Sachs-Wolfe effect in some anisotropic models

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Abstract. In this work it is shown that there are some spatially homogeneous but anisotropic models (Kantowski-Sachs and Bianchi type-III), with a positive cosmological constant, for which the inhomogeneities in the distribution of matter on the surface of the last scattering produce anisotropies (in large angular scales $\vartheta \gtrsim 10°$) that do not differ from the ones produced in Friedmann-Lemaître-Robertson-Walker (FLRW) models, if the density parameters are finely tuned. Namely, for adiabatic initial conditions, the Sachs-Wolfe effect in these anisotropic models is equal to the one obtained for isotropic models: $\frac{\delta T}{T} = \frac{1}{3}\Psi_e + 2\int\limits_e^r \frac{\partial \Psi}{\partial \eta} dw$. This result confirms the idea developed in previous works that with the present cosmological tests we cannot distinguish these anisotropic models from the FLRW models, if the Hubble parameters along the orthogonal directions are assumed to be approximately equal.

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1. Introduction

The task of proving the homogeneity and isotropy of the Universe at large scales is not a simple one. It is generally accepted that the Universe is spatially homogeneous as a result of the so called Copernican principle, that is, if we assume that we live in a typical place, and since the isotropy of Cosmic Microwave Background Radiation (CMBR) can be used to prove that the Universe is locally isotropic, then (even though we cannot find a proper homogeneity test), we must conclude that the Universe is spatially homogeneous and isotropic. This conclusion reduces drastically the space of solutions of Einstein equations, and the number of possible cosmological models.

In this way we are led to the so called FLRW cosmological models to describe our observations. Despite the high level of isotropy, some authors have worked on spatially homogeneous and anisotropic models and proved that they might agree with present observations. For instance, if the classical tests of cosmology are applied to a simple Kantowski-Sachs metric and the results compared with those obtained for the standard model, the observations will not be able to distinguish between these models if the Hubble parameters along the orthogonal directions are assumed to be approximately equal [1]. Following along the same lines, we made a qualitative study [2] of three axially symmetric metrics (Kantowski-Sachs, Bianchi type-I and Bianchi type-III), with a cosmological constant, to analyze which were physically permitted, when we assume them to be bound by a high degree of isotropy, that is, although our models were assumed anisotropic they could be considered to be almost FLRW, as far the shear parameter is concerned, from the epoch of the last scattering to the present. Recall that one defines to be 'close' to a FLRW model when both parameters

$$\Sigma^2 = \frac{\sigma_{ab}\sigma^{ab}}{6H^2}, \quad \mathcal{W}^2 = \frac{E_{ab}E^{ab} + H_{ab}H^{ab}}{6H^4}, \tag{1}$$

are almost zero, as was stated in [3]. Here, σ_{ab} is the shear tensor, H the Hubble parameter, E_{ab} and H_{ab} the electric and magnetic part of the Weyl tensor, respectively (see Appendix E). From this analysis we concluded that these models are good candidates for the description of the observed Universe, provided that the Hubble parameters are approximately equal at the last scattering. In other words, the vanishing of the first parameter, $\Sigma^2 \approx 0$, is sufficient to assure a FLRW-like behavior.

Historically, the detection of the CMBR has led to constrains in theoretical models in the field of Cosmology, and lent a hand to the Big Bang solutions. Indeed, the observed level of isotropy of the CMBR, first detected by Penzias & Wilson [4], provides strong evidence for the large-scale isotropy of the Universe, and is the best argument in favor of an isotropically expanding Universe. Later, more precise experiments proved that this radiation has temperature fluctuations, or anisotropies. These small anisotropies are thought to give rise to the observed galaxies, and large-scale structures in the Universe.

In 1992, the COBE (COsmic Background Explorer) satellite [5], [6] observed the CMBR with unprecedented precision and revealed for the first time that the level of the CMBR temperature fluctuations on large scales is as small as $\frac{\Delta T}{T} \simeq 10^{-5}$ [7, 8]. After COBE many other ground and balloon born experiments [9], with higher angular resolution, confirmed this result and allowed us to probe the level of the anisotropies on a large range of scales.

On large angular scales, the CMBR anisotropies ($\frac{\Delta T}{T}$), are dominated by Sachs-Wolfe effect. This phenomenon, already deduced theoretically by Sachs & Wolfe [10], was used to compute the first-order perturbations in a FLRW universe with a flat 3-space filled either with dust or radiation. This is just one of the various possible sources of anisotropy, which occurs when there are inhomegeneities in the distribution of matter on the surface of the last scattering, that may produce anisotropies by the redshift or blueshift of photons. In this paper we compute the Sachs-Wolfe effect [10] for some anisotropic but homogeneous models (Kantowski-Sachs and Bianchi type-III) and find that under the assumption $H_a \simeq H_b$ these models give rise to the same Sachs-Wolfe effect obtained for FLRW universes. This is an interesting result witch tells us that CMBR observations on large angular scales will not be able to distinguish these anisotropic models from FLRW ones.

2. The method

As Collins and Hawking [11] pointed out, the number of cosmological solutions which demonstrate exact isotropy well after the Big Bang origin of the Universe is a small fraction of the set of allowable solutions to the Einstein equations. It is therefore prudent to take seriously the possibility that the Universe is expanding anisotropically and to investigate what effect anisotropic expansion will have on the angular distribution of background radiation [9]. In this work we show that, for large angular scales ($\vartheta \gtrsim 10°$), there exist homogeneous but anisotropic models, where the photons traveling to an observer from the last scattering surface encounter metric perturbations which cause them to change frequency, just like in the case of FLRW models.

The metrics we consider are the Kantowski-Sachs and Bianchi type-III, given by

$$d\tilde{s}^2 = -dt^2 + a^2(t)dr^2 + b^2(t)(d\theta^2 + f^2(\theta)d\phi^2), \tag{2}$$

where

$$f(\theta) = \begin{cases} \sin\theta & \text{for Kantowski-Sachs} \\ \sinh\theta & \text{for Bianchi type-III} \end{cases}$$

We evaluate the Sachs-Wolfe effect [10, 12], assuming small perturbations in the previous metrics, and then integrating the geodesic equations for the CMBR photons along their paths, from the Last Scattering Surface (LSS) to the observer. In this work we account for the "kinematics effects" undergone by the free propagating radiation from the last scattering, in a perturbed universe, and for the "intrinsic effects" originated by the set of physical and microphysical processes related to the density perturbations in the LSS.

For simplicity, it is common to perform a conformal transformation § of the previous metrics and to work with the following metric forms

$$d\bar{\tilde{s}}^2 = -d\eta^2 + dr^2 + \frac{b^2(\eta)}{a^2(\eta)}(d\theta^2 + f^2(\theta)d\phi^2) \tag{3}$$

such that $d\bar{\tilde{s}}^2$: $d\tilde{s}^2 = a^2(\eta)d\bar{\tilde{s}}^2$, since the null geodesics are preserved by this transformation. Afterwards, the results are transported to $d\tilde{s}^2$ metric. The metric $d\bar{\tilde{s}}^2$ is perturbed in the following way

$$\begin{aligned} d\bar{s}^2 = & -(1+h_{00})d\eta^2 + (1+h_{11})dr^2 + \frac{b^2(\eta)}{a^2(\eta)}\Big[(1+h_{22})d\theta^2 \\ & +(1+h_{33})f^2(\theta)d\phi^2\Big] - (h_{01}+h_{10})d\eta dr - \frac{b(\eta)}{a(\eta)}(h_{02}+h_{20})d\eta d\theta \\ & - \frac{b(\eta)}{a(\eta)}f(\theta)(h_{03}+h_{30})d\eta d\phi + \frac{b(\eta)}{a(\eta)}(h_{12}+h_{21})dr d\theta \\ & + \frac{b(\eta)}{a(\eta)}f(\theta)(h_{13}+h_{31})dr d\phi + \frac{b^2(\eta)}{a^2(\eta)}f(\theta)(h_{23}+h_{32})d\theta d\phi, \end{aligned} \tag{4}$$

where h_{ab} are functions of time and position and such that $h_{ab} \ll 1$.

Considering the geodesic equation for the photons

$$\frac{dU_a}{dw} = \frac{1}{2}\bar{g}_{bc,a}U^bU^c, \tag{5}$$

where U_a represents the photon 4-vector velocity components and w the affine parameter associated to its trajectory, we may calculate this 4-velocity integrating the previous equation

$$U_a = \frac{1}{2}\int \bar{g}_{bc,a}U^bU^c dw + {}_{(0)}U_a. \tag{6}$$

The term ${}_{(0)}U_a$ represents the non perturbed photon 4-velocity components in the covariant form.

A material observer, moving with some 3-velocity $\vec{V}$, in a perturbed universe, has a 4-velocity given by (see Appendix A)

$$V^a \simeq \left(1 - \frac{1}{2}h_{00}, V^1, V^2, V^3\right). \tag{7}$$

This observer measures a photon energy $\bar{E}_\gamma$ proportional to U_aV^a,

$$\bar{E}_\gamma \propto \left(\frac{1}{2}\int \bar{g}_{bc,a}U^bU^c dw + {}_{(0)}U_a\right)\left(1 - \frac{1}{2}h_{00}, V^i\right). \tag{8}$$

Decomposing the 4-velocity in Taylor series and conserving only the first order terms $U^a \simeq {}_{(0)}U^a + {}_{(1)}U^a$ (the calculation of ${}_{(0)}U^a$ is in the Appendix B) and neglecting terms like $h_{00(1)}U^a$ (because they are second order terms), the expression becomes

$$\begin{aligned} \bar{E}_\gamma \propto & \frac{1}{2}\int \bar{g}_{bc,0}U^bU^c dw - \frac{1}{4}h_{00}\int \bar{g}_{bc,0(0)}U^b{}_{(0)}U^c dw + {}_{(0)}U_0 \\ & - \frac{1}{2}{}_{(0)}U_0 h_{00} + \vec{U}\cdot\vec{V} + \frac{1}{2}\int \bar{g}_{bc,i(0)}U^b{}_{(0)}U^c dw V^i. \end{aligned} \tag{9}$$

§ Owing to this transformation, η is usually called the conformal time, and it is related to cosmic time by $dt^2 = d\eta^2 a^2(\eta)$.

The last term of right hand side vanishes, because for $i = r$ and $i = \phi \Rightarrow \bar{g}_{bc,i} = 0$ and also because $U^{\phi} = 0$. The term

$$\frac{1}{4}h_{00}\int \bar{g}_{bc,0(0)}U^{b}{}_{(0)}U^{c}dw = \frac{1}{4}h_{00}\int \bar{\bar{g}}_{bc,0(0)}U^{b}{}_{(0)}U^{c}dw$$
$$= \frac{1}{4}h_{00}\int \bar{\bar{g}}_{\theta\theta,\eta}\left({}_{(0)}U^{\theta}\right)^{2}dw = \frac{1}{2}h_{00}\int \frac{b^{2}}{a^{2}}\left(\frac{\dot{b}}{b} - \frac{\dot{a}}{a}\right)\left({}_{(0)}U^{\theta}\right)^{2}dw,$$

may be numerically computed. By convention we use $(\dot{\ }) \equiv \frac{d}{d\eta}$. If we choose accurately the values of density parameters $\Omega_M + \Omega_\Lambda$ (see Aguiar & Crawford [2]) $\Omega_M + \Omega_\Lambda \simeq 1$ this integral may be neglected, because it is a second order term (see Appendix C). The first term in the right hand side may be decomposed by the following way

$$\frac{1}{2}\int \bar{g}_{bc,0}U^{b}U^{c}dw$$
$$= \frac{1}{2}\int \bar{g}_{bc,0(0)}U^{b}{}_{(0)}U^{c}dw + \frac{1}{2}\int \bar{g}_{bc,0(0)}U^{b}{}_{(1)}U^{c}dw + \frac{1}{2}\int \bar{g}_{bc,0(1)}U^{b}{}_{(0)}U^{c}dw$$
$$= \frac{1}{2}\int \bar{g}_{bc,0(0)}U^{b}{}_{(0)}U^{c}dw + \int \bar{\bar{g}}_{\theta\theta,\eta(0)}U^{\theta}{}_{(1)}U^{\theta}dw.$$

But

$$\int \bar{\bar{g}}_{\theta\theta,\eta(0)}U^{\theta}{}_{(1)}U^{\theta}dw = 2\int \frac{b^{2}}{a^{2}}\left(\frac{\dot{b}}{b} - \frac{\dot{a}}{a}\right){}_{(0)}U^{\theta}{}_{(1)}U^{\theta}dw,$$

may also be neglected because the reason pointed previously. Thus,

$$\bar{E}_{\gamma} \propto \frac{1}{2}\int \bar{g}_{bc,0(0)}U^{b}{}_{(0)}U^{c}dw + {}_{(0)}U_{0} - \frac{1}{2}{}_{(0)}U_{0}h_{00} + \vec{U}\cdot\vec{V}. \tag{10}$$

Calculating the ratio of energies in the emission instant (e) and in the reception instant (r) we obtain

$$\frac{\bar{E}_{\gamma_e}}{\bar{E}_{\gamma_r}} = \frac{(U_a V^a)|_e}{(U_a V^a)|_r} = \frac{-1 + \frac{1}{2}\left.h_{00}\right|_e + \left.(\vec{U}\cdot\vec{V})\right|_e}{-1 + \frac{1}{2}\left.h_{00}\right|_r + \left.(\vec{U}\cdot\vec{V})\right|_r + \frac{1}{2}\int_e^r \bar{g}_{bc,0(0)}U^{b}{}_{(0)}U^{c}dw}, \tag{11}$$

(the symbol $X|_A$ means that X is being evaluated at point A). Considering the approximation $(1+X)^{-1} \simeq 1 - X$ we have

$$\frac{\bar{E}_{\gamma_e}}{\bar{E}_{\gamma_r}} = 1 + \frac{1}{2}\left[h_{00}\right]_e^r + \left[\vec{U}\cdot\vec{V}\right]_e^r + \frac{1}{2}\int_e^r \bar{g}_{bc,0(0)}U^{b}{}_{(0)}U^{c}dw, \tag{12}$$

where $[X]_A^B \equiv X(B) - X(A)$. The obtained results in $d\bar{s}^2$ may be transported to ds^2 using the relation $E(t) = \frac{1}{a(t)}\bar{E}(w)$, as we see in Appendix D. The photons redshifted from the last scattering (in e) until being observed (in r) may be calculated by the ratio of measured energies in emission and reception,

$$z + 1 = \frac{\lambda_r}{\lambda_e} = \frac{E(t_e)}{E(t_r)} = \frac{a_r}{a_e}\frac{\bar{E}(w_e)}{\bar{E}(w_r)}, \tag{13}$$

because the wavelength of the photons is inversely proportional to its energy. On the other side the redshift is equal to the ratio between the black body associated temperatures in the emission and reception instants,

$$\frac{T_e}{T_r} = z + 1. \tag{14}$$

Relating the previous equation and the Equation (13) and considering newly a linear approximation we obtain

$$
\begin{aligned}
T_r &\simeq \frac{a_e}{a_r} T_e \left[1 - \frac{1}{2} \left[h_{00} \right]_e^r - \left[\vec{U} \cdot \vec{V} \right]_e^r - \frac{1}{2} \int_e^r \bar{g}_{bc,0(0)} U^b{}_{(0)} U^c dw \right] \\
&= \frac{a_e}{a_r} T_e \left(1 + \frac{\delta T_r}{T_r} \right),
\end{aligned}
\tag{15}
$$

therefore,

$$
\frac{\delta T_r}{T_r} = -\frac{1}{2} \left[h_{00} \right]_e^r - \left[\vec{U} \cdot \vec{V} \right]_e^r - \frac{1}{2} \int_e^r \bar{g}_{bc,0(0)} U^b{}_{(0)} U^c dw. \tag{16}
$$

Now it's time to define the perturbations h_{ab}. These fluctuations are *gauge* dependent. This correspond to the hypersurfaces choosing where these fluctuations are defined [13, 14]. We will choose a *Newtonian gauge* to allow a intuitive understanding. Considering only scalar perturbations the non zero quantities are,

$$
h_{00} = 2\Psi; \quad h_{11} = h_{22} = h_{33} = 2\Phi. \tag{17}
$$

As we have stated, these scalar quantities are function of time and position, $\Psi = \Psi(\eta, x^i)$, $\Phi = \Phi(\eta, x^i)$ and may be interpreted as Newtonian potential and a spatial curvature perturbation potential, respectively [13, 14].

For our models, with a proper choice of density parameters, these universes are approximately flat ($\Omega_0 + \Omega_{\Lambda_0} \simeq 1$) and one may show that if one neglect the pressure ($p = 0$) one gets $\Psi = -\Phi$. Writing (16) in the Newtonian gauge we obtain,

$$
\begin{aligned}
\frac{\delta T_r}{T_r} = &- \left[\Psi \right]_e^r - \left[\vec{U} \cdot \vec{V} \right]_e^r - \frac{1}{2} \int_e^r \left\{ \left[-h_{00,0} ({}_{(0)}U^\eta)^2 + h_{11,0} ({}_{(0)}U^r)^2 \right.\right. \\
&\left.\left. + \frac{b^2}{a^2} h_{22,0} ({}_{(0)}U^\theta)^2 \right] dw \right\} - \int_e^r (1 + h_{22}) \frac{b^2}{a^2} \left(\frac{\dot{b}}{b} - \frac{\dot{a}}{a} \right) ({}_{(0)}U^\theta)^2 dw.
\end{aligned}
\tag{18}
$$

The last term of right hand side may be neglected, as we show in Appendix C. If we consider vanishing pressure ($\Psi = -\Phi$) we get

$$
\begin{aligned}
\frac{\delta T_r}{T_r} = &- \left[\Psi \right]_e^r - \left[\vec{U} \cdot \vec{V} \right]_e^r \\
&- \int_e^r \left\{ -\frac{\partial \Psi}{\partial \eta} ({}_{(0)}U^\eta)^2 + \frac{\partial \Phi}{\partial \eta} \left[({}_{(0)}U^r)^2 + \frac{b^2}{a^2} ({}_{(0)}U^\theta)^2 \right] \right\} dw,
\end{aligned}
\tag{19}
$$

or yet,

$$
\frac{\delta T_r}{T_r} = - \left[\Psi \right]_e^r - \left[\vec{U} \cdot \vec{V} \right]_e^r + 2 \int_e^r \frac{\partial \Psi}{\partial \eta} dw. \tag{20}
$$

Note that the terms in square brackets of Equation (19)are equal to 1 as showed in Appendix B. Now, we should spell out the physical interpretation of each one of three factors of the right hand side of previous equation. When matter and radiation decoupled, free CMBR photons, climbing the gravitational potential generated by density perturbations, undergo a gravitational redshift, with corresponding loss of energy. The photon energy variation in this process is given by the term $[\Psi]_e^r \equiv \Psi(r) - \Psi(e)$. The second term, $\left[\vec{U}\cdot\vec{V}\right]_e^r \equiv \vec{U}\cdot\left(\vec{V}(r) - \vec{V}(e)\right)$, corresponds to the Doppler effect induced by the relative motion of the observer in the emission and reception events. The last term tells us that the perturbing potential may vary between the emission and reception instants.

The Doppler term has an observational meaning of a dipolar anisotropy on CMBR temperature and is usually removed from the equation to be treated aside. The last two terms are usually called the *Sachs-Wolfe effect* or also the *integrated Sachs-Wolfe effect*. For flat models of FLRW without cosmological constant, Ψ is time constant [12, 13], so the last term in the right han side of Equation (20) vanishes. In our case the cosmological constant is not vanishing, and indeed Λ will play an important role in our analysis. We consider values such that $\Omega_0 + \Omega_{\Lambda_0} \simeq 1$, taking into account recent observations [15, 16] which suggest $\Omega_0 \sim 0.3$ and $\Omega_{\Lambda_0} \sim 0.7$.

Equation (20) does not contain all physical processes which may generate fluctuations in CMBR temperature. It only accounts for kinematical effects undergone by the photons during their free propagation in a perturbed universe. So, we must also account for the intrinsic temperature fluctuations $\frac{\delta T_e}{T_e}$, originated by the set of physical and microphysical processes, associated to density perturbations in LSS. Despite its youth, the Universe is already highly isotropic (shear $\sigma \approx 0$). Then, for simplicity, we assume in this section that the Universe might be characterized by a flat FLRW model. Because the density fluctuations are very small, we may treat them in the context of linear theory of perturbations using Stefan-Boltzmann law

$$\rho_\gamma = \sigma_{SB} T^4, \tag{21}$$

where σ_{SB} is a constant and ρ_γ is the radiation density. Differentiating this equation one easily obtain

$$\frac{\delta T_e}{T_e} = \frac{1}{4}\frac{\delta\rho_\gamma}{\rho_\gamma}. \tag{22}$$

At this time, when the Universe is very young, the total energy density is not only due to radiation. The baryonic matter play an identical role, and so the matter density ρ_m is related with ρ_γ by

$$\frac{\delta\rho_m}{\rho_m} - \frac{3}{4}\frac{\delta\rho_\gamma}{\rho_\gamma} = 0, \tag{23}$$

if the perturbation mode is adiabatic and on scales larger than the horizon at this time [17]. Nevertheless, if the perturbations are isothermal, the matter distribution is perturbed without making significant changes in the radiation density [17]. In this case and while the perturbations remain outward the horizon, we have

$$\frac{\delta\rho_\gamma}{\rho_\gamma} \simeq 0, \tag{24}$$

so, the temperature inside a perturbed region with a dimension greater than the horizon remain with constant temperature ($\delta T_e \simeq 0$). In summary, for perturbations on scales greater than the horizon we may write

$$\frac{\delta T_e}{T_e} = \begin{cases} \frac{1}{3}\frac{\delta\rho_m}{\rho_m} & \Leftarrow \text{adiabatic pert.} \\ 0 & \Leftarrow \text{isothermal pert.} \end{cases} . \tag{25}$$

From the last expression we see that for adiabatic perturbations, the over-density (under-density) regions are intrinsically hotter (colder) than the LSS mean temperature. According with [12, 13], $\delta\rho_m/\rho_m = -2\Psi + \mathcal{O}[(k/H)^2]$, where k is the momentum associated to perturbation scale and H the Hubble parameter‖. The larger is the scale, the smaller is k, so, for perturbations greater than the horizon $k \ll H$, the over-density locals coincide with the potential well, because,

$$\frac{\delta T_e}{T_e} \simeq -\frac{2}{3}\Psi_e. \tag{26}$$

The measured temperature in a LSS point $T_e = T_e(\eta, x^i)$ may be related with the mean temperature $< T_e >$,

$$T_e =< T_e > \left(1 + \frac{\delta T_e}{T_e}\right). \tag{27}$$

Using the expression $T_r = \frac{a_e}{a_r} T_e \left(1 + \frac{\delta T_r}{T_r}\right)$ and substituting here the previous equation and retaining the first terms only, we obtain

$$T_r = \frac{a_e}{a_r} < T_e > \left(1 + \frac{\delta T_r}{T_r} + \frac{\delta T_e}{T_e}\right), \tag{28}$$

then, the total observed temperature fluctuation in r is

$$\frac{\delta T}{T} = \frac{\delta T_r}{T_r} + \frac{\delta T_e}{T_e}. \tag{29}$$

Substituting Equations (20) and (26) in (29) we have

$$\frac{\delta T}{T} = \frac{1}{3}\Psi_e - \vec{U} \cdot (\vec{V}_r - \vec{V}_e) + 2\int_e^r \frac{\partial\Psi}{\partial\eta} dw, \tag{30}$$

where without loss of generality we put $\Psi_r = 0$. The previous equation is valid if the observation is made for regions with angular scales containing the horizon ($\vartheta \gtrsim 3°$) in recombination epoch. If the perturbations are isothermal, the temperature fluctuations coincide with $\frac{\delta T_r}{T_r}$ and are given by (20).

Note that it is not necessary to include the second term of right hand side of Equation (30), since the dipolar anisotropy of the observer's motion is usually removed from the observation data. Finally we obtain

$$\frac{\delta T}{T} = \frac{1}{3}\Psi_e + 2\int_e^r \frac{\partial\Psi}{\partial\eta} dw, \tag{31}$$

which is the same expression obtained for FLRW universes for the same order of approximation, and for adiabatic initial conditions.

‖ Nevertheless our models have two Hubble parameters H_a and H_b, they have practically the same value $H_a \simeq H_b \simeq H$ due to our parameters choose.

3. Concluding Remarques

We stress once more we should bear in mind that the assumption $H_a \simeq H_b$ does not imply, by itself, an isotropic or even an almost isotropic metric, as is expressed by the growth of Weyl term in Equation (1), when we go back in time to the last scattering epoch. Although the Σ^2 term remains at a low value from the present ($\Sigma_0^2 \simeq 0$) to the last scattering epoch ($\Sigma_{ls}^2 \simeq 3 \times 10^{-13}$), the Weyl term, as we computed in Appendix E, grows from $\mathcal{W}_0^2 \simeq 10^{-19}$ at the present to $\mathcal{W}_{ls}^2 \sim 10^{-1}$ over the same period of time. This shows the anisotropic character of these models in the past. Even though we impose a high level of isotropy at present time, its anisotropic behavior comes forward as we go back in time. Nevertheless, the growth of $\mathcal{W}^2$ term does not affect decisively the first order computation of $\delta T/T$ term.

Because the obtained expression (for Sachs-Wolfe effect) is the same as the one given for FLRW models, we may conclude that, these anisotropic models are also good candidates to the description of observed Universe provided we may assume $H_a \simeq H_b$ and a particular choice of the density parameters: $\Omega_0 + \Omega_{\Lambda_0} \simeq 1$, from the last scattering to the present, see Appendix C). This is another step taken in the same direction as in [2]. This is also in agreement with a known result: it is not possible to distinguish a Kantowski-Sachs model from the FLRW models, with the classical tests of Cosmology, if the Hubble parameters along the orthogonal directions are assumed to be approximately equal [1].

There are now many CMBR experiments in preparation that will allow to make much higher detailed observations than presently. Experiments of particular importance are satellites (MAP [18], PLANK [19]), and interferometers (AMIBA [20], AMI [21], CBI [22]). The first, have the great advantage of mapping the sky globally, but interferometers can achieve higher resolution and therefore probe to angular power spectrum to very high ℓ. PLANK satellite best resolution is 5 arc minutes and present interferometers can now reach the 1 arc minute scale.

In conclusion, observation of Sachs-Wolfe effect plateau does not permit to distinguish between FLRW models and these anisotropic ones. To investigate this in more detail, it is necessary to consider and process the data from MAXIMA [23] and BOOMERANG [24] projects to regions smaller than the horizon at the last scattering ($\ell > 100$, $\vartheta < 1°$). Within this region of multipoles, perturbations are model dependent. Only with this information we may conclude finally whether our Universe goes through or not by a real anisotropic phase. This will be the purpose of further work.

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Appendix A. Estimate of an observer 4-velocity in a perturbed universe

We will consider a material observer which is moving along his worldline with a 4-velocity components

$$V^a = \frac{dx^a}{d\tau}, \tag{A.1}$$

where τ represents the proper time. Because he(she) is like a material particle, then he (she) is under condition $V^a V_a = -1$. So, considering the absolute value we obtain

$$\left|\bar{g}_{ab} V^a V^b\right| = 1, \tag{A.2}$$

or also

$$\begin{aligned} &\Big| -(1+h_{00})(V^0)^2 + (1+h_{11})(V^1)^2 + (1+h_{22})\frac{b^2}{a^2}(V^2)^2 \\ &+ (1+h_{33})\frac{b^2}{a^2} f^2(\theta)(V^3)^2 - (h_{01}+h_{10})V^0V^1 - \frac{b}{a}(h_{02}+h_{20})V^0V^2 \\ &- \frac{b}{a} f(\theta)(h_{03}+h_{30})V^0V^3 + \frac{b}{a}(h_{12}+h_{21})V^1V^2 + \frac{b}{a} f(\theta)(h_{13}+h_{31})V^1V^3 \\ &+ \frac{b^2}{a^2} f(\theta)(h_{23}+h_{32})V^2V^3 \Big| = 1. \end{aligned} \tag{A.3}$$

Making a first order approximation and since $V^i \ll 1$ and $h_{ab} \ll 1$ (with $i = 1, 2, 3$ and $a, b = 0, 1, 2, 3$), we may neglect products involving h_{ab} and V^i and also $(V^i)^2$, resulting

$$\left|-(1+h_{00})(V^0)^2\right| \simeq 1, \tag{A.4}$$

then

$$V^0 \simeq 1 - \frac{1}{2} h_{00} \tag{A.5}$$

thus, the 4-velocity vector of an observer in a perturbed universe may be written in first order approximation by

$$V^a \simeq \left(1 - \frac{1}{2} h_{00}, V^1, V^2, V^3\right). \tag{A.6}$$

Appendix B. Determination of photons 4-velocity in a non-perturbed universe

The photons 4-velocity in a non-perturbed universe ${}_{(0)}U^a$ may be reached recurring to geodesics equation in the metric $d\bar{\bar{s}}$

$$\frac{d_{(0)}\bar{U}_a}{dw} = \frac{1}{2} \bar{\bar{g}}_{bc,a\,(0)}\bar{U}^b{}_{(0)}\bar{U}^c. \tag{B.1}$$

For ϕ coordinate we get

$$\frac{d_{(0)}\bar{U}_\phi}{dw} = \frac{1}{2} \bar{\bar{g}}_{bc,\phi\,(0)}\bar{U}^b{}_{(0)}\bar{U}^c = 0, \tag{B.2}$$

because $\bar{\bar{g}}_{bc,\phi} = 0$, so ${}_{(0)}\bar{U}_\phi = const. \Rightarrow \frac{b^2}{a^2}\sin^2\theta \; {}_{(0)}\bar{U}^\phi = const.$ For $\theta = 0 \Rightarrow const. = 0 \Rightarrow {}_{(0)}\bar{U}^\phi = 0$. For θ coordinate we get ¶

$$\frac{d_{(0)}\bar{U}_\theta}{dw} = \frac{1}{2}\bar{\bar{g}}_{bc,\theta\,(0)}\bar{U}^b{}_{(0)}\bar{U}^c = \frac{b^2}{a^2}\sin\theta\cos\theta({}_{(0)}\bar{U}^\phi)^2 = 0, \tag{B.3}$$

because ${}_{(0)}\bar{U}^\phi = 0$. Then, ${}_{(0)}\bar{U}_\theta = const. = \sqrt{\beta} \Rightarrow {}_{(0)}\bar{U}^\theta = a^2/b^2\sqrt{\beta}$.
For r coordinate we get

$$\frac{d_{(0)}\bar{U}_r}{dw} = \frac{1}{2}\bar{\bar{g}}_{bc,r\,(0)}\bar{U}^b{}_{(0)}\bar{U}^c = 0, \tag{B.4}$$

because $\bar{\bar{g}}_{bc,r} = 0$, so ${}_{(0)}\bar{U}_r = const. = \sqrt{\alpha} \Rightarrow {}_{(0)}\bar{U}^r = \sqrt{\alpha}$. For a photon ${}_{(0)}\bar{U}_{a\,(0)}\bar{U}^a = 0 \Rightarrow ({}_{(0)}\bar{U}^\eta)^2 = ({}_{(0)}\bar{U}^r)^2 + b^2/a^2({}_{(0)}\bar{U}^\theta)^2 \Rightarrow {}_{(0)}\bar{U}^\eta = \sqrt{\alpha + a^2/b^2\beta}$. Thus the components $\bar{U}^a$ of 4-vector are the following $\left(\sqrt{\alpha + a^2/b^2\beta}, \sqrt{\alpha}, a^2/b^2\sqrt{\beta}, 0\right)$, and $\bar{U}_a$ will have the components $\left(-\sqrt{\alpha + a^2/b^2\beta}, \sqrt{\alpha}, \sqrt{\beta}, 0\right)$.

Determination of photons trajectory in a non-perturbed universe

Let us consider that at the emission instant the photon has coordinates $(\bar{x}^0_E = \eta_E,\ \bar{x}^1_E,\ \bar{x}^2_E,\ \bar{x}^3_E,\ w = 0)$ and at the reception instant has $(\bar{x}^0_E = \eta_R,\ \bar{x}^1_R = 0,\ \bar{x}^2_R = 0,\ \bar{x}^3_R = 0,\ w = \eta_R - \eta_E)$. In a general way we may consider the integral

$$\bar{x}^0 = \int_0^w {}_{(0)}\bar{U}^0 dw' + A, \tag{B.5}$$

where A is an integration constant that will be fixed by initial conditions of the problem. So, we get from previous equation that for the emission instant $(w = 0)$, $A = \eta_E$. Adding the reception data and substituting the ${}_{(0)}\bar{U}^0$ value, the integral becomes

$$\eta_R - \eta_E = \int_0^{\eta_R - \eta_E} \sqrt{\alpha + \frac{a^2}{b^2}\beta} dw. \tag{B.6}$$

During the photons travel from LSS the Hubble parameters preserve the relation $\dot{a}/a \simeq \dot{b}/b$, that is $a \propto b$. Thus

$$\sqrt{\alpha + \frac{a^2}{b^2}\beta} = const. \tag{B.7}$$

But by Equation (B.6) the constant must be 1, and ${}_{(0)}\bar{U}^0 = 1$ and ${}_{(0)}\bar{U}_0 = -1$. In conclusion, ${}_{(0)}\bar{U}^a = (1, \sqrt{\alpha}, a^2/b^2\sqrt{\beta}, 0)$ and ${}_{(0)}\bar{U}_a = (-1, \sqrt{\alpha}, \sqrt{\beta}, 0)$.

¶ For Bianchi type-III, we will have $\sinh\theta\cosh\theta$ instead of $\sin\theta\cos\theta$.

Appendix C. Numerical computation of an integral

The Einstein equations for Kantowski-Sachs and Bianchi type-III metrics with perfect fluid, vanishing pressure and cosmological constant Λ, are

$$2\,\frac{\dot{a}}{a}\,\frac{\dot{b}}{b}+\frac{\dot{b}^2}{b^2}+\frac{k}{b^2}=8\pi G\rho+\Lambda, \tag{C.1}$$

$$2\,\frac{\ddot{b}}{b}+\frac{\dot{b}^2}{b^2}+\frac{kc^2}{b^2}=\Lambda, \tag{C.2}$$

$$\frac{\ddot{a}}{a}+\frac{\ddot{b}}{b}+\frac{\dot{a}}{a}\,\frac{\dot{b}}{b}=\Lambda, \tag{C.3}$$

where ρ represents the baryonic matter density, G the gravitational constant and k takes on the values ± 1 for Kantowski-Sachs or Bianchi type-III, respectively. The first integral of Equation (C.2) leads to

$$\frac{\dot{b}^2}{b^2}=\frac{M_1}{b^3}-\frac{k}{b^2}+\frac{\Lambda}{3}, \tag{C.4}$$

where M_1 is an integration constant. We define the Hubble parameters corresponding to scale functions $a(\eta)$ and $b(\eta)$ as

$$H_a\equiv\dot{a}/a \quad \text{e} \quad H_b\equiv\dot{b}/b, \tag{C.5}$$

which may be used to define the density parameters, in analogy with which it is usely done in the FLRW models,

$$\frac{M_1}{b^3H_b^2}\equiv\Omega_M, \quad -\frac{k}{b^2H_b^2}\equiv\Omega_k \quad \text{e} \quad \frac{\Lambda}{3H_b^2}\equiv\Omega_\Lambda. \tag{C.6}$$

From (C.4) we get a conservation equation

$$\Omega_M+\Omega_k+\Omega_\Lambda=1. \tag{C.7}$$

Substituting (C.4) in (C.1) we get another relation that can be expressed, also, like a conservation condition

$$\Omega_\rho-\Omega_M+2\Omega_\Lambda=2\frac{H_a}{H_b}, \tag{C.8}$$

where $\Omega_\rho=M_\rho/(ab^2H_b^2)$, being M_ρ a constant proportional to matter of the Universe (for details see Aguiar & Crawford [2]). The matter density parameter Ω may be obtained from the previous by

$$\Omega=\frac{\Omega_\rho}{1+2\frac{H_a}{H_b}}. \tag{C.9}$$

From equations (C.1), (C.4), (C.7) and (C.8) we also obtain the differential equation

$$\frac{dx}{dy}=\frac{\frac{\Omega_{M_0}}{2}(1-\frac{x}{y})+\Omega_{\Lambda_0}(-1+xy^2)+\frac{H_{a_0}}{H_{b_0}}}{\Omega_{M_0}(1-y)+\Omega_{\Lambda_0}(y^3-y)+y} \tag{C.10}$$

where $x=a/a_0$, $y=b/b_0$ and the under script 0 denotes that the respective quantities are measured in actual epoch (see Aguiar & Crawford [2]).

Table C1. Density parameters, relative difference between H_a and H_b, and integral computation (I), for Kantowski-Sachs (KS) and Bianchi type-III (BIII) models.

	Ω_{M_0}	Ω_{Λ_0}	$\Omega_0 + \Omega_{\Lambda_0}$	$I/\frac{a_0^2}{b_0^2} \leq$	$\frac{\Delta H}{H_a}$
KS	1	$\lesssim 1.7 \times 10^{-8}$	$1 + 5.6 \times 10^{-09}$	$+5.9 \times 10^{-6}$	-1.6×10^{-6}
KS	$\lesssim 2 \times 10^{-15}$	1	$1 + 6.7 \times 10^{-16}$	$+6.8 \times 10^{-7}$	-1.4×10^{-6}
KS	$\lesssim 0.3 + 7.0 \times 10^{-9}$	0.7	$1 + 2.3 \times 10^{-09}$	$+3.5 \times 10^{-5}$	-1.7×10^{-6}
KS	0.3	$\lesssim 0.7 + 7.0 \times 10^{-9}$	$1 + 2.3 \times 10^{-09}$	$+3.5 \times 10^{-5}$	-1.7×10^{-6}
BIII	$1 - 10^{-10}$	$\gtrsim 9.9 \times 10^{-11}$	$1 - 3.3 \times 10^{-13}$	-1.4×10^{-5}	$+1.3 \times 10^{-6}$
BIII	$\gtrsim 9.8 \times 10^{-14}$	$1 - 10^{-13}$	$1 - 6.7 \times 10^{-16}$	-6.9×10^{-7}	$+1.3 \times 10^{-6}$
BIII	$\gtrsim 0.3 - 10^{-11}$	0.7	$1 - 3.3 \times 10^{-12}$	-1.1×10^{-5}	$+1.8 \times 10^{-6}$
BIII	0.3	$\gtrsim 0.7 - 10^{-11}$	$1 - 3.3 \times 10^{-12}$	-1.1×10^{-5}	$+1.8 \times 10^{-6}$

We are assuming that the two Hubble parameters along orthogonal directions are presently indistinguishable ($H_{a_0} = H_{b_0}$). With this restriction and using the previous equation we expect to compute numerically the integral

$$I = \int \frac{b^2}{a^2} \left(\frac{\dot{b}}{b} - \frac{\dot{a}}{a} \right) \left({}_{(0)}U^\theta \right)^2 dw. \tag{C.11}$$

From Appendix B we take ${}_{(0)}U^\theta$ + and as $d\eta/dw = {}_{(0)}U^\eta = 1$ we get

$$\begin{aligned} I = \beta \int \frac{a^2}{b^2} \left(\frac{\dot{b}}{b} - \frac{\dot{a}}{a} \right) dw &\leq \int \frac{a^2}{b^2} \left(1 - \frac{\dot{a}/a}{\dot{b}/b} \right) \frac{\dot{b}}{b} dw \\ &\leq \frac{a_0^2}{b_0^2} \left[\int \frac{x^2}{y^3} \left(1 - \frac{dx}{dy} \frac{y}{x} \right) dy \right]. \end{aligned} \tag{C.12}$$

The numerical integration is made between 1/1000 to 1.

If we choose the density parameters $\Omega_{M_0} + \Omega_{\Lambda_0}$ very near the unity, (under or over one), we get low values for the integral I, for Kantowski-Sachs or Bianchi type-III, respectively. In the box below we can see the numerical integration result for several combinations of density parameters Ω_{M_0} and Ω_{Λ_0}, without the multiplicative constant a_0^2/b_0^2. We computed also the percentual difference between Hubble parameters at $z = 1000$ ($\Delta H/H_a \equiv (H_a - H_b)/H_a$).

With the obtained values, if $a_0 \approx b_0$, the integral I is second order, then we neglect it, so the quantities $\frac{1}{2}h_{00}I$ and $2{}_{(1)}U^\theta I$ must also be neglected.

+ The β value may vary between 0 to 1 and tells us about the velocity percentage that a particle has in an angular direction (in this case the photon). Thus, if the velocity is purely radial $\beta = 0$, by opposition when the velocity is purely angular we have $\beta = 1$.

Appendix D. Relations between 4-vectors expressed in a conformal and non-conformal metrics

Let us consider the metrics $d\tilde{s}^2$ and $d\bar{\tilde{s}}^2$ (equations (2) and (3)). As we have $d\tilde{s}^2 = a^2 d\bar{\tilde{s}}^2$, then

$$g_{00} = \bar{g}_{00} \quad \text{and} \quad g_{ii} = a^2 \bar{g}_{ii},$$

with $i = 1, 2, 3$. On the other hand, the conformal parameter obeys to $dt = ad\eta$. This allows us to establish the relation between the photon 4-vectors for these frames. Labeling $x^a(\xi)$ and $\bar{x}^a(w)$ the photon coordinates for these two frames, then the respective 4-vectors will be

$$U^a = \frac{dx^a}{d\xi} \quad \text{and} \quad \bar{U}^a = \frac{d\bar{x}^a}{dw} \tag{D.1}$$

thus,

$$U^0 = \frac{dx^0}{d\xi} = \frac{dt}{d\xi} \tag{D.2}$$

and

$$\bar{U}^0 = \frac{d\bar{x}^0}{dw} = \frac{d\eta}{dw} = \frac{1}{a}\frac{dt}{d\xi}\frac{d\xi}{dw} = \frac{1}{a}\frac{d\xi}{dw}U^0, \tag{D.3}$$

and the covariant component is

$$\bar{U}_0 = \bar{g}_{00}\bar{U}^0 = g_{00}\frac{1}{a}\frac{d\xi}{dw}U^0 = \frac{1}{a}\frac{d\xi}{dw}U_0. \tag{D.4}$$

As $x^i(\xi) = \bar{x}^i(w)$ and $U^i = \frac{dx^i}{d\xi}$, it comes to

$$\bar{U}^i = \frac{d\bar{x}^i}{dw} = \frac{dx^i}{d\xi}\frac{d\xi}{dw} = \frac{d\xi}{dw}U^i \tag{D.5}$$

and

$$\bar{U}_i = \bar{g}_{ii}\bar{U}^i = \frac{1}{a^2}g_{ii}\frac{d\xi}{dw}U^i = \frac{1}{a^2}\frac{d\xi}{dw}U_i. \tag{D.6}$$

Thus,

$$\begin{aligned}\bar{U}^a\bar{U}_a &= \bar{U}^0\bar{U}_0 + \bar{U}^i\bar{U}_i = \frac{1}{a}\frac{d\xi}{dw}U^0\frac{1}{a}\frac{d\xi}{dw}U_0 + \frac{d\xi}{dw}U^i\frac{1}{a^2}\frac{d\xi}{dw}U_i \\ &= \frac{1}{a^2}\left(\frac{d\xi}{dw}\right)^2(U^0U_0 + U^iU_i) = \frac{1}{a^2}\left(\frac{d\xi}{dw}\right)^2 U^aU_a.\end{aligned} \tag{D.7}$$

Although the photon 4-vectors must have null norms in the two frames, its ratio is defined by

$$\frac{\bar{U}^a\bar{U}_a}{U^aU_a} = \frac{1}{a^2}\left(\frac{d\xi}{dw}\right)^2. \tag{D.8}$$

It is easily seen that the temporal components*, in the conformal and expansion frames, we should have

$$\bar{U}^0 = \frac{d\eta}{dw} = 1 \quad \text{and} \quad U^0 = \frac{dt}{d\xi} = \frac{1}{a},$$

* Note that using the geodesic equation $\frac{dU_a}{d\xi} = \frac{1}{2}g_{bc,a}U^bU^c$, we obtain $U_r = \sqrt{\alpha}$, $U_\theta = \sqrt{\beta}$ and as $U^aU_a = 0$ we get $U^a = (1/a, \sqrt{\alpha}/a^2, \sqrt{\beta}/b^2, 0)$ and $U_a = (-1/a, \sqrt{\alpha}, \sqrt{\beta}, 0)$, because $\sqrt{\alpha + \frac{a^2}{b^2}\beta} = 1$.

thus, the η and w parameters are both conformal, and because this frame has no expansion, the photon 4-vector temporal component ($\bar{U}^0$) has no changes, in opposition, in the expansion frame this component decrease in an expansion phase. From the below relations and from (D.3) we obtain

$$\frac{d\xi}{dw} = a^2. \tag{D.9}$$

Let us now obtain the relationship between the 4-velocity vectors in these two frames, for material particles.

Let it be V^a and $\bar{V}^a$ the respective 4-vectors for the expansion and conformal frames. For material particles, these 4-vectors have the same scalar product, which is then frame independent

$$V^a V_a = -1 = \bar{V}^a \bar{V}_a, \tag{D.10}$$

thus,

$$g_{00}(V^0)^2 + g_{ii}(V^i)^2 = \bar{g}_{00}(\bar{V}^0)^2 + \bar{g}_{ii}(\bar{V}^i)^2. \tag{D.11}$$

As $g_{00} = \bar{g}_{00}$,

$$g_{00}(V^0)^2 + g_{ii}(V^i)^2 = g_{00}(\bar{V}^0)^2 + \frac{1}{a^2} g_{ii}(\bar{V}^i)^2. \tag{D.12}$$

The relation below allows us to conclude that if

$$V^0 = \bar{V}^0 \quad \text{then} \quad V^i = \frac{1}{a}\bar{V}^i. \tag{D.13}$$

Indeed, as we know

$$dx^0 = a d\bar{x}^0 \quad \text{and} \quad dx^i = d\bar{x}^i. \tag{D.14}$$

Then, we may write

$$V^a = \frac{dx^a}{d\tau_1} \quad \text{and} \quad \bar{V}^a = \frac{d\bar{x}^a}{d\tau_2}, \tag{D.15}$$

where τ_1 and τ_2 are, respectively, the proper time for the material particles in the considered frames. Using relation (D.10) one gets

$$-d\tau_1^2 = -(dx^0)^2 + a^2(dx^1)^2 \quad \text{and} \quad -d\tau_2^2 = -(d\bar{x}^0)^2 + a^2(d\bar{x}^1)^2. \tag{D.16}$$

By Equation (D.14) and calculating the ratio of τ_1 and τ_2 we obtain

$$\frac{d\tau_1^2}{d\tau_2^2} = \frac{-a^2(d\bar{x}^0)^2 + a^2(d\bar{x}^1)^2}{-(d\bar{x}^0)^2 + (d\bar{x}^1)^2} \tag{D.17}$$

or also

$$d\tau_1 = a d\tau_2, \tag{D.18}$$

the choosing of the positive sign is obvious. Considering the first equation of (D.14), we may write

$$\frac{dx^0}{d\tau_1}\frac{d\tau_1}{d\tau_2} = a\frac{d\bar{x}^0}{d\tau_2}, \tag{D.19}$$

or

$$V^0 a = a\bar{V}^0 \quad \Rightarrow \quad V^0 = \bar{V}^0 \tag{D.20}$$

as we claimed above. So,

$$V_0 = \bar{V}_0 \quad \text{and} \quad V_i = g_{ii}V^i = \bar{g}_{ii}a^2\frac{1}{a}\bar{V}^i = a\bar{V}_i. \tag{D.21}$$

An observer measures the photon energy for these two frames as the time component of the photon 4-vector in his proper frame,

$$\begin{aligned} E(t) &\propto U^a V_a = U^0 V_0 + U^i V_i \\ &= \frac{1}{a}\bar{U}^0\bar{V}_0 + \frac{1}{a^2}\bar{U}^i a\bar{V}_i \\ &= \frac{1}{a}\bar{U}^a\bar{V}_a \propto \frac{1}{a}\bar{E}(w), \end{aligned} \tag{D.22}$$

as the proportional constant is the same, then

$$E(t) = \frac{1}{a}\bar{E}(w). \tag{D.23}$$

Appendix E. Computation of Σ^2 and $\mathcal{W}^2$

For Kantowski-Sachs and Bianchi type-III models, the shear tensor is such that

$$\sigma_{ab}\sigma^{ab} = 2\sigma^2 = \frac{2}{3}(H_b - H_a)^2 \tag{E.1}$$

as usually, H_a and H_b are the Hubble parameters in orthogonal directions. So, taking the definition given in Equation (1)

$$\Sigma^2 \simeq \frac{1}{9}\left(\frac{H_b - H_a}{H_a}\right)^2, \tag{E.2}$$

because $H \simeq H_a \simeq H_b$. As we see in Table C1 (Appendix C) $\left(\frac{\Delta H}{H_a}\right)_{ls} \sim 1.7 \times 10^{-6}$ for last scattering and $\left(\frac{\Delta H}{H_a}\right)_0 \sim 0$ presently. Thus, $\Sigma^2_{ls} \sim 3 \times 10^{-13}$ and $\Sigma^2_0 \sim 0$.

Given a 4-velocity field, U^a, of an observer set, one may split the Weyl tensor C_{abcd} in the electric (E_{ab}) and magnetic (H_{ab}) parts,

$$E_{ac} = C_{abcd}U^bU^d, \tag{E.3}$$

$$H_{ac} = \frac{1}{2}C^*_{abcd}U^bU^d, \tag{E.4}$$

where C^*_{abcd} is the dual of C_{abcd}. For a comoving observer, the magnetic part is null and the electric part is such that

$$E_{ab}E^{ab} = \frac{1}{6}\left(-\frac{\ddot{a}}{a} + \frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{\ddot{b}}{b} - \frac{k}{b^2} - \frac{\dot{b}^2}{b^2}\right)^2. \tag{E.5}$$

Using Equations (C.2) and (C.3) we get

$$E_{ab}E^{ab} = \frac{2}{3}\left(\frac{\ddot{a}}{a} - \frac{\ddot{b}}{b}\right)^2, \tag{E.6}$$

redefining the scale factors as $x = a/a_0$ and $y = b/b_0$ and using Equations (2.12) and (2.22) of Aguiar & Crawford [2] we derived $\dot{x}$ and $\dot{y}$ and after some algebraic manipulation we get

$$E_{ab}E^{ab} = \frac{1}{6}H_{b_0}^4 \left[\Omega_{M_0}\left(\frac{3}{y^3} - \frac{1}{xy^2}\right) + 2\Omega_{\Lambda_0}\frac{1}{xy^2} - 2\frac{H_{a_0}}{H_{b_0}}\frac{1}{xy^2}\right]^2. \tag{E.7}$$

For t_0, $x = y = 1$ and as $H_{a_0} = H_{b_0}$ we have

$$E_{ab}E^{ab} = \frac{2}{3}H_{b_0}^4(\Omega_{M_0} + \Omega_{\Lambda_0} - 1)^2. \tag{E.8}$$

We chose Ω_{M_0} and Ω_{Λ_0} parameters such that its sum was near the unit, in order to have high a level of isotropy

$$|\Omega_{M_0} + \Omega_{\Lambda_0} - 1| \simeq 10^{-9}. \tag{E.9}$$

Thus, for t_0 we have

$$\mathcal{W}_0^2 = \frac{1}{9}(\Omega_{M_0} + \Omega_{\Lambda_0} - 1)^2 \sim 10^{-19}, \tag{E.10}$$

and for the last scattering epoch $x \simeq y \simeq 1/1000$ we get

$$\mathcal{W}_{ls}^2 = \frac{1}{9}\left(\frac{\Omega_{M_0} + \Omega_{\Lambda_0} - 1}{10^{-9}}\right)^2 \sim 10^{-1}. \tag{E.11}$$

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