Topic	Estimating the epicenters of local and regional seismic sources, using the circle and chord method (Tutorial with exercise by hand and movies)
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1 Aim

This is a tutorial with related exercise. It aims at making you familiar with the basic "circle and chord" method used for determining the epicenter of near seismic sources when a "flat Earth" approach is appropriate. The method is applied both to sources inside and outside of the recording networks. With respect to phase names and essential phase features one should also consult section 2.6.1 in Chapter 2 and record examples in DS 11.2.

2 Data

- Available are two sections of vertical component short-period records of stations of the former Potsdam seismic network from a local earthquake inside the network (see Figure 2) and a strong rock-burst in a mine located outside of the network (see Figure 3).

- Travel-time curves of the main crustal phases Pn, Pg, Sn, Sg and Lg from a near surface source up to an epicentral distance of 400 km (see Figure 4). These curves are reasonably good average curves for Central Europe. For any stations in this exercise at distances beyond 400 km, you may linearly extrapolate the curve without much error.

- Map with the positions of the recording stations and a distance scale (see Figure 5).

3 Procedure

- Identify the seismic phases in short-period records of near seismic sources.

- By means of a local travel-time curve for near-surface events determine the source distance **d** from the best fit with the identified seismic phases.

- If no local travel-time curves are available, a first rough estimate of the hypocenter distance **d** or of the epicentral distance D (both in km) may be found using the following "rules-of-thumb":

$$\mathbf{d \approx t(Sg - Pg) \times 8} \quad \text{or} \tag{1}$$

$$\mathbf{D \approx t(Sn - Pn) \times 10} \tag{2}$$

with t as the travel-time difference in seconds between the respective seismic phases. These rules are approximations for a single layer crust with an average Pg-wave velocity of 5.9 km/s, a sub-Moho velocity of about 8 km/s and a velocity ratio $v_s/v_p = \sqrt{3}$. If in your area of study the respective average P- and S-wave crustal velocities v_p and v_s deviate significantly from these assumptions you may calculate d more accurately from the relationship:

$$\mathbf{d = t(S_g - P_g)\ (v_p\ v_s)/(v_p - v_s)}. \qquad (3)$$

- Draw circles with a compass around each station $\mathbf{S_i}$, which is marked on a distance-true map projection, with the radius $\mathbf{d_i}$ determined from the records of each station.
- The circles will usually cross at two points, not one point (the thought epicenter) thus forming an area of overlap (see Figure 1, shaded area) within which the epicenter most probably lies.
- Usually, it is assumed, that the best estimate of the epicenter position is the "center of gravity" of this shaded area of overlap. The best estimate of the epicenter is found by drawing so-called "chords", i.e., straight lines connecting the two crossing points of each pair of circles. The crossing point (or smaller area of overlap) of the chords should be the best estimate of the epicenter (see Figure 1).

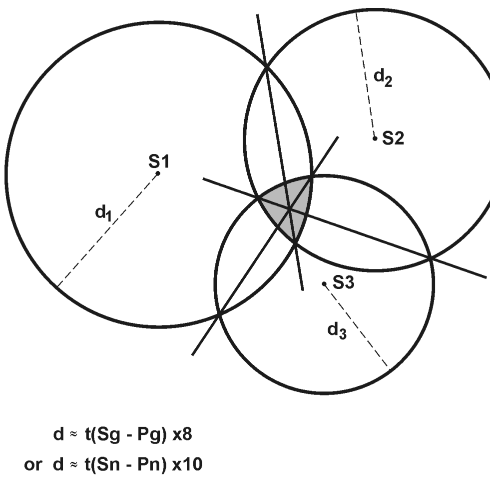

Figure 1 Principle of epicenter estimation by using the "circle and chord" method. S – station sites, d – hypocentral distance of the event determined for each station according to travel-time curves (as given in the Figure 4) or the "rules-of-thumb" (as Equations (1) and (2)).

Notes:

1) In the absence of independent information on the source depth and depth-dependent travel-times the distance d determined as outlined above is not the epicenter but the *hypocenter distance*. Therefore, for sources at depth the circles will necessarily overshoot, the more so the deeper the focus. Iterative reduction of the overshoot permits to estimate the source depth (see section 7 with linked movie).
2) However, even for a surface source, an ideal crossing of circles at a single point would require that all phases have been properly identified, their onset times been picked without error and that the travel-time curves/model for the given area (including the

effects of lateral variations) are exactly known and taken into account. This, however, isl rarely the case. Therefore, do not expect your circles to cross all at one point.

3) Yet, despite note 2, the circles should at least come close to each other in some area, overlapping or not, within about 10 to 20 km, if the epicenter is expected to lie within the network and the hypocenter within the crust. If not, one should check again the phase interpretation and resulting distance estimates and also compare for all stations the consistency of related estimates of origin time (see tasks below). Any obvious outliers should be re-evaluated.
4) For seismic sources outside the network the circle crossing will be worse, the error in epicenter estimation larger, particularly in the direction perpendicular to the azimuth of the connection line (great circle) between the network center and the source. However, the distance control, based on travel-time differences S-P, is better than the azimuth control. Azimuth estimates are more reliable if the source is surrounded by stations on at least three sides, i.e., with a maximum azimuthal gap less than 180°.
5) With only two stations one gets two possible solutions for the three unknowns (epicenter coordinates λ and φ and origin time OT) unless the source direction can be independently determined from polarity readings in three-component records of each station (see EX 11.2). If more than 3 stations are available, the estimates of both epi- and hypocenter will improve.

4 Tasks, data and approaches

4.1 Phase identification and travel-time fit (consult also section 5 for guidance)

- Identify the main local phases Pn, Pg, Sn and/or Sg/Lg in the records of the Potsdam seismic network (Figures 2 and 3) by using the travel-time curves given in Figure 4.

- Use plots of the t-D travel-time curves on transparent sheet and of the record section on paper with identical time resolution (e.g., 1 mm/s, as for the originally analog records). Overlay the t-D curves on the records so that the D(km) axix is strictly perpendicular and the t(s) axis parallel to the record trace zero lines!

- Move the t(s) axis over a given station record trace until you find a best fit for the first arrival and onsets of several later wave groups characterized by significant change/increase in amplitude and/or period. Mark with pencil the best fitting onset times with dots on the record together with their (assumed) phase names.

Notes:

1) When searching for the best fit remember that the beginning of the later wave group with the largest amplitudes in the record is usually the onset of Sg, whereas in the early parts of the record it is Pg that is usually the largest wave. For distances < 400 km Pn is usually much smaller than Pg, although for deeper crustal earthquakes with appropriate rupture orientation and for distances > 400 km Pn may become strong or even the dominating P-wave amplitude (see Figs. 11.44 to 11.46 in Chapter 11 and several network record sections in DS 11.1).
2) From the onset-time differences Sg-Pg and/or Sn-Pn you may roughly estimate the distance of the event by using the "rules-of-thumb" (Equations (1) to (3) above). If your rough estimate from S-P is d < 150 km then the first arrival should never be interpreted as Pn but rather as Pg (unless it is a deeper crustal event or the crust is less than 30 km thick). If d > 150 km, try to get the best fit to the onsets by assuming that

the first arrival is Pn, however remember that its amplitude is usually smaller than that of the following stronger Pg for d < 400 km.

3) The above said is true for near-surface events in a single-layer crust with average P-wave velocity of 5.9 km/s and sub-Moho velocity of 8 km/s. The **cross-over distance** $\mathbf{x_{co}}$ beyond which Pn becomes the first arrival is then approximately $\mathbf{x_{co} \approx 5\ z_m}$ with z_m as the Moho depth. In the case of different average crustal and sub-Moho P-wave velocities, $\overline{v}_c$ and v_m, you may use the relation $\mathbf{x_{co} = 2\ z_m\{(v_m + \overline{v}_c)/(v_m - \overline{v}_c)\}^{1/2}}$ to calculate the cross-over distance of Pn. However, be aware that for deeper crustal events Pn may over-take already at smaller distances!

4.2 Estimation of distance and origin time (consult also section 5 for guidance)

- Write down on the record plots for each station, next to the identified main wave onsets, its assumed phase name as well as the hypocentral distance corresponding to your S-P time reading, respectively the best travel-time fit. Mark on each record the estimated origin time, which is the time of the D-axis position on the record for your best phase travel-time fit.

- Check, whether your marks for the estimated origin times on the different records are roughly the same (in vertical line) for all stations. This is a good check of the accuracy and reproducibility of your phase identifications and estimated distances. For any “outliers” check the phase identification and distance estimate again until you get agreement between the origin times within about ± 3s.

- Compare your best estimate of origin time OT (average of all origin times estimated from the records of each station) with the OT computer solution given in the head lines of Figures 2 and 3.

- If your average OT deviates by more than about 3 s from the computer solution reconsider your interpretation.

4.3 Epicenter location

- Take a compass and draw circles around each station position (see Figure 5) with the radius d_i in km as determined for the distance of the source from the station S_i. Use the distance scale given on the station map.

- Connect the crossing points of each pair of circles by chords. Estimate the coordinates λ and φ (in decimal units of degree) from the chord crossings.

- Compare your coordinates with the ones given in the headlines of Figures 2 and 3. If your solutions deviate by more than 0.2° for the earthquake within the network and by more than 0.4° for the mining rock-burst outside the network, reconsider your phase interpretation, distance estimates and circle-drawings.

- Compare your solutions for phase interpretations, station distances, circle plots and estimated epicenter positions with the solutions derived by the authors when applying the same procedure (see section 6).

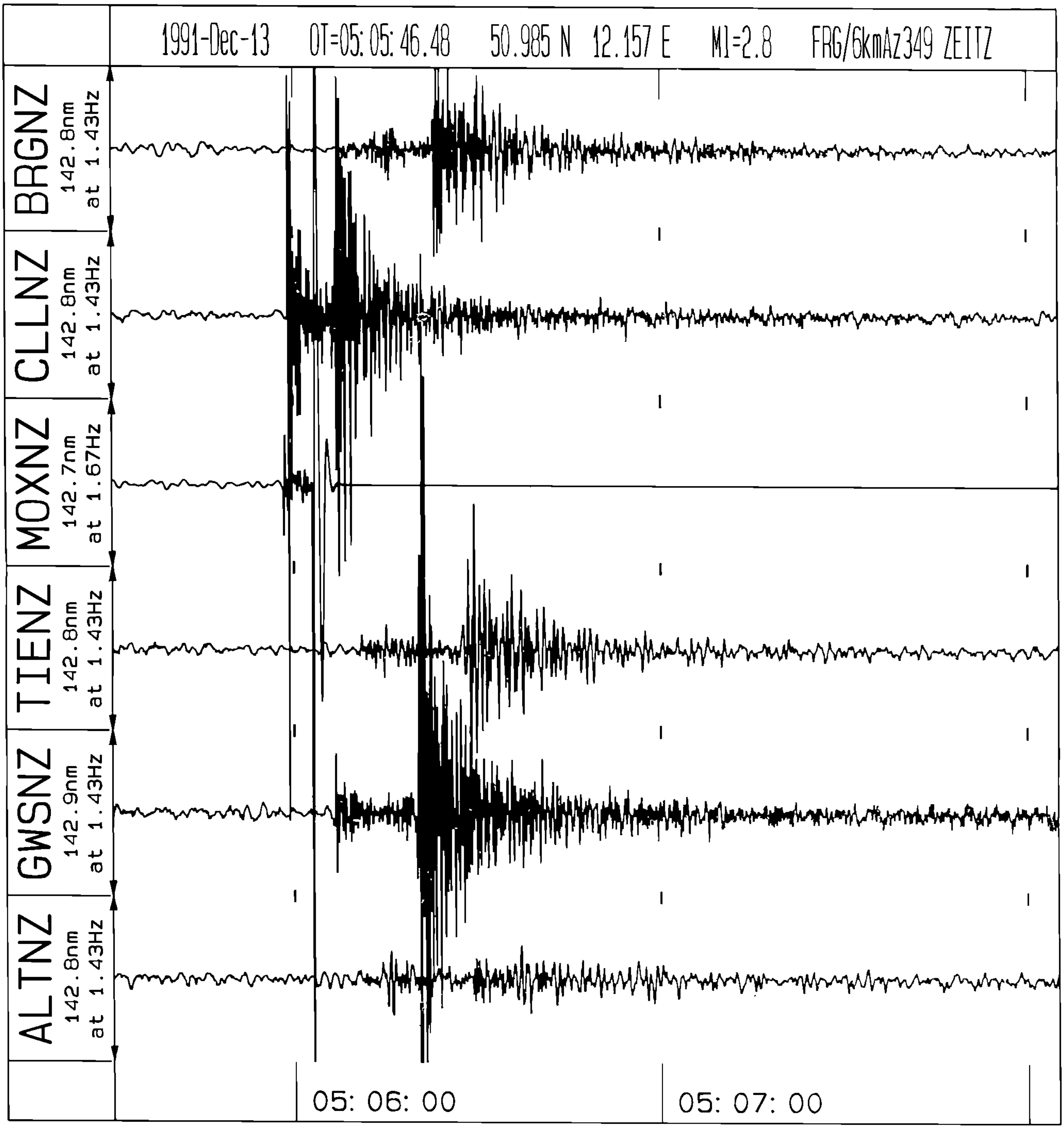

Figure 2 Recordings of a near earthquake situated within the seismic network of stations shown in Figure 5. The record time scale in hh:mm:ss is given below. Note that the second strong onset in the record of station MOX has a shape and frequency that strongly differs from all other records. It is not a natural wave onset but a malfunction of the seismograph, which responds to the strong Sg onset with its own impulse eigen response and then stops recording.

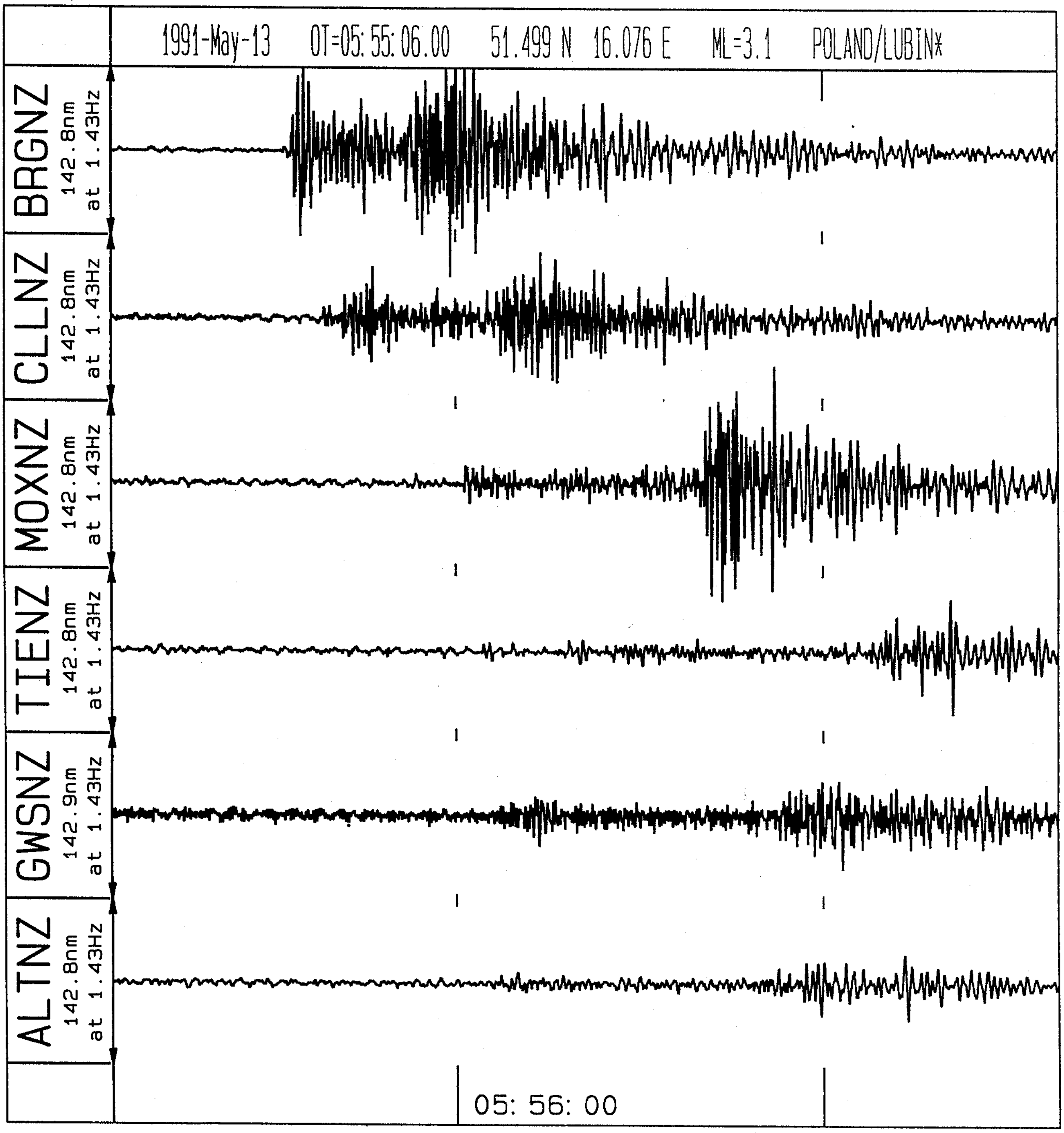

Figure 3 Recordings at regional distances from a strong mining rock-burst situated outside the seismic network of stations shown in Figure 5. Time and minute intervall are given on the abszissa.

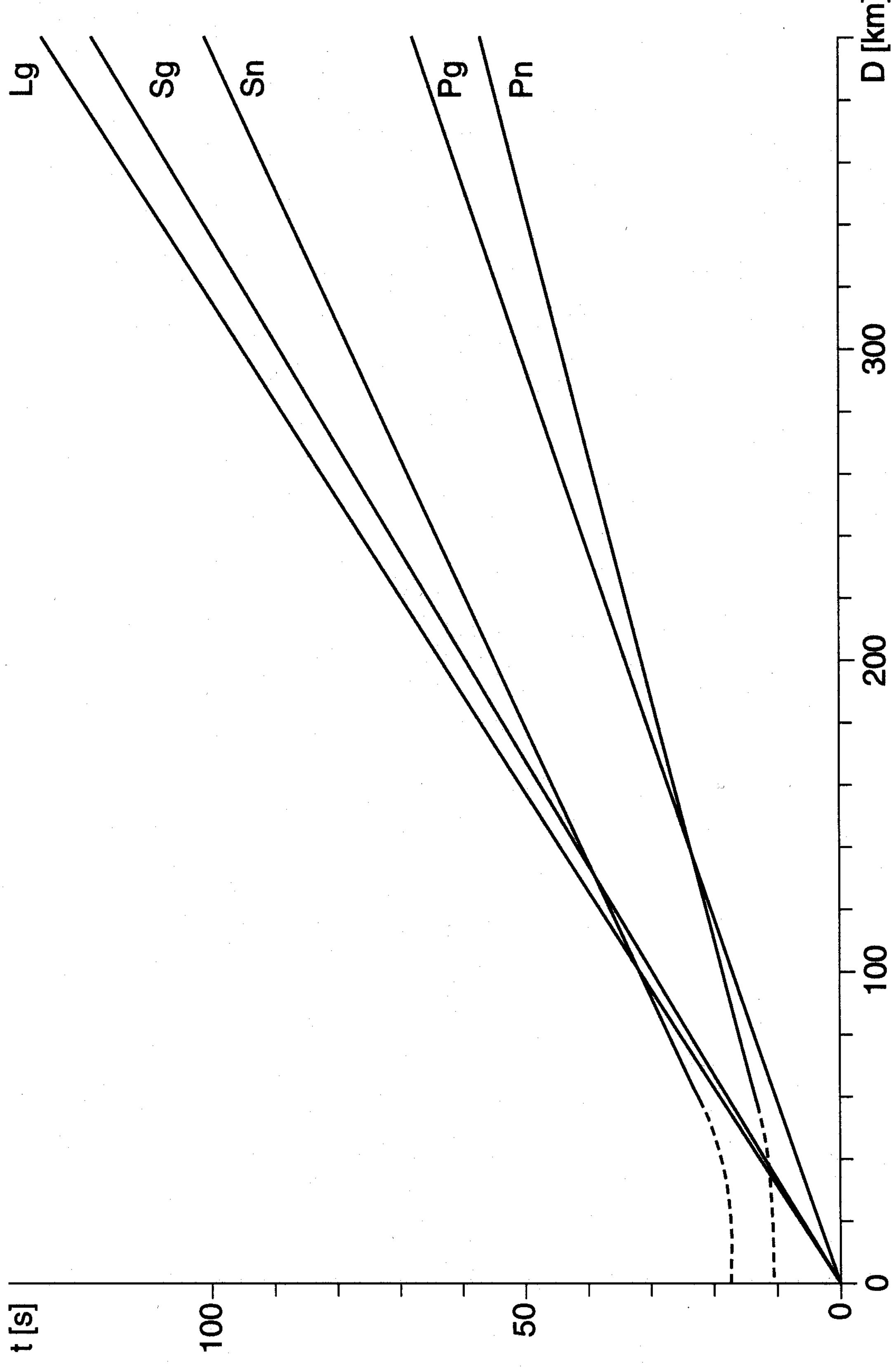

Figure 4 Travel-time curves for the main phases observed in records of near-surface local and near regional seismic sources. They are good average curves for Central Europe with a crustal thickness of about 30 km.

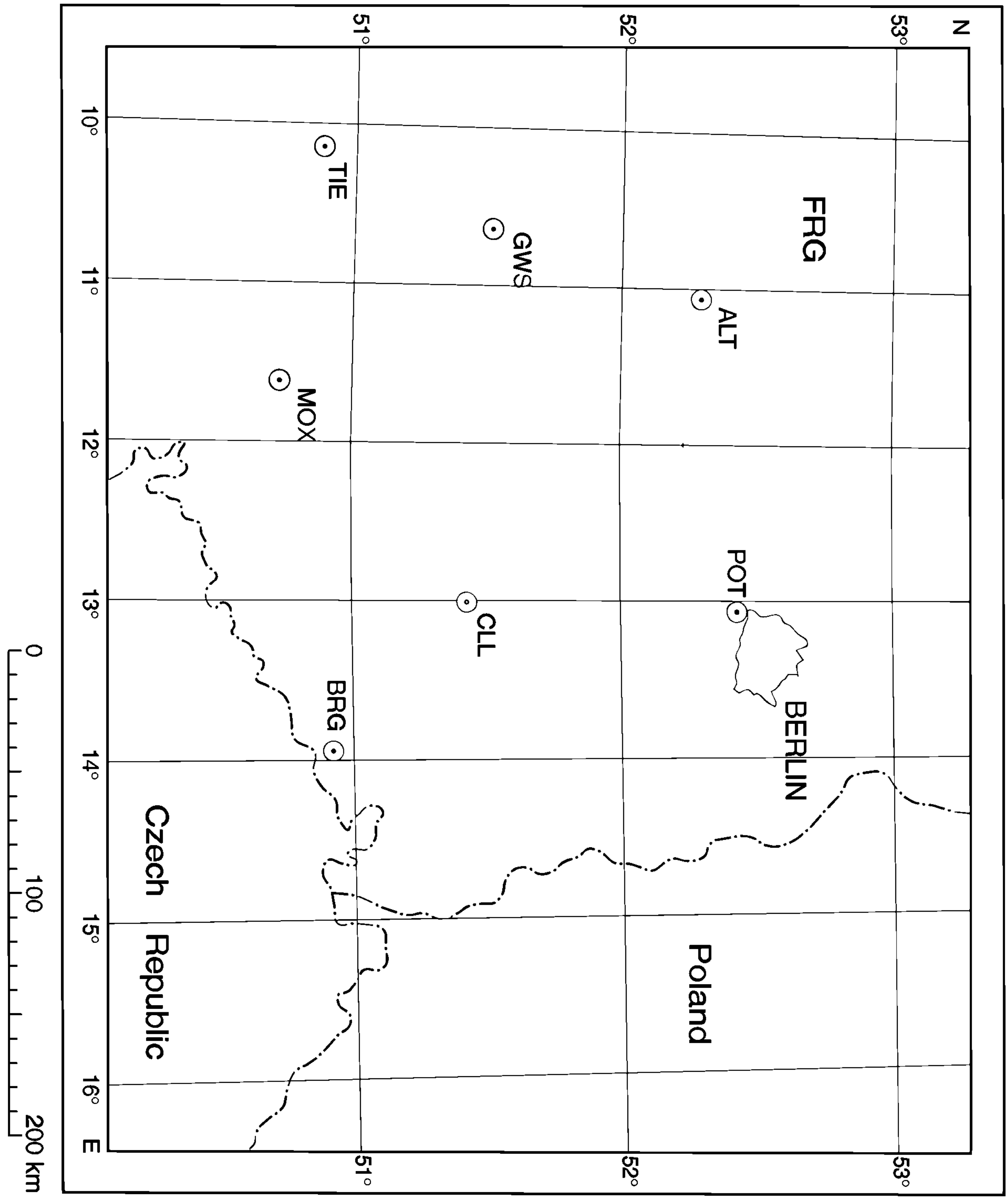

Figure 5 Map of parts of Central Europe with codes and positions (circles) of the seismic stations that recorded the seismograms shown in Figures 2 and 3 (on the map projection all distances are true).

5 Some additional guidance

5.1 Phase identification and distance estimates by matching a seismic record with travel-time curves or using the S-P time difference

Figure 6 illustrates the identification of seismic phases in a record of a shallow local event by stepwise improved matching of recognizable (although sometimes small) wave onsets with travel-time curves and the estimation of the epicentral distance D (when source depth is assumed to be zero) from the best-fitting match.

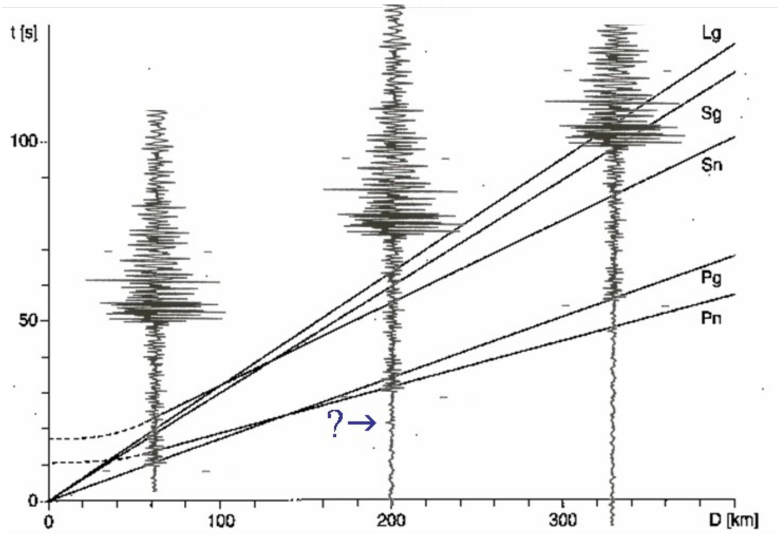

Figure 6 Stepwise movement of a seismic record over a local travel-time curve until a best fit for three recognizable onsets and the Lg wave group is achieved at a distance D ≈ 327 km. For discussion, see text.

At the first two positions we related only the first clear record onset in the early part of the seismogram with the first P-wave travel-time curve. This would be the Pg curve at 60 km distance, respectively the Pn curve at 200 km distance. But then there is no match of the onset of the largest wave group in the later part of the record with any of the well established travel-time curves. Thus, for a better match one has to move the record further to larger distances. However, as outlined above, at distances about 5 times crustal thickness, Pn is expected to arrive as the first P phase, although (at distances < 400 km) with usually much smaller amplitudes than that of Pg. Therefore, we may hypothezise that the small onset marked with ? → in the second position maybe Pn. Although its SNR < 2, its waveform differs clearly from that of the preceding noise). Indeed, with this assumption one gets a good fit for Pn, Pg and Sg, as well as the following Lg-wave maximum, at position 3. The distance read at this position is D ≈ 327 km.

When considering that at distances < 400 km Pg and Sg are usually the two most distinct first arrivals in the earlier and later part of the record one would already get - according to Equation (1) – for the record in Figure 5 with t(Sg-Pg) ≈ 40 s a distance of ≈ 320 km. This is already a good first estimate and requires to search for a possible Pn arrival prior to Pg. If, however, in records made under normal crustal conditions, t(Sg-Pg) < 18 s then one can be rather sure that these have all been recorded at distances d < ≈ 140 km. No other phases can then be expected to be first arrivals in the P- and S-wave groups in this near distance range.

Figures 7 and 8 illustrate for the two seismic events to be located by the same small network these two situations. Distance estimates are made only with the rule-of-thumb (1). On such a record section the first distance estimate should be made only from a record with undoubtedly clear onsets of Pg and Sg. Deducting from the onset-time of Pg its travel-time according to the t-D curve in Figure 6 for the determined distance one gets a first estimate of the origin-time OT. It should be marked on the record trace by a bar. The line along this bar and perpendicular to all other record traces then guides fairly well phase identification, OT and distance estimates on the other station records. Just place the D axis of the travel-time curves, corresponding to t = 0 from a surface source, on/respectively close to this first OT reference line and move the t-D diagram up and down until you get a best fit also with the most distinct phase onsets in the other records. Mark also their respective OT estimates. The may differ from the first one within a few seconds due to reading errors, worse SNR and/or significant lateral velocity differences.

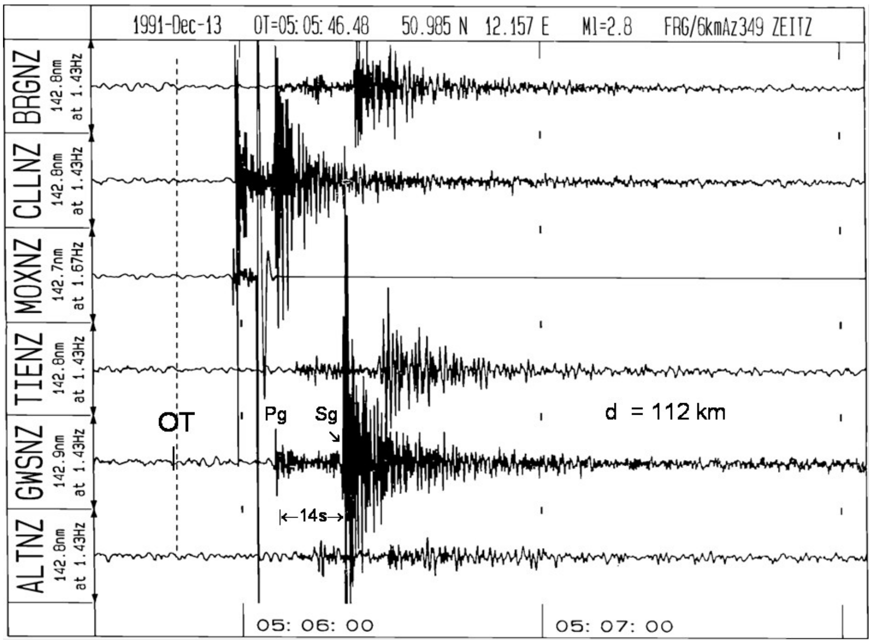

Figure 7 Section of records from a local seismic event in the Pg-Sg distance range. Note the marking of the first origin time OT prior to the clearest Pg at distance of 112 km. This guides also phase identification and d estimates at the other stations when using the t-D curve in Figure 6. Note the distinct wave group between Pg and Sg in records of BRGNZ, TIENZ and ALTNZ. This is not generally present and not explained by the simple travel-time model of Figure 6. Travel-time modeling for an average crustal model makes it likely that this is an SmP wave, i.e., an S-to-P conversion at an overcritical reflection angle from the Moho.

In contrast to Figure 7, Figure 8 shows a record section from another event outside of the network. All stations were at distances larger than 140 km when Pn is already the first arrival. This is easily found out by a quick check of the time differences t(S-P) between the two most distinct phases, Pg and Sg, which is everywhere > 18 s. The follow-up procedure of more detailed phase identification and d determination by moving the travel-time curve of Figure 6 over the records along the first OT line estimate is the same as described above. For more record examples in the local and regional distance range see section 11.5.1 in Chapter 11 and DS 11.1.

Digital records now permit to stretch the time and amplitude scales at will. This may significantly ease phase identification and allows much more accurate picking of onset times (see IS 11.4).

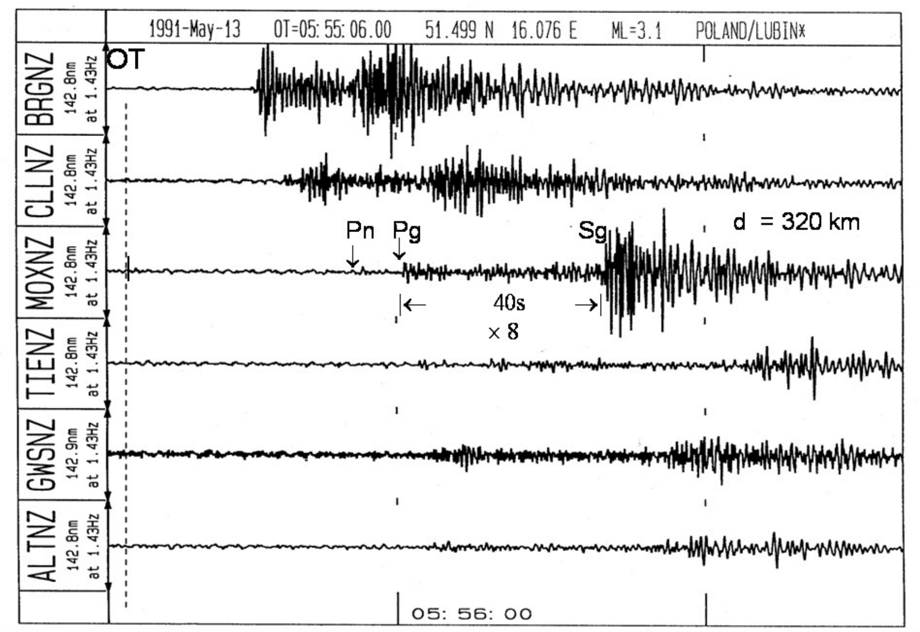

Figure 8 Section of records from a seismic event in the Pn distance range. Note the rather vague onsets of Pg and Sg in the lower three record traces. They will allow only rough estimates of d.

Solutions

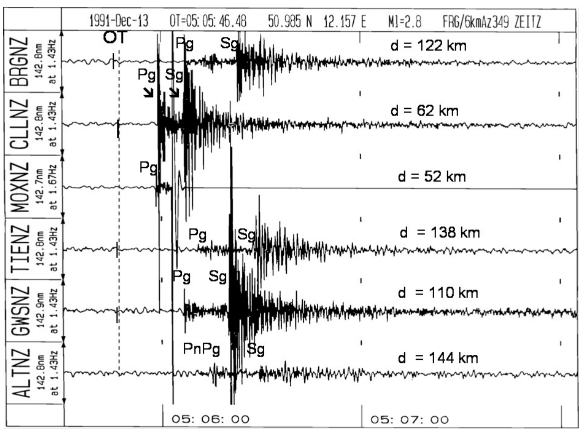

Figure 9 OTs, identified phases and calculated distances for a seismic event in the local Pg-Sg distance range.

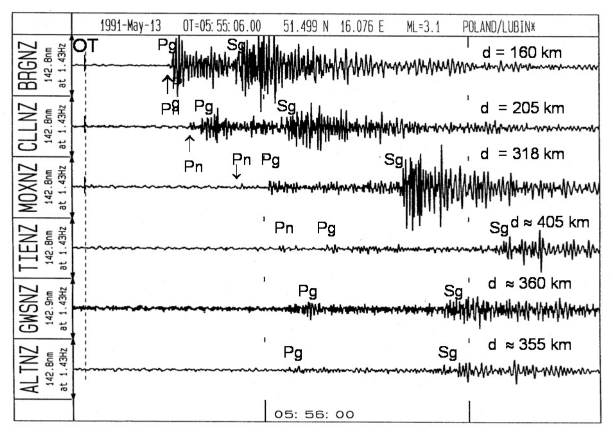

Figure 10 The same as in Figure 9 for an event in the Pn distance range. Note that in the record at 405 km Pn and Pg have already about the same amplitude, whereas at shorter distances Pn is much smaller or not even

recognizable above the noise.

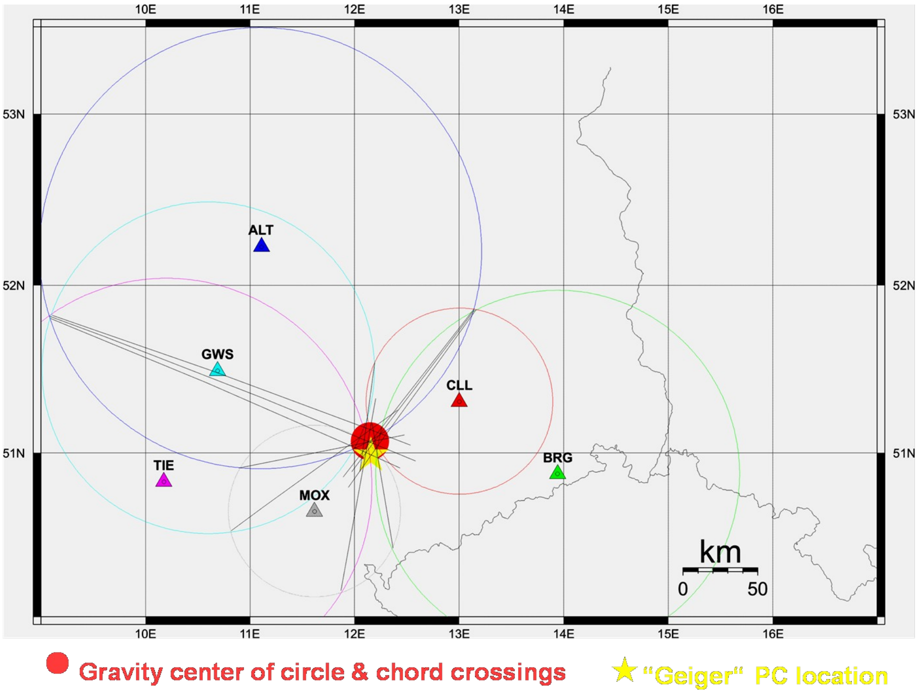

Figure 11 **Location** by the circle and chord method for the event **within the network**, using only records in the local Pg-Sg distance range and the d values given in Figure 9, determined by the "rule of thumb" (1). Note the good agreement between this rough solution by hand and the PC assisted record analysis, phase picks and event location.

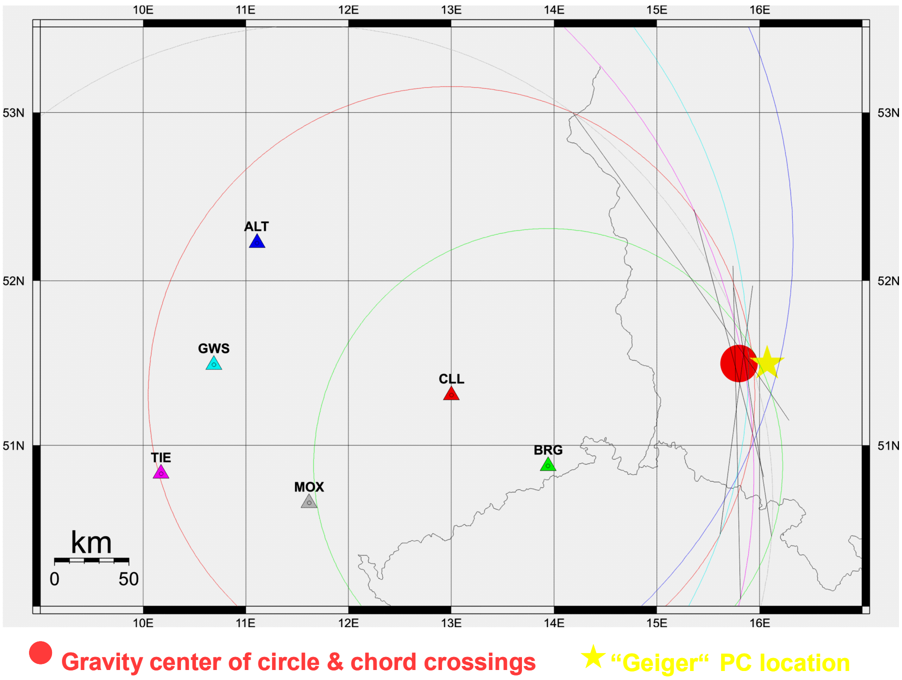

Figure 12 **Location** by the circle and chord method for the event **outside the network**, using records in the Pn distance range and the d values given in Figure 10. Note the much worse circle crossing. It results in larger location uncertainty (long axis of the error ellipse) in N-S

direction, which is perpendicular to the connection line between the network center and the source. This applies to both the manual and the PC solution. Again the two agree rather well.

6 Determination of source depth

The above exercise was run on the assumption of a (near) surface source for which effects of source depth h are negligible. However, as mentioned earlier, the overshoot of the distance circles drawn around the stations is largely due to the negligence of h and the fact that for $h > 0$ it holds d(= hypocentral distance) > D(= epicentral distance).

However, the records in Figure 9 relate to a local earthquake of larger source depth, in contrast to the records in Figure 10, which result from a more shallow strong mining explosion in Poland. Therefore, the overlap of circles in Figure 11 is larger than in Figure 12, although quite some uncertainties are introduced in the latter case due to more difficult phase identification, often worse SNR, less accurate onset time picking as well as pronounced lateral velocity differences towards the boundary of the East European Platform, which are not accounted for by the used simple one-dimensional travel-time model.

If, however, large circle overshoot hints to h >> 0 km one should re-calculate the station radii d_i by stepwise increasing h until for d_{imin} the "true" (better: the most likely) source depth h_{true} can approximately be estimated via the relationship

$$h_{true} = (R_i^2 - d_{imin}^2)^{-1/2}.$$

The circles drawn around the stations with d_{imin} are expected to have zero (or negligible) overshoot, provided that the wave propagation conditions are sufficiently homogeneous and well represented by the assumed velocity model and further that the station reading errors of phase onset times are negligible. This is illustrated by the 3 plots in Figure 13 for another local earthquake.

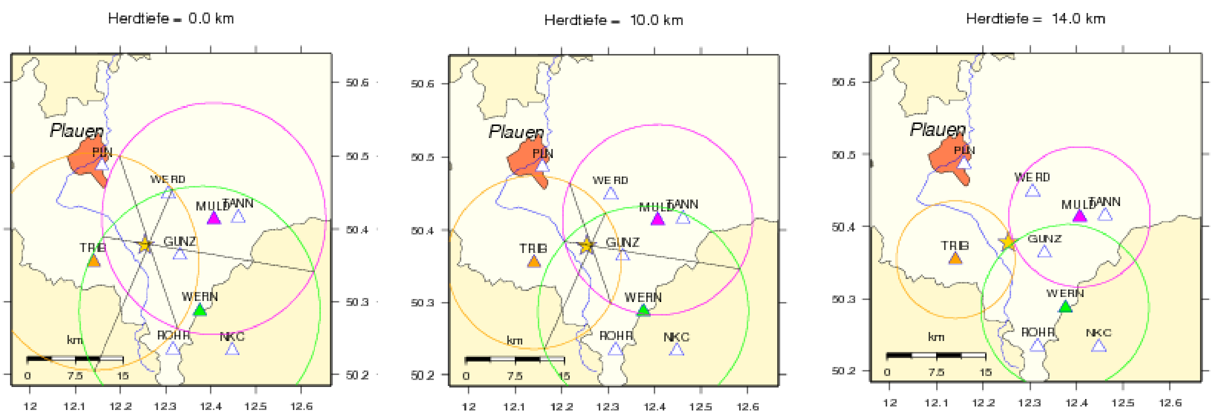

Figure 13 Sequence of circle & chord plots for an earthquake in the Vogtland swarm earthquake region of South-Eastern Germany. Assumed are different source depths for fixed hypocentral distances di of the i stations as calculated from the S-P travel-time differences. For h = 14 km the circles cross almost ideally in one point at the epicenter.

Allowing for finite source depth also means that the travel-time curves for the direct P and S waves would no longer be straight lines with zero travel time at D = 0 km. Rather, P and S would have bended travel-time curves with different finite travel times also at the epicenter. Accordingly, for constant S-P times and thus d the epicentral distance D decreases with growing depth until one gets at the epicenter position D = 0 that d = h. This also explains why reliable depth estimates require records from stations in the near source area at D < h.

Both effects are well illustrated by the movies **3D-wave-prop_travel-times_qt** and **circles_depth_qt**. You can activate these movies by right mouse click on the respective file name in bold blue. Note in the final travel-time-record plot shown in the first movie that actual phase onset times at some of the stations may differ be several tenths of a second from the theoretically calculated ones for an average 1-D crustal velocity model of the considered area.

The movies can also be downloaded via the summary listing *Download Programs & Files* (see Overview on the NMSOP-2 cover page and follow related instructions).