:<001>[∞][∞]
tiph × apeirogon; squat × trat; trat × apeirogon × apeirogon
Honeycells: triangular-square duoprisms
Verf: square-hexagonal duopyramid, hexagon edges of length 1, all other edges of length $\sqrt{2}$

This tetracomb has three different uniform gyrations ([12], [84], and [86] below).

[3] **Square-hexagonal duoprismatic tetracomb**

[3,6]:001[4,4]:(001), [3,6]:110[4,4]:(001), P_3:111[4,4]:(001), [3,6]:001[4,4]:010, [3,6]:110[4,4]:010, P_3:111[4,4]:010, 3,6]:001[4,4]:101, [3,6]:110 [4,4]:101, P_3:111 [4,4]:101, [3,6]:001[∞][∞], [3,6]:110[∞][∞], P_3:111[∞][∞]
hiph × apeirogon; squat × hexat; hexat × apeirogon × apeirogon
Honeycells: square-hexagonal duoprisms
Verf: triangular-square duopyramid, triangle edges of length $\sqrt{3}$, all other edges of length $\sqrt{2}$

[4] **Elongated triangular-square duoprismatic tetracomb**

[3,6]:100[4,4]:(001)e, P_3:<001>[4,4]:(001)e, [3,6]:100[4,4]:010e, P_3:<001>[4,4]:010e, [3,6]:100[4,4]:101e, P_3:<001>[4,4]:101e, [3,6]:100[∞][∞]e, P_3:<001>[∞][∞]e, [3,6]:100e[4,4]:(001), P_3:<001>e[4,4]:(001), [3,6]:100e[4,4]:010, P_3:<001>e[4,4]:010, [3,6]:100e[4,4]:101, P_3:<001>e[4,4]:101, [3,6]:100e[∞][∞], P_3:<001>e[∞][∞]
etoph × apeirogon; squat × etrat; etrat × apeirogon × apeirogon
Honeycells: triangular-square duoprisms and tesseracts
Verf: square-irregular pentagonal duopyramid, pentagon edges of lengths 1, 1, 1, $\sqrt{2}$, $\sqrt{2}$, all other edges of length $\sqrt{2}$

Like [2], this tetracomb has two uniform gyrations ([13] and [85] below). Note that in this case elongation commutes across the Cartesian product, so that this tetracomb may also be considered a square-elongated-triangular duoprismatic tetracomb, etc.

[5] **Trihexagonal-square duoprismatic tetracomb**

[3,6]:010[4,4]:(001), P_3:<011>[4,4]:(001), [3,6]:010[4,4]:010, P_3:<011>[4,4]:010, 3,6]:010[4,4]:101, P_3:<011>[4,4]:101, [3,6]:010[∞][∞], P_3:<011>[∞][∞]
thiph × apeirogon; squat × that; that × apeirogon × apeirogon
Honeycells: triangular-square duoprisms and square-hexagonal duoprisms
Verf: square-rectangular duopyramid, rectangle edges of length 1, $\sqrt{3}$, 1, $\sqrt{3}$, all other edges of length $\sqrt{2}$

[6] **Tomosquare-square duoprismatic tetracomb**

[4,4]:(011)[4,4]:(001), [4,4]:111[4,4]:(001), [4,4]:(011)[4,4]:010, [4,4]:111[4,4]:010, [4,4]:(011)[4,4]:101, [4,4]:111[4,4]:101, [4,4]:(011)[∞][∞], [4,4]:111[∞][∞]
tassiph × apeirogon; squat × tosquat; tosquat × apeirogon × apeirogon
Honeycells: tesseracts and square-octagonal duoprisms

Verf: isosceles-triangular-square duopyramid, isosceles triangle edges of lengths $\sqrt{2}$, $\sqrt{(2+\sqrt{2})}$, $\sqrt{(2+\sqrt{2})}$, all other edges of length $\sqrt{2}$

[7] **Tomohexagonal-square duoprismatic tetracomb**

[3,6]:011[4,4]:(001), [3,6]:011[4,4]:010, [3,6]:011[4,4]:101, [3,6]:011[∞][∞]
thaph × apeirogon; squat × toxat; toxat × apeirogon × apeirogon
Honeycells: triangular-square duoprisms and square-dodecagonal duoprisms
Verf: isosceles-triangular-square duopyramid, isosceles triangle edges of lengths 1, $\sqrt{(2+\sqrt{3})}$, $\sqrt{(2+\sqrt{3})}$, all other edges of length $\sqrt{2}$

[8] **Rhombitrihexagonal-square duoprismatic tetracomb**

[3,6]:101[4,4]:(001), [3,6]:101[4,4]:010, [3,6]:101[4,4]:101, [3,6]:101[∞][∞]
rothaph × apeirogon; squat × rothat; rothat × apeirogon × apeirogon
Honeycells: triangular-square duoprisms, tesseracts, and square-hexagonal duoprisms
Verf: trapezoidal-square duopyramid, trapezoid edges of lengths 1, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{2}$, all other edges of length $\sqrt{2}$

[9] **Omnitruncated-trihexagonal-square duoprismatic tetracomb**

[3,6]:111, [3,6]:111[4,4]:010, [3,6]:111[4,4]:101, [3,6]:111[∞][∞]
otathat × apeirogon; squat × othat; otathat × apeirogon × apeirogon
Honeycells: tesseracts, square-hexagonal duoprisms, and square-dodecagonal duoprisms
Verf: scalene-triangular-square duopyramid, triangle edges of lengths $\sqrt{2}$, $\sqrt{3}$, $\sqrt{(2+\sqrt{3})}$, all other edges of length $\sqrt{2}$

[10] **Simosquare-square duoprismatic tetracomb**

[4,4]:s[4,4]:(001), [4,4]:s[4,4]:010, [4,4]:s[4,4]:101, [4,4]:s[∞][∞]
sassiph × apeirogon; squat × snasquat; snasquat × apeirogon × apeirogon
Honeycells: triangular-square duoprisms and tesseracts
Verf: square-irregular-pentagonal duopyramid, pentagon edges 1, 1, $\sqrt{2}$, 1, $\sqrt{2}$, all other edges of length $\sqrt{2}$

[11] **Simotrihexagonal-square duoprismatic tetracomb**

[3,6]:s[4,4]:(001), [3,6]:s[4,4]:010, [3,6]:s[4,4]:101, [3,6]:s[∞][∞]
snathaph × apeirogon; squat × snathat; snathat × apeirogon × apeirogon
Honeycells: triangular-square duoprisms and square-hexagonal duoprisms
Verf: square-irregular-pentagonal duopyramid, pentagon edges 1, 1, 1, 1, $\sqrt{3}$, all other edges of length $\sqrt{2}$

[12] **Gyrated triangular-square duoprismatic tetracomb**

[3,6]:100[4,4]:(001)g, P_3:<001>[4,4]:(001)g, [3,6]:100[4,4]:010g, P_3:<001>[4,4]:010g, [3,6]:100[4,4]:101g, P_3:<001>[4,4]:101g, [3,6]:100[∞][∞]g, P_3:<001>[∞][∞]g**,** [3,6]:010[∞]g[∞], P_3:<011>[∞]g[∞]

gytoph × apeirogon
Honeycells: triangular-square duoprisms
Verf: bipyramid based on the gytoph verf and whose two apices join to it with edges of length $\sqrt{2}$

This stacking of triangular duoprisms employs the "meson" gyration, in which the duoprisms assume two different orientations in alternating laminae. It is a gyration of [2] above. The other gyration of [2], with three different orientations, is [84]. In each gyrated lamina, the duoprisms are reflected relative to the laminae above and below in a mirror realm defined by the center of a lateral square and a diagonal plane of the opposite cubic cell. Each lamina comprises all the duoprisms, half pointing up and half pointing down, that are sandwiched between two consecutive realms of cubic honeycombs.

[13] **Elongated gyrated triangular-square duoprismatic tetracomb**

[3,6]:100[4,4]:(001)ge, P_3:<001>[4,4]:(001)ge, [3,6]:100[4,4]:010ge, P_3:<001>[4,4]:010ge, [3,6]:100[4,4]:101ge, P_3:<001>[4,4]:101ge, [3,6]:100[∞][∞]ge, P_3:<001>[∞][∞]ge**,** [3,6]:010[∞]g[∞]e, P_3:<011>[∞]g[∞]e, [3,6]:010[∞]ge[∞], P_3:<011>[∞]ge[∞], [3,6]:100e[4,4]:(001)g, P_3:<001>e[4,4]:(001)g, [3,6]:100e[4,4]:010g, P_3:<001>e[4,4]:010g, [3,6]:100e[4,4]:101g, P_3:<001>e[4,4]:101g, [3,6]:100e[∞][∞]g, P_3:<001>e[∞][∞]g
gyetaph × apeirogon
Honeycells: triangular-square duoprisms and tesseracts
Verf: same as [4]

This tetracomb is [12] with laminae of tesseracts in between the gyrated laminae of triangular-square duoprisms. Another gyration of this tetracomb is [85].

[14] **Truncated-cubic prismatic tetracomb**

[4,3,4]:(0011)[∞], h[4,3,4]:1100[∞]
tich × apeirogon
Honeycells: octahedral prisms and truncated cubic prisms
Verf: bipyramid based on an equilateral-triangular pyramid, base edges of length $\sqrt{2}$, lateral edges of length $\sqrt{(2+\sqrt{2})}$; all other edges of length $\sqrt{2}$

[15] **Rectified-cubic prismatic tetracomb**

[4,3,4]:(0010)[∞], h[4,3,4]:0100[∞], h[4,3,4]:0011[∞], P_4:<0101>[∞]
rich × apeirogon
Honeycells: octahedral prisms and cuboctahedral prisms
Verf: bipyramid based on a square prism, base edges of length $\sqrt{2}$, height $\sqrt{3}$; all other edges of length $\sqrt{2}$

[16] **Bitruncated-cubic prismatic tetracomb**

[4,3,4]:0110[∞], h[4,3,4]:0111[∞], P_4:1111[∞]
batch × apeirogon

Honeycells: truncated-octahedral prisms
Verf: bipyramid based on a tetragonal disphenoid, two opposite edges of length $\sqrt{3}$, four lateral edges of length $\sqrt{2}$; all other edges of length $\sqrt{2}$

[17] **Cantellated-cubic prismatic tetracomb**
[4,3,4]:(0101)[∞], h[4,3,4]:1011[∞]
srich × apeirogon
Honeycells: tesseracts, cuboctahedral prisms, and rhombicuboctahedral prisms
Verf: bipyramid based on a rectangular wedge, base a rectangle with edges of length 1, $\sqrt{2}$, 1, $\sqrt{2}$, lateral faces alternately equilateral triangles with edges of length $\sqrt{2}$ and trapezoids with edges of lengths 1, $\sqrt{2}$, $\sqrt{2}$, $\sqrt{2}$; all other edges of length $\sqrt{2}$

[18] **Cantitruncated-cubic prismatic tetracomb**
[4,3,4]:(0111)[∞], h[4,3,4]:1111[∞]
grich × apeirogon
Honeycells: tesseracts, truncated-octahedral prisms, and truncated-cuboctahedral prisms
Verf: bipyramid based on an equilateral-triangular pyramid, base edge of length $\sqrt{2}$, lateral edges of lengths $\sqrt{3}$, $\sqrt{3}$, $\sqrt{(2+\sqrt{2})}$; all other edges of length $\sqrt{2}$

[19] **Runcitruncated-cubic prismatic tetracomb**
[4,3,4]:(1011)[∞]
prich × apeirogon
Honeycells: tesseracts, square-octagonal duoprisms, truncated-cubic prisms, and rhombicuboctahedral prisms
Verf: bipyramid based on a trapezoidal pyramid, base edges of lengths 1, $\sqrt{2}$, $\sqrt{2}$, $\sqrt{2}$, lateral edges of lengths $\sqrt{2}$, $\sqrt{2}$, $\sqrt{(2+\sqrt{2})}$, $\sqrt{(2+\sqrt{2})}$ ordered so that edges 1, $\sqrt{(2+\sqrt{2})}$, $\sqrt{(2+\sqrt{2})}$ form a lateral isosceles triangle; all other edges of length $\sqrt{2}$

[20] **Omnitruncated-cubic prismatic tetracomb**
[4,3,4]:1111[∞]
otch × apeirogon
Honeycells: square-octagonal duoprisms and truncated-cuboctahedral prisms
Verf: bipyramid based on an irregular tetrahedron, three zigzag edges of lengths $\sqrt{(2+\sqrt{2})}$, $\sqrt{3}$, $\sqrt{(2+\sqrt{2})}$, other three zigzag edges all of length $\sqrt{2}$; all other edges of length $\sqrt{2}$

This exhausts the list of Wythoffian derivatives of the cubic honeycomb. The remaining eight honeycombs derive from the other Euclidean space groups.

[21] **Alternated-cubic prismatic tetracomb**

h[4,3,4]:00|01|[∞], P_4:<0001>[∞]
octet × apeirogon
Honeycells: tetrahedral prisms and octahedral prisms
Verf: bipyramid based on a cuboctahedron, edges of length 1; all other edges of length $\sqrt{2}$

[22] **Gyrated-alternated-cubic prismatic tetracomb**
h[4,3,4]:00|01|g[∞], P_4:<0001>g[∞], h[4,3,4]:00|01|[∞]g, P_4:<0001>[∞]g
gytoh × apeirogon
Honeycells: tetrahedral prisms and octahedral prisms
Verf: bipyramid based on a triangular gyrobicupola, edges of length 1; all other edges of length $\sqrt{2}$

[23] **Elongated-alternated-cubic prismatic tetracomb**
h[4,3,4]:00|01|e[∞], P_4:<0001>e[∞], h[4,3,4]:00|01|[∞]e, P_4:<0001>[∞]e
etoh × apeirogon
Honeycells: tetrahedral prisms, octahedral prisms, and triangular-square duoprisms
Verf: bipyramid based on a triangular cupola, edges of length 1, joined at base hexagon to hexagonal pyramid, lateral edges of length $\sqrt{2}$; all other edges of length $\sqrt{2}$

[24] **Gyrated-elongated-alternated-cubic prismatic tetracomb**
h[4,3,4]:00|01|ge[∞], P_4:<0001>ge[∞], h[4,3,4]:00|01|g[∞]e, P_4:<0001>g[∞]e, h[4,3,4]:00|01|[∞]ge, P_4:<0001>[∞]ge
gyetoh × apeirogon
Honeycells: tetrahedral prisms, octahedral prisms, and triangular-square duoprisms
Verf: same as [23]

[25] **Truncated-alternated-cubic prismatic tetracomb**
h[4,3,4]:01|01|[∞], P_4:<0111>[∞]
tatoh × apeirogon
Honeycells: cuboctahedral prisms, truncated-tetrahedral prisms, and truncated-octahedral prisms
Verf: bipyramid based on a rectangular pyramid, base edges of lengths 1, $\sqrt{2}$, 1, $\sqrt{2}$, lateral edges of lengths $\sqrt{3}$; all other edges of length $\sqrt{2}$

[26] **Runcinated-alternated-cubic prismatic tetracomb**
h[4,3,4]:10|01|[∞]
ratoh × apeirogon
Honeycells: tetrahedral prisms, tesseracts, and rhombicuboctahedral prisms
Verf: bipyramid based on a frustum of an equilateral-triangular pyramid, upper base with edges of length 1, lower base with edges of length $\sqrt{2}$, lateral

faces trapezoids with edges of length 1, $\sqrt{2}$, $\sqrt{2}$, $\sqrt{2}$; all other edges of length $\sqrt{2}$

[27] **Quarter-cubic prismatic tetracomb**

P_4:<0011>[∞]
batatoh × apeirogon
Honeycells: tetrahedral prisms and truncated-tetrahedral prisms
Verf: bipyramid based on an elongated equilateral-triangular antiprism, base edges of length 1, lateral edges of length $\sqrt{3}$; all other edges of length $\sqrt{2}$

[28] **Runcicantic-cubic prismatic tetracomb**

h[4,3,4]:11|01|[∞]
gratoh × apeirogon
Honeycells: truncated-tetrahedral prisms, truncated-cubic prisms, and truncated-cuboctahedral prisms
Verf: bipyramid based on a tetrahedron, two opposite edges of lengths 1, $\sqrt{2}$, four lateral edges of lengths $\sqrt{3}$, $\sqrt{3}$, $\sqrt{(2+\sqrt{2})}$, $\sqrt{(2+\sqrt{2})}$, ordered so that its unit edge belongs to two different isosceles triangles; all other edges of length $\sqrt{2}$

The honeycells of a uniform prismatic tetracomb are always uniform prisms based on uniform polyhedra. Sometimes the bases are themselves uniform prisms based on regular *p*-gons, which makes them square-*p*-gonal duoprisms. Although a tesseract is a regular polychoron, it is also a cubic prism and a square duoprism, so it would not be excluded from appearing as a honeycell of a prismatic or a duoprismatic tetracomb. The verf of a uniform prismatic tetracomb is always a bipyramid based on the verf of the underlying uniform honeycomb. Its lateral edges, being the verfs of squares, are always of length $\sqrt{2}$.

The honeycells of a uniform duoprismatic tetracomb are always uniform *duoprisms*, which are the Cartesian products of two regular polygons of the same edge length in absolutely perpendicular planes, concentric with the planes' point of intersection. If both polygons of a duopyramid are congruent, we name it after that polygon; otherwise we list both polygons in its name, with the polygon of fewer sides first if there is one. The verf of a duoprismatic tetracomb is always a duopyramid whose base polygons are the verfs of the two tilings whose Cartesian product is the tetracomb. A *duopyramid* is formed from a *p*-gon and a *q*-gon in absolutely perpendicular planes (more generally, from polygons in any two planes that intersect only in a point). Each pair of edges, one from either polygon, become the opposite edges of a tetrahedron that is a cell of the duopyramid; so such a duopyramid has *pq* tetrahedral cells altogether. If the polygons are regular, have the same edge length, and are centered on the point of intersection of the two absolutely perpendicular planes, then the duopyramid is the dual of the uniform duoprism that is the Cartesian product of the same polygons. If (as is the case with the verfs of all duoprismatic tetracombs) the two polygons (regular or not) are vertex-inscribed in unit circles centered on the intersection point of their planes, the distance between any vertex of one polygon and any vertex of the other polygon is always $\sqrt{2}$. If both polygons of a duopyramid are the same, we name it after that polygon; otherwise we list both polygons in its name.

The first eleven tetracombs listed above are duoprismatic, being the Cartesian products of the eleven uniform tilings with the square tiling. Now we list the remaining 55 duoprismatic tetracombs. Their honeycells are various combinations of duoprisms. The prefixes “tomo-” and “simo-,” suggested by Norman Johnson and denoting “truncated” and “snub,” respectively, are used to shorten awkward names. Their designations commute across the Cartesian product, so there is no need to list both versions of each designation:

[29] **Triangular duoprismatic tetracomb**

[3,6]:100[3,6]:100, P_3:<001>P_3:<001>, [3,6]:100P_3:<001>
trat × trat
Honeycells: triangular duoprisms
Verf: hexagonal duopyramid, both hexagons with edges of length 1; all other edges of length $\sqrt{2}$

This tetracomb may be elongated by inserting laminae of triangular-square duoprisms in between the laminae of triangular duoprisms. The resulting tetracomb is [31], trat × etrat. That tetracomb may itself be elongated in a second direction by inserting laminae of triangular-square duoprisms and tesseracts in between the laminae of triangular duoprisms and triangular-square duoprisms. The resulting tetracomb is [48], etrat × etrat.

[30] **Triangular-hexagonal duoprismatic tetracomb**

[3,6]:100[3,6]:001, P_3:<001>[3,6]:001, [3,6]:100[3,6]:110, P_3:<001>[3,6]:110, [3,6]:100P_3:111, P_3:<001>P_3:111
trat × hexat
Honeycells: triangular-hexagonal duoprisms
Verf: triangular-hexagonal duopyramid, triangle edges of length $\sqrt{3}$, hexagon edges of length 1; all other edges of length $\sqrt{2}$

This tetracomb may be elongated by inserting laminae of square-hexagonal duoprisms in between the laminae of triangular-hexagonal duoprisms. The resulting tetracomb is [40], hexat × etrat.

[31] **Elongated triangular duoprismatic tetracomb**

[3,6]:100[3,6]:100e, P_3:<001>P_3:<001>e, [3,6]:100P_3:<001>e
trat × etrat
Honeycells: triangular duoprisms and triangular-square duoprisms
Verf: irregular-pentagonal-hexagonal duopyramid, hexagon edges of length 1, pentagon edges of lengths 1, 1, 1, $\sqrt{2}$, $\sqrt{2}$; all other edges of length $\sqrt{2}$

This is an elongation of the triangular duoprismatic tetracomb [29] with triangular-square duoprisms alternated between laminae of triangular duoprisms. It may be further elongated in a different direction to yield [48], etrat × etrat.

[32] **Triangular-trihexagonal duoprismatic tetracomb**

[3,6]:100[3,6]:010, P_3:<001>[3,6]:010, [3,6]:100P_3:<011>, P_3:<001>P_3:<011>
trat × that
Honeycells: triangular duoprisms and triangular-hexagonal duoprisms
Verf: rectangular-hexagonal duopyramid, rectangle edges of lengths 1, $\sqrt{3}$, 1, $\sqrt{3}$, hexagon edges of length 1; all other edges of length $\sqrt{2}$

This tetracomb may be elongated by inserting laminae of triangular-square and square-hexagonal duoprisms in between the laminae of triangular and triangular-hexagonal duoprisms. The resulting tetracomb is [49], etrat × that.

[33] **Triangular-tomosquare duoprismatic tetracomb**

[3,6]:100[4,4]:(011), P_3:<001>[4,4]:(011), [3,6]:100[4,4]:111, P_3:<001>[4,4]:111
trat × tosquat
Honeycells: triangular-square duoprisms and triangular-octagonal duoprisms
Verf: isosceles-triangular-hexagonal duopyramid, triangle edges of lengths $\sqrt{2}$, $\sqrt{2+\sqrt{2}}$, $\sqrt{2+\sqrt{2}}$, hexagon edges of length 1; all other edges of length $\sqrt{2}$

This tetracomb may be elongated by inserting laminae of tesseracts and square-octagonal duoprisms in between the laminae of triangular-square and triangular-octagonal duoprisms. The resulting tetracomb is [50], etrat × tosquat.

[34] **Triangular-tomohexagonal duoprismatic tetracomb**

[3,6]:100[3,6]:011, P_3:<001>[3,6]:011
trat × toxat
Honeycells: triangular duoprisms and triangular-dodecagonal duoprisms
Verf: isosceles-triangular-hexagonal duopyramid, triangle edges of lengths 1, $\sqrt{2+\sqrt{3}}$, $\sqrt{2+\sqrt{3}}$, hexagon edges of length 1; all other edges of length $\sqrt{2}$

This tetracomb may be elongated by inserting laminae of triangular-square duoprisms and square-dodecagonal duoprisms in between the laminae of triangular and triangular-dodecagonal duoprisms. The resulting tetracomb is [51], etrat × toxat.

[35] **Triangular-rhombitrihexagonal tetracomb**

[3,6]:100[3,6]:101, P_3:<001>[3,6]:101
trat × rothat
Honeycells: triangular duoprisms, triangular-square duoprisms, and triangular-hexagonal duoprisms
Verf: trapezoidal-hexagonal duopyramid, trapezoid edges of lengths 1, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{2}$, hexagon edges of length 1; all other edges of length $\sqrt{2}$

This tetracomb may be elongated by inserting laminae of triangular-square duoprisms, tesseracts, and square-hexagonal duoprisms in between the laminae of triangular duoprisms, triangular-square duoprisms, and triangular-hexagonal duoprisms. The resulting tetracomb is [52], etrat × rothat.

[36] **Triangular-omnitruncated-trihexagonal tetracomb**

[3,6]:100[3,6]:101, P_3:<001>[3,6]:101
trat × othat
Honeycells: triangular-square duoprisms, triangular-hexagonal duoprisms, and triangular-dodecagonal duoprisms
Verf: scalene-triangular-hexagonal duopyramid, scalene triangle edges of lengths $\sqrt{2}$, $\sqrt{3}$, $\sqrt{(2+\sqrt{3})}$, hexagon edges of length 1; all other edges of length $\sqrt{2}$

This tetracomb may be elongated by inserting laminae of tesseracts, square-hexagonal duoprisms, and square-dodecagonal duoprisms in between the laminae of triangular-square duoprisms, triangular-hexagonal duoprisms, and triangular-dodecagonal duoprisms. The resulting tetracomb is [53], etrat × othat.

[37] **Triangular-simosquare tetracomb**

[3,6]:100[4,4]:s, P_3:<001>[4,4]:s
trat × snasquat
Honeycells: triangular duoprisms and triangular-square duoprisms
Verf: irregular-pentagonal-hexagonal duopyramid, pentagon edges of lengths 1, 1, $\sqrt{2}$, 1, $\sqrt{2}$, hexagon edges of length 1; all other edges of length $\sqrt{2}$

This tetracomb may be elongated by inserting laminae of tesseracts and triangular-square duoprisms in between the laminae of triangular duoprisms and triangular-square duoprisms. The resulting tetracomb is [54], etrat × snasquat.

[38] **Triangular-simotrihexagonal tetracomb**

[3,6]:100[3,6]:s, P_3:<001>[3,6]:s
trat × snathat
Honeycells: triangular duoprisms and triangular-hexagonal duoprisms
Verf: irregular-pentagonal-hexagonal duopyramid, pentagon edges of lengths 1, 1, 1, 1, $\sqrt{3}$, hexagon edges of length 1; all other edges of length $\sqrt{2}$

Although a snathat is chiral, this tetracomb is not. The tetracomb that is the Cartesian product of the triangular tiling with a left-handed snathat may simply be rotated by a half-turn to become the tetracomb that is the Cartesian product of the triangular tiling with a right-handed snathat. This tetracomb may be elongated by inserting laminae of triangular-square duoprisms and square-hexagonal duoprisms in between the laminae of triangular duoprisms and triangular-hexagonal duoprisms. The resulting tetracomb is [55], etrat × snathat.

[39] **Hexagonal duoprismatic tetracomb**

[3,6]:001[3,6]:001, [3,6]:110[3,6]:001, P_3:111[3,6]:001, [3,6]:110[3,6]:110, P_3:111[3,6]:110, P_3:111P_3:111
hexat × hexat

Honeycells: hexagonal duoprisms
Verf: triangular duopyramid, both triangles with edges of length $\sqrt{3}$; all other edges of length $\sqrt{2}$

[40] **Elongated triangular-hexagonal duoprismatic tetracomb**
[3,6]:100e[3,6]:001, P_3:<001>e[3,6]:001, [3,6]:100e[3,6]:110, P_3:<001>e[3,6]:110, [3,6]:100eP_3:111, P_3:<001>eP_3:111, [3,6]:100[3,6]:001e, P_3:<001>[3,6]:001e, [3,6]:100[3,6]:110e, P_3:<001>[3,6]:110e, [3,6]:100P_3:111e, P_3:<001>P_3:111e
hexat × etrat
Honeycells: triangular-hexagonal duoprisms and square-hexagonal duoprisms
Verf: triangular-irregular-pentagonal duopyramid, triangle edges of length $\sqrt{3}$, pentagon edges of lengths 1, 1, 1, $\sqrt{2}$, $\sqrt{2}$; all other edges of length $\sqrt{2}$

This tetracomb is an elongation of [30], trat × hexat.

[41] **Hexagonal-trihexagonal duoprismatic tetracomb**
[3,6]:001[3,6]:010, [3,6]:110[3,6]:010, P_3:111[3,6]:010, [3,6]:001P_3:<011>, [3,6]:110P_3:<011>, P_3:111P_3:<011>
hexat × that
Honeycells: triangular-hexagonal duoprisms and hexagonal duoprisms
Verf: triangular-rectangular duopyramid, triangle edges of length $\sqrt{3}$, rectangle edges of lengths 1, $\sqrt{3}$, 1, $\sqrt{3}$; all other edges of length $\sqrt{2}$

[42] **Hexagonal-tomosquare duoprismatic tetracomb**
[3,6]:001[4,4]:(011), [3,6]:110[4,4]:(011), P_3:111[4,4]:(011), [3,6]:001[4,4]:111, [3,6]:110[4,4]:111, P_3:111[4,4]:111
hexat × tosquat
Honeycells: square-hexagonal duoprisms and hexagonal-octagonal duoprisms
Verf: triangular-isosceles-triangular duopyramid, triangle edges of length $\sqrt{3}$, isosceles triangle edges of lengths $\sqrt{2}$, $\sqrt{(2+\sqrt{2})}$, $\sqrt{(2+\sqrt{2})}$; all other edges of length $\sqrt{2}$

[43] **Hexagonal-tomohexagonal duoprismatic tetracomb**
[3,6]:001[3,6]:011, [3,6]:110[3,6]:011, P_3:111[3,6]:011
hexat × toxat
Honeycells: triangular-hexagonal duoprisms and hexagonal-dodecagonal duoprisms
Verf: triangular-isosceles-triangular duopyramid, triangle edges of length $\sqrt{3}$, isosceles triangle edges of lengths 1, $\sqrt{(2+\sqrt{3})}$, $\sqrt{(2+\sqrt{3})}$; all other edges of length $\sqrt{2}$

[44] **Hexagonal-rhombihexagonal duoprismatic tetracomb**
[3,6]:001[3,6]:101, [3,6]:110[3,6]:101, P_3:111[3,6]:101
hexat × rothat

Honeycells: triangular-hexagonal duoprisms, square-hexagonal duoprisms, and hexagonal duoprisms
Verf: triangular-trapezoidal duopyramid, triangle edges of length $\sqrt{3}$, trapezoid edges of lengths 1, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{2}$; all other edges of length $\sqrt{2}$

[45] **Hexagonal-omnitruncated-trihexagonal duoprismatic tetracomb**
[3,6]:001[3,6]:111, [3,6]:110[3,6]:111, P_3:111[3,6]:111
hexat × othat
Honeycells: square-hexagonal duoprisms, hexagonal duoprisms, and hexagonal-dodecagonal duoprisms
Verf: triangular-scalene-triangular duopyramid, triangle edges of length $\sqrt{3}$, scalene-triangle edges of lengths $\sqrt{2}$, $\sqrt{3}$, $\sqrt{(2+\sqrt{3})}$; all other edges of length $\sqrt{2}$

[46] **Hexagonal-simosquare duoprismatic tetracomb**
[3,6]:001[4,4]:s, [3,6]:110[4,4]:s, P_3:111[4,4]:s
hexat × snasquat
Honeycells: triangular-hexagonal duoprisms and square-hexagonal duoprisms
Verf: triangular-irregular-pentagonal duopyramid, triangle edges of length $\sqrt{3}$, pentagon edges of lengths 1, 1, $\sqrt{2}$, 1, $\sqrt{2}$; all other edges of length $\sqrt{2}$

[47] **Hexagonal-simotrihexagonal duoprismatic tetracomb**
[3,6]:001[3,6]:s, [3,6]:110[3,6]:s, P_3:111[3,6]:s
hexat × snathat
Honeycells: triangular-hexagonal duoprisms and hexagonal duoprisms
Verf: triangular-irregular-pentagonal duopyramid, triangle edges of length $\sqrt{3}$, pentagon edges of lengths 1, 1, 1, 1, $\sqrt{3}$; all other edges of length $\sqrt{2}$

Although a snathat is chiral, this tetracomb is not. The tetracomb that is the Cartesian product of the hexagonal tiling with a left-handed snathat may simply be rotated a half-turn to become the tetracomb that is the Cartesian product of the hexagonal tiling with a right-handed snathat.

[48] **Bielongated triangular duoprismatic tetracomb**
[3,6]:100e[3,6]:100e, [3,6]:100eP_3:<001>e, [3,6]:100[3,6]:100ee, [3,6]:100P_3:<001>ee, P_3:<001>eP_3:<001>e, P_3:<001>P_3:<001>ee
etrat × etrat
Honeycells: triangular duoprisms, triangular-square duoprisms, and tesseracts
Verf: irregular-pentagonal duopyramid, both pentagons with edges of lengths 1, 1, 1, $\sqrt{2}$, $\sqrt{2}$; all other edges of length $\sqrt{2}$

This tetracomb results from elongating [31], trat × etrat, or from elongating [29], trat × trat, in two different directions.

[49] **Elongated triangular-trihexagonal duoprismatic tetracomb**
[3,6]:100e[3,6]:010, P_3:<001>e[3,6]:010, [3,6]:100eP_3:<011>,

P_3:<001>eP_3:<011> [3,6]:100[3,6]:010e, P_3:<001>[3,6]:010e, [3,6]:100P_3:<011>e, P_3:<001>P_3:<011>e

etrat × that

Honeycells: triangular duoprisms, triangular-square duoprisms, triangular-hexagonal duoprisms, and square-hexagonal duoprisms

Verf: rectangular-irregular-pentagonal duopyramid, rectangle edges of lengths 1, $\sqrt{3}$, 1, $\sqrt{3}$, pentagon edges of lengths 1, 1, 1, $\sqrt{2}$, $\sqrt{2}$; all other edges of length $\sqrt{2}$

This tetracomb results from elongating [32], trat × that.

[50] **Elongated triangular-tomosquare duoprismatic tetracomb**

[3,6]:100e[4,4]:(011), P_3:<001>e[4,4]:(011), [3,6]:100e[4,4]:111, P_3:<001>e[4,4]:111, [3,6]:100[4,4]:(011)e, P_3:<001>[4,4]:(011)e, [3,6]:100[4,4]:111e, P_3:<001>[4,4]:111e

etrat × tosquat

Honeycells: triangular-square duoprisms, tesseracts, triangular-octagonal duoprisms, and square-octagonal duoprisms

Verf: isosceles-triangular-irregular-pentagonal duopyramid, isosceles triangle edges of length $\sqrt{2}$, $\sqrt{(2+\sqrt{2})}$, $\sqrt{(2+\sqrt{2})}$, pentagon edges of lengths 1, 1, 1, $\sqrt{2}$, $\sqrt{2}$; all other edges of length $\sqrt{2}$

This tetracomb results from elongating [33], trat × tosquat.

[51] **Elongated triangular-tomohexagonal duoprismatic tetracomb**

[3,6]:100e[3,6]:011, P_3:<001>e[3,6]:011, [3,6]:100[3,6]:011e, P_3:<001>[3,6]:011e

etrat × toxat

Honeycells: triangular duoprisms, triangular-square duoprisms, triangular-dodecagonal duoprisms, and square-dodecagonal duoprisms

Verf: isosceles-triangular-irregular-pentagonal duopyramid, isosceles triangle edges of length 1, $\sqrt{(2+\sqrt{3})}$, $\sqrt{(2+\sqrt{3})}$, pentagon edges of lengths 1, 1, 1, $\sqrt{2}$, $\sqrt{2}$; all other edges of length $\sqrt{2}$

This tetracomb results from elongating [34], trat × toxat.

[52] **Elongated triangular-rhombitrihexagonal tetracomb**

[3,6]:100e[3,6]:101, P_3:<001>e[3,6]:101, [3,6]:100[3,6]:101e, P_3:<001>[3,6]:101e

etrat × rothat

Honeycells: triangular duoprisms, triangular-square duoprisms, triangular-hexagonal duoprisms, tesseracts, and square-hexagonal duoprisms

Verf: trapezoidal-irregular-pentagonal duopyramid, trapezoid edges of length 1, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{2}$, pentagon edges of lengths 1, 1, 1, $\sqrt{2}$, $\sqrt{2}$; all other edges of length $\sqrt{2}$

This tetracomb results from elongating [35], trat × rothat.

[53] **Elongated triangular-omnitruncated-trihexagonal tetracomb**

[3,6]:100e[3,6]:101, P_3:<001>e[3,6]:101, [3,6]:100[3,6]:101e, P_3:<001>[3,6]:101e

etrat × othat

Honeycells: triangular-square duoprisms, triangular-hexagonal duoprisms, triangular-dodecagonal duoprisms, tesseracts, square-hexagonal duoprisms, and square-dodecagonal duoprisms

Verf: scalene-triangular-irregular pentagonal duopyramid, triangle edges of lengths $\sqrt{2}$, $\sqrt{3}$, $\sqrt{(2+\sqrt{3})}$, pentagon edges of lengths 1, 1, 1, $\sqrt{2}$, $\sqrt{2}$; all other edges of length $\sqrt{2}$

This tetracomb results from elongating [36], trat × othat.

[54] **Elongated triangular-simosquare tetracomb**

[3,6]:100e[4,4]:s, P_3:<001>e[4,4]:s, [3,6]:100[4,4]:se, P_3:<001>[4,4]:se

etrat × snasquat

Honeycells: triangular duoprisms, triangular-square duoprisms, and tesseracts

Verf: irregular-bipentagonal duopyramid, first pentagon edges of lengths 1, 1, $\sqrt{2}$, 1, $\sqrt{2}$, second pentagon edges of lengths 1, 1, 1, $\sqrt{2}$, $\sqrt{2}$; all other edges of length $\sqrt{2}$

This tetracomb results from elongating [37], trat × snasquat.

[55] **Elongated triangular-simotrihexagonal tetracomb**

[3,6]:100e[3,6]:s, P_3:<001>e[3,6]:s, [3,6]:100[3,6]:se, P_3:<001>[3,6]:se

etrat × snathat

Honeycells: triangular duoprisms, triangular-square duoprisms, triangular-hexagonal duoprisms, and square-hexagonal duoprisms

Verf: irregular-bipentagonal duopyramid, first pentagon edges of lengths 1, 1, 1, 1, $\sqrt{3}$, second pentagon edges of lengths 1, 1, 1, $\sqrt{2}$, $\sqrt{2}$; all other edges of length $\sqrt{2}$

Although a snathat is chiral, this tetracomb is not. The tetracomb that is the Cartesian product of the elongated triangular tiling with a left-handed snathat may simply be rotated a half-turn to become the tetracomb that is the Cartesian product of the elongated triangular tiling with a right-handed snathat. This tetracomb results from elongating [38], trat × snathat.

[56] **Trihexagonal duoprismatic tetracomb**

[3,6]:010[3,6]:010, P_3:<011>[3,6]:010, P_3:<011>P_3:<011>

that × that

Honeycells: triangular duoprisms, triangular-hexagonal duoprisms, and hexagonal duoprisms

Verf: rectangular duopyramid, both rectangles with edges of lengths 1, $\sqrt{3}$, 1, $\sqrt{3}$; all other edges of length $\sqrt{2}$

Note that in this rectangular duopyramid the bases are two identical rectangles in absolutely perpendicular planes, concentric with the point of intersection of the planes.

[57] **Trihexagonal-tomosquare duoprismatic tetracomb**

[3,6]:010[4,4]:(011), P_3:<011>[4,4]:(011), [3,6]:010[4,4]:111, P_3:<011>[4,4]:111

that × tosquat
Honeycells: triangular-square duoprisms, triangular-octagonal duoprisms, square-hexagonal duoprisms, and hexagonal-octagonal duoprisms
Verf: isosceles-triangular-rectangular duopyramid, triangle edges of lengths $\sqrt{2}$, $\sqrt{(2+\sqrt{2})}$, $\sqrt{(2+\sqrt{2})}$, rectangle edges of lengths 1, $\sqrt{3}$, 1, $\sqrt{3}$; all other edges of length $\sqrt{2}$

[58] **Trihexagonal-tomohexagonal duoprismatic tetracomb**
[3,6]:010[3,6]:011, P_3:<011>[3,6]:011
that × toxat
Honeycells: triangular duoprisms, triangular-hexagonal duoprisms, triangular-dodecagonal duoprisms, and hexagonal-dodecagonal duoprisms
Verf: isosceles-triangular-rectangular duopyramid, triangle edges of lengths 1, $\sqrt{(2+\sqrt{3})}$, $\sqrt{(2+\sqrt{3})}$, rectangle edges of lengths 1, $\sqrt{3}$, 1, $\sqrt{3}$; all other edges of length $\sqrt{2}$

[59] **Trihexagonal-rhombitrihexagonal duoprismatic tetracomb**
[3,6]:010[3,6]:101, P_3:<011>[3,6]:101
that × rothat
Honeycells: triangular duoprisms, triangular-square duoprisms, triangular-hexagonal duoprisms, square-hexagonal duoprisms, and hexagonal duoprisms
Verf: rectangular-trapezoidal duopyramid, trapezoid edges of lengths 1, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{2}$, rectangle edges of lengths 1, $\sqrt{3}$, 1, $\sqrt{3}$; all other edges of length $\sqrt{2}$

[60] **Trihexagonal-omnitruncated-trihexagonal duoprismatic tetracomb**
[3,6]:010[3,6]:111, P_3:<011>[3,6]:111
that × othat
Honeycells: triangular-square duoprisms, triangular-hexagonal duoprisms, triangular-dodecagonal duoprisms, square-hexagonal duoprisms, hexagonal duoprisms, and hexagonal-dodecagonal duoprisms
Verf: scalene-triangular-rectangular duopyramid, triangle edges of lengths $\sqrt{2}$, $\sqrt{3}$, $\sqrt{(2+\sqrt{3})}$, rectangle edges of lengths 1, $\sqrt{3}$, 1, $\sqrt{3}$; all other edges of length $\sqrt{2}$

[61] **Trihexagonal-simosquare duoprismatic tetracomb**
[3,6]:010[4,4]:s, P_3:<011>[4,4]:s
that × snasquat
Honeycells: triangular duoprisms, triangular-square duoprisms, triangular-hexagonal duoprisms, and square-hexagonal duoprisms
Verf: rectangular-irregular-pentagonal duopyramid, pentagon edges of lengths 1, 1, $\sqrt{2}$, 1, $\sqrt{2}$, rectangle edges of lengths 1, $\sqrt{3}$, 1, $\sqrt{3}$; all other edges of length $\sqrt{2}$

[62] **Trihexagonal-simotrihexagonal duoprismatic tetracomb**

[3,6]:010[3,6]:s, P_3:<011>[3,6]:s
that × snathat
Honeycells: triangular duoprisms, triangular-hexagonal duoprisms, and hexagonal duoprisms
Verf: rectangular-irregular-pentagonal duopyramid, pentagon edges of lengths 1, 1, 1, 1, √3, rectangle edges of lengths 1, √3, 1, √3; all other edges of length √2

Although a snathat is chiral, this tetracomb is not. The tetracomb that is the Cartesian product of the trihexagonal tiling with a left-handed snathat may simply be rotated by a half-turn to become the tetracomb that is the Cartesian product of the trihexagonal tiling with a right-handed snathat.

[63] **Tomosquare duoprismatic tetracomb**

[4,4]:(011)[4,4]:(011), [4,4]:111[4,4]:(011), [4,4]:111[4,4]:111
tosquat × tosquat
Honeycells: tesseracts, square-octagonal duoprisms, and octagonal duoprisms
Verf: Isosceles-triangular duopyramid, both triangles with edges of lengths √2, √(2+√2), √(2+√2); all other edges of length √2

[64] **Tomosquare-tomohexagonal duoprismatic tetracomb**

[4,4]:(011)[3,6]:011, [4,4]:111[3,6]:011
tosquat × toxat
Honeycells: triangular-square duoprisms, triangular-octagonal duoprisms, square-dodecagonal duoprisms, and octagonal-dodecagonal duoprisms
Verf: Isosceles-bitriangular duopyramid, one triangle with edges of lengths 1, √(2+√3), √(2+√3), other triangle with edges of lengths √2, √(2+√2), √(2+√2); all other edges of length √2

[65] **Tomosquare-rhombitrihexagonal duoprismatic tetracomb**

[4,4]:(011)[3,6]:101, [4,4]:111[3,6]:101
tosquat × rothat
Honeycells: triangular-square duoprisms, triangular-octagonal duoprisms, tesseracts, square-hexagonal duoprisms, square-octagonal duoprisms, and hexagonal-octagonal duoprisms
Verf: Isosceles-triangular-trapezoidal duopyramid, trapezoid edges of lengths 1, √2, √3, √2, triangle edges of lengths √2, √(2+√2), √(2+√2); all other edges of length √2

[66] **Tomosquare-omnitruncated-trihexagonal duoprismatic tetracomb**

[4,4]:(011)[3,6]:111, [4,4]:111[3,6]:111
tosquat × othat
Honeycells: tesseracts, square-hexagonal duoprisms, square-octagonal duoprisms, square-dodecagonal duoprisms, hexagonal-octagonal

duoprisms, and octagonal-dodecagonal duoprisms
Verf: Scalene-triangular-isosceles-triangular duopryamid, scalene triangle edges of lengths $\sqrt{2}$, $\sqrt{3}$, $\sqrt{(2+\sqrt{3})}$, isosceles triangle edges of lengths $\sqrt{2}$, $\sqrt{(2+\sqrt{2})}$, $\sqrt{(2+\sqrt{2})}$; all other edges of length $\sqrt{2}$

[67] **Tomosquare-simosquare duoprismatic tetracomb**
[4,4]:(011)[4,4]:s, [4,4]:111[4,4]:s
tosquat × snasquat
Honeycells: triangular-square duoprisms, triangular-octagonal duoprisms, tesseracts, and square-octagonal duoprisms
Verf: Isosceles-triangular-irregular-pentagonal duopyramid, pentagon edges of lengths 1, 1, $\sqrt{2}$, 1, $\sqrt{2}$, triangle edges of lengths $\sqrt{2}$, $\sqrt{(2+\sqrt{2})}$, $\sqrt{(2+\sqrt{2})}$; all other edges of length $\sqrt{2}$

[68] **Tomosquare-simotrihexagonal duoprismatic tetracomb**
[4,4]:(011)[3,6]:s, [4,4]:111[3,6]:s
tosquat × snathat
Honeycells: triangular-square duoprisms, triangular-octagonal duoprisms, square-hexagonal duoprisms, and hexagonal-octagonal duoprisms
Verf: Isosceles-triangular-irregular-pentagonal duopyramid, pentagon edges of lengths 1, 1, 1, 1, $\sqrt{3}$, triangle edges of lengths $\sqrt{2}$, $\sqrt{(2+\sqrt{2})}$, $\sqrt{(2+\sqrt{2})}$; all other edges of length $\sqrt{2}$

Although a snathat is chiral, this tetracomb is not. The tetracomb that is the Cartesian product of the truncated square tiling with a left-handed snathat may simply be rotated by a half-turn to become the tetracomb that is the Cartesian product of the truncated square tiling with a right-handed snathat.

[69] **Tomohexagonal duoprismatic tetracomb**
[3,6]:011[3,6]:011
toxat × toxat
Honeycells: triangular duoprisms, triangular-dodecagonal duoprisms, and dodecagonal duoprisms
Verf: Isosceles-triangular duopyramid, both triangles edges of lengths 1, $\sqrt{(2+\sqrt{3})}$, $\sqrt{(2+\sqrt{3})}$; all other edges of length $\sqrt{2}$

[70] **Tomohexagonal-rhombirihexagonal duoprismatic tetracomb**
[3,6]:011[3,6]:101
toxat × rothat
Honeycells: triangular duoprisms, triangular-square duoprisms, triangular-hexagonal duoprisms, triangular-dodecagonal duoprisms, square-dodecagonal duoprisms, and hexagonal-dodecagonal duoprisms
Verf: Isosceles-triangular-trapezoidal duopyramid, trapezoid edges of lengths 1, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{2}$, triangle edges of lengths 1, $\sqrt{(2+\sqrt{3})}$, $\sqrt{(2+\sqrt{3})}$; all other edges of length $\sqrt{2}$

[71] **Tomohexagonal-omnitruncated-trihexagonal duoprismatic tetracomb**

[3,6]:011[3,6]:111

toxat × othat

Honeycells: triangular-square duoprisms, triangular-hexagonal duoprisms, triangular-dodecagonal duoprisms, square-dodecagonal duoprisms, hexagonal-dodecagonal duoprisms, and dodecagonal duoprisms

Verf: Scalene-triangular-isosceles-triangular duopyramid, scalene triangle edges of lengths $\sqrt{2}$, $\sqrt{3}$, $\sqrt{(2+\sqrt{3})}$, isosceles triangle edges of lengths 1, $\sqrt{(2+\sqrt{3})}$, $\sqrt{(2+\sqrt{3})}$; all other edges of length $\sqrt{2}$

[72] **Tomohexagonal-simosquare duoprismatic tetracomb**

[3,6]:011[4,4]:s

toxat × snasquat

Honeycells: triangular duoprisms, triangular-square duoprisms, triangular-dodecagonal duoprisms, and square-dodecagonal duoprisms

Verf: Isosceles-triangular-irregular-pentagonal duopyramid, pentagon edges of lengths 1, 1, $\sqrt{2}$, 1, $\sqrt{2}$, triangle edges of lengths 1, $\sqrt{(2+\sqrt{3})}$, $\sqrt{(2+\sqrt{3})}$; all other edges of length $\sqrt{2}$

[73] **Tomohexagonal-simotrihexagonal duoprismatic tetracomb**

[3,6]:011[3,6]:s

toxat × snathat

Honeycells: triangular duoprisms, triangular-hexagonal duoprisms, triangular-dodecagonal duoprisms, and hexagonal-dodecagonal duoprisms

Verf: Isosceles-triangular-irregular-pentagonal duopyramid, pentagon edges of lengths 1, 1, 1, 1, $\sqrt{3}$, triangle edges of lengths 1, $\sqrt{(2+\sqrt{3})}$, $\sqrt{(2+\sqrt{3})}$; all other edges of length $\sqrt{2}$

Although a snathat is chiral, this tetracomb is not. The tetracomb that is the Cartesian product of the truncated hexagonal tiling with a left-handed snathat may simply be rotated by a half-turn to become the tetracomb that is the Cartesian product of the truncate hexagonal tiling with a right-handed snathat.

[74] **Rhombitrihexagonal duoprismatic tetracomb**

[3,6]:101[3,6]:101

rothat × rothat

Honeycells: triangular duoprisms, triangular-square duoprisms, triangular-hexagonal duoprisms, tesseracts, square-hexagonal duoprisms, and hexagonal duoprisms

Verf: trapezoidal duopyramid, both trapezoids with edges of lengths 1, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{2}$; all other edges of length $\sqrt{2}$

[75] **Rhombitrihexagonal-omnitruncated-trihexagonal duoprismatic tetracomb**

[3,6]:101[3,6]:111

rothat × othat

Honeycells: triangular-square duoprisms, triangular-hexagonal duoprisms, triangular-dodecagonal duoprisms, tesseracts, square-hexagonal duoprisms, square-dodecagonal duoprisms, hexagonal duoprisms, and hexagonal-dodecagonal duoprisms

Verf: scalene-triangular-trapezoidal duopyramid, triangle edges of lengths $\sqrt{2}$, $\sqrt{3}$, $\sqrt{(2+\sqrt{3})}$, trapezoid edges of lengths 1, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{2}$; all other edges of length $\sqrt{2}$

[76] **Rhombitrihexagonal-simosquare duoprismatic tetracomb**

[3,6]:101[4,4]:s

rothat × snasquat

Honeycells: triangular duoprisms, triangular-square duoprisms, triangular-hexagonal duoprisms, tesseracts, and square-hexagonal duoprisms

Verf: trapezoidal-irregular-pentagonal duopyramid, pentagon edges of lengths 1, 1, $\sqrt{2}$, 1, $\sqrt{2}$, trapezoid edges of lengths 1, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{2}$; all other edges of length $\sqrt{2}$

[77] **Rhombitrihexagonal-simotrihexagonal duoprismatic tetracomb**

[3,6]:101[3,6]:s

rothat × snathat

Honeycells: triangular duoprisms, triangular-square duoprisms, triangular-hexagonal duoprisms, square-hexagonal duoprisms, and hexagonal duoprisms

Verf: trapezoidal-irregular-pentagonal duopyramid, pentagon edges of lengths 1, 1, 1, 1, $\sqrt{3}$, trapezoid edges of lengths 1, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{2}$; all other edges of length $\sqrt{2}$

Although a snathat is chiral, this tetracomb is not. The tetracomb that is the Cartesian product of the rhombitrihexagonal tiling with a left-handed snathat may simply be rotated by a half-turn to become the tetracomb that is the Cartesian product of the rhombitrihexagonal tiling with a right-handed snathat.

[78] **Omnitruncated-trihexagonal duoprismatic tetracomb**

[3,6]:111[3,6]:111

othat × othat

Honeycells: tesseracts, square-hexagonal duoprisms, square-dodecagonal duoprisms, hexagonal duoprisms, hexagonal-dodecagonal duoprisms, and dodecagonal duoprisms

Verf: scalene-triangular duopyramid, both triangles with edges of lengths $\sqrt{2}$, $\sqrt{3}$, $\sqrt{(2+\sqrt{3})}$; all other edges of length $\sqrt{2}$

Vertex figure is chiral, but both *dextro* and *laevo* versions occur equally throughout the tetracomb. *Dextro* version is duopyramid in which both triangles have the same handedness, *laevo* version is duopyramid in which the triangles have opposite handedness.

[79] **Omnitruncated-simosquare duoprismatic tetracomb**

[3,6]:111[4,4]:s

othat × snasquat

Honeycells: triangular-square duoprisms, triangular-hexagonal duoprisms, triangular-dodecagonal duoprisms, tesseracts, square-hexagonal duoprisms, and square-dodecagonal duoprisms

Verf: scalene-triangular-irregular-pentagonal duopyramid, triangle edges of lengths $\sqrt{2}$, $\sqrt{3}$, $\sqrt{(2+\sqrt{3})}$, pentagon edges of lengths 1, 1, $\sqrt{2}$, 1, $\sqrt{2}$; all other edges of length $\sqrt{2}$

[80] **Omnitruncated-simotrihexagonal duoprismatic tetracomb**

[3,6]:111[3,6]:s

othat × snathat

Honeycells: triangular-square duoprisms, triangular-hexagonal duoprisms, triangular-dodecagonal duoprisms, square-hexagonal duoprisms, hexagonal duoprisms, and hexagonal-dodecagonal duoprisms

Verf: scalene-triangular-irregular-pentagonal duopyramid, triangle edges of lengths $\sqrt{2}$, $\sqrt{3}$, $\sqrt{(2+\sqrt{3})}$, pentagon edges of lengths 1, 1, 1, 1, $\sqrt{3}$; all other edges of length $\sqrt{2}$

Although a snathat is chiral, this tetracomb is not. The tetracomb that is the Cartesian product of the omnitruncated trihexagonal tiling with a left-handed snathat may simply be rotated by a half-turn to become the tetracomb that is the Cartesian product of the omnitruncated trihexagonal tiling with a right-handed snathat.

[81] **Simosquare duoprismatic tetracomb**

[4,4]:s[4,4]:s

snasquat × snasquat

Honeycells: triangular duoprisms, triangular-square duoprisms, and tesseracts

Verf: irregular-pentagonal duopyramid, both pentagons with edges of lengths 1, 1, $\sqrt{2}$, 1, $\sqrt{2}$; all other edges of length $\sqrt{2}$

[82] **Simosquare-simotrihexagonal duoprismatic tetracomb**

[4,4]:s[3,6]:s

snasquat × snathat

Honeycells: triangular duoprisms, triangular-square duoprisms, triangular-hexagonal duoprisms, and square-hexagonal duoprisms

Verf: irregular-bipentagonal duopyramid, first pentagon edges of lengths 1, 1, 1, 1, $\sqrt{3}$, second pentagon edges of lengths 1, 1, $\sqrt{2}$, 1, $\sqrt{2}$; all other edges of length $\sqrt{2}$

Although a snathat is chiral, this tetracomb is not. The tetracomb that is the Cartesian product of the snub square tiling with a left-handed snathat may simply be rotated by a half-turn to become the tetracomb that is the Cartesian product of the snub square tiling with a right-handed snathat.

[83] **Simotrihexagonal duoprismatic tetracomb**

[3,6]:s[3,6]:s

snathat × snathat

Honeycells: triangular duoprisms, triangular-hexagonal duoprisms, and hexagonal duoprisms

Verf: irregular-pentagonal duopyramid, pentagon edges of lengths 1, 1, 1, 1, $\sqrt{3}$; all other edges of length $\sqrt{2}$

This tetracomb is chiral, the two mirror images being (1) the Cartesian product of two snathats of the same handedness and (2) the Cartesian product of two snathats of opposite handedness. The notations for the distinct chiral versions would be [3,6]:sd[3,6]:sd and [3,6]:sd[3,6]:sl; [3,6]:sl[3,6]:sl has the same chirality as [3,6]:sd[3,6]:sd.

Two additional duoprismatic tetracombs exist whose honeycells are the same as those of other tetracombs but occur in different arrangements. These tetracombs have symmetry groups that cannot be expressed as ordinary Cartesian products of lower-dimensional tessellations.

[84] **Bigyrated triangular-square duoprismatic tetracomb**

[3,6]:100[4,4]:(001)gg, P_3:<001>[4,4]:(001)gg, [3,6]:100[4,4]:010gg, P_3:<001>[4,4]:010gg, [3,6]:100[4,4]:101gg, P_3:<001>[4,4]:101gg, [3,6]:100[∞][∞]gg, P_3:<001>[∞][∞]gg**,** [3,6]:010[∞]g[∞]g, P_3:<011>[∞]g[∞]g, [3,6]:010[∞]g[∞]g, P_3:<011>[∞]g[∞]g

Honeycells: triangular-square duoprisms

Verf: same as [12]

In this tetracomb, the triangular-square duoprisms form a “hadron” gyration, in which a pattern of three consecutive gyrated laminae of duoprisms is repeated endlessly (rather than two alternately, as in [12]). In each gyrated lamina, the duoprisms are rotated 120° relative to the laminae above and below about an axis plane defined by the center of a lateral square and a long diagonal of the opposite cubic cell. Each lamina comprises all the duoprisms, half pointing up and half pointing down, that are sandwiched between two consecutive realms of cubic honeycombs.

[85] **Elongated bigyrated triangular-square duoprismatic tetracomb**

[3,6]:100[4,4]:(001)gge, P_3:<001>[4,4]:(001)gge, [3,6]:100[4,4]:010gge, P_3:<001>[4,4]:010gge, [3,6]:100[4,4]:101gge, P_3:<001>[4,4]:101gge, [3,6]:100[∞][∞]gge, P_3:<001>[∞][∞]gge**,** [3,6]:010[∞]g[∞]ge, P_3:<011>[∞]g[∞]ge, [3,6]:010[∞]ge[∞]g, P_3:<011>[∞]ge[∞]g, [3,6]:100e[4,4]:(001)gg, P_3:<001>e[4,4]:(001)gg, [3,6]:100e[4,4]:010gg, P_3:<001>e[4,4]:010gg, [3,6]:100e[4,4]:101gg, P_3:<001>e[4,4]:101gg, [3,6]:100e[∞][∞]gg, P_3:<001>e[∞][∞]gg

Honeycells: triangular-square duoprisms and tesseracts

Verf: same as [4]

In this tetracomb, the three “hadron” gyrated laminae of the preceding tetracomb are separated by laminae of tesseracts. The other elongated gyrated duoprismatic tetracomb employing these honeycells is [13].

[86] **Prismatoelongated gyrated triangular-square duoprismatic tetracomb**

[3,6]:100[4,4]:(001)p, P_3:<001>[4,4]:(001)p, [3,6]:100[4,4]:010p, P_3:<001>[4,4]:010p, [3,6]:100[4,4]:101p, P_3:<001>[4,4]:101p, [3,6]:100[∞][∞]p, P_3:<001>[∞][∞]p

Honeycells: triangular-square duoprisms

Verf: same as [12]

In this tetracomb, the alternating laminae of [12] are spread apart and new laminae of triangular duoprisms, gyrated with respect to both kinds of laminae of [12], are inserted between them. This produces a uniform isochoric tetracomb in which the triangular-square duoprisms are gyrated all three “hadron” ways, but one way (of the inserted duoprisms) appears twice as frequently as the other two, in the pattern …-1-2-1-3-1-2-1-3-…. (First identified as uniform by Wendy Krieger and Andrew Weimholt.)

This completes the list of known prismatic and duoprismatic tetracombs, that is, those tetracombs whose honeycells are exclusively uniform duoprisms or polyhedral prisms. There remain for examination the uniform panoploid tetracombs derived from the five irreducible four-dimensional space groups (see Coxeter’s *Regular Polytopes*, for example), denoted by [4,3,3,4], h[4,3,3,4], [3,3,4,3], P_5, and Q_5. (The group h[4,3,3,4] is also sometimes denoted S_5.) Each has a maximum of $2^5-1=31$ Wythoffian derivatives, but many, such as palindromes and cyclic and other permutations, are duplicates. Four anomalous (non-Wythoffian) tetracombs are in addition derivable from some of these by snubbing or schmoozing (see [133], [141], [142], and [143] below). All the Wythoffian combinations are listed and associated with their corresponding tetracombs. The apparently remote possibility that there may be other anomalous tetracombs is difficult to eradicate and keeps us from a proof of completeness of our table, both in three-space and in four-space.

IV. Uniform panoploid tesseractic tetracombs

The following tetracombs are the remaining Wythoffian derivatives of the tesseractic tetracomb. The vertex figures of many of these are uncomplicated but rather unfamiliar polychora that require extended descriptions. The combinations [4,3,3,4]:(00001), [4,3,3,4]:10001, and h[4,3,3,4]:10000, of course, represent the tesseractic tetracomb itself, [1], which is also both a prismatic and a duoprismatic tetracomb. Just as the regular tesseractic tetracomb [1] is first encountered here as a prismatic tetracomb, the regular icositetrachoric tetracomb is first encountered here as a tesseractic tetracomb, [88]. It has its own Wythoffian derivatives, the distinct ones of which are tabulated following the tesseractic and demitesseractic tetracombs.

In a typical Wythoffian [4,3,3,4] tetracomb, the tesseracts of [1] are replaced by related uniform polychora with tesseractic symmetry (the “body” honeycells). These may require a second set of uniform polychora with tesseractic symmetry to fill in the gaps where the vertices of [1] were ("corner" honeycells), a third set of uniform polychora with octahedral-prismatic symmetry to fill in the gaps where the edges of [1] were ("edge" honeycells), a fourth set of uniform polychora with square-duoprismatic symmetry to fill in the gaps where the faces of [1] were ("face" honeycells), and even a fifth set of polychora with octahedral-prismatic symmetry to fill in the gaps where the cells of [1] were ("cell" honeycells). In a tesseractic tetracomb, a body honeycell must adjoin 16 corner honeycells, 32 edge honeycells, 24 face honeycells, and eight cell honeycells, if any of these are present. If there are no cell honeycells, each body honeycell adjoins eight other body honeycells. These numbers follow from the numbers of the elements of a tesseract.

Sometimes the body honeycells and corner honeycells are identical, so that each "body and corner" honeycell has 24 identical neighbors (possibly connected by identical edge and cell honeycells). This occurs, for example, when the binary digit string in the notation is palindromic, and creates additional symmetries in the tetracomb. In principle, in the tesseractic tetracombs it is arbitrary which honeycells are the body and which are the corner honeycells, since both have the same symmetries; likewise the edge and cell honeycells. But when the body and corner honeycells are different, we may designate the larger honeycells the body honeycells. Then the cell honeycells, if present, would be prisms that connect two body cells via their bases, and the edge honeycells, if present, would likewise connect two corner honeycells. Among the prismatic and duoprismatic tetracombs, listed above, there is of course no meaningful distinction among body, corner, edge, face, and cell honeycells.

[87] **Rectified tesseractic tetracomb**

[4,3,3,4]:(00010), h[4,3,3,4]:00011, h[4,3,3,4]:01000, Q_5:0|0011|
tesseractihexadecachoric-hexadecachoric tetracomb
rectified demitesseractic tetracomb
Honeycells: tesseractihexadecachora (body) and hexadecachora (corner)
Verf: octahedral prism, base edges of length 1, height $\sqrt{2}$

This tetracomb has its vertices on the midpoints of the edges of the tesseractic tetracomb, so its honeycells are (1) rectified tesseracts, or tesseractihexadecachora, and (2) verfs of the tesseractic tetracomb, or hexadecachora.

[88] **Icositetrachoric tetracomb**

[4,3,3,4]:00100, h[4,3,3,4]:00100, [3,3,4,3]:00001, [3,3,4,3]:01000, Q_5:10000
birectified tesseractic tetracomb
rectified hexadecachoric tetracomb
rectified alternated tesseractic tetracomb
Honeycells: icositetrachora (body and corner)
Verf: hexadecachoron, edges of length $\sqrt{2}$

This is the well-known regular tetracomb of icositetrachora, three around a common face, 16 at every vertex. As a regular tetracomb, it has its own set of Wythoffian derivatives, which appear in this list below. Some of those are duplicates of other tesseractic tetracomb derivatives. It derives from the tesseractic tetracomb by having its vertices located at the centers of the tesseractic tetracomb's square faces, and it has three times the symmetry of a typical tesseractic tetracomb.

[89] **Truncated tesseractic tetracomb**

[4,3,3,4]:(00011), h[4,3,3,4]:11000
truncated alternated tesseractic tetracomb
Honeycells: truncated tesseracts (body) and hexadecachora (corner)
Verf: octahedral pyramid, octahedral edges of length 1, lateral edges of length $\sqrt{(2+\sqrt{2})}$

[90] **Small prismatotesseractic tetracomb**

[4,3,3,4]:(00101), h[4,3,3,4]:10100
Honeycells: prismatotesseractihexadecachora (body), icositetrachora (corner), and octahedral prisms (edge)
Verf: cubic wedge, cells a cube adjoining four square wedges at a girdle of four squares and two square pyramids at the other two (opposite) squares, cube edges of length 1, wedge edge and all other edges of length $\sqrt{2}$ (wedge edge is parallel to four edges of cubic base and symmetrically positioned with respect to base)

[91] **Small diprismatotesseractic tetracomb**

[4,3,3,4]:(01001)
Honeycells: diprismatotesseractihexadecachora (body), tesseractihexadecachora (corner), cuboctahedral prisms (edge), and, tesseracts (face)
Verf: triangular-antipodial antifastigium, cells a triangular prism, base edge of length 1 and height $\sqrt{2}$; two triangular antipodia, small base edges of length 1, large base edges of length $\sqrt{2}$, lateral edges of length $\sqrt{2}$; three rectangular pyramids, base edges of lengths 1, $\sqrt{2}$, 1, $\sqrt{2}$, lateral edges of length $\sqrt{2}$; and three regular tetrahedra, edges of length $\sqrt{2}$

[92] **Bitruncated tesseractic tetracomb**

[4,3,3,4]:(00110), h[4,3,3,4]:00111, h[4,3,3,4]:01100, Q_5:1|0011|
Honeycells: truncated-octahedral tesseractihexadecachora (body) and truncated hexadecachora (corner)
Verf: square double pyramid, base a square with edges of length 1, axis-element an edge of length $\sqrt{2}$, all other edges of length $\sqrt{3}$

Cells of the vertex figure are two congruent right square pyramids adjoined at their base squares (square edges of length 1, lateral edges of length $\sqrt{3}$), and four digonal disphenoids defined by opposite edges of lengths 1 and $\sqrt{2}$ connected by four edges of length $\sqrt{3}$; the axis-element is their common edge of length $\sqrt{2}$.

[93] **Rectified icositetrachoric tetracomb**

[4,3,3,4]:01010, h[4,3,3,4]:01011, [3,3,4,3]:00010, [3,3,4,3]:10100, Q_5:01111
Honeycells: disicositetrachora (body and corner) and tesseracts (face)
Verf: right tetrahedral prism, base edges of length √2, height 1

The vertices of this tetracomb lie on the midpoints of the edges of the icositetrachoric tetracomb. When this tetracomb is considered as an icositetrachoric tetracomb, the disicositetrachora are all body honeycells and the tesseracts are corner honeycells.

[94] **Great prismatotesseractic tetracomb**

[4,3,3,4]:(00111), h[4,3,3,4]:11100
Honeycells: great prismatotesseractihexadecachora (body), truncated hexadecachora (corner), and octahedral prisms (edge)
Verf: square double pyramid, less symmetric than verf of [92]; base square edges of length 1, axis-element of length √(2+√2), edges from one vertex of axis-element of length √2, edges from other vertex of axis-element of length √3

Cells of the vertex figure are two unequal right square pyramids adjoined at their base squares (square edges of length 1, lateral edges of length √2 for one pyramid, √3 for the other pyramid), and four digonal disphenoids defined by opposite edges of lengths 1 and √(2+√2) connected by two edges of length √2 and √3 so that one face is an isosceles triangle with edges of lengths 1, √2, √2; the axis-element is their common edge of length √(2+√2).

[95] **Small tomocubic-diprismatotesseractic tetracomb**

[4,3,3,4]:(01011), h[4,3,3,4]:11011
Honeycells: truncated-cubic diprismatotesseractihexadecachora (body), disicositetrachora (corner), cuboctahedral prisms (edge), and square-octagonal duoprisms (face)
Verf: tilted pyramid based on a right triangular prism; prism with base edges of length √2, height 1, two lateral edges of pyramid with lengths √(2+√2), all other edges with lengths √2, arranged so that the two long lateral edges form an isosceles triangle with a unit edge of the prism

Cells of the vertex figure are the base prism, one right rectangular pyramid, two tilted rectangular pyramids, and two sphenoids.

[96] **Tomotesseractic-diprismatotesseractic tetracomb**

[4,3,3,4]:(10011)
Honeycells: truncated tesseracts (body), diprismatotesseractihexadecachora (corner), tesseracts (edge), square-octagonal duoprisms (face), and truncated-cubic prisms (cell)
Verf: tilted triangular-antipodial pyramid, antipodium with equilateral triangles of edge lengths and √2 as bases, all lateral edges of length √2; lateral edges of pyramid of length √(2+√2) connected to small triangular base, all other edges of length √2

Cells of the vertex figure are the base antipodium and eight lateral tetrahedra of four different kinds: an equilateral-triangular pyramid, base edge of length 1, lateral edges of lengths √(2+√2); a regular tetrahedron, edges of length √2; three digonal disphenoids with two opposite edges of lengths √2 and √(2+√2) and all other edges of length √2; and three isosceles-triangular pyramids, base triangles with edges of lengths 1, √(2+√2), √(2+√2) and lateral edges all of length √2.

[97] **Rhombitesseractic-diprismatotesseractic tetracomb**

[4,3,3,4]:(01101), h[4,3,3,4]:10111

Honeycells: rhombicuboctahedral diprismatotesseractihexadecachora (body), truncated-octahedral tesseractihexadecachora (corner), truncated-octahedral prisms (edge), and tesseracts (face)

Verf: trapezoidal double pyramid, base a trapezoid with edges of lengths 1, √2, √2, √2, axis-element an edge of length √2, edges of length √3 joining ends of axis element to trapezoidal edge of length 1; all other edges of length √2

Cells of the vertex figure are two congruent trapezoidal pyramids, a regular tetrahedron, a digonal disphenoid with opposite edges of lengths 1 and √2 connected by four edges of length √3, and two digonal disphenoids with opposite edges of lengths 1 and √3 connected by four edges of length √2.

[98] **Small rhombitesseractic-prismatotesseractic tetracomb**

[4,3,3,4]:10101

Honeycells: prismatotesseractihexadecachora (body and corner), rhombicuboctahedral prisms (edge and cell), and tesseracts (face)

Verf: tetrahedral prismoid, base a tetrahedron with edges of length √2, peak a square ridge with edges of length 1 positioned above the midplane of the tetrahedron with its edges parallel to two opposite edges of the tetrahedron; lateral edges all of length √2

Cells of the vertex figure are the base tetrahedron, four lateral tilted trapezoidal pyramids, and two lateral wedges that adjoin at the peak square. This tetracomb has twice the symmetry of a typical tesseractic tetracomb.

[99] **Truncated icositetrachoric tetracomb**

[4,3,3,4]:01110, h[4,3,3,4]:01111, [3,3,4,3]:00011, [3,3,4,3]:11100, Q_5:11111

Honeycells: truncated icositetrachora (body and corner) and tesseracts (face)

Verf: regular-tetrahedral right pyramid, base a tetrahedron of edge length √2, lateral cells isosceles equilateral-triangular pyramids with base triangle edge length √2 and lateral edges of length √3

. This tetracomb has three times the symmetry of a typical tesseractic tetracomb.

[100] **Great diprismatotesseractic tetracomb**

[4,3,3,4]:(01111), h[4,3,3,4]:11111

Honeycells: great diprismatotesseractihexadecachora (body), truncated icositetrachora (corner), truncated-octahedral prisms (edge), and square-

octagonal duoprisms (face)
Verf: irregular pentachoron, pattern of edges when projected into a pentagon is: one outer edge of length √3, the other four of length √2, diagonals of pentagon cycle as a pentagram, beginning with edge of length √(2+√2) connecting two outer edges of lengths √2 and √3, then edge of length √2, edge of length √3 connecting the other pair of outer edges of lengths √2 and √3, then an edge of length √3, and finally an edge of length √2

Cells of vertex figure include two great prismatotesseractihexadecachoric vertex figures, and one vertex figure for each of the other three kinds of cells (all the cells are tetrahedra of various shapes).

[101] **Great rhombitesseractic-prismatotesseractic tetracomb**

[4,3,3,4]:(10111)
Honeycells: great prismatotesseractihexadecachora (body), rhombicuboctahedral diprismatotesseractihexadecachora (corner), rhombicuboctahedral prisms (edge), square-octagonal duoprisms (face), and truncated-cuboctahedral prisms (cell)
Verf: tilted trapezoidal double pyramid, base a trapezoid with edges of lengths 1, √2, √2, √2, axis-element an edge of length √(2+√2), edges of length √2 and √3 joining the ends of the axis-element to the unit trapezoidal edge (edges of the same length occur at each axis-element end); all other edges of length √2

Cells of the vertex figure are two different trapezoidal pyramids and four tetrahedra, two bilaterally symmetric and two *dextro-laevo* images of each other. The bilaterally symmetric tetrahedra have either base of the trapezoid as one of their edges.

[102] **Great tomocubic-diprismatotesseractic tetracomb**

[4,3,3,4]:11011
Honeycells: truncated-cubic diprismatotesseractihexadecachora (body and corner), truncated-cubic prisms (edge and cell), and octagonal duoprisms (face)
Verf: antialigned rectangular double pyramid, base a rectangle with edges of lengths 1, √2, 1, √2, axis-element an edge of length √2, two edges of length √2 join either axis-element endpoint to one of the unit rectangle edges, two edges of length √(2+√2) join the same endpoint to the other of the unit rectangle edges, so that isosceles triangles of edges 1, √(2+√2), √(2+√2) and 1, √2, √2 adjoin at either unit rectangular edge

This tetracomb has twice the symmetry of a typical tesseractic tetracomb.

[103] **Omnitruncated tesseractic tetracomb**

[4,3,3,4]:11111
Honeycells: great diprismatotesseractihexadecachora (body and corner), truncated-cuboctahedral prisms (edge and cell), and octagonal duoprisms (face)

Verf: irregular pentachoron, pattern of edges when projected into a pentagon is: one outer edge of length $\sqrt{3}$, the other four of length $\sqrt{2}$, diagonals of pentagon cycle as a pentagram, beginning with edge of length $\sqrt{3}$ connecting two outer edges of lengths $\sqrt{2}$ and $\sqrt{3}$, then edge of length $\sqrt{(2+\sqrt{2})}$, then edge of length $\sqrt{2}$ connecting the other pair of outer edges of lengths $\sqrt{2}$ and $\sqrt{3}$, then an edge of length $\sqrt{(2+\sqrt{2})}$, and finally an edge of length $\sqrt{2}$

Vertex figure is chiral, but both *dextro* and *laevo* versions occur equally throughout the tetracomb. This tetracomb has twice the symmetry of a typical tesseractic tetracomb.

V. Uniform panoploid demitesseractic tetracombs

This exhausts the list of Wythoffian derivatives of the tesseractic tetracomb, symmetry group [4,3,3,4]. A different kind of uniform panoploid tetracomb may be constructed by using alternate vertices of the tesseractic tetracomb: the demitesseractic tetracomb, h[4,3,3,4]:000|01|. (This, incidentally, turns out to be the third regular tetracomb, [104]. Its honeycells are all regular hexadecachora, so it is better named the hexadecachoric tetracomb. Indeed, all three regular tetracombs are among the Wythoffian derivatives of the demitesseractic symmetry group. As with the other two regular tetracombs, we encounter the hexadecachoric tetracomb in advance of the table of tetracombs sharing its most inclusive symmetry group, [3,3,4,3].) The fundamental region for the demitesseractic symmetry group is not an orthoscheme, and its Coxeter-Dynkin diagram has one trivalent node. In the compressed Coxeter-Dynkin notation used here, the first two (leftmost) digits correspond to the nodes at the 4-edge, the third digit corresponds to the trivalent node, and the fourth and fifth digits correspond to the end nodes of the short branches. The fourth and fifth digits may be interchanged without changing the corresponding tetracomb, so when they are different they are bracketed by vertical strokes.

In a typical demitesseractic tetracomb, symmetry group h[4,3,3,4], the tesseracts of the tesseractic tetracomb, [1], are replaced alternately by two kinds of body honeycells, so that either kind of honeycell is a neighbor (sometimes separated by a cell honeycell) to eight of the other kind. Corner, edge, and/or cell honeycells may be required to fill in the gaps as in the tesseractic tetracombs above. The edge honeycells generally have tetrahedral-prismatic symmetry: half the symmetry of the corresponding honeycells in a tesseractic tetracomb. Likewise the corner honeycells, which appear with demitesseractic symmetry. Wythoff's construction often calls for the two body honeycells to be identical. In such cases, the tetracomb acquires an extra symmetry and occurs among the Wythoff derivatives of the tesseractic symmetry group. If one or both body honeycells and the corner cells are identical, the tetracomb has even more symmetry and occurs among the icositetrachoric tetracombs. This is why there aren't many new tetracombs here (just eight) and their notations all end with |01|, and why none of them has cell honeycells, which can appear only when the last two digits are both 1.

[104] **Hexadecachoric tetracomb**

h[4,3,3,4]:000|01|, [3,3,4,3]:10000, Q_5:0|0001|
demitesseractic tetracomb
alternated tesseractic tetracomb
Honeycells: hexadecachora (body and corner)
Verf: icositetrachoron, all edges of length 1

This is the well-known regular tetracomb of hexadecachora, three around a common face, 24 at every vertex, and it has six times the symmetry of a general demitesseractic tetracomb. Both kinds of body honeycells and the corner honeycells are hexadecachora. Wythoff derivatives of this tetracomb, with symmetry group [3,3,4,3], are numerous, and are tabulated following the demitesseractic tetracombs.

[105] **Truncated hexadecachoric tetracomb**

h[4,3,3,4]:001|01|, [3,3,4,3]:11000, Q_5:1|0001|
Honeycells: truncated hexadecachora (body_1 and corner) and icositetrachora (body_2)
Verf: cubic pyramid, base cube with edge length 1, lateral cells six square pyramids with base edges of length 1, lateral edges of length $\sqrt{3}$

This tetracomb has icositetrachoric/hexadecachoric tetracomb symmetry. When considered as an icositetrachoric-hexadecachoric tetracomb, the truncated hexadecachora are body honeycells and the icositetrachora are corner honeycells.

[106] **Birectified demitesseractic tetracomb**

h[4,3,3,4]:010|01|, [3,3,4,3]:00100, Q_5:0|0111|
birectified icositetrachoric tetracomb
birectified hexadecachoric tetracomb
birectified icositetrachoric-hexadecachoric tetracomb
Honeycells: disicositetrachora (body_1) and tesseractihexadecachora (body_2 and corner)
Verf: unequal-equilateral-triangular duoprism, one triangle with edges of length 1, the other triangle with edges of length $\sqrt{2}$

Cells of the vertex figure are three equilateral-triangular prisms with base edges of length 1 and height $\sqrt{2}$ and three more equilateral-triangular prisms with base edges of length $\sqrt{2}$ and height 1. This tetracomb has icositetrachoric/hexadecachoric tetracomb symmetry. When considered as an icositetrachoric-hexadecachoric tetracomb, the disicositetrachora are body honeycells and the tesseractihexadecachora are corner honeycells.

[107] **Bitruncated demitesseractic tetracomb**

h[4,3,3,4]:011|01|, [3,3,4,3]:01100, Q_5:1|0111|
bitruncated hexadecachoric tetracomb
Honeycells: truncated icositetrachora (body_1) and truncated-octahedral tesseractihexadecachora (body_2 and corner)
Verf: equilateral-triangular double pyramid, base equilateral triangle with edge of

length $\sqrt{2}$, axis-element of length 1, all other edges of length $\sqrt{3}$

Cells of vertex figure are two equilateral-triangular pyramids adjoining at the base triangle and three digonal disphenoids joining the axis-element to each of the three base edges. This tetracomb has icositetrachoric/hexadecachoric tetracomb symmetry. When considered as an icositetrachoric-hexadecachoric tetracomb, the truncated icositetrachora are body honeycells and the truncated-octahedral tesseractihexadecachora are corner honeycells.

[108] **Small diprismatodemitesseractic tetracomb**

h[4,3,3,4]:100|01|

Honeycells: diprismatotesseractihexadecachora ($body_1$), tesseracts ($body_2$), hexadecachora (corner), and tetrahedral prisms (edge)

Verf: tetrahedral-octahedral antiprism, bases a regular octahedron with edge length 1 and a regular tetrahedron with edge length $\sqrt{2}$, lateral cells four equilateral-triangular pyramids with lateral faces isosceles triangles with edges of lengths 1, $\sqrt{2}$, $\sqrt{2}$ adjoining alternate faces of the octahedron, and four equilateral-triangular antipodia adjoining the other four faces of the octahedron and the four faces of the tetrahedron, each with equilateral triangular bases with edges of length 1 and $\sqrt{2}$ and lateral faces alternating between isosceles triangles with edges of lengths 1, $\sqrt{2}$, $\sqrt{2}$ and equilateral triangles with edge length $\sqrt{2}$

The vertex figure is a deformed version of the uniform dispentachoron, whose cells are five regular octahedra and five regular tetrahedra.

[109] **Small prismatodemitesseractic tetracomb**

h[4,3,3,4]:101|01|

Honeycells: rhombicuboctahedral diprismatotesseractihexadecachora ($body_1$), prismatotesseractichexadecachora ($body_2$), truncated hexadecachora (corner), and truncated-tetrahedral prisms (edge)

Verf: tilted square-wedge pyramid, base a square wedge with base edges of length 1, wedge edge and all other edges of length $\sqrt{2}$, apex connected to square by edges of length $\sqrt{3}$ and to wedge edge by edges of length $\sqrt{2}$

Cells of vertex figure are base square wedge and five pyramids, including one square pyramid, two triangular pyramids, and two trapezoidal pyramids.

[110] **Great prismatodemitesseractic tetracomb**

h[4,3,3,4]:110|01|

Honeycells: truncated-cubic diprismatotesseractihexadecachora ($body_1$), truncated tesseracts ($body_2$), tesseractihexadecachora (corner), and tetrahedral prisms (edge)

Verf: tilted triangular-prismatic pyramid, base an equilateral-triangular prism, base edges of length 1, height $\sqrt{2}$, apex connected to one base by edges all of length $\sqrt{(2+\sqrt{2})}$ and to the other base by edges all of length $\sqrt{2}$

Cells of vertex figure are the base triangular prism and altogether four lateral pyramids, two triangular and three rectangular.

[111] **Great diprismatodemitesseractic tetracomb**

h[4,3,3,4]:111|01|

Honeycells: great diprismatotesseractihexadecachora (body$_1$), great prismatotesseractihexadecachora (body$_2$), truncated-octahedral tesseractihexadecachora (corner), and truncated-tetrahedral prisms (edge)

Verf: irregular pentachoron, pattern of edges when projected into a pentagon is: one outer edge of length $\sqrt{2}$, next to an edge of length $\sqrt{(2+\sqrt{2})}$, the other three edges of length $\sqrt{3}$, diagonals of pentagon cycle as a pentagram, beginning with edge of length $\sqrt{3}$ connecting outer edges of lengths $\sqrt{2}$ and $\sqrt{(2+\sqrt{2})}$, then three edges of length $\sqrt{2}$, then an edge of length 1 that does not touch the edge of length $\sqrt{(2+\sqrt{2})}$

VI. Uniform panoploid icositetrachoric-hexadecachoric tetracombs

This exhausts the Wythoffian derivatives of the demitesseractic tetracomb symmetry group. The demitesseractic tetracomb itself, however, is the regular hexadecachoric tetracomb [3,3,4,3]:10000, which admits a separate set of Wythoffian derivatives that are not all duplicates of ones already listed, with symmetry group [3,3,4,3]. The demitesseractic tetracomb symmetry group h[4,3,3,4] is a subgroup of index 6 in [3,3,4,3], so we may say that [3,3,4,3] has six times the symmetry of h[4,3,3,4]—and three times the symmetry of [4,3,3,4]. In the hexadecachoric tetracomb, incidentally, the honeycells are all corner cells. In the related (or “dual”) icositetrachoric tetracomb, the honeycells are all body cells, and so we call the tetracombs of this group the “icositetrachoric-hexadecachoric tetracombs.”

In a typical icositetrachoric-hexadecachoric tetracomb, the icositetrachora of [3,3,4,3]:00001 are replaced by related body honeycells with icositetrachoric symmetry. The gaps are filled by corner honeycells of tesseractic symmetry, edge honeycells of tetrahedral-prismatic symmetry, face honeycells of triangular-duoprismatic symmetry, and/or cell honeycells of octahedral-prismatic symmetry. A body honeycell adjoins 24 corner honeycells, 96 edge honeycells, 96 face honeycells, and 24 cell honeycells, if any of these are present. If there are no cell honeycells, each body honeycell adjoins 24 other body honeycells. These numbers follow from the numbers of the elements of an icositetrachoron. Tetracomb [133], the remarkable snub icositetrachoric tetracomb, has a symmetry group, the “ionic” icositetrachoric tetracomb group, that is a subgroup of index 2 of the [3,3,4,3] group.

[112] **Small prismatodisicositetrachoric tetracomb**

[3,3,4,3]:00101

Honeycells: prismatodisicositetrachora (body), tesseractihexadecachora (corner), and tetrahedral prisms (edge)

Verf: equilateral-triangular-prismatic wedge, base a triangular prism with base edges of length 1, height $\sqrt{2}$, wedge edge of length 1 symmetrically positioned parallel to height of base prism and connected to base by edges all of length $\sqrt{2}$

[113] **Small tetracontaoctachoric tetracomb**
[3,3,4,3]:00110
bitruncated icositetrachoric tetracomb
Honeycells: tetracontaoctachora (body) and truncated tesseracts (corner)
Verf: irregular pentachoron with one edge of length 1, edges of opposite triangle all of length 1, and all other edges of length $\sqrt{(2+\sqrt{2})}$

[114] **Great prismatodisicositetrachoric tetracomb**
[3,3,4,3]:00111
Honeycells: great prismatodisicositetrachora (body), truncated tesseracts (corner), and tetrahedral prisms (edge)
Verf: irregular pentachoron, one triangle equilateral with edges of length 1, opposite edge of length $\sqrt{3}$, one end of opposite edge connected to triangle with three edges of length $\sqrt{2}$, other end of opposite edge connected to triangle with three edges of length $\sqrt{(2+\sqrt{2})}$

Cells of vertex figure include a short equilateral-triangular pyramid, a tall equilateral-triangular pyramid, and three identical isosceles-triangular pyramids (the latter all share the opposite edge of length $\sqrt{3}$.

[115] **Small prismatotetracontaoctachoric tetracomb**
[3,3,4,3]:01001
Honeycells: prismatotetracontaoctachora (body), icositetrachora (corner), octahedral prisms (edge), and triangular duoprisms (face)
Verf: square-antiprismatic antifastigium, cells a cube with edge of length 1; two elongated square antiprisms, base edges of length 1, lateral edges of length $\sqrt{2}$; four square pyramids, base edges of length 1, lateral edges of length $\sqrt{2}$; and four tetragonal disphenoids, two opposite edges of length 1, lateral edges of length $\sqrt{2}$

[116] **Grand prismatodisicositetrachoric tetracomb**
[3,3,4,3]:01010
Honeycells: prismatodisicositetrachora (body), prismatotesseractihexadecachora (corner), and triangular duoprisms (face)
Verf: tilted disphenoidal prism, bases being tetragonal disphenoids with two opposite edges of length 1 and lateral edges all of length $\sqrt{2}$; two lateral edges of length 1 and one short edge of either disphenoid making a square, the other two lateral edges of length $\sqrt{2}$ making a rectangle with the other short edges of either disphenoid

Cells of vertex figure are the two tetragonal disphenoids, two square wedges adjoined at their squares, and two rectangular wedges adjoined at their rectangles. The wedges have identical lateral trapezoids and adjoin one another along them, and identical lateral isosceles triangles at which they adjoin the disphenoids.

[117] **Small diprismatodisicositetrachoric tetracomb**

[3,3,4,3]:01011

Honeycells: diprismatodisicositetrachora (body), prismatotesseractihexadecachora (corner), octahedral prisms (edge), and triangular-hexagonal duoprisms (face)

Verf: square-wedge pyramid, base a square wedge with square edges of length 1, wedge edge and all other edges of length √2, apex connected to the four edges of length 1 by edges of length √2 and to the wedge edge by two edges of length √3

Cells of vertex figure are the base wedge, two digonal disphenoids, and two trapezoidal pyramids.

[118] **Great diprismatodisicositetrachoric tetracomb**

[3,3,4,3]:01101

Honeycells: diprismatodisicositetrachora (body), truncated-octahedral tesseractihexadecachora (corner), truncated-tetrahedral prisms (edge), and triangular duoprisms (face)

Verf: trapezoidal double pyramid, base a trapezoid with edges of lengths 1, √2, √2, √2, axis-element an edge of length 1; both ends of axis-element connected to ends of unit trapezoid edge by edges of length √2, to other two trapezoid vertices by edges of length √3

Cells of vertex figure are two congruent trapezoidal pyramids joined at their trapezoids, tetragonal disphenoid, digonal disphenoid, and two congruent isosceles-triangular pyramids.

[119] **Great grand prismatodisicositetrachoric tetracomb**

[3,3,4,3]:01110

Honeycells: great prismatodisicositetrachora (body), great prismatotesseractihexadecachora (corner), and triangular duoprisms (face)

Verf: irregular pentachoron, tilted pyramid based on bilaterally symmetric tetrahedron with two opposite edges of length 1 and √3 connected by edges of length √2 from one end of the edge of length √3 and of length √(2+√2) from the other end of that edge, apex connected to edge of length 1 by edges of length √2, to the √3 end of the opposite edge by an edge of length 1, and to the √(2+√2) end of that edge by an edge of length √3

[120] **Cantellated icositetrachoric tetracomb**

[3,3,4,3]:01111

Honeycells: Great prismatotetracontaoctachora (body), great prismatotesseractihexadecachora (corner), truncated-tetrahedral prisms (edge), and triangular-hexagonal duoprisms (face)

Verf: bilaterally symmetric pentachoron, one cell a tetrahedron with edge of length 1 opposite edge of length $\sqrt{(2+\sqrt{2})}$ with two edges of length $\sqrt{3}$ connecting unit edge to one end of $\sqrt{(2+\sqrt{2})}$ edge, and two edges of length $\sqrt{2}$ connecting unit edge to other end of $\sqrt{(2+\sqrt{2})}$ edge, fifth corner connected to the three corners of the 1, $\sqrt{3}$, $\sqrt{3}$ triangle by edges of length $\sqrt{2}$ and to the other corner by an edge of length $\sqrt{3}$

[121] **Runcinated icositetrachoric tetracomb**

[3,3,4,3]:10001

runcinated hexadecachoric tetracomb

runcinated icositetrachoric-hexadecachoric tetracomb

Honeycells: icositetrachora (body), hexadecachora (corner), tetrahedral prisms (edge), triangular duoprisms (face), and octahedral prisms (cell)

Verf: cubic-octahedral antiprism, bases a cube and an octahedron with edges of length 1 positioned symmetrically in parallel realms, with edges all of length $\sqrt{2}$ joining each corner of one base to its corresponding face in the other

Cells of the vertex figure are the base cube and octahedron, eight triangular pyramids whose bases are the octahedral faces and whose apices are the cubic vertices, six square pyramids whose bases are the cubic faces and whose apices are the octahedral vertices, and twelve tetragonal disphenoids, whose opposite edges are the cubic edges and the corresponding octahedral edges.

[122] **Small disicositetrachoric tetracomb**

[3,3,4,3]:10010

Honeycells: disicositetrachora (body), diprismatotesseractihexadecachora (corner), triangular duoprisms (face), and cuboctahedral prisms (cell)

Verf: triangular-antipodial antifastigium, cells a triangular prism, base edge of length $\sqrt{2}$ and height 1; two triangular antipodia, small base edges of length 1, large base edges of length $\sqrt{2}$, lateral edges of length $\sqrt{2}$; three rectangular pyramids, base edges of lengths 1, $\sqrt{2}$, 1, $\sqrt{2}$, lateral edges of length $\sqrt{2}$; and three tetragonal disphenoids, opposite edges of length 1, lateral edges of length $\sqrt{2}$

[123] **Tomoicositetrachoric-diprismatotesseractic tetracomb**

[3,3,4,3]:10011

Honeycells: truncated icositetrachora (body), diprismatotesseractihexadecachora (corner), tetrahedral prisms (edge), triangular-hexagonal duoprisms (face), and truncated-octahedral prisms (cell)

Verf: equilateral-triangular antipodial pyramid, base an equilateral-triangular antipodium with small equilateral-triangular base with edges of length 1, large equilateral-triangular base with edges of length $\sqrt{2}$, lateral edges all of length √2, apex connected to small base by edges all of length 1, and to large base by edges all of length √3

Cells of vertex figure are base antipodium, two right equilateral-triangular pyramids of different sizes, three digonal disphenoids, and three isosceles-triangular pyramids.

[124] **Great disicositetrachoric tetracomb**

[3,3,4,3]:10101

Honeycells: prismatodisicositetrachora (body), disicositetrachora (corner), cuboctahedral prisms (edge), triangular duoprisms (face), and rhombicuboctahedral prisms (cell)

Verf: equilateral-triangular-prismatic wedge, base an equilateral-triangular prism with base edges of length √2, height 1, wedge edge of length 1 symmetrically positioned with respect to and parallel to one rectangular face of the base; ends of wedge edge connected to other two rectangular faces of base by edges of length √2

Cells of vertex figure are base triangular prism, rectangular wedge adjoining base along one rectangle, two rectangular pyramids adjoining base along other two rectangles, two trapezoidal pyramids adjoining wedge at trapezoidal faces, and tetragonal disphenoid adjoining all four pyramids.

[125] **Great tetracontaoctachoric tetracomb**

[3,3,4,3]:10110

Honeycells: tetracontaoctachora (body), truncated-cubic diprismatotesseractihexadecachora (corner), triangular duoprisms (face), and truncated-cubic prisms (cell)

Verf: rectangular double pyramid, base a rectangle with edges of lengths 1, √2, 1, √2, axis-element an edge of length 1, both ends of axis-element connected to the ends of one short edge of the rectangle by edges of length √2 and to the ends of the other short edge by edges of length √(2+√2)

Cells of vertex figure are two identical rectangular pyramids adjoined at their bases, their lateral faces connected by two different digonal disphenoids and two identical isosceles-triangular pyramids.

[126] **Runcicantic icositetrachoric tetracomb**

[3,3,4,3]:10111

Honeycells: great prismatodisicositetrachora (body), truncated-cubic diprismatotesseractihexadecachora (corner), cuboctahedral prisms (edge), triangular-hexagonal duoprisms (face), and truncated-cuboctahedral prisms (cell)

Verf: rectangular double pyramid, base a rectangle with edges of lengths 1, √2, 1, √2, axis-element an edge of length √3, one end of axis-element connected to all four base vertices by edges of length √2, other end of axis-element connected to ends of one short rectangle edge by edges of length $\sqrt{2}$, to ends of other short rectangle edge by edges of length $\sqrt{(2+\sqrt{2})}$

[127] **Great prismatotetracontaoctachoric tetracomb**

[3,3,4,3]:11001

Honeycells: prismatotetracontaoctachora (body), truncated hexadecachora (corner), truncated-tetrahedral prisms (edge), triangular-hexagonal duoprisms (face), and octahedral prisms (cell)

Verf: Tilted elongated-square-antiprismatic pyramid, base an elongated square antiprism with both bases squares with edge length of 1, lateral isosceles triangles with edges of lengths 1, $\sqrt{2}$, $\sqrt{2}$, apex connected to one square by edges of length $\sqrt{2}$ and the other square by edges of length $\sqrt{3}$

Cells of vertex figure are base square antiprism, a short and a tall right square pyramid, four digonal disphenoids (two edges of lengths 1 and $\sqrt{3}$ connected by edges of length $\sqrt{2}$), and four isosceles-triangular pyramids (base edges of lengths 1, $\sqrt{3}$, $\sqrt{3}$, lateral edges all of length $\sqrt{2}$).

[128] **Prismatodiprismatodisicositetrachoric tetracomb**

[3,3,4,3]:11010

Honeycells: prismatodisicositetrachora (body), rhombicuboctahedral diprismatodisicositetrachora (corner), triangular-hexagonal duoprisms (face), and cuboctahedral prisms (cell)

Verf: rectangular-wedge pyramid, base a rectangular wedge with rectangle with edges of lengths 1, $\sqrt{2}$, 1, $\sqrt{2}$, wedge edge of length 1, and lateral edges all of length $\sqrt{2}$, apex connected to ends of wedge edge by edges of length $\sqrt{3}$, all other lateral edges of length $\sqrt{2}$

Cells of vertex figure are the base wedge, two identical tilted trapezoidal pyramids, one rectagular pyramid, and two digonal disphenoids.

[129] **Biruncinated icositetrachoric tetracomb**

[3,3,4,3]:11011

biruncinated hexadecachoric tetracomb

Honeycells: diprismatodisicositetrachora (body), rhombicuboctahedral diprismatotesseractihexadecachora (corner), truncated-tetrahedral prisms (edge), hexagonal duoprisms (face), and truncated-octahedral prisms (cell)

Verf: trapezoidal double prism, base a trapezoid with edges of lengths 1, $\sqrt{2}$, $\sqrt{2}$, $\sqrt{2}$, axis-element an edge of length $\sqrt{2}$, one end of the axis-element connected to the ends of the unit trapezoid edge by a pair of edges of length 1 and to the ends of the opposite trapezoid edge by a pair of edges of length $\sqrt{3}$, the other end of the axis-element connected to the same trapezoid edges *vice versa*

[130] **Runcicantic hexadecachoric tetracomb**

[3,3,4,3]:11101
Honeycells: diprismatodisicositetrachora (body), truncated icositetrachora (corner), truncated-octahedral prisms (edge), triangular-hexagonal duoprisms (face), and rhombicuboctahedral prisms (cell)
Verf: tilted trapezoidal double pyramid, base trapezoid with edges of lengths 1, $\sqrt{2}$, $\sqrt{2}$, $\sqrt{2}$, axis-element an edge of length $\sqrt{3}$, both ends of axis element connected to trapezoid short edge by edges of length $\sqrt{2}$, one end of axis element connected to long edge of trapezoid opposite short edge by edges of length $\sqrt{2}$, other end by edges of length $\sqrt{3}$

[131] **Cantellated hexadecachoric tetracomb**
[3,3,4,3]:11110
Honeycells: great prismatodisicositetrachora (body), great diprismatotesseractihexadecachora (corner), triangular-hexagonal duoprisms (face), and truncated-cubic prisms (cell)
Verf: bilaterally symmetric pentachoron, an irregular-tetrahedral pyramid based on a tetrahedron with edge of length 1 opposite an edge of length $\sqrt{3}$, one end of the long edge connected to the ends of the short edge by edges of length $\sqrt{2}$, the other end by edges of length $\sqrt{(2+\sqrt{2})}$; apex connected to the isosceles triangle with edges 1, $\sqrt{(2+\sqrt{2})}$, $\sqrt{(2+\sqrt{2})}$ by edges of length $\sqrt{2}$ and to the fourth vertex by an edge of length $\sqrt{3}$

[132] **Omnitruncated icositetrachoric tetracomb**
[3,3,4,3]:11111
omnitruncated hexadecachoric tetracomb
Honeycells: great prismatotetracontaoctachora (body), great diprismatotesseractihexadecachora (corner), truncated-octahedral prisms (edge), hexagonal duoprisms (face), and truncated-cuboctahedral prisms (cell)
Verf: irregular pentachoron, pattern of edges when projected into a pentagon is: one outer edge of length $\sqrt{3}$, the other four of length $\sqrt{2}$, diagonals of pentagon cycle as a pentagram, beginning with edge of length $\sqrt{(2+\sqrt{2})}$ connecting two outer edges of lengths $\sqrt{3}$ and $\sqrt{2}$, then edge of length $\sqrt{3}$, then edge of length $\sqrt{2}$ connecting the other pair of outer edges of lengths $\sqrt{2}$ and $\sqrt{3}$, then an edge of length $\sqrt{3}$, and finally an edge of length $\sqrt{2}$

Vertex figure is chiral, but both *dextro* and *laevo* versions occur equally throughout the tetracomb.

[133] **Snub icositetrachoric tetracomb**
[3,3,4,3]:s
Honeycells: snub icositetrachora (body), hexadecachora (corner), and pentachora (edge)
Verf: irregular decachoron with an axis of tetrahedral symmetry, whose cells are

(1) an octahedron, (2) five tetrahedra, and (3) four tridiminished icosahedra; the octahedron adjoins the four tetrahedra along alternate faces and the four tridiminished icosahedra along its other four alternate faces; each tridiminished icosahedron adjoins the octahedron at the sole triangle that does not adjoin a pentagon, and adjoins four tetrahedra at its other four triangles; each tridiminished icosahedron adjoins the other three tridiminished icosahedra at its three pentagons

This peculiar non-Wythoffian tetracomb was discovered by Thorold Gosset around 1900. Each body honeycell is a snub icositetrachoron with half the symmetries of a regular icositetrachoron, and each edge honeycell is a regular pentachoron with tetrahedral-pyramidal symmetry, half the symmetry of a tetrahedral prism. And the corner hexadecachora of course have demitesseractic symmetry, half their usual symmetry.

VII. Uniform panoploid simplectic tetracombs

The symmetry group Q_5, whose fundamental region is represented by the Coxeter-Dynkin diagram that has one node connected to four other nodes by unnumbered edges, alas generates no new tetracombs. All nine of its Wythoffian derivatives are among the simpler Wythoffian derivatives of the hexadecachoric and icositetrachoric tetracombs (see designations in above listings). This leaves just the symmetry group P_5, whose fundamental region is represented by a Coxeter-Dynkin graph that is a pentagon, or a cycle of five unnumbered edges, to be considered. All seven of its Wythoffian derivatives, wonderfully, are tetracombs not yet listed. They exist by virtue of the geometric properties of the regular pentachoron and are not related to the tesseractic, demitesseractic, and icositetrachoric-hexadecachoric tetracombs. In this they are unlike their three-space analogues, which, because a regular tetrahedron is a demicube, mostly duplicate honeycombs derived from other groups. Inasmuch as all the honeycells of these tetracombs share pentachoric symmetry, there is little need to distinguish body, corner, edge, face, and cell honeycells among them. There is a P_{n+1} group for every *n*-space, and we might call these the “simplectic” *n*-tessellations. Three more non-Wythoffian tetracombs occur as elongations and schmoozes of the pentachoric-dispentachoric tetracomb.

[134] **Pentachoric-dispentachoric tetracomb**

P_5:<00001>
Honeycells: pentachora and dispentachora
Verf: prismatodecachoron with edges of length 1

Cells of vertex figure are ten tetrahedra and 20 triangular prisms, corresponding to the ten pentachora and 20 dispentachora that meet at each vertex. In this tetracomb, all the vertices lie in parallel realms in which they form alternated cubic honeycombs, the tetrahedra being either the “tops” of the dispentachora or the bases of the pentachora, and the octahedra being the “bottoms” of the dispentachora. The tops of the dispentachora adjoin the bases of the pentachora, and *vice versa*, in adjacent laminae; but alternating laminae may be inverted so that the tops of the dispentachora adjoin the tops of dispentachora and the bases of the pentachora adjoin the bases of other pentachora. This inversion results in the non-Wythoffian uniform panoploid tetracomb [142], described below. Octahedral and tetrahedral prisms may be inserted in between alternate laminae as well, resulting in the further non-Wythoffian elongated tetracombs [141] and [143], also described below.

[135] **Small truncated-pentachoric tetracomb**

P_5:<00011>

Honeycells: truncated pentachora, decachora, and pentachora

Verf: elongated tetrahedral antiprism, bases two congruent regular tetrahedra with edges of length 1, lateral edges all of length $\sqrt{3}$: a "stretched" hexadecachoron

Cells of the vertex figure are the two tetrahedral bases, eight equilateral-triangular pyramids, and six tetragonal disphenoids.

[136] **Small prismatodispentachoric tetracomb**

P_5:<00101>

Honeycells: prismatodispentachora, prismatodecachora, and dispentachora

Verf: triangular elongated-antiprismatic prism, bases two identical equilateral-triangular antiprisms with base edges of length 1 and lateral eges all of length $\sqrt{2}$, lateral cells two uniform triangular prisms with edges of length 1 and six isosceles-triangular prisms with base-triangle edges of lengths 1, $\sqrt{2}$, $\sqrt{2}$ and height 1

This is the uniform rectification of the pentachoric-dispentachoric tetracomb, in which each pentachoron is replaced by the dispentachoron whose vertices lie at the midpoints of its edges, each dispentachoron is replaced by the prismatodispentachoron whose vertices lie at the midpoints of its edges, and the vertices are replaced by their vertex figures, the prismatodecachora.

[137] **Great truncated-pentachoric tetracomb**

P_5:<00111>

Honeycells: great prismatodispentachora, truncated pentachora, and prismatodecachora

Verf: triangular elongated-antiprismatic pyramid, base an equilateral-triangular antiprism with base edges of length 1 and lateral eges all of length $\sqrt{2}$, apex connected to all vertices of the base by edges of length $\sqrt{3}$

This is the uniform truncation of the pentachoric-dispentachoric tetracomb, in which each pentachoron and dispentachoron of the latter is replaced by its truncation, and its vertices are replaced by their vertex figures, the prismatodecachora. Cells of the vertex figure include the base antiprism, two right equilateral-triangular pyramids, and six isosceles-triangular pyramids.

[138] **Great prismatodispentachoric tetracomb**

P_5:<01011>

Honeycells: great diprismatodispentachora, prismatodispentachora, and decachora
Verf: triangular-prismatic antifastigium, cells two identical isosceles-triangular prisms with base-triangle edges of lengths 1, $\sqrt{2}$, $\sqrt{2}$ and height 1 adjoined antialigned at their square faces, a tetragonal disphenoid connecting the two edges of the prisms opposite their common square, and four congruent tilted rectangular pyramids connecting the corners of the disphenoid with the rectangular faces of the prisms, each pyramid based on a 1, $\sqrt{2}$, 1, $\sqrt{2}$ rectangle with apex connected to one short edge by two edges of length $\sqrt{2}$ and to the other short edge by two edges of length $\sqrt{3}$

[139] **Grand prismatodispentachoric tetracomb**
P_5:<01111>
Honeycells: great diprismatodispentachora, great prismatodispentachora, and great prismatodecachora
Verf: tilted rectangular double pyramid, cells two congruent tilted pyramids based on a 1, $\sqrt{2}$, 1, $\sqrt{2}$ rectangle with apex connected to one short edge by two edges of length $\sqrt{2}$ and to the other short edge by two edges of length $\sqrt{3}$, adjoined antialigned at their bases, axis-element an edge of length $\sqrt{3}$, and four irregular bilaterally symmetric tetrahedra that connect the two pyramids at corresponding lateral faces

[140] **Great-prismatodecachoric tetracomb**
P_5:11111
omnitruncated pentachoric-dispentachoric tetracomb
Honeycells: great prismatodecachora
Verf: irregular pentachoron with five edges of length $\sqrt{2}$ and five edges of length $\sqrt{3}$, arranged so that two of each edge adjoin at each vertex

In this tetracomb, the honeycells that adjoin at each truncated octahedron and the honeycells that adjoin at each hexagonal prism are antialigned across the common cell. This is perhaps the closest four-dimensional analogue of the truncated-octahedral honeycomb in three-space. The vertex figure is chiral, and *dextro* and *laevo* versions appear evenly throughout the tetracomb. The vertex figure may be projected into a pentagon so that the pentagon's outer edges are all the same length and the inner pentagram's edges are all the same length. This isohoneycelled tetracomb has a nontrivial isohoneycelled analogue in Euclidean n-space, $n \geq 2$.

The vertices of tetracomb [134] form alternated cubic honeycombs in parallel realms, in between which lie the honeycells in their entirety. This allows [134] to be elongated by inserting laminae of tetrahedral and octahedral prisms in between the laminae of pentachora and dispentachora. In addition, alternate laminae of pentachora and dispentachora may be relatively inverted so that pentachora adjoin pentachora and dispentachora adjoin dispentachora along their tetrahedral cells, rather than pentachora always adjoining dispentachora. This polyhedral inversion in a line has no short common name, so we invent the term *schmooze* for it, and append the suffix “i,” for “inverted,” to the notations for the unschmoozed tetracombs (“s,” for “schmoozed,” is already in use for “snub”). The alternately schmoozed [134] can also be uniformly elongated. These operations provide altogether three more uniform panoploid tetracombs, all non-Wythoffian:

[141] **Elongated pentachoric-dispentachoric tetracomb**

P_5:<00001>e

Honeycells: pentachora, dispentachora, octahedral prisms, and tetrahedral prisms

Verf: one half of a prismatodecachoron with edges of length 1 disjoined along an equatorial cuboctahedron (“tetrahedral bicupola”), joined along its cuboctahedral base to a cuboctahedral pyramid, lateral edges all of length $\sqrt{2}$

[142] **Schmoozed pentachoric-dispentachoric tetracomb**

P_5:<00001>i

Honeycells: pentachora and dispentachora

Verf: prismatodecachoron disjoined along an equatorial cuboctahedron, one half schmoozed along an axis through the centers of the cuboctahedron and the tetrahedral cell parallel to the cuboctahedron, then rejoined to the other half

As for tetracomb [134], cells of vertex figure are ten tetrahedra and 20 triangular prisms, corresponding to the ten pentachora and 20 dispentachora that meet at each vertex. But for this tetracomb, the vertex figure is reflectionally symmetric in the realm of the splitting cuboctahedron.

[143] **Elongated schmoozed pentachoric-dispentachoric tetracomb**

P_5:<00001>ie

Honeycells: pentachora, dispentachora, octahedral prisms, and tetrahedral prisms

Verf: Same as [141]

The difference between this tetracomb and [141] is that here the alternate laminae are schmoozed as in [142], whereas they are not in [141].

This exhausts the uniform panoploid tetracombs obtainable by Wythoff’s construction on the fundamental Euclidean pentachora.

VIII. Higher-dimensional uniform panoploid tessellations

In higher-dimensional spaces, the number of uniform panoploid tessellations rapidly increases. For example, in five-space, there are up to 63 (instead of 31) Wythoffian derivatives of each irreducible Euclidean group to examine, and fewer, if any, will be duplicates. Even though we lose the analogues of the Euclidean group [3,3,4,3], the derivatives of the demipenteractic group h[4,3,3,3,4] should somewhat compensate for their absence. We expect that the demipenteractic derivatives will duplicate fewer, if any, pentacombs among the derivatives of the group [4,3,3,3,4], because the demipenteract is a distinct uniform polyteron, not merely a five-dimensional cross polytope. The group Q_6 (the "quarter-penteractic" group) should yield distinct pentacombs as well, instead of merely duplicating others. In addition, each of the 143 uniform tetracombs gives rise to a distinct prismatic or duoprismatic pentacomb by Cartesian product with the apeirogon, and more duoprismatic pentacombs arise from the 270 Cartesian products of the 10 non-square tilings with the 27 non-cubic honeycombs. There are also new ways to gyrate, schmooze, and/or elongate in five-space. So we can expect some six or seven hundred uniform panoploid pentacombs altogether to exist. In spaces of six through eight dimensions the Gosset groups add hundreds more uniform panoploid hypercombs to those available from the analogues of the tetracomb groups. There may also be subgroups of the usual space groups that give rise to unsuspected anomalous hypercombs in the higher-dimensional spaces. By the 24th dimension, say, the number of Wythoff combinations to be examined and described numbers in the tens of millions, so most likely humans will not bother to catalogue the uniform panoploid hypercombs of spaces of dimension much more than that. ❑

Glossary

Listed below are informal working definitions of some of the less familiar names and terms used in this essay, and a few others.

apeirogon—the unique uniform tessellation of a line into equal line segments; if the segments are not equal, the apeirogon is "irregular"

Cartesian product—the set of points $P \times Q$ in Euclidean n-space each of whose coordinates are the concatenation of the coordinates of any point of the set P in Euclidean p-space followed by the coordinates of any point of the set Q in Euclidean q-space, for $0<p\leq q<n$ and $p+q=n$; that is, the set of points $P \times Q$ of n-space whose coordinates are $\{a_1, a_2, ..., a_p, b_1, b_2, ..., b_q\}$, when a point a in P has coordinates $\{a_1, a_2, ..., a_p,\}$ and a point b in Q has coordinates $\{b_1, b_2, ..., b_q\}$

dispentachoron—a uniform polychoron with five each of two kinds of cells, namely, tetrahedra and octahedra

duoprism—a polytope in n-space that is the Cartesian product of two nonprismatic polytopes of dimension p and q, where $1<p\leq q<n-1$ and $p+q=n$; for n=4, p and q must both be 2, and a duoprism in four-space is the Cartesian product of two polygons; in five-space a duoprism is the Cartesian product of a polygon and a polyhedron, and in six-space there are two kinds of duoprisms, the Cartesian product of two polyhedra and the Cartesian product of a polygon and a polychoron

dyad—a 1-dimensional polytope, whose two facets are called endpoints; a dyad that includes its interior is called a line segment

elements of a polytope—the k-dimensional polytopes, $-1<k<n$, that occur in a polytope in n-space; sometimes it is convenient to regard the polytope itself as its one and only n-dimensional element (called a body) and the nullitope as its one and only (-1)-dimensional element; the others these are called (in increasing order of k) corners or vertices, edges, faces, cells, terons, petons, exons, ..., peaks, ridges, and facets (elements for $k>3$ are named for the Greek numbers with a letter or two removed)

endpoints—the two facets of a dyad, the ends of a line segment

facets—the $(n-1)$-dimensional elements, or flat sides, of a polytope in n-space

hexacomb—a tessellation of a six-dimensional space or hexrealm

hexadecachoron—a polychoron having 16 cells; if the cells are all regular tetrahedra, the hexadecachoron is regular, having in addition eight corners, 24 edges, and 32 triangular faces; its compressed Coxeter-Dynkin symbol is [3,3,4]:0001

holeycomb—a tessellation that leaves some regions of its space uncovered (the “holes” of the holeycomb)

honeycells—the polytopes and/or $(n-1)$-dimensional tessellations that make up a tessellation

honeycomb—a tessellation of a three-dimensional space, or realm; some workers use this term interchangeably with tessellation

icositetrachoron—a polychoron having 24 cells; if the cells are all regular octahedra, the icositetrachoron is regular, having in addition 24 corners, 96 edges, and 96 triangular faces; its compressed Coxeter-Dynkin symbol is [3,4,3]:(0001) or [3,3,4]:0100

isometry—a mapping of a metric space onto itself that preserves the distance between any two points

line segment—a dyad together with its interior; as an element of a polytope, a line segment is called an edge or side (if the polytope is a polygon)

monad—a point, regarded as the polytope in a 0-dimensional space

nullitope—the empty set, regarded as the unique polytope in (-1)-dimensional space panoploid—covering an entire *n*-space with honeycells just once, without overlapping

pentachoron—a polychoron with five cells and five corners; the cells must all be tetrahedra, and if they are all regular, the pentachoron is regular as well; the compressed Coxeter-Dynkin symbol for a regular pentachoron is [3,3,3]:(0001)

pentacomb—a tessellation of a five-dimensional space, or pentrealm

polychoron—a polytope in four-space, whose facets are called cells, ridges are called faces, and peaks are called edges; its 0-dimensional elements are called corners or vertices

polygon—a plane polytope, whose facets are called edges or sides and whose ridges are called corners or vertices

polyhedron–a polytope in three-space, whose facets are called faces, ridges are called edges, and peaks are called corners or vertices

polyploid—not panoploid

polytope—a closed, flat-sided geometric figure in *n*-space, constructed by adjoining a finite number of ($n-1$)-dimensional polytopes (the facets) along their (n-2)-dimensional elements (the ridges) so that each ridge belongs to exactly two facets (this closes the polytope), and no subset of the facets itself forms a polytope; for $n=-1$, a polytope is the empty set, or *nullitope*, for n=0, a polytope is a single point, or *monad*, whose only facet is the nullitope, and for n=1, a polytope is a pair of points separated by a distance, or *dyad*; for n>1, polytopes are known as polygons, polyhedra, polychora, polyterons, polyexons, and so forth

prism—a polytope in *n*-space constructed as the Cartesian product of an ($n-1$)-dimensional polytope with a line segment; likewise, a polytope in *n*-space constructed by translating an ($n-1$)-dimensional polytope outside its hyperplane and connecting corresponding elements of the two images (the bases of the prism) with edges and various kinds of prisms (the lateral elements of the prism); prisms in *n*-space for n>3 are sometimes called “hyperprisms”

prismatotesseractihexadecachoron—a uniform polychoron that has eight cells in the realms of a tesseract, 16 more cells in the realms of a hexadecachoron, and 24 more cells that are triangular prisms; the eight tesseractic cells are rhombicuboctahedra and the 16 hexadecachoric cells are octahedra; the rhombicuboctahedra adjoin along their cubic squares, the octahedra adjoin four rhombicuboctahedra along their triangles and adjoin four prisms along their bases, and the rhombicuboctahedra adjoin twelve prisms along their rhombisquares

schmooze—polyhedral inversion in a line in four-space

symmetry—an isometry that permutes the elements of a figure, such as a polytope or a tessellation

symmetry group—the set of all symmetries of a particular figure; these form a group under the operation "followed by," as in "rotation *r* followed by reflection *s*"

tessellation—sometimes written *n*-tessellation; a collection of bounded polytopes and/or ($n-1$)-dimensional tessellations arranged in *n*-space so that they adjoin exactly along their facets (or honeycells, regarded as facets of an ($n-1$)-tessellation), each facet belonging to exactly two polytopes or ($n-1$)-tessellations, with no subcollection of these itself forming a tessellation; the polytopes and/or ($n-1$)-tessellations of a tessellation are called its honeycells

tesseract—the four-dimensional analogue of a cube; it has 16 corners, 32 edges, 24 square faces, and eight cubic cells; its compressed Coxeter-Dynkin symbol is [3,3,4]:0001

tesseractihexadecachoron—the four-dimensional analogue of a cuboctahedron; it has 32 corners (located at the midpoints of the edges of a tesseract), 96 edges, 64 triangular and 24 square faces, and 16 tetrahedral and eight cuboctahedral cells; its compressed Coxeter-Dynkin symbol is [3,3,4]:0010

tetracomb—a tessellation of a four-dimensional space, or tetrealm

tiling—a tessellation of a two-dimensional space, or plane; some workers use this term for any panoploid tessellation

truncated hexadecachoron—the uniform polychoron that is a regular hexadecachoron with its corners cut off, so that its triangles become regular hexagons; it has 48 vertices, 120 edges, 64 triangular faces, 32 hexagonal faces, 16 truncated-tetrahedral cells, and eight octahedral cells; its compressed Coxeter-Dynkin symbol is [3,3,4]:1100

truncated tesseract—the uniform polychoron that is a tesseract with its corners cut off, so that its squares become regular octagons; it has 64 vertices, 128 edges, 64 triangular faces, 24 octagonal faces, 16 tetrahedral cells, and eight truncated-cubic cells; its compressed Coxeter-Dynkin symbol is [3,3,4]:0011

Wythoff's construction—a method of locating the vertices of a uniform polytope relative to its center and hyperplanes of symmetry

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