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# Herd Behavior and Contagion in Financial Markets

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# Herd Behavior and Contagion in Financial Markets*

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## Abstract

We study a sequential trading financial market where there are gains from trade, that is, where informed traders have heterogeneous private values. We show that an informational cascade (i.e., a complete blockage of information) arises and prices fail to aggregate information dispersed among traders. During an informational cascade, all traders with the same preferences choose the same action, following the market (herding) or going against it (contrarianism). We also study financial contagion by extending our model to a two-asset economy. We show that informational cascades in one market can be generated by informational spillovers from the other. Such spillovers have pathological consequences, generating long-lasting misalignments between prices and fundamentals.

**KEYWORDS:** herd behavior, financial contagion, social learning, informational cascades, financial crises

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# 1 Introduction

The last two decades have witnessed a series of major international financial crises, for example, in Mexico in 1995, Southeast Asia in 1997-8, Russia in 1998 and Brazil in 1998-9. These episodes have revived interest among economists in the study of the financial system fragility. A common finding in much of the empirical work on financial crises (see, e.g., Kaminsky, 1999) is that the fundamentals of an economy help to predict when a crisis will occur, but crises may occur even when the fundamentals are sound or may not occur even when the fundamentals are weak. A possible explanation for why sound fundamentals may not be reflected in asset prices is that information about these fundamentals is spread among investors, and prices may fail to aggregate it. In particular, this would happen if investors, instead of acting according to their own private information, simply decided to herd.

The herd-like behavior of market participants is often linked to another feature of financial markets, that is, the strong co-movements among seemingly unrelated financial assets. In 1997, for instance, financial asset prices plunged in most emerging markets, following the financial crisis that hit some Asian economies. This high degree of co-movement across markets that differ in size, structure and location around the world is not a peculiarity of the Asian crisis. Indeed, it is a common and well-documented regularity of financial markets.

In this paper, we show that, in contrast with previous findings in the literature, informational cascades (i.e., situations in which every agent chooses the same action regardless of his own private information) can indeed arise in financial markets. As a consequence, financial markets can fail to aggregate private information efficiently, and misalignments of the price with respect to the fundamental can occur. Furthermore, we show that informational cascades can spread from one market to another, thus generating financial contagion. While the case of financial crises serves to motivate our work, we do not consider our study descriptive of any particular episode of crisis. The central aim of the paper is instead to offer a theoretical contribution by showing how informational cascades can occur in a market and transmit from one market to another.

To discuss informational cascades and financial contagion, we study an economy à la Glosten and Milgrom (1985) in which privately informed traders, in sequence, trade an asset with a market maker. Traders are heterogenous, for example, because of differences in endowments or intertemporal preferences. Therefore, trading can be mutually beneficial; that is, there are gains from trade. Traders trade for two reasons: they have an informational advantage over the market maker (due to private information), and they have a gain to trading. We find that, eventually, as trades accumulate, the gain from trade overwhelms the importance of the informational advantage; therefore, traders choose their action independently of their information on the asset value, that is, an informational cascade occurs. During an informational cascade, all informed traders choose the same action; they either follow the market (herding) or go against it (contrarianism). Given that agents do not use their own information, private information is not aggregated, and prices may not reflect the true value of the assets.

After illustrating our argument for informational cascades, we discuss how cascades can lead to financial contagion. We study an economy in which traders trade two correlated assets. The history of trades in one market can permanently affect the price path of the other; as a result, the price converges to a different value from that to which it would have otherwise converged. Informational spillovers are to be expected between correlated asset markets. With gains from trade, however, these informational spillovers can have pathological outcomes. Informational cascades in one market generate cascades in another, pushing the prices, even in the long run, far from the fundamentals. This long-lasting spillover represents a form of contagion: a crisis or a boom in one market transmits itself to the other without regard to the fundamentals.

Our paper is part of the theoretical literature on social learning. Several models of social learning have shown that herding is not necessarily an irrational phenomenon, and cascades can indeed occur in societies. Their explanation, however, cannot be directly applied to financial markets. The theoretical research on herd behavior started with the seminal papers by Banerjee (1992), Bikhchandani et al. (1992), and Welch (1992).[1] These papers do not discuss herd behavior in financial markets, but in an abstract environment, where agents with private information make their decisions in sequence. They show that, after a finite number of agents have chosen their actions, all following agents disregard their own private information (i.e.,

[1]Note that here we only consider models of informational herding. We do not discuss models of herding due to reputational reasons (see, e.g., Scharfstein and Stein, 1990) or payoff externalities. In a recent paper, Dasgupta and Prat (2008) show that reputational concerns generate herding in a financial market with sequential trading.

an informational cascades arises) and herd.[2] This is an important result because it provides a rationale for the imitating behavior that we observe in consumers' and investors' decisions. In these first models of herding, however, the cost of taking an action (e.g., investing in a new project) is held constant. In other words, these models do not analyze situations where the price of a good changes when agents make the decisions to buy or sell it. Therefore, they are not appropriate for a discussion of herd behavior in financial markets, where prices are certainly flexible and react to the order flow.

More recently, Avery and Zemsky (1998) have studied herd behavior in a financial market where the price is efficiently set by a market maker according to the order flow. They show that the presence of an efficient price mechanism makes an informational cascade impossible.[3] Agents always find it optimal to trade on the difference between their own information (the history of trades and the private signal) and the commonly available information (the history of trades only). For this reason, the price aggregates the information contained in the history of past trades correctly.[4]

The difference between Avery and Zemsky's (1998) results and ours stems from a crucial assumption. In their model, informed traders are all homogeneous so the market does not help them to realize any gain from trade. In contrast, in our work, informed traders are heterogeneous

[2] This early work has been extended in many directions: for instance, Chamley and Gale (1994) allow agents to choose their position in the sequence; Smith and Sørensen (2000) generalize the results on cascades by considering different distributions of private beliefs and heterogeneous agents. For a critical review see, e.g., Gale (1996), Bikhchandani and Sharma (2001), Hirshleifer and Theo (2003), Chamley (2004), and Vives (2008).

[3] Avery and Zemsky (1998) show that, when there there is multidimensional uncertainty (i.e., uncertainty not only on the direction of a shock to the asset fundamental, but also on the existence of the shock itself), herd behavior can arise even in their framework. Their definition of herding, however, is not the standard one in the literature (see footnote 24). Even with multidimensional uncertainty, informational cascades cannot arise in their study (see their Proposition 2 and their comments at page 733). See also the considerations of Brunnermeier (2001, p. 179), Chari and Kehoe (2004, p. 144) and of Hirshleifer and Teoh (2003, pp. 39-40).

[4] The theoretical contributions on informational herding in financial markets are few (see, e.g., Lee, 1998; Chari and Kehoe, 2004; Décamps and Lovo, 2006; and Park and Sabourian, 2008). For empirical evidence, see, e.g., Lakonishok et al. (1992), Grinblatt et al. (1995), Wermers (1999), Sias (2004), Cipriani and Guarino (2006) and the other references in Hirshleifer and Teoh (2003). For experimental evidence, see Cipriani and Guarino (2005a, 2005b, 2008a, 2008b), Drehmann et al. (2005, 2007) and Park and Sgroi (2008).

because they value the asset differently. Traders' private values originate from differences in time preferences or from liquidity and hedging reasons to trade. As a result of traders' heterogeneity, there are gains from trade that can be realized in the market; the presence of gains from trade makes the aggregation of private information inefficient.

Our paper also offers a contribution to the literature on financial contagion. This literature tries to explain why, in financial markets, we observe co-movements across asset prices and clustering of financial crises that are difficult to explain in terms of common shocks (such as a change in the level of the international interest rate or in the price of commodities). The closest paper to ours is that by King and Wadhwany (1990), which studies contagion due to correlated information in a rational expectation model. In their model, asset values depend on a common component and an idiosyncratic one. In the presence of asymmetric information, changes in one asset's idiosyncratic component will affect the other asset's price (since, with some probability, they will be interpreted as changes in the common component). In our paper contagion is due to information spillovers and informational asymmetries as well. In contrast to King and Wadhwany (1990), however, we study a sequential trading model. This allows us to show how, because of informational cascades, the sequence of trades matters for financial contagion: specific sequences of trades, through informational spillovers from one market to the other, generate informational cascades and have long-lasting pathological effects in the other market. Moreover, we show that such contagious effects can also occur when the fundamental values of the assets are independent, as long as there are other channels through which informational spillovers occur from one market to the other (e.g., because of correlated liquidity shocks).

Other papers have explained financial market contagion through different channels. Calvo (1999) presents a model of contagion through liquidity shocks: when agents are hit by a liquidity shock in one market, they also liquidate assets in other markets in order to meet a call for additional collateral, thus transmitting the shock to other markets. Kodres and Pritsker (2002) and Yuan (2005) study contagion in a rational expectations model of financial markets. In Kodres and Pritsker (2002), contagion happens through cross-market rebalancing: when agents are hit by a shock in one market, they need to rebalance their portfolio of assets; the presence of asymmetric information exacerbates the price co-movements resulting from this rebalancing. In Yuan (2005), contagion arises because of the interaction between asymmetric information and borrowing constraints. Kyle and Xiong (2001) show how financial contagion can also be due to wealth effects. Fostel and Geanakoplos (2008) show that financial contagion can arise as a result of the interplay between market incompleteness, agents' heterogeneity and margin requirements. Mondria (2006) shows that financial contagion can arise even when two markets are independent because of investors' shifts of attention toward regions hit by a financial crisis. Other authors (e.g., Corsetti et al., 1999; Kaminsky and Reinhart, 2000; Rigobon, 2002) focus on currency crises and study the factors (e.g., incorrect monetary or fiscal policies) that lead to simultaneous speculative attacks. Finally, the literature on financial contagion is part of the broader literature on contagion. For instance, Allen and Gale (1998, 2000, 2007) and Lagunoff and Shreft (2001) study financial system fragility due to contagious effects among financial institutions.

The structure of the paper is as follows. Section 2 presents the model for the case of a one-asset economy. Section 3 discusses the main results on informational cascades and herd behavior. Section 4 illustrates cascades and financial contagion in a two-asset economy. Section 5 concludes. The Appendix contains all the proofs.

## 2 The Model

**The Market** An asset is traded by a sequence of traders who interact with a market maker. Time is represented by a set of trading dates indexed by $t = 1, 2, 3, ....$ At each time $t$, a trader can exchange the asset with the market maker. The trader can buy, sell or decide not to trade. Each trade consists of the exchange of one unit of the asset for cash. The trader's action space is, therefore, $\mathcal{A} = \{buy, sell, no\ trade\}$. We denote the action of the trader at time $t$ by $x_t \in \mathcal{A}$. Moreover, we denote the history of trades and prices until time $t-1$ by $h_t$.

At any time $t$, the market maker sets the prices at which a trader can buy or sell the asset. When posting these prices, he must take into account the possibility of trading with agents who (as we see below) have some private information on the asset value. Therefore, he sets different prices at which he is willing to sell and to buy the asset; that is, there will be a bid-ask spread (Glosten and Milgrom, 1985). We denote the ask price (i.e., the price at which a trader can buy) at time $t$ by $a_t$ and the bid price (i.e., the price at which he can sell) by $b_t$.

**The Asset Value** Market participants assign random utility $KV$ to one share of the asset, where $V$ represents the common component (or the "fundamental") of the asset value and $K$ its private component. The common component, $V$, is a random variable taking values $v^L$ and $v^H$ ($v^H > v^L$) with probabilities $(1-p)$ and $p$.[5] [6] The private component $K$, which reflects agents' heterogeneity, is a positive discrete random variable.

The common component of the asset value reflects the present value of the security's cash flow and is realized after the trade has occurred. The private values, in contrast, are known in advance of the trade. Trading in the market can happen because of informational reasons (i.e., because traders have private information on $V$) or because of non-informational reasons (i.e., because traders and market maker have different private values).

Of course, in actual markets there can be several sources of heterogeneity (i.e., of different private values) among market participants, and these could be microfounded in different ways. For instance, heterogeneity can stem from different preferences of traders over present and future consumption, in which case $K$ would represent traders' and market maker's subjective discount factors (for a formal microfoundation, see the Appendix).

Another source of heterogeneity among traders could be differences in endowments. For instance, in Bhattacharya and Spiegel (1991) or Wang (1994), traders are endowed with non-tradable labor income or with private investment opportunities correlated with the asset payoff.[7] Therefore, they have a non-informational motive to trade, since they need to hedge the risk of the non-tradable payoff. Similarly, in Dow and Rahi (2003), traders have a non-informational reason to trade as they hedge the risk stemming from a stochastic endowment. Such a non-informational reason to trade is summarized in our model by the private component of the asset value.

Finally, we can also interpret the parameter $K$ as "the result of imperfect

[5]Note that, throughout the paper, we use capital letters to indicate random variables and lowercase letters to indicate their realizations. Note also that, although $x_t$ can equal *buy*, *sell* or *no trade*, for convenience, we treat it as the realization of a real-valued random variable $X_t$ (as if it took values in $\{-1, 0, 1\}$). The same comment applies to $h_t$.

[6]In presenting our model with only two values, we have followed much of the literature in both social learning and market microstucture. It is conceptually easy but algebraically quite costly to show the results in a set up with a finite set of values and of signals. The interested reader can find the proofs for this more general case in Cipriani and Guarino (2004).

[7]This source of heterogeneity is also studied in Décamps and Lovo (2006). They analyze the case in which heterogeneity arises from shocks to the wealth of risk-averse traders in a model similar to that of Glosten (1989).

access to capital markets or ... differential subjective assessments of the distribution of the random variable ....” (Glosten and Milgrom, 1985). For the rest of the paper, we do not restrict ourselves to any of these different interpretations, but just use the reduced form of gains from trade presented above.

**The Market Maker** As in Glosten and Milgrom (1985) and in the related literature, we assume that unmodeled potential (Bertrand) competition forces the market maker to set prices so as to make zero expected profits in each period $t$. The market maker observes the history of traders' decisions and prices until time $t-1$, $h_t$. When setting the prices, the market maker takes into account not only the information conveyed by $h_t$, but also the information conveyed by the time $t$ decision to buy or to sell the asset. Bertrand competition implies that the equilibrium bid (ask) will be the highest (lowest) price satisfying the zero expected profit condition.

As a normalization, we set the private component $K$ for the market maker equal to 1. Hence, the equilibrium bid and ask prices at time $t$ has to satisfy the following conditions:

$$b_t := \max\{b \in [v^L, v^H] : b = E(V|h_t, X_t = sell, a_t, b)\}, \tag{1}$$

$$a_t := \min\{a \in [v^L, v^H] : a = E(V|h_t, X_t = buy, a, b_t)\}. \tag{2}$$

Note that the expected values are conditioned on the bid and the ask prices themselves, since the traders' decisions (and, therefore, the informational content of a trade) depend on the bid and ask prices that the traders face.[8] Therefore, the equilibrium ask and bid prices are fixed points.

Finally, we denote the expected value of asset $V$ at time $t$, *before the trader in $t$ has traded*, by $p_t$, that is,

$$p_t = E(V|h_t). \tag{3}$$

We will refer to $p_t$ as the “price” of the asset.[9] In some cases, we will find it convenient to abstract from the bid-ask spread and discuss our results in terms of $p_t$.

[8] The market maker posts the bid and ask prices at $t$ before the trader at time $t$ makes his decision.

[9] Note that $p_t$ is the price at which a transaction takes place at time $t-1$ if at $t-1$ there was a transaction (i.e., if the trader did not abstain from trading). Moreover, as we see below, $p_t$ is always between the bid and ask price of the asset, thus resembling a mid-price.

**The Traders** There are a countably infinite number of traders. Traders act in an exogenously determined sequential order. Each trader, indexed by $t$, is chosen to take an action only once, at time $t$. Traders are of two types, informed and uninformed (or noise). The trader's type is not known publicly, that is, it is his private information. At each time $t$, with probability $\mu$, the trader arriving in the market is an informed trader and, with probability $(1-\mu)$, he is a noise trader.

***Noise traders*** Uninformed (or noise) traders trade for unmodeled (e.g., liquidity) reasons: they buy, sell or do not trade the asset with exogenously given probabilities. For convenience, we assume that in each period in which they are called to trade, they buy, sell or do not trade with equal probability.[10]

***Informed traders*** Informed traders know their own private component and have private information on the asset value's common component. If at time $t$ an informed trader is chosen to trade, he observes a private signal $S_t$ on the realization of $V$. $S_t$ is a symmetric binary signal, taking values $s^L$ and $s^H$ with precision $q > \frac{1}{2}$; that is, $\Pr(S_t = s^L|V = v^L) = \Pr(S_t = s^H|V = v^H) = q$. Note that, conditional on the realization of $V$, the random variables $S_t$ are i.i.d. In addition to his signal, an informed trader at time $t$ observes the history of trades and prices. Therefore, his expected value of the asset is $E(V|h_t, s_t)$.

For simplicity's sake, we assume that each informed trader's private component $K$ can take only two values, $l$, $g$, with $l < 1 < g$. In other words, a trader either has a gain ($K = g$) or suffers a loss ($K = l$) from holding the asset. We denote the private component $K$ of a trader trading at time $t$ by $K_t$, assume that the sequence of $K_t$ is i.i.d. and that $K_t$ equals $l$ or $g$ with equal probability.[11] Obviously, the realization of $K_t$ is not publicly known; that is, it is private information to the trader.

The informed traders' payoff function $U : \{v^L, v^H\} \times \mathcal{A} \times [v^L, v^H]^2 \times$

[10]Noise traders with inelastic demand are a common feature of market microstructure models. Their presence guarantees that the market does not collapse because of the asymmetric information between informed traders and the market maker. Note that, although in our model informed traders are heterogeneous, their demand functions are not inelastic with respect to the price (as we see below). The heterogeneity of informed traders is not sufficient to assure that the market does not collapse after any history of trades.

[11]Note that the assumption that $K_t$ are i.i.d. is introduced only for convenience. All our results are trivially robust to relaxing the assumption.

$\{l, g\} \longrightarrow \mathbf{R}^+$ is defined as[12]

$$U(v, x_t, a_t, b_t, k_t) = \begin{cases} k_t v - a_t & \text{if } x_t = buy, \\ 0 & \text{if } x_t = no\ trade, \\ b_t - k_t v & \text{if } x_t = sell. \end{cases} \tag{4}$$

Informed traders choose $x_t$ to maximize $E(U(\cdot)|h_t, s_t)$. Note that we are assuming that the event "the trader is informed" and the event "the trader has a private value $l$ or $g$" are independent of the realized value of $V$. Therefore, the knowledge of these events does not convey any information on the asset value to the trader.

An informed trader's payoff depends on his private component $K_t$. In particular, an agent with a gain $g$ from holding the asset buys whenever $gE(V|h_t, s_t) > a_t$ and sells whenever $gE(V|h_t, s_t) < b_t$. Analogously, a trader with a loss $l$ from the asset buys whenever $lE(V|h_t, s_t) > a_t$ and sells whenever $lE(V|h_t, s_t) < b_t$. Finally, if $b_t < k_t E(V|h_t, s_t) < a_t$, the trader does not trade. If a trader is indifferent between not trading and buying (i.e., $k_t E(V|h_t, s_t) = a_t$) or between not trading and selling (i.e., $k_t E(V|h_t, s_t) = b_t$), he randomizes between the two actions.

**Parametric assumptions** Throughout the paper, we make two assumptions on the parameter values. First, we assume that $v^L > 0$. The assumption guarantees that the agents' expectations are bounded away from zero. If expectations were allowed to converge to zero, the non-informational reasons to trade would vanish, and there would be no heterogeneity in the market. Second, we assume that $l, g \in (\frac{v^H + v^L}{2v^H}, \frac{v^H + v^L}{2v^L})$. With this condition, we rule out the case where there exist bid and ask prices at which the private component determines a trader's choice, no matter what the precision of his private information is.[13]

[12] In the case of a sale, the payoff can result either from the trader being endowed with the asset or from the trader being allowed to short sell the asset.

[13] To see this, let us discuss what happens when the assumption does not hold. Consider a trader with a gain $g > \frac{v^H + v^L}{2v^L}$. Even if he knew that the asset value is $v^L$, his evaluation of the asset would, nevertheless, be greater than $\frac{v^H + v^L}{2}$. Similarly, a trader with a loss $l < \frac{v^H + v^L}{2v^H}$ would value the asset less than $\frac{v^H + v^L}{2}$ even if he knew that the asset value is $v^H$. As a result, as long as the bid and ask prices are close to $\frac{v^H + v^L}{2}$,

In other words, the two assumptions exclude two extreme (and uninteresting) cases. The first assumption excludes that non-informational reasons to trade (modeled as a multiplicative parameter) vanish as the expectations converge towards one of the possible realizations of the asset value. The second excludes that non informational reasons to trade are so strong that they overwhelm informational reasons to trade, no matter how precise the private signal.

Finally, the two assumptions imply that the traders' expectations and private values are all strictly positive. As a result, an informed trader's payoff from the asset, $k_t E(V|h_t, s_t)$, is increasing in both $k_t$ and $E(V|h_t, s_t)$. This rules out both the case where some traders value the asset more if its realization is $v^L$ than if it is $v^H$ and the case where $g$ represents a gain for one realization of the asset value and a loss for the other.

## 2.1 Preliminary Results

Before proceeding to the main analysis, let us provide some results on the properties of the market prices:

**Proposition 1** ***(Existence and Uniqueness of Bid and Ask Prices)*** *At each time $t$, there exists a unique bid and ask price. Moreover, $b_t \leq p_t \leq a_t$.*

It is also useful to remark that the sequence of prices is a martingale with respect to the history of trades and prices, since $p_t$ is an expectation conditional on all public information available until time $t$.[14] This property will be important to prove some of our results.

# 3 Informational Cascades and Herd Behavior

In this section, we show how in our economy the prices fail to aggregate private information correctly. Indeed, there will be a time when information stops flowing to the market, and the prices may remain stuck at a level far from the fundamental value of the asset. This blockage of information is called an informational cascade.

In order to present our results, let us first introduce a formal definition:

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the private component always overwhelms the informational reasons to trade for any precision of the signal.

[14] The result immediately follows from the law of iterated expectations and from the fact that the price is a bounded random variable (because it is the expected value of the fundamental, which is itself bounded).

**Definition 1** *An informational cascade arises at time $t$ when all informed traders act independently of their own signal.*

During an informational cascade, an informed trader makes the same trading decision whatever signal he may receive: the probability of an action is independent of the private signal, that is, $\Pr(X_t = x|h_t, a_t, b_t, S_t = s) = \Pr(X_t = x|h_t, a_t, b_t)$ for all $x$ and all $s$. Hence, a trader's private information is not revealed by his action. As a result, the market maker will be unable to infer the traders' private information from their actions and will be unable to update his beliefs on the asset value. In other words, in an informational cascade, trades do not convey any information on the asset value.

We now show that, as more trades arrive in the market, an informational cascade arises almost surely. Early on in the process of trading, when there is a fair amount of uncertainty and, therefore, traders are relatively well-informed compared to the market maker, their expected gain from acting upon their signal is greater than their exogenous gain from trade. As a result, they follow their signal, and there is no informational cascade in the market. Over time, as the prices aggregate private information, the informational content of the signal becomes relatively less important than that of the history of trades. After a long enough sequence of trades, the valuations of the traders become so close to the bid and ask prices that the expected gain from acting upon private information becomes smaller than the gain from trade. At this point, all informed traders with a gain $g$ from the asset decide to buy independently of their signal, and all traders with a loss $l$ from holding the asset decide to sell, thus an informational cascade arises.

**Proposition 2 (Almost Sure Occurrence of Informational Cascades)** *In equilibrium, an informational cascade arises almost surely if and only if $q < 1$.*

It is instructive to outline the main steps of the proof of this proposition. We first present two lemmas. Lemma 1 shows that the price, the bid and the ask converge almost surely to the same random variable.[15] This happens because, as more information is aggregated by the market prices, the degree

[15] Following most of the literature in market microstructure and social learning, we have not explicitly characterized the probability space with respect to which the random variables are defined and almost sure convergence occurs. Because of the complexity of the model, the characterization is notationally cumbersome. We refer the interested reader to an appendix available on the authors' webpages and upon request.

of asymmetric information in the market decreases, and the bid-ask spread shrinks to zero. In Lemma 2, we show that, as long as $q < 1$, the probability that, after any history of trade, a buy or a sell order comes from an informed trader is bounded away from zero. The condition $q < 1$ rules out perfectly informative signals. If signals were perfectly informative, a trader would disregard even a very long history of trades, and his expectation would diverge from that of the market maker.[16] Finally, we use the two lemmas to prove that, if an informational cascade did not occur, over time traders' expectations would become arbitrarily close to the bid and ask prices. In such a case, however, the traders would find it optimal to stop following their private information and trade according to their gain or loss from holding the asset, which is a contradiction.[17]

Note the difference between this channel of informational cascade and that described in the standard models of informational cascades with a fixed price (Bikhchandani et al., 1992; Banerjee, 1992). In those models, a cascade occurs because private information is eventually overwhelmed by public information. In contrast, in our economy with flexible prices, private information is never overwhelmed by public information; nevertheless, a cascade occurs since private information is eventually overwhelmed by the private values (i.e., the gains from trade).

It is also useful to note that Lee (1998) showed that, in a similar financial market, transaction costs can also cause a blockage of information. With transaction costs, however, when a cascade occurs, traders stop trading. Therefore, information ceases to flow into the market only because the market shuts down. Chari and Kehoe (2004) present a model of cascades in financial markets with endogenous timing. In their work, agents make a

[16] Our condition on the signal is equivalent to saying that beliefs must be "bounded" (Smith and Sørensen, 2000), a standard condition in the social learning literature (see, e.g., Chamley, 2004).

[17] Smith and Sørensen (2000) also study informational cascades in an economy with heterogeneous types (in a setup without prices). In their model, in addition to informational cascades, another phenomenon occurs: confounded learning. Confounding learning refers to a situation where, although agents use their private information, no inferences can be made by observing their actions (because different types use private information differently). A necessary condition for confounding learning is that agents have "opposite preferences," that is, they order outcomes differently (in our setup, this would arise if some agents valued $v^H$ more than $v^L$). Such preferences are not natural in financial markets, where the common value component reflects the present value of the security's cash flow, and they are ruled out by our parametric assumptions. For this reason, confounded learning never arises in our setup.

real investment decision in addition to trading in the market. A cascade of investment or no investment occurs in their model. When a cascade occurs, however, no one trades in the market. For this reason the market does not aggregate information. In our model, in contrast to both Lee (1998) and Chari and Kehoe (2004), a cascade in the financial market occurs and information stops flowing to the market despite the fact that agents keep trading.

Since during an informational cascade a trader's action does not convey any information on the asset value, the market maker's expected value of the asset conditional on receiving a buy or a sell order is the same, and the bid-ask spread collapses to zero. Moreover, the market maker will not update his belief after observing a trade. The prices remain stuck at the level reached when the cascade started. Finally, since during a cascade agents face the same decision problem in each period, an informational cascade never ends.

**Corollary 1 (Cascades last for ever)** *Suppose an informational cascade occurs at time $t$. Then, it lasts for ever and $b_{t+i} = p_{t+i} = a_{t+i}$ for all $i = 0, 1, 2, ....$*

An informational cascade may be incorrect; that is, the prices may remain stuck at a level far from the fundamental value. We show this through a simple example.

**Example 1 (Incorrect Informational Cascade)** *Let us consider an economy in which the asset can take values $1$ or $2$ with equal probabilities, and its realization is $2$. The probability that a trader is informed is $0.5$, and the precision of the signal is $0.8$. The private values are $g$ equal to $1.05$ and $l$ equal to $0.95$. Suppose that a sequence of sell orders arrive in the market. Figure 1 shows the bid and ask prices set by the market maker (black lines), the asset valuation of a trader with a low signal and a gain from the asset (gray line), and the valuation of a trader with a high signal and a loss from holding the asset (dotted gray line).*[18]

***Phase 1*** *At times $1$ and $2$, the valuation of a trader with a high signal is higher than the ask (even when he has a loss from holding the asset), and the valuation of a trader with a low signal is smaller than the bid (even with*

[18] The valuations of a trader with a low signal and a loss from the asset and the valuation of a trader with a high signal and a gain from holding the asset are not shown in the figure because they are trivially always outside the bid-ask spread.

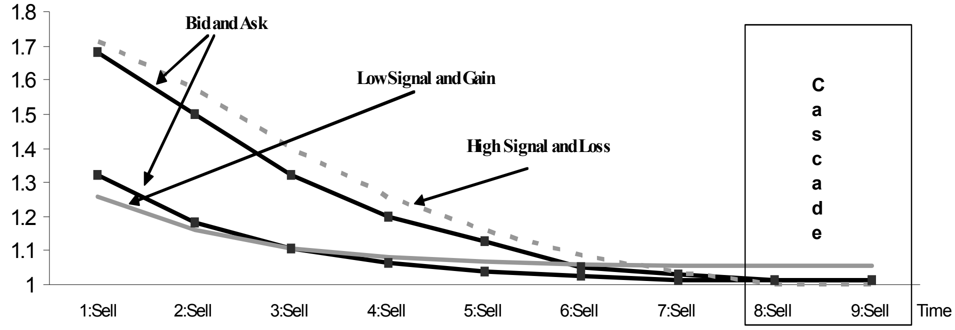


Figure 1: Informational Cascade. The figure shows the bid and ask prices, the valuations of traders with a high signal and a loss from holding the asset and those of traders with a low signal and a gain from the asset. A sequence of sell orders generates an informational cascade in the market. The parameter values for this and for all the other figures are summarized in a table at the end of the Appendix.

*a gain from the asset). As a result, all informed traders find it optimal to follow their private information. As the sell orders arrive in the market, both the traders and the market maker update their valuations downward; moreover, the valuations of the traders and those of the market maker become closer to one another.*

***Phase 2*** *Between times* 3 *and* 5*, the valuation of a trader with a low signal and a gain from the asset is higher than (or equal to) the equilibrium bid price and lower than the equilibrium ask; therefore, the trader would abstain from trading if called to trade.*[19] *Of course, the market maker takes this into account, when updating the price.*

***Phase 3*** *At times* 6 *and* 7*, the valuation of a trader with a low signal and a gain from holding the asset is higher than the equilibrium ask price; this trader would therefore buy if called to trade. Note that between times* 3 *and* 7*, the market maker still updates the price downward after a sale since a selling decision can either come from a noise trader or an informed trader*

[19] More precisely, at times 4 and 5, the trader would abstain from trade with probability 1 since his valuation is strictly higher than the bid. At time 3, instead, the equilibrium bid requires mixed strategies. The market maker sets the bid equal to the valuation of a trader with a low signal and a gain from the asset, and, at that bid, such a trader mixes between selling and not trading.

*with a low signal (all traders with high signals buy).*

***Phase 4*** *After seven sells, the valuation of a trader with a high signal and a loss from holding the asset is lower than the bid; the valuation of a trader with a low signal and a gain from the asset is higher than the ask. At this point, informed traders stop following their private information and buy or sell according to their private values: an informational cascade starts, and the bid and ask prices remain stuck forever at a level close to* 1*, far from the fundamental value of the asset,* $V = 2$*.*

In the example, the incorrect cascade occurs when enough information has accumulated in the market and market participants attach a high probability to the asset value being equal to $v^L$. As we said before, for a cascade to occur, the expected gain from acting upon private information must be smaller than the gain from trade; this happens only when the public belief that $V = v^H$ is close to zero or one. In such cases, the private signal conveys little additional information to the trader, and he finds it optimal not to use it. In a nutshell, a cascade only occurs when the public belief that $V = v^H$ has reached either an upper or a lower bound. We show this in the following proposition:

**Proposition 3 (Informational Cascades Regions)** *Let* $\widehat{p}_t$ *denote the public belief at time* $t$ *that* $V = v^H$*, that is,* $\widehat{p}_t := \Pr(V = v^H | h_t)$*. An informational cascade occurs when either*

$$\widehat{p}_t > M$$

*or*

$$\widehat{p}_t < m,$$

*where* $m := \min\{\underline{\alpha}, \underline{\beta}\}$*,* $M := \max\{\overline{\alpha}, \overline{\beta}\}$*, and* $(\underline{\alpha}, \overline{\alpha})$ *and* $(\underline{\beta}, \overline{\beta})$ *are the real roots of the following two quadratic equations in* $\widehat{p}_t$*:*[20]

$$\left( \frac{lq}{q\alpha + (1-q)(1-\alpha)} - 1 \right) \alpha = \frac{v^L(1-l)}{(v^H - v^L)} \tag{5}$$

*and*

$$\left( \frac{g(1-q)}{(1-q)\beta + q(1-\beta)} - 1 \right) \beta = \frac{v^L(1-g)}{v^H - v^L}. \tag{6}$$

[20]If both equations have complex roots, then a cascade occurs for any value of $\widehat{p}_t$. If one has real roots and the other complex roots, then only the first is relevant to define the cascade regions. Finally, the roots are relevant only if belonging to $(0, 1)$.

Note that the two equations (and, as a result, the thresholds after which a cascade arises) only depend on the parameters $g$, $l$ and $q$ (and, of course on $v^L$ and $v^H$). The reason is simple: a cascade starts when the private value (which depends on $g$ and $l$) overwhelms the informational gain from trading (which, for a given public belief, only depends on the precision of the private signal, $q$). Inspection of the two equations immediately shows that the relation between these parameters and the thresholds is what one would expect: the closer $g$ and $l$ are to 1, or the higher $q$ is, the closer the thresholds are to 0 or 1. In other words, the cascade regions shrink when the private values become less relevant and when the precision of private information becomes higher.

We now show how to compute the probability of an incorrect cascade, that is, the probability that the cascade occurs at the low threshold when the realized value is high or vice versa.

**Proposition 4 (Probability of An Incorrect Cascade)** *When $V = v^H$, the probability of an incorrect cascade (i.e., a cascade occurring at the lower threshold) is approximately equal to $\frac{(M-p)m}{p(M-m)}$. Similarly, when $V = v^L$, the probability of an incorrect cascade (i.e., a cascade occurring at the higher threshold) is approximately equal to $\frac{(p-m)(1-M)}{(M-m)(1-p)}$.*

The probability of an incorrect cascade is computed using the fact that, when the asset value is $v^H$, the likelihood ratio $\dfrac{\Pr(V = v^L|H_t)}{\Pr(V = v^H|H_t)}$ is a martingale with respect to the history $H_t$.[21] Therefore, its unconditional expected value is equal to its value at time 0, $\dfrac{1-p}{p}$. As we know from Proposition 3, when $t$ converges to infinity, $\Pr(V = v^H|H_t)$ converges either (close) to $m$ or (close) to $M$. This allows us to compute the two possible asymptotic values of the likelihood ratio. The probability of an incorrect cascade when $V = v^H$ can, therefore, be easily calculated by equating the asymptotic expected value of the likelihood ratio to its value at time 0.[22]

[21] See Doob (1953) and Cripps (2007).

[22] Our result is approximate since $m$ and $M$ are only bounds for the cascade regions. Due to the Bayesian updating, the price moves in discrete steps. A cascade may therefore start at a price slightly higher than $M$ or lower than $m$. The approximation is, obviously, very small unless very few buys or sales are enough to reach the cascade regions. As an alternative method, one can easily find an upper bound on the probability of an incorrect cascade. To explain, note that the lowest value that the public belief can reach

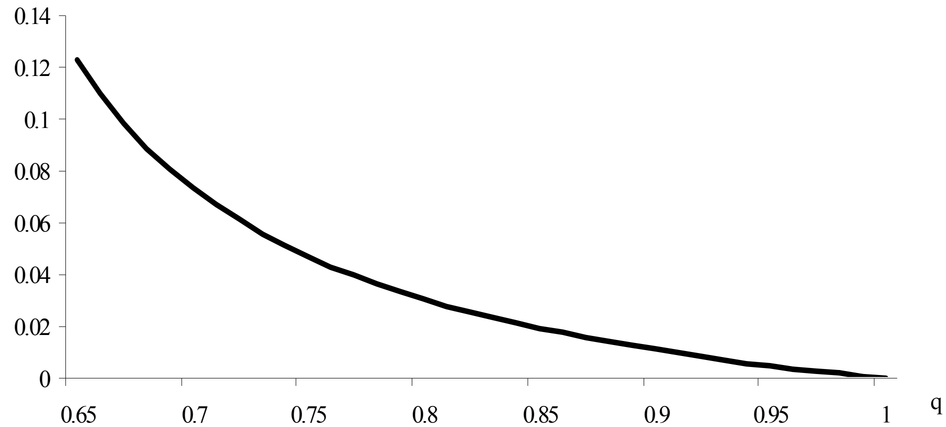


Figure 2: Probability of an incorrect cascade for different values of q.

Figure 2 shows the probability of an incorrect cascade for different values of $q$ for an economy in which $V$ is equal to 1 or 2 with equal probability, $g = 1.075$ and $l = 0.925$. The probability of the incorrect cascade is computed for the case in which $V = 2$. The probability is decreasing in the precision of the signal $q$ and, of course, tends to zero when $q$ converges to 1. Given this probability, we can compute another measure of the degree of asymptotic informational inefficiency in the market: the expected value of the distance between the asymptotic price and the fundamental value. Figure 3 shows this distance for different levels of $q$, under the same parametrization used for Figure 2: as $q$ increases from 0.65 to 0.80, the expected distance decreases from 0.30 (30 percent of the maximum distance) to 0.08.

In the Introduction we mentioned that, according to empirical analyses, fundamentals do help in predicting financial crises, but that crises may still occur even though the fundamentals are good. Similarly, in our model, fundamentals do help since the probability that the asset price is stuck at a low level (a "crisis") is lower when the asset value is high than when it is low. Nevertheless, crises happen even when the realized asset value is high.

is $\frac{\Pr(sell|V=v^H,h_t)m}{\Pr(sell|V=v^H,h_t)m+\Pr(sell|V=v^L,h_t)(1-m)} =: m'$ where $\Pr(sell|V = v^L, h_t) = \mu q + (1-\mu)\frac{1}{3}$ (in the most extreme case). One can show that the probability of reaching $m'$ conditional on $V = v^H$ is bounded above by $\frac{m'}{p}$. This result can be proven, for instance, using the "right ballpark property" discussed in Cripps (2007).

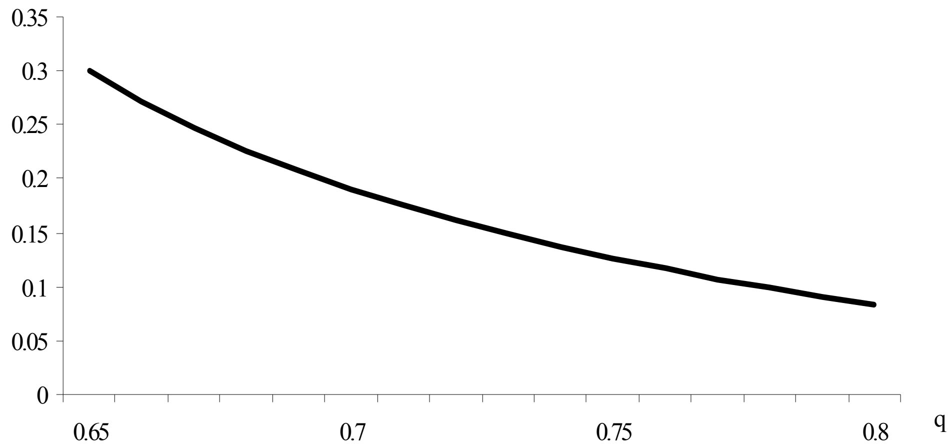


Figure 3: Expected distance between the asymptotic price and the fundamental value for different values of q.

## 3.1 Herd Behavior

In the social learning literature, a herd is said to take place if all agents in the economy choose the same action. In an economy with homogenous agents, during an informational cascade, all agents choose the same action, that is, they herd.[23] In our economy, because of trader heterogeneity, uniformity of actions only occurs for traders of the same type. In particular, during a cascade, all informed traders with the same private value choose the same action, either conforming to the established pattern of trade or going against it. In the first case, we say that informed traders act as "herders," in the second case, that they act as "contrarians."[24]

[23]The converse is not necessarily true. That is, under some conditions on the signal space, herding can also arise when there is no informational cascade (see Smith and Sørensen, 2000).

[24]The distinction between "herders" and "contrarians" based on whether traders conform to or trade against the established pattern of trade is also present in Avery and Zemsky (1998). Note, however, that Avery and Zemsky (in contrast to our notion of herding and that of most of the social learning literature) define a herder only as the informed trader who conforms to the established patter of trade *despite* his signal (the same definition is adopted by Park and Sabourian, 2008). Therefore, according to this definition, a trader who follows both his private information and the established pattern of trade would not be herding. Note also that Smith and Sørensen (2000) define herd

For instance, consider the case in which a cascade happens when the public belief reaches the higher threshold (i.e., more buys than sells have arrived in the market). In such a case, all informed traders with a positive gain from the asset $g$ buy independently of their signal. By doing so, they herd because they conform to the established pattern of trade. In contrast, all informed traders with a loss from holding the asset $l$ sell independently of their signal. Since these informed traders act against the history of past trades independently of their signal, they behave as contrarians. During the cascade, herders and contrarians execute trades of the opposite sign with the market maker. Nevertheless, since their actions are independent of their private information, no new information is aggregated by the price.[25]

Our analysis contributes to the understanding of what drives traders' decisions to act as herders or as contrarians. The prevalent view is that herders and contrarians disagree about the asset's fundamental value. Our paper shows that herding and contrarianism can arise through a different mechanism: in our economy, herders and contrarians agree about the fundamental value of the asset; the reason for their different decisions is that they have different private values.

## 4 Informational Cascades and Financial Contagion in a Two-Asset Economy

In order to study financial contagion, we now extend our model to a two-asset economy. We denote the two assets by $Y$ and $W$; their fundamental values, $V^Y$ and $V^W$, are both distributed on $\{v^L, v^H\}$. Given that we are interested in studying the informational spillovers between the two markets, we concentrate on the case in which the two random variables $V^Y$ and $V^W$ are not independent.

Assets $Y$ and $W$ are traded in two markets, market $Y$ and market $W$, respectively. In each market, there is a different market maker setting the bid and ask prices at which traders can trade. We denote the history of

behavior as conformity of actions conditional on an agent's type. According to their definition, therefore, in our economy, during a cascade all informed traders would be herding.

[25] This means that the trading activity of herders and contrarians does not contribute positively to the informational efficiency of financial markets. On the other hand, the fact that in our model herding and contrianism arise does not imply that the economy exhibits excess price volatility with respect to the fundamentals. In particular, since the price is the conditional expected value of the asset, its variance is bounded above by the variance of the asset value itself.

trades and prices in market $Y$ until time $t-1$ by $h_t^Y$. Similarly, the history of trades and prices in market $W$ is denoted by $h_t^W$, and the history in both markets is defined by $h_t := \{h_t^Y, h_t^W\}$. Both market makers observe $h_t$, which they use to form their conditional expectation.

As in the one-asset economy, there are both informed and noise traders (who are chosen to trade with probability $\mu$ and $1-\mu$). At each time $t$, a trader is exogenously chosen to trade in market $Y$ or in market $W$. We do not impose any restriction on the stochastic process according to which a trader is chosen to trade in one of the two markets. We only assume that such a process is known to the traders and to the market makers, and the event "a trade occurs in market $Y$ (or $W$)" is independent of the realized values of $V^Y$ and $V^W$.

If a noise trader is chosen to trade, he buys, sells or does not trade with equal probability. If instead an informed trader is chosen to trade, he first receives a signal on the value of the asset that he can trade and then makes a trading decision, exactly as in the one-asset economy.[26]

The time $t$ signal on the value of asset $J \in \{W, Y\}$ is represented by a random variable $S_t^J$. In particular, as in the previous setup, the signal is symmetric and binary, taking values $s^L$ and $s^H$ with precision $q^J$:

$$q^J = \Pr(S_t^J = s^L | V^J = v^L, V^I = v^H) = \Pr(S_t^J = s^L | V^J = v^L, V^I = v^L), \tag{7}$$

$$q^J = \Pr(S_t^J = s^H | V^J = v^H, V^I = v^H) = \Pr(S_t^J = s^H | V^J = v^H, V^I = v^L), \tag{8}$$

for $I, J \in \{W, Y\}$, $I \neq J$. Note that, according to the two expressions above, the conditional probability of a signal on one asset value is independent of the value of the other asset. This is the sense in which the signal is on one of the two values. Nevertheless, since the two asset values are not independent, a signal on one asset also indirectly provides some information on the value of the other.

As in the one-asset economy, there are gains from trade. For ease of exposition, we assume that the private component of traders' valuations takes value $l$ or $g$ with equal probability for both the informed traders trading in market $Y$ and those trading in market $W$.

[26]We do not allow traders to place orders in both markets contemporaneously because we are interested in the informational spillovers from one market to the other. Allowing agents to trade at the same time in both markes would make it more complicated to disentangle the spillover effects without offering further insights.

The presence of another market does not alter the characteristics of the equilibrium prices that we have already studied in the one-asset economy. In particular, it is straightforward to show that, as in the one-asset economy, in equilibrium there exists a unique bid and a unique ask in both markets. Furthermore, in each market, at any time $t$, $b_t^J \leq p_t^J \leq a_t^J$, and the sequence of prices for each asset $\{P_t^J : t = 1, 2, ...\}$ is a martingale with respect to the history $H_t$.[27]

Before we start analyzing informational cascades and financial contagion, it is worth noting that in our economy, if $V^Y$ and $V^W$ are positively correlated, after any order, the prices in the two markets move in the same way. It is never the case that one price goes up and the other goes down.[28] We formalize this result in the following proposition:

**Proposition 5 (Cross-market Updating)** *Suppose the asset values are positively correlated. If, after an action $x_t^J$ in market $J = Y, W$, $p_{t+1}^J \geq p_t^J$ ($p_{t+1}^J \leq p_t^J$), then $p_{t+1}^I \geq p_t^I$ ($p_{t+1}^I \leq p_t^I$), for $I \neq J$. In particular, after a buy order in market $J$, $p_{t+1}^I \geq p_t^I$, and after a sell order in market $J$, $p_{t+1}^I \leq p_t^I$.*

## 4.1 Informational Cascades

In a two-asset economy, an informational cascade occurs in either market for the same reasons already discussed for the one-asset economy. With one asset, however, an informational cascade never ends. In contrast, with two assets, a cascade can be broken. Even when there is an informational cascade in one market, the history of trades in the other reveals some information. As a result of the trades in the other market, the asset price moves despite the informational cascade. This can make the valuations of the traders and of the market maker diverge and break the cascade. We show this through a simple example.

**Example 2 (Broken Cascade)** *Let us consider an economy where both assets $Y$ and $W$ can take values 1 or 2 with equal probabilities. The two*

[27] Note that the market makers post a bid and an ask in each market at any time $t$. Even if a trade does not occur in a market at time $t$, the market maker updates the prices after the trade in the other market because that reveals information on the value of the asset.

[28] If, instead, the two asset values are negatively correlated, the prices always move in opposite directions. In general, all our results obtained by assuming positive correlation also hold with negative correlation (inverting the relation when it is obvious to do so).

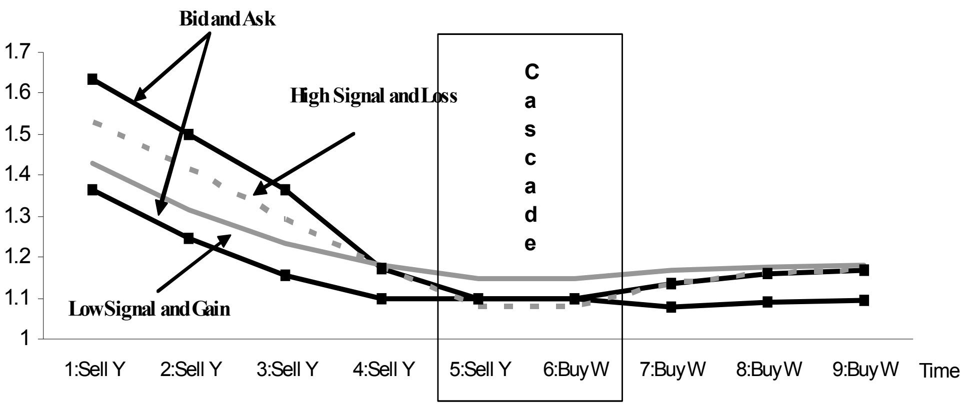


Figure 4: Broken cascade. The figure shows, for market Y, the bid and ask prices, the valuations of traders with a high signal and a loss from holding the asset and those of traders with a low signal and a gain from the asset. A buy order in market W at time 6 breaks the cascade in market Y.

*asset values are positively correlated, in particular*

$$\Pr(V^W = 1|V^Y = 1) = \Pr(V^W = 2|V^Y = 2) = 0.63.$$

*The probability of a trader being informed is* $0.74$*. The precision of the signal on the value of asset* $Y$ *is* $0.7$*, whereas the precision of the signal on the value of asset* $W$ *is* $0.97$*. The gain from holding the asset is* $g = 1.1$*, and the loss is* $l = 0.9$*. Suppose that the realized value of both assets is* $2$*, and that in the first five periods there are five sell orders in market* $Y$*. Figure* $4$ *shows, for market* $Y$*, the bid and ask prices set by the market maker (black lines), the valuation of a trader with a low signal and a gain from the asset (gray line), and the valuation of a trader with a high signal and a loss from holding the asset (dashed gray line).*

***Phase 1*** *At the beginning of trading activity, in market* $Y$*, the valuation of a trader with a high signal and a loss is within the bid-ask spread. Similarly for the valuation of a trader with a low signal and a gain from the asset. The valuations for traders with a high signal and a gain and for traders with a low signal and a loss (not shown in the figure) are both outside the spread. Since the traders' actions depend on their signal, the market maker sets a positive bid-ask spread and updates the prices downward as sell orders arrive in the market.*

***Phase 2*** *At time* 5*, the valuation of a trader with a low signal and a gain becomes greater than the ask, and that of a trader with a high signal and a loss becomes smaller than the bid. At this point, an informational cascade occurs in market* $Y$*.*

***Phase 3*** *At time* 6*, a buy order arrives in market* $W$*. The buy order conveys information to both the traders and the market maker in market* $Y$*. Since the two assets' fundamental values are positively correlated, the traders and the market maker update their expectations on asset* $Y$ *upward. As a result, the importance of informed traders' private information increases. That is, as the expectations of the traders and those of the market maker are updated upward, they also move apart. Therefore, it is no longer true that all informed traders in market* $Y$ *find it optimal to disregard their private information. The buy order in market* $W$ *has broken the informational cascade in market* $Y$*.*

It is worth mentioning that our result (i.e., a cascade in one market can be broken by the history of trades in the other) does not depend on the assumption that traders are exogenously chosen to trade in one of the two markets. Indeed, in the example above, even if traders could choose the market in which to trade, they would always find it optimal to trade the asset on which they receive the signal.[29]

At a more general level, our model gives some insights on how financial markets may recover from a crisis. In a crisis, gains from trade in a market can make trading completely uninformative. Without observing trading in the other market, the price of an asset would remain undervalued even though traders receive new, positive information. By revealing some new information, however, trading in the other market can help the market to recover. A positive history of trades in the other market leads to an increase in the price of the asset. After the price starts to rally, gains from trade cease to be binding and the normal flow of information to the market resumes.

Given this result, it is important to distinguish the case of an informational cascade in only one market from the case of an informational cascade in both markets. In the latter case, no new information will reach the markets and the cascades will last forever. We refer to the case of informational

[29]This is true although the traders have the same private values in both markets. Obviously, to check the robustness of the example to endogenous market choice, we assumed that, when choosing between the two markets, traders maximize the expected return on the asset.

cascades in both markets as an "informational breakdown."

**Definition 2** *An informational breakdown arises at time $t$ if there is an informational cascade in both markets.*

We can show that an informational breakdown occurs almost surely.

**Proposition 6 (A.s. Occurrence of Informational Breakdowns)** *In equilibrium, an informational breakdown occurs almost surely if and only if $q^J < 1$ for $J = Y, W$.*

An informational breakdown occurs for the same reasons an informational cascade occurs in either market. Over time, the valuations of the traders become so close to those of the market makers that, even in this two-asset economy, the expected gain from acting upon private information becomes smaller than the gain from trade.

Whereas in a two-asset economy an informational cascade blocks the flow of information only temporarily, the informational breakdown, once arisen, never ends; therefore, the prices remain stuck forever at the levels reached when the breakdown started. Moreover, using the same argument as in the one-asset economy case, a breakdown is incorrect with positive probability, that is, the price of either or both assets may remain stuck at a level far from the fundamental values. If the informational breakdown is incorrect, the markets can never correct their valuations and can never learn the assets' fundamental values.

## 4.2 Financial Contagion

We will now show that, in the presence of gains from trade, informational spillovers across markets can have long-lasting pathological effects on the behavior of prices. The fact that traders and market makers are able to observe the trading activity in another market can cause the price mechanism to fail in aggregating information. As a result, the flow of information to the market can stop early, and the price can remain stuck at a wrong level. We regard this as a form of contagion. More precisely, we say that there is a *contagious spillover* at time $t$ when an informational cascade occurs in a market and would have not occurred if the agents were able to observe only the history of trades in their own market. That is, the informational cascade happens only because agents observe the history of the other market.

**Definition 3** *A contagious spillover from market $J$ to market $I$ ($I, J = Y, W$; $I \neq J$) occurs at time $t$ if*

*1) an informational cascade occurs in market $I$ at time $t$ and*

*2) the cascade would not have occurred if the agents had been able to observe only the history of trades and prices in their own market.*

Recall from the previous section that during an informational cascade the probability of an action is independent of the private signal: $\Pr(X_t^I = x|h_t, a_t^I, b_t^I, S_t^I = s^I) = \Pr(X_t^I = x|h_t, a_t^I, b_t^I)$ for all $x$ and all $s$. The cascade is due to a contagious spillover when it is generated by the observation of the history in the other market. This means that, if the market were isolated from the other (i.e., only $h_t^I$ were observable), there would exist an action $x$ and a realization of the signal $s^I$ such that $\Pr(X_t^I = x|h_t^I, a_t^I, b_t^I, S_t^I = s^I) \neq \Pr(X_t^I = x|h_t^I, a_t^I, b_t^I)$.

The contagious spillover can have permanent effects. If the informational cascade in market $W$, caused by the spillover effect, happens together with an informational cascade in market $Y$, an informational breakdown arises and the price remains stuck at a wrong level.

We now present an example in which a contagious spillover arises. If traders in market $Y$ were not able to observe the history in market $W$, the price of asset $Y$ would converge to a value close to its fundamental. Given that traders are able to observe the history in both markets, however, the initial sales in market $W$ cause an informational cascade in both markets $Y$ and $W$, that is, an informational breakdown. The price of asset $Y$ is stuck forever at a level below the fundamental value, and its initial fall is not reversed even in the long run.

**Example 3 (Contagious Spillover)** *Let us consider an economy where both assets $Y$ and $W$ can take values $1$ or $2$ with equal probabilities. As in the previous example, the two asset values are positively correlated, in particular*

$$\Pr(V^W = 1|V^Y = 1) = \Pr(V^W = 2|V^Y = 2) = 0.72.$$

*The probability of a trader being informed is $0.40$. The precision of the signal on the value of asset $Y$ is $0.65$, whereas the precision of the signal on asset $W$ is $0.70$. The gain from holding the asset is $g = 1.10$ and the loss is $l = 0.90$. Suppose that the realized value of both assets is $2$ and that in the first eight periods there are eight sell orders in market $W$ followed by buy orders in market $Y$. Figure $5$ shows, for market $Y$, the bid and ask prices*

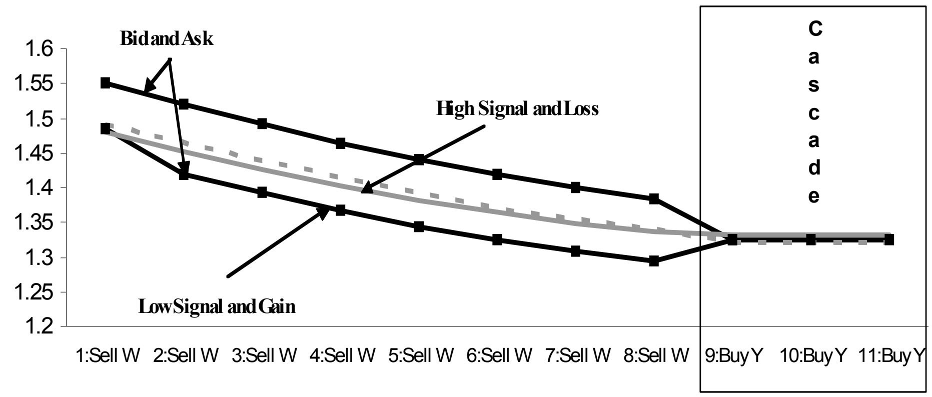


Figure 5: An example of contagious spillover. The figure shows, for market Y, the bid and ask prices, the valuations of traders with a high signal and a loss from holding the asset and those of traders with a low signal and a gain from the asset. A sequence of 8 sales in market W generates a cascade in market Y.

*set by the market maker (black lines), the valuations of a trader with a low signal and a gain from the asset (gray line), and the valuations of a trader with a high signal and a loss from holding the asset (dashed gray line).*

***Phase 1*** *As in the previous example, at the beginning of the trading activity, traders' actions in market $Y$ depend on the signal they observe. However, as sell orders arrive in market $W$, the traders and the market maker in market $Y$ revise their valuations downward; furthermore, their valuations become closer to one another.*

***Phase 2*** *At time 9, after eight sell orders in market $W$, an informational breakdown occurs. As a result, the market maker in market $Y$ does not update the price upward (towards the realized value of the fundamental) as buy orders arrive in market $Y$ from period 9 onward. That is, observing the history in market $W$ prevents the correct aggregation of private information in market $Y$.*

Figure 6 shows a simulated price path for market $Y$ under two different scenarios: when the traders and the market maker in market $Y$ are not able to observe the history of market $W$ (solid line) and when they are able

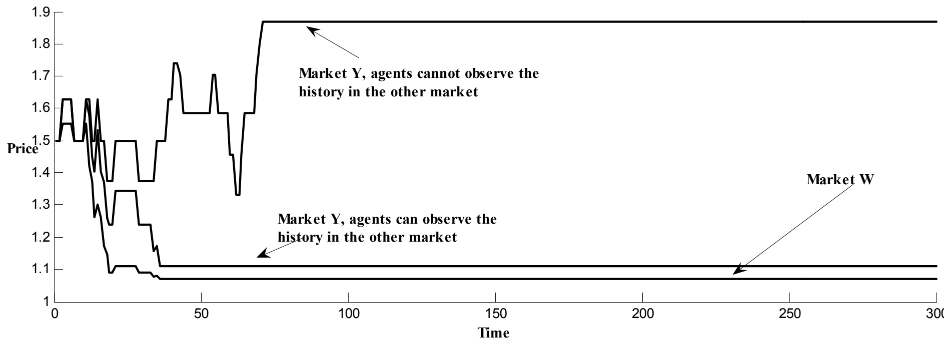


Figure 6: Contagious spillover. Simulated price path in market Y when agents in that market are (solid line) or are not (dotted line) able to observe the trading activity in market W. The figure also shows the price path in market W (dash-dotted line) when agents observe the history in both markets.

to do so (dotted line).[30] When both histories of trades are observed, the fall in asset $W$'s price makes the price of $Y$ fall to the level at which an informational breakdown arises. The price of asset $Y$ remains stuck there, far below its fundamental value, 2. On the other hand, if the traders and the market maker in market $Y$ do not observe the history in market $W$, the price of $Y$ remains above the level at which an informational cascade arises and eventually converges to 1.9, a value close to the fundamental.

It is also interesting to see how the spillover effects across market affect the ex-ante long-run distribution of prices in the two markets. As we know, in contrast to what happens in an one-asset economy, in a two-asset economy the price of an asset does not necessarily stop being updated when the market enters a cascade region. As a result, the asymptotic prices are not always close to the two thresholds illustrated for a one-asset economy. Instead, there are two intervals, one above the high threshold and one below the low threshold, where the prices can settle. In the following proposition,

[30]The dash-dotted line, instead, shows the history of prices in market $W$. The parameters of the simulation are as follows: the probability of an informed trader is 0.7; the precision of both signals is 0.7; the gains from trade are $g = 1.1$ and $l = 0.9$; finally, $p = 0.5$ and $\Pr(V^W = v^H | \ V^Y = v^H) = \Pr(V^W = v^L | \ V^Y = v^L) = 0.7$. The realized fundamental values for $V^Y$ and $V^W$ are 2 and 1, respectively.

we characterize the asymptotic levels that the public belief can take. From these, one can easily find the asymptotic levels of the asset prices, as we see below.

**Proposition 7 (Asymptotic Level of the Public Belief)** *Let $\widehat{p}_t^I$ denote the public belief at time $t$ that $V^I = v^H$, i.e., $\widehat{p}_t^I := \Pr(V^I = v^H|h_t)$, and let $m^I$ and $M^I$ be the thresholds at which an informational cascade arises in an one-asset economy. In a two-asset economy, if an informational breakdown occurs at time $t$, then:*
*a) $\widehat{p}_t^J \cong m^J$ and $\widehat{p}_t^I \cong M^I$, or*
*b) $\widehat{p}_t^J \cong m^J$ and $m^I - \Delta_L^I \leq \widehat{p}_t^I \leq m^I$, or*
*c) $\widehat{p}_t^J \cong M^J$ and $M^I \leq \widehat{p}_t^I \leq M^I + \Delta_H^I$,*
*where $\Delta_L^I = m^I(1 - m^J)$ and $\Delta_H^I = M^J(1 - M^I)$.*

When the breakdown occurs, if the beliefs in the two markets have moved in opposite directions (i.e., one above 0.5 and the other below 0.5), they remain stuck at the thresholds already illustrated in a one-asset economy (letter a of the Proposition). Otherwise (letters b and c), one of the two beliefs remains stuck at the low (high) threshold and the other in an interval adjacent to and below (above) it. By characterizing the asymptotic levels of the public beliefs, the proposition also characterizes the asymptotic behavior of the prices, given that $p_t^I = \widehat{p}_t^I(v^H - v^L) + v^L$.

In a two-asset economy, since the price can converge to different levels, the probability of an incorrect cascade by itself does not characterize the level of asymptotic informational inefficiency in the market. For such a characterization, we look, instead, at the expected distance between the price and the fundamental value. This distance is always smaller in a two-asset economy than in a one-asset economy, as illustrated in the next proposition:

**Proposition 8 (Second Market and Informational Inefficiency)** *The expected time-$t$ price of asset $I$ ($I = Y, W$) conditional to the realized value of the fundamental, $E(p_t^I|V^I)$, is closer to the fundamental value when agents in market $I$ observe the history in both markets ($h_t$), than when they only observe the history in their own market ($h_t^I$).*

Note that our result holds for any $t$ and not only asymptotically. The result shows that although contagious, pathological effects do occur for some histories of trades, informational spillovers from one market to the other are benign in expected value.

## 4.3 Excess Correlation

In the empirical literature, contagion is sometimes defined as correlation between asset prices in excess to that between fundamentals. It is important to understand whether the contagious effects shown above also lead to excess correlation. Interestingly, excess correlation is present in our sequential trading economy with gains from trade. To illustrate this point, we first offer an analytical characterization of the covariance between the asset prices and then turn to the analysis of the correlation.

**Proposition 9 (Bound on the Covariance between Prices)** *Suppose the asset values are positively correlated. The unconditional covariance of the asset prices, $Cov(P_t^Y, P_t^W)$, is monotonically increasing over time: $Cov(P_t^Y, P_t^W) \leq Cov(P_{t+1}^Y, P_{t+1}^W)$. Furthermore, it is bounded above by the covariance of the asset fundamentals: $Cov(P_t^Y, P_t^W) \leq Cov(V^Y, V^W)$ for any $t$.*

The monotonic increase of the covariance over time is a direct consequence of the prices always being updated in the same direction after any action (as shown in Proposition 5). The bound on the covariance is reminiscent of the bound on the variance of the price and stems from the fact that the asset prices are conditional expectations of the asset values.

Whereas the covariance between asset prices can be found analytically, the study of the correlation turns out to be analytically difficult. Therefore, we studied it through a simulation, whose results are described in Figure 7.[31] The three-dimensional chart shows the correlation between asset prices over time as $q$ changes and compares it with that of the fundamentals. The unconditional correlation between prices is monotonically decreasing over time and is always greater than the correlation between fundamentals.[32]

Note that, with no gains from trade, the correlation between prices would converge towards the correlation between fundamentals. This is because in such a case the prices themselves converge almost surely to the

[31] The parameters of the simulation are identical to those used for Figure 6, with the exception of $q$, which here takes values in the range $[0.65, 1]$ (we do not consider values lower than 0.65 since for those values a cascade occurs immediately at time 1). The correlation between the asset values is 0.4, since $p = 0.5$ and $\Pr(V^W = v^H | V^Y = v^H) = \Pr(V^W = v^L | V^Y = v^L) = 0.7$. The simulation was run for 300 periods and was repeated for one million runs.

[32] We have repeated the analysis also for many different values of the other parameters (the probability of an informed trader, the gains from trade, the correlation between fundamentals), and the results do not change.

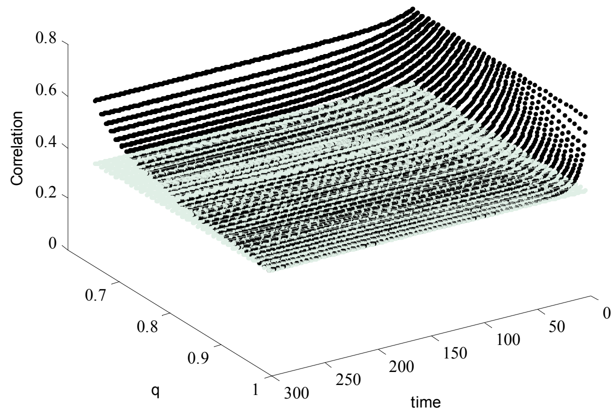


Figure 7: Price correlation. The monotonically decreasing curves represent the correlation between asset prices over time for different values of q. The correlation between fundamentals is represented by the straight lines (i.e., the plane) lying below the asset price correlation (at the level of 0.4).

fundamental values, as shown by Avery and Zemsky (1998). In contrast, when traders have gains from trade, informational breakdowns arise, and the true values of the assets are never discovered. As a result, the correlation between the prices is in excess of that between the fundamentals even in the long run. Therefore, with sequential trading and gains from trade, contagion—defined as excess correlation—can also occur as a long-run phenomenon. Many empirical studies on contagion have documented this excess correlation among financial asset prices. Our finding suggests that excess correlation is not necessarily the result of irrational behavior or frictions in the markets, but may be the result of the learning process of rational agents.

To clarify what generates excess correlation, we study what happens when the two markets are isolated from each other, that is, agents in a market do not observe what occurs in the other market. Since the two markets operate in isolation, the analysis is similar to that of Section 3 (with the qualification that the correlation between fundamentals creates asset price correlation). We are able, therefore, to find the (approximate) asymptotic distribution of the asset prices in the two markets. In particular, we know that, in each market, the price settles at one of two threshold levels. Given that the markets are isolated, the probability that the price in a market settles on either threshold is independent of what occurs in the other market. We exploit this fact to find the joint asymptotic distribution of the two prices and compute their asymptotic correlation. The next proposition shows that the asymptotic correlation is indeed lower than the fundamental correlation. For convenience, we restrict our attention to the case in which the joint distribution of asset values is symmetric (in the sense indicated below), and the gains from trade are such that the two thresholds for the cascade regions are symmetric around $\frac{1}{2}$.[33]

**Proposition 10 (Price Correlation between Independent Markets)** *Suppose that agents in each market $J = Y, W$ can only observe the history of trades and prices in their own market, $h_t^J$. Suppose the two assets take values $v^L$ and $v^H$ with equal probability, and $\Pr(V^W = v^L | V^Y = v^L) = \Pr(V^W = v^H | V^Y = v^H) > \frac{1}{2}$. Furthermore, suppose that in both markets the thresholds $m$ and $M$ defining the cascade regions are symmetric around $\frac{1}{2}$, that is, $\frac{m+M}{2} = \frac{1}{2}$. Then the asymptotic correlation between prices is lower than the asymptotic correlation between the fundamentals.*

By comparing this finding with the previous simulation result, it is clear that excess correlation is indeed due to the informational spillovers across markets. Only because traders and market makers observe the trading activity in both markets, prices are more correlated than fundamental values.

## 4.4 Other Channels of Financial Contagion

So far we have shown that when the two asset values are correlated, informational cascades can spill over from one market to the other and can have long-lasting pathological effects. It is, however, important to highlight that the mechanism identified in the paper is more general than the specific channel of contagion that we have analyzed. Gains from trade remain important irrespective of the type of shock generating informational spillovers from one market to the other. For instance, let us consider an economy in which the transmission of shocks across markets happens not because asset values are correlated, but because of the correlation in noise trading

[33] This simplifies the exposition a great deal. It is easy to show, however, that our argument extends to the general case.

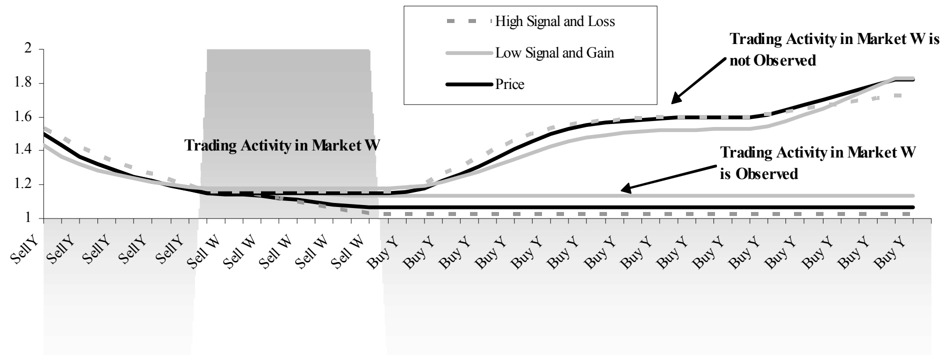


Figure 8: Contagious spillover in the case of independent asset values. The figure shows the price and the traders' valuations in market Y. A sequence of 10 sales in market Y is followed by a sequence of 10 sales in market W and then by buy orders in market Y. Note that trades on the X axis are labelled only every other period. If agents in market Y observe the trading activity in the other market, an informational breakdown occurs at time 20 and the price of asset Y remains stuck at a low level (while the fundamental value is 2). If they cannot observe the trading activity in the other market, the price converges towards the fundamental value.

activity (i.e., liquidity shocks are correlated across markets). We will now show that, in such an economy, even though the fundamental values are uncorrelated, a contagious spillover occurs.

**Example 4 (Contagious Spillover with Independent Fundamentals)** *Let us consider an economy where the two assets $Y$ and $W$ can take values* $1$ *or* $2$ *with equal probabilities. In contrast to previous sections, the asset values are independently distributed. The probability of a trader being informed is* $0.35$*. The precision of the signal on the asset value is* $0.7$ *for both assets. The gain from holding the asset is* $g = 1.1$ *and the loss is* $l = 0.9$*.*

*For the purpose of studying this different channel of contagion, the most important departure from the previous analysis is that noise traders do not buy and sell with equal probabilities. On the contrary, in each market, there are two possible noise trading regimes: "high noise buying regime" (HNB) and "high noise selling regime" (HNS), each occurring with equal probability. In HNB, when noise traders do not abstain from trading, they buy with probability* $0.7$ *and sell with probability* $0.3$*; the opposite occurs in HNS. For simplicity's sake, we assume that when one market is in HNB the other is in HNS:* $Pr(HNB^Y|HNS^W) = Pr(HNS^Y|HNB^W) = 1.$

*Suppose that the realized values of both assets are* $2$. *Suppose also that at the beginning of trading activity there are sell orders in market* $Y$, *followed by sell orders in market* $W$. *Figure* $8$ *shows, for market* $Y$, *the price set by the market maker (black line),*[34] *the valuations of a trader with a low signal and a gain from the asset (gray line), and the valuations of a trader with a high signal and a loss from holding the asset (dotted gray line). Note that the figure shows the behavior of the asset price and of the traders' valuations under two different scenarios, one in which the traders and the market makers in market* $Y$ *are able to observe the history in market* $W$ *and the other in which they are not.*

***Phase 1*** *At the beginning of the trading activity, ten sell orders in market* $Y$ *cause the price to fall. As this happens, expectations of traders and market makers become close to one another. Nevertheless, there is no informational cascade in the market, because a proportion of informed traders use their private information when trading. In particular, the valuations of a trader with a high signal and a loss are always above the equilibrium price.*

***Phase 2*** *Starting from time* $11$, *ten sell orders arrive in market* $W$. *Because of the sell orders, traders and market makers update the probability of* $HNS$ *in market* $W$ *upward. This also affects the price in market* $Y$. *In particular, since the activity of the noise traders is inversely related in the two markets, traders and market makers update the probability that we are in* $HNB$ *in market* $Y$ *upward. In doing so, they attach a higher probability to the event that the initial sales in market* $Y$ *came from informed traders and, as a result, the price in market* $Y$ *falls. As this happens, the valuations of the traders and the market maker in market* $Y$ *become closer to one another, and, at time 15, also the valuations of traders with a high signal and a loss fall below the equilibrium bid price. All informed traders in market* $Y$ *now find it optimal to follow their private value component and disregard their private signal, that is, there is an informational cascade.*[35]

***Phase 3*** *Starting from time* $21$, *buy orders arrive in market* $Y$. *Because of the informational cascade, however, the price is not updated after the buys, and remains stuck at a low level, far from the fundamental value of the asset (*$2$*).*

*The figure also shows what happens in market* $Y$ *if the traders and the market maker do not observe the trading activity in market* $W$ *(in Phase*

[34] For simplicity, we show only the asset price and not the bid and the ask.

[35] Note that the sell orders in market $W$ also generate an informational cascade in that market, i.e., the economy is in an informational breakdown.

2*). Since in Phase* 1 *the sell orders in market* $Y$ *did not start a cascade, the buy orders arriving in Phase* 3 *do convey information to the market maker and, as a result, the price is updated upward. As more and more buy orders arrive in the market, the price converges close to the fundamental value of the asset.*

The example focuses on the case in which the liquidity shock is inversely related in the two markets. Obviously, the logic applies as well to the case in which the liquidity shock affects both markets in the same way (i.e., because of a liquidity shock, some traders liquidate their positions in both markets). The correlation in liquidity shocks is what creates correlation between prices.

Our example shows that a contagious spillover similar to that described in the previous sections can occur even though the correlation between the asset values is zero. That is, in an economy with gains from trade, the cross-market spillovers can have pathological consequences (i.e., they can generate incorrect cascades) irrespective of the specific mechanism through which they occur.

# 5 Conclusion

In this paper, we have obtained two main results. The first result is on informational cascades. In financial markets, when agents are heterogeneous so that trade can be mutually beneficial, informational cascades arise. Information stops flowing to the market, which is, therefore, unable to infer traders' private information and to discover the true values of the assets. The asset prices can remain stuck at levels different from the fundamental values. The probability of these incorrect cascades depends on the precision of private information and on the degree of heterogeneity across market participants. Informational cascades imply that all informed traders choose the same action, either following the market (herding) or going against it (contrarianism).

The second result is on contagion. Because of gains from trade, the history of trades on one asset can significantly affect the price of the other. It can generate informational cascades in the other market, pushing its price far from the fundamental value even in the long run. This creates correlation between asset prices in excess of the correlation between the fundamentals. The pathological effects of spillovers across markets happen when the asset fundamental values are correlated and even when they are not, provided that there is correlation in liquidity trading activity.

# 6 Appendix

## 6.1 Microfoundation of Gains from Trade: An Example

The reduced form of gains from trade that we used in the paper can be microfounded in the following way. As is standard in the market microstructure literature, we can interpret the trading times $t = 1, 2, ...$ as the times at which traders trade during the trading day. The asset value, $V$, is realized at the end of the day. If the trader trades the asset during the day, he will obtain the payoff at the end of the day and consume it the day after. The trader has an endowment of $c$ units of cash and of 1 unit of the asset. The prices at which he can buy or sell the asset are denoted by $a$ and $b$.

The trader's utility function over realized levels of consumption is
$u(c_1, c_2) = c_1 + \rho c_2$,
where $c_1$ and $c_2$ indicate consumption in days 1 and 2.
The trader maximizes his expected utility
$E(C_1 + \rho C_2)$
subject to the following budget constraints:
$C_1 = c - \chi_{x=1}a + \chi_{x=-1}b$, and
$C_2 = (1 + x)V$,
where $C_1$ and $C_2$ are the two random variables indicating consumption, $x$ takes value $-1$ if the agent sells, 0 if he decides not to trade and 1 if he buys, and $\chi_x$ is an indicator function. The trader's maximization problem is equivalent to

$$\underset{x\in\{-1,0,1\}}{Max} E\left[c - \chi_{x=1}a + \chi_{x=-1}b + \rho(1 + x)V\right].$$

It is straightforward to see that the trader buys the asset if $\rho E(V) > a$, sells it if $\rho E(V) < b$, and does not trade if $b < \rho E(V) < a$. Moreover, he can randomize when an equality holds. In the paper, we have normalized the discount factor of the market maker to 1 and assumed that the discount factor for a trader is $\rho \in \{l, g\}$, with $l < 1 < g$. Therefore, a trader with gain from trader $l$ is more impatient than the market maker, while a trader with gains from trade $g$ is less impatient than the market maker.

## 6.2 Proof of Proposition 1

First, we prove the existence of the ask price. Because of unmodeled potential Bertrand competition, the ask price at time $t$, $a_t$, must satisfy the condition

$$a_t := \min\{a \in [v^L, v^H] : a = E(V|h_t, X_t = buy, a, b_t)\}.$$

Let us denote by $i_t$ the event that the agent buying at time $t$ is *informed*. The expected value of the asset at time $t$, given a buy order at the ask price $a_t$ is

$$E(V|h_t, X_t = buy, a_t, b_t) =$$
$$E(V|h_t, X_t = buy, a_t, b_t, i_t)\Pr(i_t|h_t, X_t = buy, a_t, b_t)+$$
$$E(V|h_t)(1 - \Pr(i_t|h_t, X_t = buy, a_t, b_t)).$$

Let us now consider the correspondence $\psi : [v^L, v^H] \rightrightarrows [v^L, v^H]$ defined as $\psi(y) := E(V|h_t, X_t = buy, A_t = y, b_t)$. The correspondence $\psi$ is piecewise constant. In particular, it is constant everywhere except for $y = lE(V|h_t, S_t = s^L)$, $y = gE(V|h_t, S_t = s^L)$, $y = lE(V|h_t, S_t = s^H)$, and $y = gE(V|h_t, S_t = s^H)$. To see this, let us order these values in an increasing way and denote them by $y^1$, $y^2$, $y^3$, and $y^4$. For $y > y^4$, $\Pr(i_t|h_t, X_t = buy, a_t, b_t) = 0$; therefore, $\psi(y) = E(V|h_t)$. For any $y^3 < y < y^4$, the correspondence takes value $E(V|h_t, X_t = buy, a_t, b_t, i_t^4)\Pr(i_t^4|h_t, X_t = buy, a_t, b_t)+ E(V|h_t)(1-\Pr(i_t^4|h_t, X_t = buy, a_t, b_t))$, where $i_t^4$ denotes the event that the agent in $t$ is informed with valuation of the asset $y^4$. A similar analysis applies to the other cases. When $y = y^i$ (for $i = 1, 2, 3, 4$), an informed trader with valuation $y^i$ can randomize between buying and not trading. Hence, $\psi(y)$ takes all the values belonging to the interval connecting $E(V|h_t, X_t = buy, A_t = y^i - \varepsilon, b_t)$ and $E(V|h_t, X_t = buy, A_t = y^i + \varepsilon, b_t)$ for a small $\varepsilon$.

From these observations, it is immediate to see that the correspondence $\psi(y)$ is non-empty, convex-valued and has a closed graph. By Kakutani's fixed point theorem, the correspondence has a (finite number of) fixed points. The smallest fixed point is the equilibrium ask price.

The proof of the existence of the bid price is analogous.

Now we prove that $b_t \leq p_t \leq a_t$. We prove that $p_t \leq a_t$; the proof that $b_t \leq p_t$ is analogous.

We can write the ask and the price at time $t$ as follows:

$$a_t = E(V|h_t, X_t = buy, a_t, b_t) =$$
$$\Pr(V = v^H|h_t, X_t = buy, a_t, b_t)(v^H - v^L) + v^L,$$

and

$$p_t = E(V|h_t) = \Pr(V = v^H|h_t)(v^H - v^L) + v^L.$$

Therefore, in order to prove the proposition, we only need to show that

$$\Pr(V = v^H|h_t, X_t = buy, a_t, b_t) \geq \Pr(V = v^H|h_t).$$

Now,

$$\Pr(V = v^H|h_t, X_t = buy, a_t, b_t) =$$
$$\frac{\Pr(X_t = buy|V = v^H, h_t, a_t, b_t)\Pr(V = v^H|h_t, a_t, b_t)}{\Pr(X_t = buy|h_t, a_t, b_t)}$$

and

$$\Pr(V = v^H|h_t) = \Pr(V = v^H|h_t, a_t, b_t).$$

Therefore, we must show that

$$\Pr(X_t = buy|V = v^H, h_t, a_t, b_t) \geq \Pr(X_t = buy|h_t, a_t, b_t),$$

which is true if and only if, for any given history, the probability of observing a buy is not lower if the asset value is high than if the asset value is low.

We have that

$$\Pr(X_t = buy|V = v^H, h_t, a_t, b_t) =$$
$$q\Pr(X_t = buy|S_t = s^H, h_t, a_t, b_t) + (1-q)\Pr(X_t = buy|S_t = s^L, h_t, a_t, b_t)$$

and, similarly,

$$\Pr(X_t = buy|V = v^L, h_t, a_t, b_t) =$$
$$(1-q)\Pr(X_t = buy|S_t = s^H, h_t, a_t, b_t) + q\Pr(X_t = buy|S_t = s^L, h_t, a_t, b_t).$$

Since $q > \frac{1}{2}$, $\Pr(V = v^H|S_t = s^H, h_t) \geq \Pr(V = v^H|S_t = s^L, h_t)$. It is immediate to see that this implies that $\Pr(X_t = buy|S_t = s^H, h_t, a_t, b_t) \geq \Pr(X_t = buy|S_t = s^L, h_t, a_t, b_t)$. Hence, $\Pr(X_t = buy|V = v^H, h_t, a_t, b_t) \geq \Pr(X_t = buy|V = v^L, h_t, a_t, b_t)$. This concludes the proof.

## 6.3 Proof of Proposition 2

To prove the Proposition, we first prove two lemmata.

**Lemma 1** The ask, the bid and the price, $A_t$, $B_t$ and $P_t$, converge almost surely to the same random variable (i.e., the bid-ask spread converges almost surely to 0).

**Proof of Lemma 1**

The proof of this lemma can be found in Glosten and Milgrom (1985, pp. 86-88).

In order to introduce Lemma 2, let us define the random variable $U_t$ that takes value 1 if at time $t$ an uninformed trader arrives in the market and value 0 if at time $t$ an informed trader arrives in the market.

**Lemma 2** At any time $t$, if $q < 1$, the probability that, after any history of trade, a buy or a sell order comes from an informed trader is bounded away from zero:

$$\Pr(S_t = s^H, U_t = 0 | h_t, X_t = buy, a_t, b_t) \geq \frac{1}{2}\mu(1-q) > 0,$$
$$\Pr(S_t = s^L, U_t = 0 | h_t, X_t = sell, a_t, b_t) \geq \frac{1}{2}\mu(1-q) > 0.$$

**Proof of Lemma 2**

Let us prove the first part:

$$\Pr(S_t = s^H, U_t = 0 | h_t, X_t = buy, a_t, b_t) \geq \frac{1}{2}\mu(1-q).$$

By Bayes's rule,

$$\Pr(S_t = s^H, U_t = 0 | h_t, X_t = buy, a_t, b_t) =$$

$$\frac{\Pr(X_t = buy | h_t, S_t = s^H, a_t, b_t, U_t = 0) \Pr(S_t = s^H, U_t = 0 | h_t, a_t, b_t)}{\Pr(X_t = buy | h_t, a_t, b_t)}.$$

First, note that

$\Pr(S_t = s^H, U_t = 0 | h_t, a_t, b_t) = \Pr(S_t = s^H | h_t, a_t, b_t, U_t = 0)\mu =$

$\mu(q \Pr(V = v^H | h_t, a_t, b_t, U_t = 0) + (1-q) \Pr(V = v^L | h_t, a_t, b_t, U_t = 0)) \geq \mu(1-q)$, which is greater than 0 since $q < 1$.

We now show that $\Pr(X_t = buy|h_t, S_t = s^H, a_t, b_t, U_t = 0)$ is also bounded away from 0.

Indeed,

$\Pr(X_t = buy|h_t, S_t = s^H, a_t, b_t, U_t = 0) =$

$\Pr(X_t = buy|h_t, S_t = s^H, a_t, b_t, U_t = 0, K_t = g)\frac{1}{2}+$

$\Pr(X_t = buy|h_t, S_t = s^H, a_t, b_t, U_t = 0, K_t = l)\frac{1}{2}$.

Note that $\Pr(X_t = buy|h_t, S_t = s^H, a_t, b_t, U_t = 0, K_t = g) = 1$ since the market maker's zero profit condition implies that $E(V|h_t, S_t = s^H) > a_t$. Hence,

$\Pr(X_t = buy|h_t, S_t = s^H, a_t, b_t, U_t = 0) \geq \frac{1}{2}$, and

$\Pr(S_t = s^H, U_t = 0|h_t, X_t = buy, a_t, b_t) \geq \frac{1}{2}\mu(1-q)$.

Analogous steps prove that

$$\Pr(X_t = sell|h_t, S_t = s^L, a_t, b_t, U_t = 0) \geq \frac{1}{2}\mu(1-q).$$

This concludes the proof of Lemma 2.

Now, we prove Proposition 2. Let us first note that at time $t$, there is an informational cascade if the following two conditions hold:

1) Condition 1:

$$a_t - gE(V|h_t, S_t = s^L) \leq 0.$$

and, if the above expression holds as an equality, in equilibrium an informed trader with a gain $g$ from holding the asset and a signal $s^L$ buys with probability one.

2) Condition 2:

$$lE(V|h_t, S_t = s^H) - b_t \leq 0.$$

and, if the above expression holds as an equality, in equilibrium an informed trader with a loss $l$ from holding the asset and a signal $s^H$ sells with probability one.

Conditions 1 and 2 imply that, whatever his signal, a trader always follows his gain from trade (i.e., buys when $K_t = g$ and sells when $K_t = l$) because buying is strictly preferred to any other action or, in the case of indifference, because it is the strategy that the trader plays in equilibrium. In equilibrium, during an informational cascade, $a_t = b_t = p_t$, since buying or selling is independent of a trader's private information. As a result, it is easy to prove that a cascade occurs if and only if:

1) Condition $(A1)$:

$$p_t - gE(V|h_t, S_t = s^L) \leq 0.$$

and, if the above expression holds as an equality, in equilibrium an informed trader with a gain $g$ from holding the asset and a signal $s^L$ buys with probability one.

2) Condition $(A2)$:

$$lE(V|h_t, S_t = s^H) - p_t \leq 0.$$

and, if the above expression holds as an equality, in equilibrium an informed trader with a loss $l$ from holding the asset and a signal $s^H$ sells with probability one.

Let us write the conditions for the equilibrium ask and bid prices at time $t$:

$$(a_t - E(V|h_t)) \Pr(U_t = 1|h_t, X_t = buy, a_t, b_t) + \sum_{s=s^H, s^L} (a_t - E(V|h_t, S_t = s)) \Pr(S_t = s, U_t = 0|h_t, X_t = buy, a_t, b_t) = 0,$$

$$(E(V|h_t) - b_t) \Pr(U_t = 1|h_t, X_t = buy, a_t, b_t) + \sum_{s=s^H, s^L} (E(V|h_t, S_t = s) - b_t) \Pr(S_t = s, U_t = 0|h_t, X_t = sell, a_t, b_t) = 0.$$

As we have proved in Lemma 1, $B_t$, $P_t = E(V|H_t)$ and $A_t$ converge almost surely to the same random variable. Therefore, it is immediate to show that, for every $\varepsilon > 0$, there exists a time $T$ such that, for any $t > T$,

$-\sum_{s=s^H,\, s^L} (p_t - E(V|h_t, S_t = s)) \Pr(S_t = s, U_t = 0|h_t, X_t = buy, a_t, b_t) < \varepsilon$

and

$-\sum_{s=s^H,\, s^L} (E(V|h_t, S_t = s) - p_t) \Pr(S_t = s, U_t = 0|h_t, X_t = sell, a_t, b_t) < \varepsilon$.

This implies that

$(E(V|h_t, S_t = s^H) - p_t)(\Pr(S_t = s^H, U_t = 0|h_t, X_t = buy, a_t, b_t) - \Pr(S_t = s^H, U_t = 0|h_t, X_t = sell, a_t, b_t)) +$

$(E(V|h_t, S_t = s^L) - p_t)(\Pr(S_t = s^L, U_t = 0|h_t, X_t = buy, a_t, b_t) - \Pr(S_t = s^L, U_t = 0|h_t, X_t = sell, a_t, b_t)) < 2\varepsilon$

Note that both terms on the left of the inequality are positive.

The proof is by contradiction. Let us consider now the case in which condition $(A2)$ is violated. This means that a trader with a high signal never sells; therefore, $\Pr(S_t = s^H, U_t = 0|h_t, X_t = sell, a_t, b_t) = 0$.[36]

From Lemma 2, $\Pr(S_t = s^H, U_t = 0|h_t, X_t = buy, a_t, b_t) \geq \frac{1}{2}\mu(1-q) > 0$. From the above inequality, $E(V|h_t, S_t = s^H) - p_t < \frac{2\varepsilon}{\frac{1}{2}\mu q}$, which can be written as $\dfrac{E(V|h_t, S_t = s^H)}{p_t} < \dfrac{4\varepsilon}{\mu q p_t} + 1$.

Note that $p_t \geq v^L > 0$. By choosing $\varepsilon \leq \frac{1}{4}\left(\frac{1}{l} - 1\right) v^L \mu q$, the inequality becomes $\dfrac{E(V|h_t, S_t = s^H)}{p_t} < \frac{1}{l}$. This implies that $(A2)$ holds, a contradiction. Similar contradictions are obtained if $(A1)$ is violated or both $(A1)$ and $(A2)$ are.

Until now, we proved that, if $q < 1$, an informational cascade arises almost surely. To conclude the proof, we must show that, if instead $q = 1$, it is never the case that all informed traders disregard their private information. If $q = 1$, for any history of trades, $E(V|h_t, S_t = s^H) = v^H$ and $E(V|h_t, S_t = s^L) = v^L$. Let us consider a trader with a loss $l$ from holding the asset. Obviously, whatever the bid and ask prices are, if the trader receives a signal $s^L$, he sells. If, instead, the trader receives a signal $s^H$, his private valuation of the asset is $lv^H$. Since $l > \dfrac{v^H + v^L}{2v^H}$, the trader's valuation is greater than or equal to $\left(\dfrac{v^H + v^L}{2v^H}\right) v^H = \dfrac{v^H + v^L}{2}$; therefore, the trader would sell (thus disregarding his own private information) only if the bid price were greater than $\dfrac{v^H + v^L}{2}$.

Let us now consider a trader with a gain $g$ from holding the asset. Obviously, whatever the bid and ask prices are, if the trader receives a

[36] Observe that condition $(A2)$ is also violated when a trader with a high signal and a loss from the asset has a valuation equal to the price $p_t$ and sells with a positive probability lower than 1. This, however, cannot occur in equilibirum. It is easy to show that, if at time $t$ either $(A1)$ or $(A2)$ is violated, the bid-ask spread is strictly positive (i.e., $b_t < p_t < a_t$). As a result, a trader with a high signal and loss from holding the asset whose valuation equals the price will not sell with positive probability. Therefore, if $(A2)$ is violated, it is always the case that
$\Pr(S_t = s^H, U_t = 0|h_t, X_t = sell, a_t, b_t) = 0$.

signal $s^H$, he buys. If the trader receives a signal $s^L$, his private valuation of the asset is $gv^L$. Since $g < \frac{v^H + v^L}{2v^L}$, the trader's valuation is smaller than or equal to $\left(\frac{v^H + v^L}{2v^L}\right) v^L = \frac{v^H + v^L}{2}$; therefore, the trader would buy (thus disregarding his own private information) only if the ask price were smaller than $\frac{v^H + v^L}{2}$. Since the ask is always greater than the bid, for any price either the action of a trader with a loss or the action of a trader with a gain depends on his private signal. Therefore, it is never the case that an informational cascade arises.

## 6.4 Proof of Corollary 1

The proof that in a cascade $a_t = p_t = b_t$ is already contained in the proof of Proposition 2. When conditions $(A1)$ and $(A2)$ in the proof of Proposition 2 hold at time $t$, the expectations of the market maker and of the traders (for any signal realization) remain unchanged at time $t + j$ for $j = 0, 1, 2...$, which implies that $(A1)$ and $(A2)$ also hold at $t + j$.

## 6.5 Proof of Proposition 3

As shown in the proof of Proposition 2, an informational cascade occurs when

$$lE(V|h_t, s^H) < p_t \text{ and}$$
$$gE(V|h_t, s^L) > p_t.$$

Recall that

$$E(V|h_t, s^H) = v^H \Pr(V = v^H|h_t, s^H) + v^L \Pr(V = v^L|h_t, s^H) =$$
$$(v^H - v^L) \Pr(V = v^H|h_t, s^H) + v^L,$$

and note that $p_t = (v^H - v^L)\widehat{p}_t + v^L$.
Then, the first inequality can be written as

$$l(v^H - v^L) \Pr(V = v^H|h_t, s^H) + lv^L < (v^H - v^L)\widehat{p}_t + v^L,$$

i.e.,

$$l(v^H - v^L)\frac{q\widehat{p}_t}{q\widehat{p}_t + (1-q)(1-\widehat{p}_t)} + lv^L < (v^H - v^L)\widehat{p}_t + v^L$$

or

$$\left(\frac{lq}{q\widehat{p}_t+(1-q)(1-\widehat{p}_t)}-1\right)\widehat{p}_t<\frac{v^L(1-l)}{(v^H-v^L)}.$$

It is straightforward to check that this inequality is quadratic in $\widehat{p}_t$. Similarly, the second inequality can be written as

$$g(v^H-v^L)\Pr(V=v^H|h_t,s^L)+gv^L>(v^H-v^L)\widehat{p}_t+v^L,$$

i.e.,

$$\left(\frac{g(1-q)}{(1-q)\widehat{p}_t+q(1-\widehat{p}_t)}-1\right)\widehat{p}_t>\frac{v^L(1-g)}{v^H-v^L}$$

which is again quadratic in $\widehat{p}_t$. If the associated quadratic equations only have complex roots, then the inequalities are always satisfied. If, instead, one or both have real roots, then the solutions are those indicated in the proposition, as it is immediate to check.

When $\widehat{p}_t$ is higher than $M:=\max\{\overline{\alpha},\overline{\beta}\}$ or lower than $m:=\min\{\underline{\alpha},\underline{\beta}\}$, the market is in an informational cascade. Note that, since the public belief moves in discrete steps, $m$ and $M$ are only bounds for the cascade regions.

## 6.6 Proof of Proposition 4

We compute the probability that the public belief $\widehat{p}_t$ reaches the cascade region $[0,m)$ given that $V=v^H$.

To this aim, we first prove the following lemma:

**Lemma** Given $V=v^H$, $\dfrac{\Pr(V=v^L|H_t)}{\Pr(V=v^H|H_t)}$ is a martingale with respect to the history $H_t$.

**Proof of lemma**

Let us express $\widehat{p}_{t+1}$ as a function of $\widehat{p}_t$ using Bayes's rule

$$\widehat{p}_{t+1}=\Pr(V=v^H|h_{t+1})=$$
$$\Pr(V=v^H|h_t,x_t)=\frac{\Pr(x_t|h_t,V=v^H)\widehat{p}_t}{\Pr(x_t|h_t)}.$$

Therefore,

$$1-\widehat{p}_{t+1}=1-\frac{\Pr(x_t|h_t,V=v^H)\widehat{p}_t}{\Pr(x_t|h_t)}=$$
$$\frac{\Pr(x_t|h_t)-\Pr(x_t|h_t,V=v^H)\widehat{p}_t}{\Pr(x_t|h_t)}=\frac{\Pr(x_t|h_t,V=v^L)(1-\widehat{p}_t)}{\Pr(x_t|h_t)}$$

Hence,

$$\frac{1-\widehat{p}_{t+1}}{\widehat{p}_{t+1}} = \frac{\Pr(x_t|h_t, V=v^L)(1-\widehat{p}_t)}{\Pr(x_t|h_t, V=v^H)\widehat{p}_t}.$$

Finally,

$$E(\frac{1-\widehat{P}_{t+1}}{\widehat{P}_{t+1}}|H_t, V=v^H) = \sum_{x_t} \Pr(x_t|H_t, V=v^H)\frac{\Pr(x_t|H_t, V=v^L)(1-\widehat{P}_t)}{\Pr(x_t|H_t, V=v^H)\widehat{P}_t} =$$

$$\sum_{x_t} \frac{\Pr(x_t|H_t, V=v^L)(1-\widehat{P}_t)}{\widehat{P}_t} = \frac{(1-\widehat{P}_t)}{\widehat{P}_t}\sum_{x_t} \Pr(x_t|H_t, V=v^L) = \frac{1-\widehat{P}_t}{\widehat{P}_t},$$

which ends the proof.

Given the lemma, the following equality must be satisfied

$$(1-\Pr(\widehat{P}_t = m|H_t, V=v^H))\frac{1-M}{M} + \Pr(\widehat{P}_t = m|H_t, V=v^H)\frac{1-m}{m} = \frac{1-p}{p}.$$

whose solution gives the expression in the proposition.

## 6.7 Proof of Proposition 5

We first prove the following lemma:

**Lemma** For any history $h_t$, $\Pr(V^Y = v^H|V^W = v^H, h_t) > \Pr(V^Y = v^H|V^W = v^L, h_t)$.

**Proof of lemma** We prove the lemma in a recursive way.

For $h_1 = \varnothing$, the result is clearly true by assumption.

Take now $h_2 = x_1^Y$. Note that we consider the action in market $Y$ because if it were in market $W$ the action would be uninformative, given the value of $V^W$, and the result would hold by assumption. Then:

$$\Pr(V^Y = v^H|V^W = v^H, x_1^Y) =$$

$$\frac{\Pr(x_1^Y|V^W=v^H, V^Y=v^H)\Pr(V^Y=v^H|V^W=v^H)}{\Pr(x_1^Y|V^W=v^H, V^Y=v^H)\Pr(V^Y=v^H|V^W=v^H)+\Pr(x_1^Y|V^W=v^H, V^Y=v^L)\Pr(V^Y=v^L|V^W=v^H)} =$$

$$\frac{\Pr(x_1^Y|V^Y=v^H)\Pr(V^Y=v^H|V^W=v^H)}{\Pr(x_1^Y|V^Y=v^H)\Pr(V^Y=v^H|V^W=v^H)+\Pr(x_1^Y|V^Y=v^L)\Pr(V^Y=v^L|V^W=v^H)}.$$

and

$$\Pr(V^Y = v^H | V^W = v^L, x_1^Y) =$$
$$\frac{\Pr(x_1^Y|V^W=v^L,V^Y=v^H)\Pr(V^Y=v^H|V^W=v^L)}{\Pr(x_1^Y|V^W=v^L,V^Y=v^H)\Pr(V^Y=v^H|V^W=v^L)+\Pr(x_1^Y|V^W=v^L,V^Y=v^L)\Pr(V^Y=v^L|V^W=v^L)} =$$
$$\frac{\Pr(x_1^Y|V^Y=v^H)\Pr(V^Y=v^H|V^W=v^L)}{\Pr(x_1^Y|V^Y=v^H)\Pr(V^Y=v^H|V^W=v^L)+\Pr(x_1^Y|V^Y=v^L)\Pr(V^Y=v^L|V^W=v^L)}.$$

It is immediate to see that the second probability is lower than the first, since we know from the previous step that $\Pr(V^Y = v^H|V^W = v^L) > \Pr(V^Y = v^H|V^W = v^L)$. The same procedure applies to the next steps.

Now we prove the proposition for the case of a buy order. We must show that $\Pr(V^Y = v^H|h_t, buy_t^W) > \Pr(V^Y = v^H|h_t)$.

Note that

$$\Pr(V^Y = v^H|h_t, buy_t^W) =$$
$$\frac{\Pr(buy_t^W|V^Y = v^H, h_t)\Pr(V^Y = v^H|h_t)}{\Pr(buy_t^W|V^Y = v^H, h_t)\Pr(V^Y = v^H|h_t) + \Pr(buy_t^W|V^Y = v^L, h_t)(1 - \Pr(V^Y = v^L|h_t))}.$$

Hence, $\Pr(V^Y = v^H|h_t, buy_t^W) > \Pr(V^Y = v^H|h_t)$ if and only if $\Pr(buy_t^W|V^Y = v^H, h_t) > \Pr(buy_t^W|V^Y = v^L, h_t)$.

The left hand side of the inequality is equal to:

$$\Pr(buy_t^W|V^Y = v^H, h_t) =$$

$$\Pr(buy_t^W|V^Y = v^H, V^W = v^H, h_t)\Pr(V^W = v^H|V^Y = v^H, h_t)+$$
$$\Pr(buy_t^W|V^Y = v^H, V^W = v^L, h_t)\Pr(V^W = v^L|V^Y = v^H, h_t) =$$
$$\Pr(buy_t^W|V^W = v^H, h_t)\Pr(V^W = v^H|V^Y = v^H, h_t)+$$
$$\Pr(buy_t^W|V^W = v^L, h_t)\Pr(V^W = v^L|V^Y = v^H, h_t).$$

Similarly,

$$\Pr(buy_t^W|V^Y = v^L, h_t) =$$

$$\Pr(buy_t^W|V^W = v^H, h_t)\Pr(V^W = v^H|V^Y = v^L, h_t)+$$
$$\Pr(buy_t^W|V^W = v^L, h_t)\Pr(V^W = v^L|V^Y = v^L, h_t).$$

Since $\Pr(buy_t^W|V^W = v^H, h_t) > \Pr(buy_t^W|V^W = v^L, h_t)$, and we also know that $\Pr(V^Y = v^H|V^W = v^H, h_t) > \Pr(V^Y = v^H|V^W = v^L, h_t)$ (as we have shown in the proof of Result 1 above), the result immediately follows. The proof in the case of a sell or a no trade follows identical steps.

## 6.8 Proof of Proposition 6

By following the same steps as in the proof of Proposition 2, we can prove that there exists a time $T$ such that almost surely for all $t \geq T$,

$$E(V^Y|h_t, X_t^Y = buy^Y) - gE(V^Y|h_t, S_t^Y = s^L) \leq 0.$$

$$lE(V^Y|h_t, S_t^Y = s^H) - E(V^Y|h_{t,}X_t^Y = sell^Y) \leq 0$$

$$E(V^W|h_t, X_t^W = buy^W) - gE(V^W|h_t, S_t^W = s^L) \leq 0.$$

$$lE(V^W|h_t, S_t^W = s^H) - E(V^W|h_{t,}X_t^W = sell^W) \leq 0$$

When these conditions are satisfied, in both markets a trader with a gain $g$ from the asset buys and a trader with a loss $l$ sells independently of their signals. Therefore, in both markets, the probability of a trade is independent of the realization of the signal. This means that there is an informational cascade in both markets; that is, an informational breakdown occurs.

## 6.9 Proof of Proposition 7

Let us first prove letter a. We know from Proposition 4 that, when $\widehat{p}_t^I \leq m^I$ or $\widehat{p}_t^I \geq M^I$, market $I$ is in an informational cascade. We want to prove that if the breakdown occurs when the public beliefs in the two markets are one below 0.5 and the other above 0.5, then the breakdown occurs at the same (approximate) thresholds that we computed for the one-asset economy. Without loss of generality, suppose that a cascade starts in market $I$ at time $T$ when $\widehat{p}_T^I \cong M^I$, and a breakdown occurs at a later time $T' > T$, when $\widehat{p}_{T'}^J < 0.5$. By using the same logic of the one-asset economy, it must be that $\widehat{p}_{T'}^J \cong m^J$. Moreover, by definition, $\widehat{p}_t^J$ does not change after $T'$. Hence, to prove our result, it only remains to show that the price in market $I$ does not increase between $T$ and $T'$; that is, $\widehat{p}_{T'}^I \leq \widehat{p}_T^I$. To prove this, note that

$$\widehat{p}_T^I = \Pr(V^I = v^H | h_T) =$$
$$\Pr(V^I = v^H | h_T, V^J = v^H) \Pr(V^J = v^H | h_T) +$$
$$\Pr(V^I = v^H | h_T, V^J = v^L) \Pr(V^J = v^L | h_T),$$

and

$$\widehat{p}_{T'}^I = \Pr(V^I = v^H | h_{T'}) =$$
$$\Pr(V^I = v^H | h_{T'}, V^J = v^H) \Pr(V^J = v^H | h_{T'}) +$$
$$\Pr(V^I = v^H | h_{T'}, V^J = v^L) \Pr(V^J = v^L | h_{T'}).$$

Since between $T$ and $T'$ market $I$ is in a cascade, the only informative actions are those in market $J$. Hence, $\Pr(V^I = 2 | h_T, V^J = v^H) = \Pr(V^I = 2 | h_{T'}, V^J = v^H)$ and $\Pr(V^I = v^H | h_T, V^J = v^L) = \Pr(V^I = v^H | h_{T'}, V^J = v^L)$. Furthermore, since market $J$ is in a cascade at time $T'$ but not at time $T$, $\Pr(V^J = v^H | h_{T'}) \leq m^J < \Pr(V^J = v^H | h_T)$. It is, therefore, immediate to see that $\widehat{p}_T^I \geq \widehat{p}_{T'}^I$.

We now prove letter c. The proof for letter b is analogous. Suppose that $\widehat{p}_{T-1}^I < M^I$ and $\widehat{p}_T^I \geq M^I$. If at time $T$ market $J$ is in a cascade too, then the economy is in a breakdown and $\widehat{p}_t^I$ remains stuck forever close to $M^I$, which proves the first inequality. If, instead, at $T$ market $J$ is not in a cascade, the history in market $J$ affects the price in market $I$. In particular, for some sequences of decisions in market $J$, $\widehat{p}_t^I$ keeps increasing above the threshold $M^I$ until market $J$ itself is in a cascade. To find an upper bound on the level that $\widehat{p}_t^I$ can reach, let us consider a history in which market $I$ remains in the cascade for $t > T$; let us denote the time when market $J$ reaches its cascade region too by $T'$ $(T' > T)$. Then, note that

$$\Pr(V^I = v^H | h_{T'})$$
$$\Pr(V^I = v^H | h_{T'}, V^J = v^H) \Pr(V^J = v^H | h_{T'}) +$$
$$\Pr(V^I = v^H | h_{T'}, V^J = v^L) \Pr(V^J = v^L | h_{T'}) =$$
$$\Pr(V^I = v^H | h_{T'}, V^J = v^H) M^J + \Pr(V^I = v^H | h_{T'}, V^J = v^L)(1 - M^J).$$

Moreover,

$$\Pr(V^I = v^H | h_{T'}, V^J = v^H) \leq 1,$$
$$\Pr(V^I = v^H | h_{T'}, V^J = v^L) \leq M^I,$$

where the second inequality comes from the fact that the asset values are positively correlated.

Therefore,

$$\Pr(V^I = v^H|h_{T'}) \leq M^J + M^I(1 - M^J)$$

and

$$\Delta_H^I = M^J + M^I(1 - M^J) - M^I = (1 - M^I)M^J,$$

which ends the proof.

## 6.10 Proof of Proposition 8

We will prove the proposition for the case in which $V^I = v^H$. Consider a particular realization for the sequence of random variables determining, in market $I$, for each time $t$, whether the trade occurs in the market, whether the trader is informed or not, his private signal and private value if he is informed, and his decision if he is uninformed. Consider the set of times when a decision occurs in market $I$ and denote this set by $T^I$. Let us compute the sequence of prices in $T^I$ assuming that the traders and the market maker in the market only observe $h_t^I$, and denote it by $\left\{p_t^{I*}\right\}_{t\in T^I}$. Now, let us consider the price at time $t' \in T^I$ for the case in which agents in market $I$ observe the history in both markets, $h_{t'}$ (i.e., $p_{t'}^I = E(V^I|h_{t'})$). Let us compute the expected value of the price conditional on $V^I = v^H$ and $h_{t'}^I$, namely, $E(p_{t'}^I|V^I = v^H, h_{t'}^I)$ by integrating over all possible histories in market $J$. This expectation is always higher than $p_{t'}^{I*}$ since, for any $t < t'$ and $t \notin T^I$, in expectation, the decision in market $J$ increases the price of asset $J$ and, as a result, of asset $I$. Note that this inequality will hold for any $h_{t'}^I$. Hence, $E(p_t^I|V^I = v^H) > p_t^{I*}$. The proof for the case $V^I = v^L$ is analogous.

## 6.11 Proof of Proposition 9

First we prove that the covariance between prices is bounded by the covariance between fundamentals.

Note that

$$E(E(V^Y|H_t)E(V^W|H_t)) =$$
$$\sum_{h_t}\left(\sum_{v^Y} v^Y \Pr(v^Y|h_t) \sum_{v^W} v^W \Pr(v^Y|h_t)\right)\Pr(h_t).$$

By the lemma in the proof of Proposition 5, it is easy to show that, for any history $h_t$, $Cov(V^Y, V^W|h_t) > 0$. Therefore,

$$\sum_{h_t} \left( \sum_{v^Y} v^Y \Pr(v^Y|h_t) \sum_{v^W} v^W \Pr(v^Y|h_t) \right) \Pr(h_t) \leq$$
$$\sum_{h_t} \left( \sum_{v^Y} v^Y v^W \Pr(v^Y, v^W|h_t) \right) \Pr(h_t) = E(V^Y V^W),$$

which ends the proof.

Now let us shows that the covariance is increasing over time. Let us define $\Delta P_t^J =: P_t^Y - P_{t-1}^Y$. We prove the statement in two steps.

**Step 1** $E(\Delta P_t^Y \Delta P_t^W) \geq 0$.

**Proof of Step 1** Note that

$$E(\Delta P_t^Y \Delta P_t^W) =$$
$$\sum_{x_t^Y} \Pr(x_t^Y|h_h)[E(V^Y|h_t, x_t^Y) - E(V^Y|h_t)][E(V^W|h_t, x_t^Y) - E(V^W|h_t)].$$

We know from Proposition 5 that

$$sign[E(V^Y|h_t, x_t^Y) - E(V^Y|h_t)] = sign[E(V^W|h_t, x_t^Y) - E(V^W|h_t)].$$

Therefore, all the terms in the above sum are greater than zero and $E(\Delta P_t^Y \Delta P_t^W) \geq 0$.

**Step 2** $Cov(P_t^Y, P_t^W)$ is greater than zero and increasing.

**Proof of Step 2**

$$Cov(P_t^Y, P_t^W) = Cov(P_0^Y + \sum_{i=1}^{t} \Delta P_i^Y, P_0^W + \sum_{i=1}^{t} \Delta P_i^W) =$$
$$E\left[ (P_0^Y + \sum_{i=1}^{t} \Delta P_i^Y)(P_0^W + \sum_{i=1}^{t} \Delta P_i^W) \right] -$$
$$E(P_0^Y + \sum_{i=1}^{t} \Delta P_i^Y) E(P_0^W + \sum_{i=1}^{t} \Delta P_i^W) =$$
$$= E(\sum_{i=1}^{t} \Delta P_i^Y \Delta P_i^W) =$$
$$= \sum_{i=1}^{t} E(\Delta P_i^Y \Delta P_i^W), \tag{A9}$$

where the second equality holds because prices are martingales; therefore, $E(\Delta P_t^Y) = 0$, $E(\Delta P_t^W) = 0$, and $E(\Delta P_i^Y \Delta P_{i+k}^W) = 0$ (for $k \neq 0$).

Since, as shown in Step 1, all the terms in the sum are greater than zero, $Cov(P_t^Y, P_t^W)$ is greater than zero and increasing.

## 6.12 Proof of Proposition 10

Recall that the public belief $\widehat{P}_t^I$ is a linear transformation of the price ($\widehat{P}_t^I = \frac{P_t^I - v^L}{v^H - v^L}$), and, as a result, the correlation between $\widehat{P}_t^Y$ and $\widehat{P}_t^W$ is the same as that between $P_t^Y$ and $P_t^W$. Therefore, to prove the proposition, we must prove that the correlation between the asymptotic values of $\widehat{P}_t^Y$ and $\widehat{P}_t^W$ is lower than that between the fundamentals. Let us define $r := \Pr(V^W = v^L | V^Y = v^L) = \Pr(V^W = v^H | V^Y = v^H)$, which we know by assumption to be greater than $\frac{1}{2}$. It is straightforward to see that the correlation between $V^Y$ and $V^W$ is equal to $2r - 1$.

Moreover, since, in both markets, the thresholds $m$ and $M$ are symmetric around $\dfrac{1}{2}$ and since $\widehat{P}_t^I$ is a martingale, the same logic used in Proposition 3 implies that $\widehat{P}_t^I$ converges to $m$ and $M$ with equal probability. This means that, in both markets, the asymptotic variance of $\widehat{P}_t^I$ is equal to $\frac{1}{2}(m^2 + M^2) - \frac{1}{4}$. Furthermore, the asymptotic covariance between $\widehat{P}_t^Y$ and $\widehat{P}_t^W$ is equal to $m^2\frac{\pi}{2} + (1-\pi)mM + M^2\frac{\pi}{2} - \frac{1}{4}$, where $\frac{\pi}{2}$ is the probability that both beliefs converge to $m$ (or both to $M$). Using the fact that $M = 1 - m$, it is easy to show that the asymptotic correlation between beliefs is equal to $2\pi - 1$. Since the agents in each market can only observe the history of trades and prices in their own market, given the realizations of $V^Y$ and $V^W$, the probability that the belief in a market converges to $M$ (or $m$) is independent of where the price converges in the other market. Therefore, the value of $\pi$ can be computed by using the same methodology used in Proposition 4 to find the probability of an incorrect cascade in a one-asset economy. In this way, one finds that $\pi$ is equal to $\frac{r}{2}(2m)^2 + (1-\frac{r}{2})(1-2m)^2 + \frac{r}{2}(1-\frac{r}{2})(2m)(1-2m)$, which is lower than $r$. As a result, the correlation between beliefs (equal to that between asset prices), is $2\pi - 1$, lower than that between the fundamentals, $2r - 1$.

### 6.13 Parameter Values

| | $v^H$ | $v^L$ | $p$ | $\mu$ | $q$ | $g$ | $l$ | $r$ |
|---|---|---|---|---|---|---|---|---|
| Figure 1 | 1 | 2 | 0.5 | 0.5 | 0.8 | 1.05 | 0.95 | |
| Figure 2 | 1 | 2 | 0.5 | | | 1.075 | 0.925 | |
| Figure 3 | 1 | 2 | 0.5 | | | 1.075 | 0.925 | |
| Figure 4 | 1 | 2 | 0.5 | 0.74 | 0.7/0.97 | 1.1 | 0.9 | 0.63 |
| Figure 5 | 1 | 2 | 0.5 | 0.4 | 0.65/0.7 | 1.1 | 0.9 | 0.72 |
| Figure 6 | 1 | 2 | 0.5 | 0.7 | 0.7 | 1.1 | 0.9 | 0.7 |
| Figure 7 | 1 | 2 | 0.5 | 0.7 | | 1.1 | 0.9 | 0.7 |
| Figure 8 | 1 | 2 | 0.5 | 0.35 | 0.7 | 1.1 | 0.9 | 0 |

When two values are indicated for $q$, the first refers to the signal on asset $Y$ and the second to that on asset $W$. Furthermore, for Figures 4 to 7, $r := \Pr(V^W = v^L | V^Y = v^L) = \Pr(V^W = v^H | V^Y = v^L)$, while for Figure 8 $r := \Pr(HNB^Y | HNS^W) = \Pr(HNS^Y | HNB^W)$.

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