Chapter 2

Difference and Analytical Engines

Introduction

The development of a successful computing machine requires the provision of mechanism for at least two basic functions—the storage and arithmetic manipulation of numbers, and some control mechanism whereby a series of arithmetic operations may be combined to produce the result of a desired calculation. The previous chapter described the development of such mechanisms – the commercially successful machines of Thomas de Colmar in the 1820s being the first to combine a practical design with an effective method of manufacture. In these machies the control function was provided by the human operator.

Developments were slow, however, and it was not until the 1880s and 1890s that "scientific" machines (capable of multiplication and division) following the designs of de Colmar or Odhner and "commercial" machines (capable of addition, or addition and subtraction only) by Felt and Tarrant, Burroughs, and others began to appear in quantity. The need for these was spurred by the demands of the larger businesses bred by the Industrial Revolution. In turn the Industrial Revolution made possible the economic mass production of the calculators themselves.

Devices for the automatic control of mechanisms have a much longer history, leading back to the ancient Greek civilizations.

Automatic mechanisms were extensively developed in the great astronomical clocks of the Renaissance and the automata of the eighteenth century. Two devices of particular importance to our story, the pin barrel music box and the punched card Jacquard loom, were well established by the early nineteenth century. Of considerable importance, but in a more abstract intellectual sense, was the development by Stephenson of the ball governor for the steam engine and the idea of feedback that it embodies.

By the early nineteenth century, therefore, the basic mechanisms and ideas existed from which an automatic calculating machine could be developed. This was done by the English mathematician Charles Babbage, who developed, single-handedly, most of the basic ideas inherent in the logical design of modern digital computers—an intellectual tour de force seldom equalled in the history of science and technology. Babbage's ideas were embodied in the design of two calculating machines, the Difference Engine and the Analytical Engine, which form the main topics of this chapter.

Charles Babbage

Charles Babbage was born in south London on December 26, 1791, the son of Benjamin Babbage, a London banker. Charles was a somewhat sickly youth whose education was irregular and mainly conducted at the hands of private tutors. As a youth he was his own instructor in algebra, of which he was passionately fond, and was well-read in the continental mathematics of his day, particularly the calculus of Leibniz.

Upon entering Trinity College, Cambridge, in 1811, Babbage found himself in mathematics far in advance of his tutors who, along with most English mathematicians, were stultified by an overstrict adherence to the unfortunate notations of the calculus of Newton and to geometrical forms of argument in general. As an undergraduate, with John Herschel, Peacock, and others, Babbage founded the Analytical Society for promoting continental mathematics—the "D'ism of Leibniz in opposition to the Dot'age of the University." In time this campaign was successful and played an important role in the revitalization of English mathematics in the mid-nineteenth century.

In his twenties Babbage worked as a mathematician. He was elected a Fellow of the Royal Society in 1816 and played a prominent part in the formation of the Astronomical Society of London (later the Royal Astronomical Society) in 1820. It was about this time that Babbage first acquired the interest in calculating machinery that became his consuming passion for the remainder of his life. From this time he did no more serious mathematical work.

Throughout his life Babbage worked in many intellectual fields and made contributions that would have assured his fame irrespective of the Difference and Analytical Engines. His interests are well-represented by his published works. He wrote *A Comparative View of the Various Institutions for the Assurance of Lives* (1826) concerning the actuarial principles underlying life insurance. His *Table of Logarithms of the Natural Numbers from 1 to 108,000* (1827) was a paradigm of accuracy and was extensively used into the twentieth century. *Reflections on the Decline of Science in England* (1830) is the best known of Babbage's many polemics against the scientific institutions of his day and fueled much debate at the time and after. Babbage's interest in this area is also seen in his important role in the establishment of the Association for the Advancement of Science (a direct outgrowth of the publication of *Decline of Science*) and the Statistical Society (later the Royal Statistical Society), and in his extensive contacts with continental scientific institutions. *On the Economy of Machinery and Manufactures* (1832), the best known of Babbage's books, is a masterly study of the manufacturing techniques of his day and their economic base. It is seen by some as laying the foundations of the study of operations research. The *Ninth Bridgewater Treatise* (1837) is the most curious of Babbage's works. It was written as Babbage's unsolicited addendum to the *Bridgewater Treatises*, which aimed to prove the existence of God through the richness of natural phenomena. By analogy with his machine, Babbage postulated the existence of a hierarchy of natural laws (an idea that rose to prominence in the twentieth century with the development of relativistic and quantum mechanics) and used this idea to provide a rational explanation of natural miracles. The autobiographical *Passages from the Life of a Philosopher* (1864) is a charming though idiosyncratic view of nineteenth-century life. It contains the most extensive accounts in Babbage's hand of the principles and capabilities of his machines though, unfortunately, written at a very elementary level.

Despite his many achievements, the failure to construct his calculating machines and, in particular, the failure of the government to support his work (as we shall describe later), left Babbage in his declining years a disappointed and embittered man. His bitter, but well-warranted, campaign against street musicians became an easy cliché of the last years of his life and the basis of the "irascible genius" myth that so poorly represents the strengths of his personality and his stupendous achievements. Babbage died at his home on Dorset Street, London, on October 18, 1871.

The Genesis of the Difference Engine

The idea of an automatic calculating machine first came to Babbage about 1820. In one account, written many years later, he describes how he was engaged with his friend, the astronomer John Herschel, in proofreading a set of tables prepared for astronomical calculations. In a moment of exasperation with the errors they found, Babbage remarked, "I wish to God these calculations had been executed by steam." Herschel's reply, "It is quite possible," set Babbage thinking and in a short time, a few days, he had formulated the general idea of the machine that later became known as the Difference Engine.

The idea of the method of differences, which underlies Babbage's first automatic calculating machine, was much in vogue at that time. Consider the formula

$$T = x^2 + x + 41$$

of the variable x. It generates a sequence of values for T—many of which happen to be prime numbers, as seen in the table in Figure 2.1. If we take the differences between successive values of T, the column labeled Δ in the table, these so-called first differences follow quite a simple rule. If we take the differences between the differences, known as the second differences, the result is even more striking—the second difference, Δ^2, is a constant. With this knowledge, the table can be built up in a very simple way, as shown by the box in the table. Take the second difference, 2, and add it to the first difference to form a new first difference

$$4 + 2 = 6.$$

Take this new first difference and add it to the tabular value to form a new tabular value

$$47 + 6 = 53.$$

By simply repeating this process the table of the function T may be extended indefinitely using no other mathematical operation than simple addition (Figure 2.1).

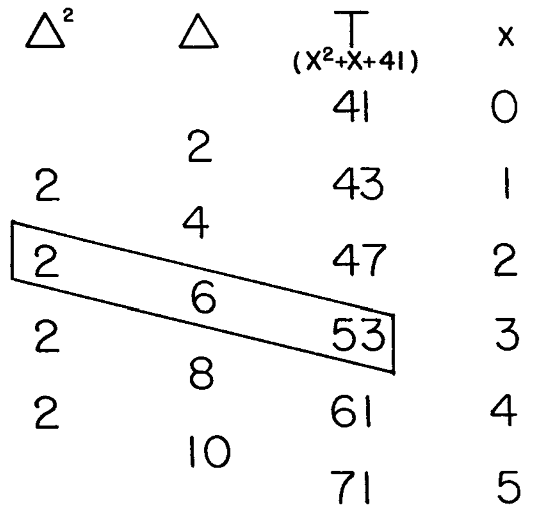

Figure 2.1. Tabulation of a quadratic using the method of differences. The box shows the successive updating of the differences required to form one new tabular value.

The process can be generalized. In our example the second difference is constant because the function T is a quadratic, i.e., a polynomial of degree 2. If the function T were a cubic, such as

$$T = x^3,$$

the second difference would vary, but the third difference, the difference between successive second differences, would be constant. In general a polynomial of degree n will have a constant nth difference and each successive new value of the function can be obtained by n simple additions.

The usefulness of difference techniques is greatly increased by the fact that any section of a well-behaved continuous function can be approximated by a polynomial. The shorter the section and the higher the degree of the polynomial the closer the approximation. So if we wished to tabulate a function, such as a sine or the time of sunset, it is only necessary to divide the function into short enough intervals and find a suitable approximating polynomial for each interval. (Mathematical techniques for doing this were much improved later in the nineteenth century.) The method of differences can then be used to tabulate the function throughout the interval. This process is known as *sub-tabulation.*

Babbage realized that a machine could carry out this sub-tabulation process. First, he needed a mechanism for storing, separately, the numbers corresponding to the values of the tabular value, T, the first difference, Δ, the second difference, Δ^2, etc. and a mechanism to add each difference to the value of the preceding difference. A quadratic, for example, could be tabulated by the machine shown schematically in Figure 2.2.

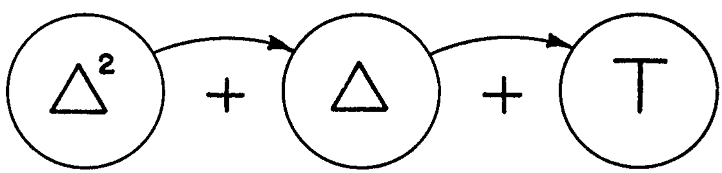

Figure 2.2. Schematic arrangement of the operations required to tabulate a quadratic using the method of differences. The mechanism of Babbage's Difference Engine corresponds exactly to this schematic.

By early 1822 Babbage had constructed just such a machine as this and applied it to the tabulation of

$$T = x^2 + x + 41$$

among other functions—the first thirty values being tabulated in two and a half minutes. Unfortunately, Babbage's first Difference Engine has not survived and his notes and drawings are lost. The only details we have of it are contained in several short letters he wrote in 1822. It was probably similar in general arrangement and design to the later design of 1830. It is important to note that this model was a working machine, though undoubtedly very limited in its numerical capacity. Ideas for a more general range of calculating machines, for extracting roots of equations, multiplying, and computing primes, had occurred to Babbage at this time and had been partially worked out, but no details remain.

A complete Difference Engine requires, in addition to the mechanisms shown in Figure 2.1, a means for controlling the actions of the various parts so that they are performed in the correct sequence. It seems that Babbage's original model acted automatically in this way, but we have no idea of the mechanisms employed.

From the beginning, Babbage was concerned with producing accurate mathematical, astronomical, and other tables. Performing the necessary calculations is only half of the problem. The other half is to transfer the calculated results to the printed page, which, if done manually and with the movable type of the day, is another great source of error. Babbage, therefore, proposed that the Difference Engine should be made to prepare mechanically the type or plates needed for printing. No mechanism for this was included in the first model but Babbage was carrying out independent experiments with such mechanisms at the same time.

The Project to Build the Difference Engine

Babbage recognized his model of 1822 to be just that, a model from which a final production machine could be developed, given the substantial resources in time, effort, and money that would be required. In that the manner of government support for research and development with which we are now familiar did not exist in Babbage's day, he commenced by communicating news of his model and the possibilities it opened to him to the scientific community, most notably in an open letter to Sir Humphrey Davy, president of the Royal Society.

Babbage's achievements brought him immediate acclaim. He was awarded the first Gold Medal of the Astronomical Society of London. Some of this acclaim must have been due to his ingenuity in reducing the mental task not just of arithmetic but of an extended sequence of arithmetic operations to a mechanical mechanism. Mostly, however, it was due to the perceived importance of the Difference Engine in the preparation of mathematical tables.

In the early nineteenth century tables were the only common aid to calculation—the Thomas de Colmar calculating machine was just starting production in 1822, and slide rules, with their limited accuracy, were rarely found outside such specialist applications as the calculation of excise duty on spirits. Any means to economize the production and, especially, ensure the accuracy of tables was of major importance. Nowhere was this so evident as in the preparation of the astronomical tables to aid navigation at sea. These tables had to be recomputed annually, and the consequences of errors in tables, translated into errors in navigation at sea, could be most serious. Therefore, Babbage's project was of major importance to a nation, such as Britain, that relied for much of its wealth on overseas trade.

Babbage's letter to Sir Humphrey Davy, and the evident importance of the Difference Engine as assessed by the scientific men of his day, led to support from the British government towards the development of the Difference Engine for the preparation of practical tables. The grant, initially fifteen hundred pounds but rising eventually to around seventeen thousand pounds, was never clearly formulated to embody the commitments and expectations of either Babbage or the government. It was probably seen by the government as an *ex gratia* grant-in-aid to Babbage, a grant without commitment or expectation, but was certainly seen by Babbage as a commitment to the construction project by the government.

This lack of a formal arrangement led to difficulties; Babbage considered that the government reneged on its agreement. The government gave no further support to the construction after 1833 when Babbage's relationship with the engineer Joseph Clement, who was building the Engine, reached an impasse. It is probably not coincidental that 1832 marked the passage of the Great Reform Bill and the first extension of the voting franchise in Britain. From that time, government patronage, of the sort that had supported Babbage, was no longer politically viable, though it was not until 1842 that the termination of government support was made explicit to Babbage.

Of Babbage's relationship with Clement we have less direct evidence. The construction of the Difference Engine was a very demanding piece of precision engineering for its day, though the existing portions of the calculating mechanism are proof that the necessary precision could be obtained by the development of appropriate and specialized tools and skill in their application. It seems, however, that precision was carried to extremes and applied in areas, such as the decorative finish of support columns, to which it was irrelevant. It seems also that Clement had grasped the potential for profit of an open-ended government job and Babbage felt exploited by this.

Babbage, on the other hand, treated Clement as his servant and seems not to have grasped that in the decade that the Difference Engine was being built it had declined from being the major part of Clement's work to one job among many of a successful engineering workshop. It is not surprising then that Babbage's demands that Clement relocate his workshop to better suit the Difference Engine project were countered by huge financial demands. This impasse stopped construction work on the Difference Engine in March 1833, and it was never resumed.

The demands for precision in the manufacture of the Difference Engine had a major influence in the development of the British machine tool industry. Joseph Whitworth, the leading machine-tool maker in the mid-nineteenth century, had been employed by Clement on the Difference Engine work. Whitworth's developments of standardized screw threads, for example, are traceable to Clement's work in the same direction for the Difference Engine. There seems much truth in the observation that "Babbage made Clement, and Clement made Whitworth." Late in Babbage's life the government received evidence from prominent engineers that the investment in the Difference Engine had been amply repaid by its spin-off into British industry.

It is a great pity that when work on the Difference Engine ceased it was close to completion. Henry Babbage later estimated that only a further five hundred pounds would have sufficed. Babbage could readily have found the funds; however his feelings and attitudes to both the government and Clement could not at the time have countenanced his doing so. Indeed, these feelings did much to form his embittered attitudes as an older man.

Within a year or two, Babbage's mind had moved a long way towards the much more complex and intellectually rewarding Analytical Engine. There was then no way he would have returned to the original Difference Engine design and brought it to completion, even had events made that feasible.

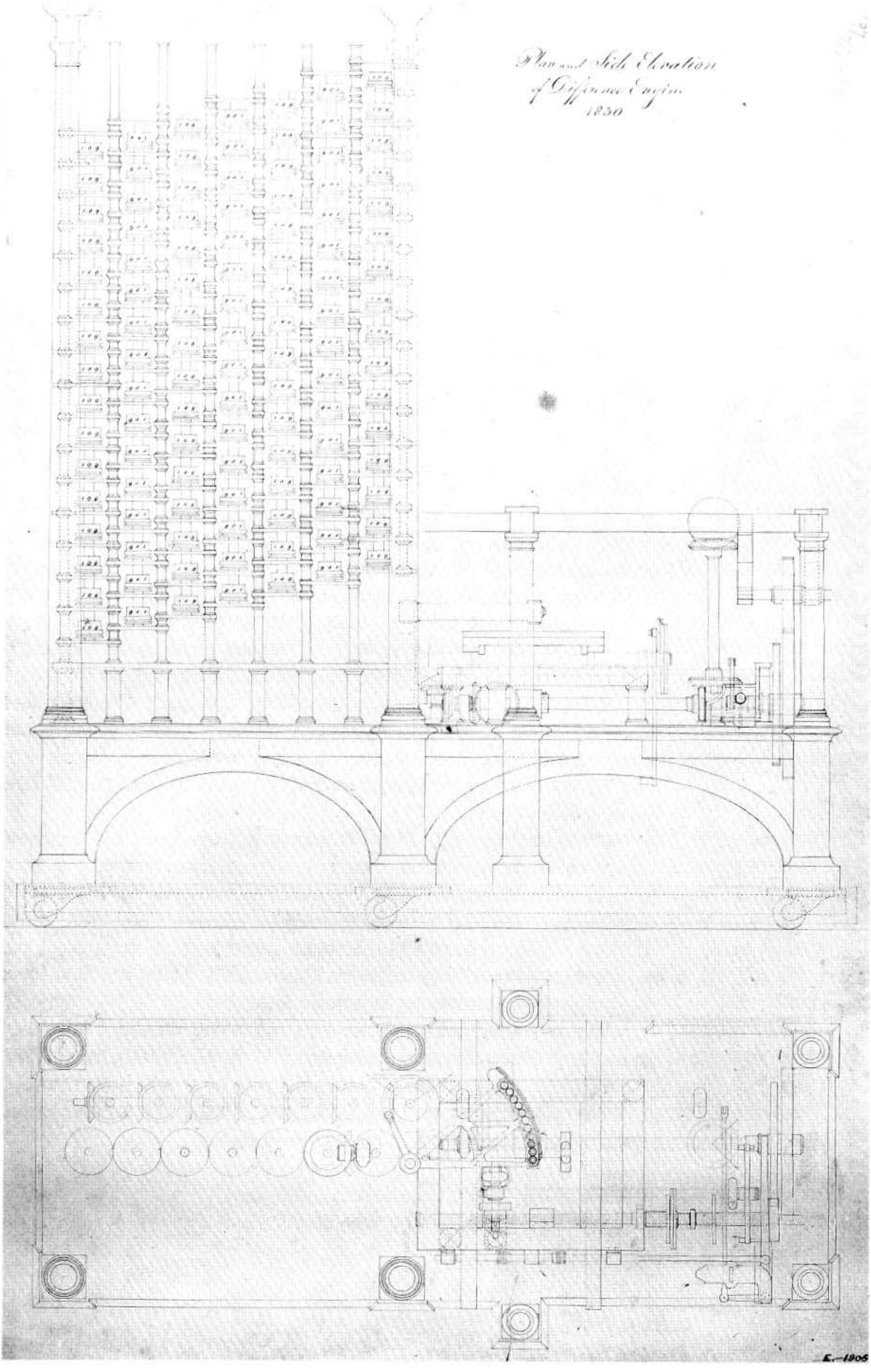
Plan and Side Elevation
of Difference Engine
1830
E.-1905

Figure 2.3. Elevation and plan drawings of Babbage's Difference Engine as planned about 1830. The calculating mechanism is on the left; the axes of figure wheels for the tabular value (far right) and six differences are clearly visible. The printing mechanism is on the right, and the moving table carrying the stereotype printing plate and the sector carrying the digit-type punches are visible in the center of both drawings.

The Design of the Difference Engine

The Difference Engine consisted of two major parts—the calculating mechanism and the printing and control mechanism. These are clearly seen in Figure 2.3, which shows the general arrangement of the Difference Engine as planned about 1830. A portion of the calculating mechanism was assembled in 1832 to demonstrate to a committee of the Royal Society that the project was proceeding satisfactorily. That portion, shown in Figure 2.4, is about one-third of the height and one-half of the width, or about one-seventh of the entire calculating mechanism. Almost all of the parts of the entire calculating mechanism had been made, but not assembled, when work on the project stopped early in 1833.

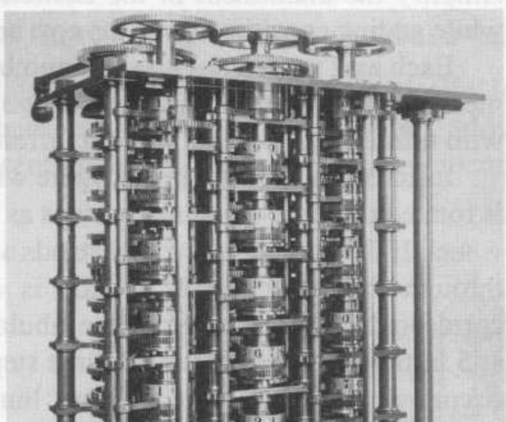

Figure 2.4. The portion of the calculating mechanism of the Babbage's Difference Engine assembled in 1832. Records of nearly a hundred functions tabulated by Babbage with this portion have survived.

Digits are represented in the Difference Engine by the rotational position of horizontal gear wheels. A number is made up of a series of these figure wheels rotating about a common vertical axis. The bottommost wheel represents units, the next tens, the next hundreds, and so on. (A user can imagine a decimal point located between any pair of figure wheels provided this is done consistently throughout the Difference Engine.) Babbage used the term *Axis* to mean a stack of figure wheels that together store a number as a collection of decimal digits. The entire Difference Engine consists of an axis for the tabular value of the function, another axis for the difference, a third axis for the second difference, and so on for as many orders of differences as are desired. These axes stand beside one another, as shown in Figure 2.3, with the axis of the tabular value nearest to the printing mechanism.

The Difference Engine is built on quite a large scale, with figure wheels about 6 inches (15 centimeters) in diameter spaced vertically about 3 inches (7.5 centimeters) apart on the axes. (These wheels are behind the numbered wheels visible in Figure 2.4.) No calculating machine before or since Babbage has used such large components. The large scale in Babbage's designs is possibly traceable to anticipated government expectations based on the proportions common in naval equipment. The large size probably did little to simplify the attainment of the desired accuracy of machined parts while adding considerably to the cost and manufacturing difficulties.

Each axis served not just as a number store but also as an adding mechanism. Addition occurred in two steps that will be explained with reference to adding the first difference to the tabular value.

Inside each first difference figure wheel there is a mechanism that is rotated through just as many steps as the value stored by the figure wheel. If the units figure wheel stands at 3, the mechanism will move through three steps. This motion is conveyed by gearing to the corresponding figure wheel of the tabular value axis. If the latter stood at 5 initially, it will be moved three steps to stand at 8. This process occurs simultaneously in the tens, hundreds, thousands, and other digit positions.

It may happen that addition to a figure wheel will generate a carry that must be propagated or added into the next higher digit position. If the units digit of the tabular value were initially 8 and 3 is added, it will move forward three places and come to stand at 1, but a carry must also be propagated into the tens figure wheel of the tabular value. Carry propagation is complicated by the fact that if the tens figure wheel already stands at 9 it will be moved forward by the carry to stand at 0 and a new carry will be propagated into the hundreds figure wheel. In the Difference Engine these consecutive carries may propagate, as on occasion they must, from the units up through the most significant figure wheel.

Each addition, therefore, consists of two distinct steps—the simultaneous addition of all figures of the first difference to the corresponding figures of the tabular value, and the consecutive propagation of carries from the units up to the most significant digits as required.

Tabulation of a function involves the repetition of this basic addition process for each of the orders of difference involved. As each axis is also an adding mechanism the tabulation of a cubic function from third differences, for example, requires six steps for each tabular value produced:

1. Addition of third difference digits to second difference digits
2. Carry propagation among second difference digits
3. Addition of second difference digits to first difference digits
4. Carry propagation among first difference digits
5. Addition of first difference digits to tabular value digits
6. Carry propagation among tabular value digits.

This process is shown schematically in Figure 2.5. Negative numbers may be handled with no additional mechanism by representing them as their ten's complements.

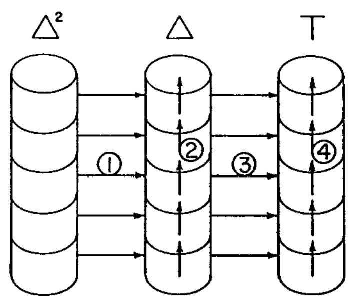

Figure 2.5. Tabulation of a cubic, showing sequential updating of the differences.

This scheme is readily extended to higher order differences. Tabulation from sixth differences, as planned for the Difference Engine, would require twelve steps for each tabular value produced. Babbage found a way to rearrange the calculation so that only four steps were required for each tabular value produced irrespective of the number of differences involved. This is characteristic of the sophisticated logical considerations underlying Babbage's designs.

Babbage observed that when the first difference is added to the tabular value, in steps five and six, both the third difference and second difference axes are idle. He could thus add the third difference to the second difference, steps one and two, at the same time as the first difference is added to the tabular value. Steps one and two overlap steps five and six. Thus only four units of time, for steps three to six, are needed for each tabular value produced. This rearranged manner of doing the calculation is shown in Figure 2.6. In modern terminology we would call the arrangement of hardware to perform a calculation in this way a *pipeline*.

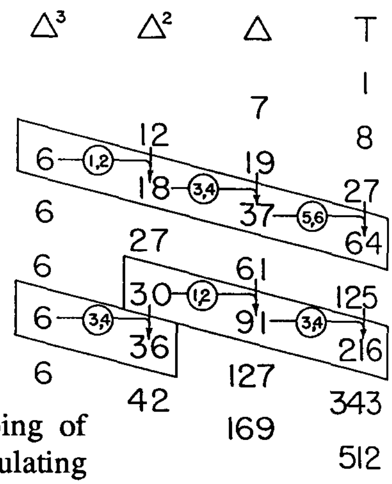

Figure 2.6. Tabulation of a cubic, showing the overlapping of updating used in the Difference Engine so that the calculating time is independent of the number of differences used.

The overlapping idea can be extended to higher differences and a new tabular value can always be produced in four steps. In general, all the even differences are added to the odd differences in two steps and all the odd differences are then added to the even differences (and the first difference is added to the tabular value) in two further steps. Not only does this rearranged form of the calculation save considerable time but it also makes the arrangements for driving the calculating mechanism much simpler.

The calculating mechanism of the Difference Engine is really quite straightforward as demonstrated by the early date at which Babbage produced his first demonstration model. Some complication is added by the (very necessary) apparatus that Babbage added to ensure that the machine would calculate accurately with great reliability. Although Babbage later found much simpler mechanisms for addition and carriage, the calculating part would have been perfectly successful if its construction had been completed as planned. The success of the portion shown in Figure 2.4 proves that.

There are among Babbage's papers a number of tabulations of short sections of the logarithm function with this small portion of the Difference Engine. These show that Babbage understood how to obtain rounded values for printing without any additional calculation.[1] Had the calculating part been completed, Babbage might well have discovered some new tabulation techniques because he always expected that there would be this kind of feedback on analysis once an automatic calculating machine was available. The twentieth century proved him right.

If the calculating mechanism of the Difference Engine is straightforward, the printing and control mechanism is not. Its sophistication and the considerable intellectual effort expended by Babbage on its refinement did much to lay the foundations for the Analytical Engine and make its very rapid development possible. It may also have delayed the completion of the Difference Engine to a fatal extent.

The Difference Engine was intended to print an entire page of tables automatically from the initial setting of the differences. Figure 2.7 shows a sample of seven-figure logarithm tables typical of those the Difference Engine was intended to prepare.

1550

No.		0	1	2	3	4	5	6	7	8	9	Diff.
1550	190	3317	3597	3877	4157	4438	4718	4998	5278	5558	5838	
51		6118	6398	6678	6958	7238	7518	7798	8078	8357	8637	
52		8917	9197	9477	9757	0036	0316	0596	0876	1155	1435	
53	191	1715	1994	2274	2553	2833	3113	3392	3672	3951	4231	279
54		4510	4790	5069	5348	5628	5907	6187	6466	6745	7025	1 28
55		7304	7583	7862	8142	8421	8700	8979	9259	9538	9817	2 56
56	192	0096	0375	0654	0933	1212	1491	1770	2049	2328	2607	3 84 4 112
57		2886	3165	3444	3723	4002	4281	4559	4838	5117	5396	5 140
58		5675	5953	6232	6511	6789	7068	7347	7625	7904	8183	6 167
59		8461	8740	9018	9297	9575	9854	0132	0411	0689	0968	7 195
60	193	1246	1524	1803	2081	2359	2638	2916	3194	3473	3751	8 223 9 251
1561		4029	4307	4585	4864	5142	5420	5698	5976	6254	6532	
62		6810	7088	7366	7644	7922	8200	8478	8756	9034	9312	
63		9590	9868	0145	0423	0701	0979	1257	1534	1812	2090	
64	194	2367	2645	2923	3200	3478	3756	4033	4311	[illegible]		
65		5143	5421	5698	5976	6253	6531	[illegible]				
66		7918	8195	8472	[illegible]							
67	195	0690	[illegible]									
68		[illegible]										

Figure 2.7. A sample of seven-figure mathematical tables laid out in a manner possible with the Difference Engine.

The full seven-figure logarithm is printed only for the first entry of the line. Other entries show only the four less significant digits. The three most significant digits are printed only if they changed during the preceding line (hence they appear on line 1553) or in the calculation of the first entry for the present line (line 1556). The columns and rows are not evenly spaced but rather an additional gap is left, after every fifth column and fifth row, to guide the reader's eye. All of these features could be obtained with the Difference Engine.

Each digit was printed by punching a type into a soft metal stereotype printing plate. The particular digits of the tabular function printed, and their number, was determined by selecting and counting wheels in the printing mechanism. Another counting wheel determined the number of table columns and their separations. This was actually a wheel of sixty positions, so that ten columns could be obtained by repeating the control pattern six times. By suitably programming this wheel, six, twelve, or fifteen columns, for example, could be obtained for the printing of trigonometric tables. A similar wheel controlled the spacing of columns, and it was possible to print table entries by columns instead of rows if desired. The printing of the leading three digits was controlled by a trip mechanism activated by the appropriate carry propagation in the tabular value axis.[2]

The printing and control mechanism underwent a major redesign about 1832, so that in the final design it would have been very much longer than shown in Figure 2.3. This redesign marks a major advance in Babbage's understanding of control ideas. In the earlier design the various counting wheels acted directly to carry out their program function themselves. In the later design they always put into gear a connection from the main drive to carry out the function. The control mechanism, therefore, transmits very little power and the weight of the mechanism to be driven does not limit the complexity of the control apparatus. This idea was extended to the control of the calculating mechanism (which was overlapped so far as possible with the printing), resulting in a design very similar to the barrels later employed in the Analytical Engine.

The Origins of the Analytical Engine

Despite the utility of the Difference Engine as a practical aid to table making it is, in the mathematical sense, an extremely limited instrument for it is only directly capable of handling polynomials. The powers of the engine are increased greatly if it is arranged so that the high-order differences can be effected by the tabular function or low-order differences. For example, the sine function satisfies the difference equation

$$\Delta^2 \sin(x) = - k \sin(x).$$

If we could make the second difference relate to the tabular value in this way, the sine function could be tabulated exactly, without the use of polynomial approximations.[3] Alas, the Difference Engine cannot carry out the multiplication required by the above formula.

In late 1822, after the completion of the first model of the Difference Engine, Babbage commenced exploring the sorts of feedback functions that could be calculated by the Difference Engine. These have definitions similar to

$$\Delta^2 u = \textit{units digit of } u.$$

What motivated Babbage to explore these functions we do not know, but despite their analytic intractability they came to exercise a considerable fascination upon him. Two brief accounts of the functions were published by Babbage in 1822; the Difference Engine included additional transfer gearing to enable it to calculate such functions, and the model assembled in 1832 had additional facilities for this purpose. About fifty of these feedback functions, calculated with the model, are tabulated in Babbage's notebooks, and the concept provided the basis for his arguments about miracles in the *Ninth Bridgewater Treatise*.

When construction of the Difference Engine ceased in 1833, Babbage returned in earnest to feedback functions, such as the sine, which he characterized strikingly as "the Engine eating its own tail." Although Babbage's exact train of thought at this stage is unknown, it seems that he first realized that multiplication could be carried out by repeated addition if the ability to step numbers up or down on the axes (multiply or divide by ten) was provided. Division can be performed as repeated subtraction but it is a "tentative" process, for we need to be able to examine the result of one subtraction, whether positive or negative, before knowing what to do next. Babbage also found a substantially faster method of carry propagation, the Anticipating Carriage, that made carry propagation a parallel rather than a sequential process. The complexity of this new mechanism forced the abandonment of the arrangements of the Difference Engine where each storage axis is also an adder. In the Analytical Engine there is a separate "store" for numbers and a "mill," or arithmetic unit, where calculations are made.

Babbage's ideas developed very quickly, aided by the commanding knowledge of control mechanisms that he had gained from the printing part of the Difference Engine. By late-1834 the basic plans for the Analytical Engine had been formulated. The mill was separate from the store. Multiplication and division were carried out in the mill by combinations of simpler operations under the direction of one or more barrels. The sequence of operations ordered by the barrels included what today we call "loops" and by alternate sequences dependent on arithmetic results that arose during calculations (today known as "branching"). The calculation performed was directed by "super" barrels that initiated transfers of numbers between the store and mill and started the sequences of operations of the subsidiary barrels. The super barrels also included looping and branching capabilities. (In mid-1836 the super barrels were replaced by strings of Jacquard punched cards.) Within about a year of the cessation of construction of the Difference Engine, Babbage had formulated the basic design of a universal calculating machine. Most of the remainder of his life was spent in refining the details of this design.

The Analytical Engine

There was not one design of the Analytical Engine, but many. New insights of both a logical and mechanical nature continually opened up new possibilities to Babbage for his design, which was, therefore, in an almost continual state of flux.

Between 1834 and 1837 Babbage developed in outline form several possible arrangements of the basic storage and calculating units. Some of these were of considerable interest, such as one with two mills that could be used separately and in parallel for calculations on 30-digit numbers or linked together as a single mill for calculations on 60-digit numbers, but none were developed beyond a preliminary stage. However, by working with these early designs, Babbage gained the facility with logical design ideas that he was later to exploit so effectively.

Many-digit numbers are characteristic of Babbage's designs. Their use gives considerable accuracy, and also a large dynamic range for number values. Babbage did not consider the use of a floating point number representation, whose complexity the designers of early electronic digital computers also avoided.

By 1837 Babbage had settled on a straightforward arrangement, with a single mill and store, that is very similar to early electronic digital computers. This arrangement was altered little in the following years, but the design was refined and elaborated to a considerable extent. By 1847, when this design work ended, there was little doubt that an Analytical Engine could have been built had the necessarily considerable resources been available. Here we describe the design as it stood in the middle years between 1838 and 1840. It may be considered representative of Babbage's plans.

Figure 2.8 shows the general arrangement of Babbage's Plan 16, dated August 1840. The figure is actually a plan drawing of how the mechanism would have appeared from above. It also serves very effectively as a logical diagram of the functional parts of the mechanism and their interconnection. This has been emphasized in the figure.

On the right of Figure 2.8 is the store. This consists of figure wheels, similar to those in the Difference Engine, arranged on vertical axes on either side of a set of "racks" or toothed bars. The racks convey numbers between the store axes and the mill on the left of the figure.

Numbers consisted of forty decimal digits. Negative numbers, in a sign-and-magnitude representation, were indicated with a separate sign wheel on each store axis. In a mechanical calculating machine the binary number system has no especial advantage, whereas a decimal notation uses less apparatus and is easier for a human to interpret. Babbage carefully examined number bases between 2 and 100 at various stages in the design. The sign-and-magnitude representation is convenient for multiplication and division operations, but complicates addition and subtraction.

The Analytical Engine uses a much simpler mechanism than the

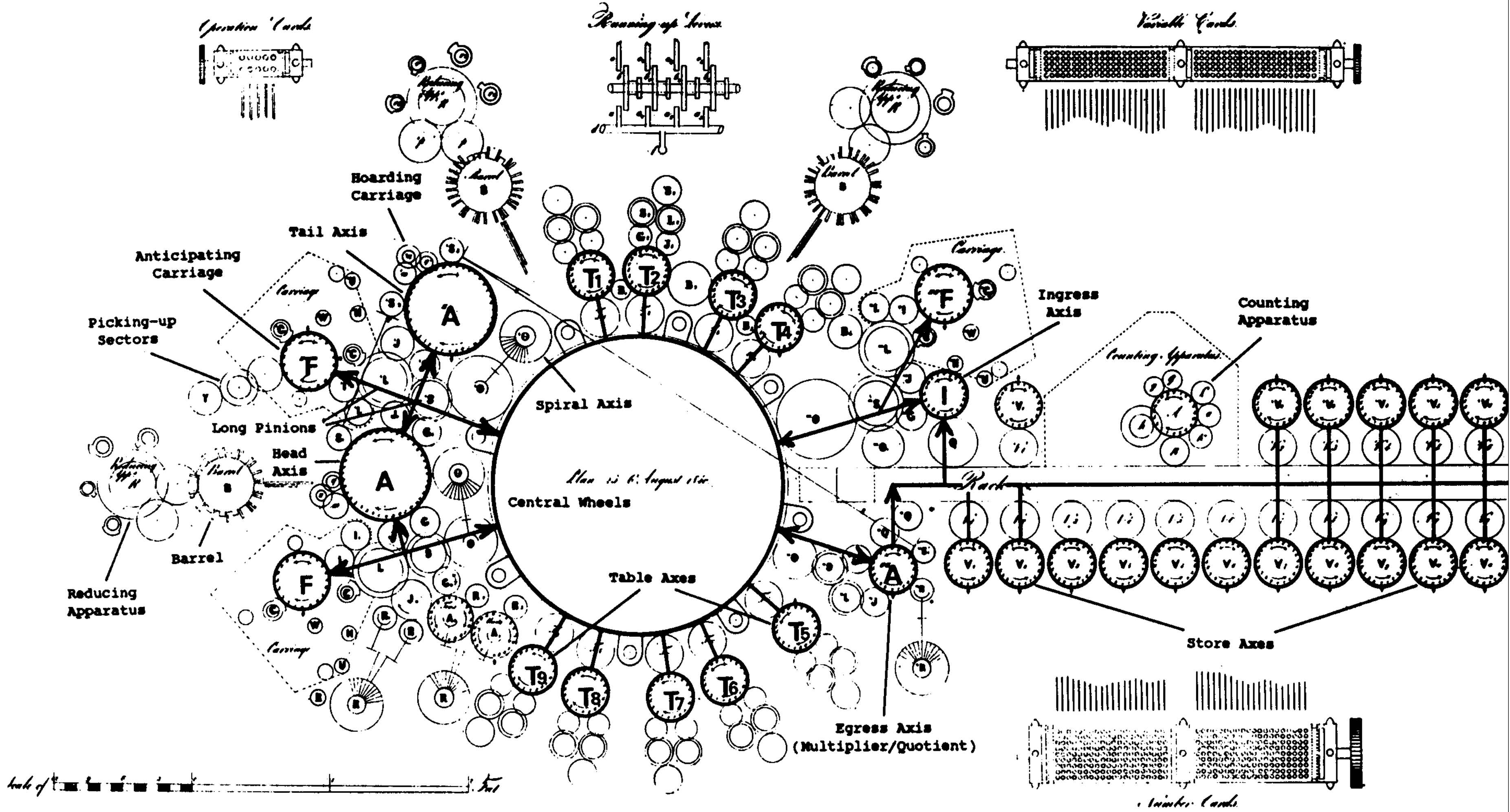

Figure 2.8. Babbage's Plan 16 for the Analytical Engine in August 1840. The original plan drawing of the mechanism has been annotated to show the functional relationships of the principal parts.

Difference Engine to store and read out numbers. To read a number each figure wheel is turned backwards to stand at its 0 position. In the process the wheel will rotate through just as many digit positions as the digit it originally stored, and this motion is conveyed via the racks to the mill. Number readout is therefore destructive, but the number read can be restored to the store axis, if desired, by leaving the store in gear with the racks as they are returned to their starting position at the end of the number transfer. In the mill the destructive readout has more complex effects and most number axes comprise a double set of figure wheels that are used alternately—one set receiving what the other set gives off.

The capacity of the store is unclear from the figure, as the racks may easily be extended further to the right. Babbage spoke at various times of from one hundred to one thousand numbers in the store. Because the Analytical Engine would have been fairly slow, one hundred numbers would probably have sufficed for all practical purposes. In that case the racks would have been about 10 feet (3 meters) long.

Babbage also proposed to have apparatus to read numbers from and punch numbers to Jacquard cards. The requisite apparatus would have communicated with the store racks, but the details are not known. Printing apparatus for making stereotype plates was also intended, but Babbage suggested that this might be a separate machine driven by punched number cards.

Quite long trains of gearing might be involved in the transfer of numbers from one place to another in the Analytical Engine. The necessary looseness and backlash in the gearing, required to ensure easy mechanical action, might have accumulated in these long transfers to such an extent as to make the operation of the machine uncertain. Babbage overcame this difficulty by a series of "lockings"—wedges that come between gear teeth to bring them accurately to their correct position. This is exactly analogous to the provision of amplification in electronic logic gates to ensure that the output voltage levels conform to a standard set of values irrespective of the particular logic circuit involved. Although the lockings added much mechanical complexity to the Analytical Engine, Babbage well understood that they were essential if the machine was to work reliably. This feature is characteristic of the sophistication of Babbage's designs and gives much confidence that the machine, if built, would have worked successfully.

On the left of Figure 2.8 is the mill, or central processing unit.

This consists of a number of axes of figure wheels arranged around a set of "central wheels." The central wheels are used to transfer numbers within the mill and play a role analogous to that of the racks in the store. The mill would have been about 6 feet (2 meters) in diameter and 15 feet (5 meters) high.

The ingress axis, *I*, and the egress axis, *"A*, are used as buffers for number transfers between the store and the mill. The head and tail axes, *A* and *'A*, together constitute a double-length (80-digit) accumulator to hold the product in multiplication and the dividend and remainder in division. *A* is also used as a single-length accumulator in addition and subtraction. The table axes, T_1 to T_9, are used in multiplication and division.

The axes *F*, *'F* and *"F* identify the three Anticipating Carry mechanisms incorporated in the Analytical Engine. As we saw, the Difference Engine used a sequential form of carry propagation. If the hundreds figure wheel, for example, stood at 9 and received a carry from the tens figure wheel, it would move forward one digit position to stand at 0 and propagate a carry to the thousands figure wheel. The thousands will not receive the carry until the hundreds figure wheel has actually moved forward. It is this delay that causes the carry propagation to be sequential and take considerable time.

The Anticipating Carry is so called because it anticipates this sequential action. The thousands may receive a carry for one of two reasons: either the hundreds figure wheel moved past 9 to 0 during the addition step and so generated a carry; or the hundreds figure wheel stands at 9 and so will propagate a carry received, by whatever cause, from the tens figure wheel. The anticipating carry incorporates a mechanism, the "carry chain," that directly implements these two logical alternatives. The anticipating carry mechanism can therefore determine, before any figure wheel is moved, which ones should receive a carry and all of these can be moved forward through one digit position simultaneously. The sequential and anticipating carry mechanisms are contrasted in Figure 2.9.

The anticipating carry was probably the idea of which Babbage was most proud. His autobiography contains a delightful story concerning its invention and describes how his principal draughtsman had thought Babbage had taken leave of his senses for even contemplating its possibility. Perhaps it was the base of the enormous confidence Babbage exhibited in developing the logical design of the Analytical Engine.

The carry chain of the anticipating carry would have demanded

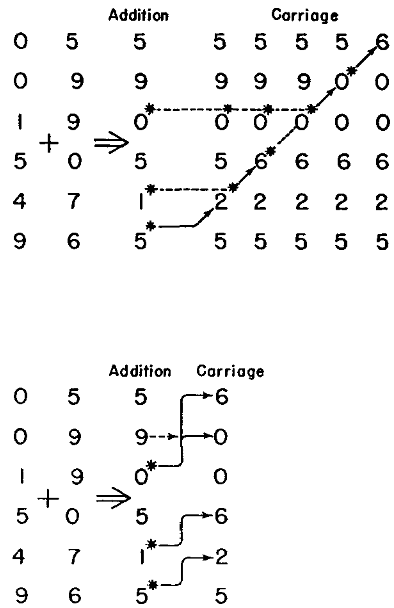

Figure 2.9. Babbage's two methods of carriage propagation in addition. The upper example shows the sequential carry mechanism used in the Difference Engine. Corresponding digits are first added simultaneously (units to units, tens to tens, and so on) and carriages warned, as shown by a star. The carriage propagation then proceeds sequentially from the units digit upward. The lower example shows the anticipating carriage used in the Analytical Engine. The addition process is unchanged, but all of the carriages are propagated simultaneously.

a very high degree of precision in its manufacture. But measurements made of parts built for the Difference Engine show that the requisite degree of accuracy had been obtained on a mass production basis by about 1830. There seems no basis for the common belief that Babbage's machines could not have been made with the technology available in his day, though doubtless it would have been expensive. Rather, it seems that after his bitter experience in attempting to build the Difference Engine, Babbage chose to concentrate on the intellectual issues raised in the design of the Analytical Engine and built only small trial models to verify his designs. Babbage's designs were very thoroughly developed and the mechanical issues carefully considered. They were much more than just pen and paper sketches of an idea.

The Methods Employed for Arithmetic Operations

The methods used for multiplication and division in the Analytical Engine are quite straightforward, although the amount of apparatus required is substantial and there are many technical refinements.

Multiplication commences by taking one of the operands from the store and repeatedly adding it to itself to form the first nine multiples, which are stored on the table axes T_1 to T_9. The multiplier is then taken one digit at a time, commencing with the units, and the corresponding multiple is selected from the table axes and added to the product, which is accumulated on the head and tail axes A and $'A$. The multiples on the table axes are all stepped up one digit position and the process is repeated with the next digit of the multiplier. The various actions are overlapped in such a manner that each whole step requires only a single addition time. The result is a double-length product that is returned to the store.

Division is similar. A table is first made of the nine multiples of the divisor. The two most significant digits of the remainder, on the head and tail axes, are compared simultaneously with the two leading digits of each of the multiples to estimate the next quotient digit. This guess will either be correct or one too large. The selected multiple is subtracted from the remainder and if this becomes negative the divisor is added back to give a new remainder. The new remainder is stepped one place and the process is repeated. In 1840 Babbage found ways of overlapping the actions so that division also took only a single addition (subtraction) time irrespective of whether the quotient digit had been correctly guessed initially.

Addition and subtraction are much more complex processes because of the sign-and-magnitude representation used in the store. In multiplication and division the signs of the operands can be ignored, the operands treated as unsigned, and the correct sign simply inserted into the result. A negative operand, however, turns an addition into a subtraction, and vice versa, and so the function performed in the mill must be changed by the sign.

Multiplication and division are slow, taking one to two minutes and one to four minutes respectively, so the overhead time in fetching operands and storing results is not important (although these are overlapped with other actions as far as possible). In addition and subtraction the fetching and storing, including the conversions from and to the sign-and-magnitude representation, take much longer than the two to three seconds required by the operation itself.

Babbage avoided these difficulties to a large extent by taking as his basic operation the addition or subtraction (in any mixture) of a whole set of operands. Any partial sums could be written to the store as required. In effect, Babbage provided residual storage in the mill (on the head axis A) of the partial sums in a complement number system. This is exactly the same as the practice in electronic digital computers, save that there the residual storage (in registers) is available for other types of operations as well.

Babbage organized addition and subtraction so that operands could be in different stages of processing simultaneously. This is shown in Figure 2.10. Each operand is first fetched from the store to the ingress axis. The value is then added or subtracted, as required, from the total on the head axis, A. This total is in a ten's complement representation. If the partial sum is to be written to the store, it is transferred to the egress axis and converted, in the process, to a sign-and-magnitude representation. Finally, the result is written to the store. The control is very ingeniously arranged so that the maximum possible throughput is achieved—the limitation being access to the store via the racks to fetch operands or store results. This was a stupendous achievement in logical design.

The Analytical Engine provided other arithmetic operations, but the complete set is not clear because Babbage did not list what we would now call the user instruction set. There were variants of multiplication and division for use when only a limited number of digits of the result were required. These were used, for example, in the early steps of finding a square root by iterative formulas. In earlier designs the square root operation had been implemented as an elementary operation. During the slow multiplication and division operations the ingress axis, I, and the anticipating carry, $''F$, could be used directly with the store as a difference engine. Possibly this was intended for calculating simple polynomial functions required by the main calculation, or sub-tabulation of functions between pivotal values calculated in the mill. It is a nice example of the use of functional parallelism.

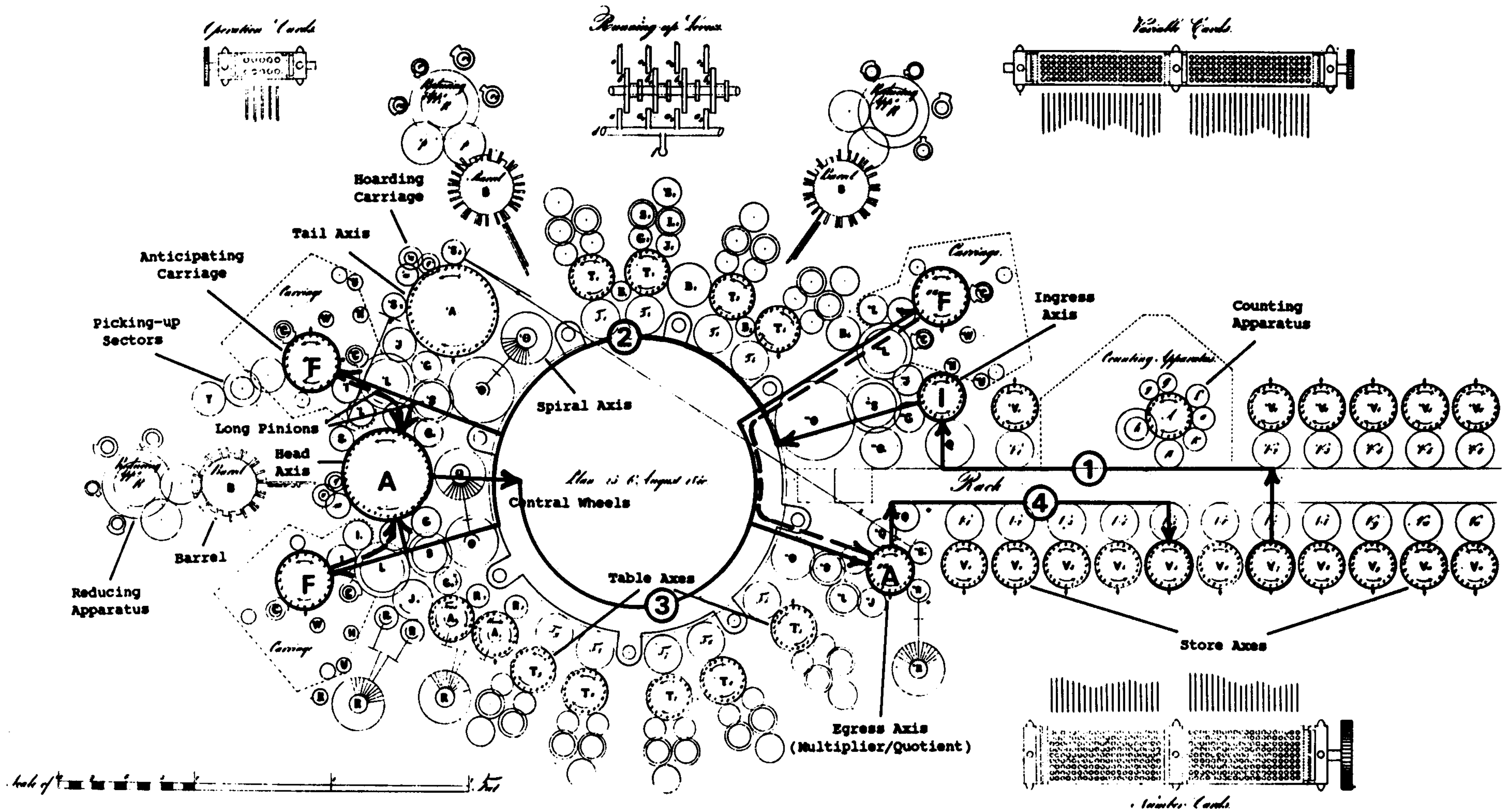

Figure 2.10. The arrangement of addition in the Analytical Engine. The four steps, which may be overlapped with one another for different operands, are described in the text.

The Control Mechanism

The algorithms used in the Analytical Engine, while simple enough in outline, are complex when examined in detail—for there are a large number of hardware components that must work together. Babbage achieved control of this machinery with a hierarchical system of mechanisms.

At the lowest level the control is exercised by "barrels" (Figure 2.11) similar to those employed in music boxes, barrel organs, and many automata familiar in Babbage's day. Studs may be screwed to the surface of the barrel in any desired pattern. When the barrel advances, by moving its axis sideways, one vertical row of studs acts by pressing against control levers in a pattern determined by the arrangement of the studs. The levers in turn engage and disengage the transmission of power from the main drive shaft to the various mechanisms of the Analytical Engine. One "vertical," or line of studs, determines the actions during one adding cycle. In practice, a barrel had from 50 to 100 verticals, each with as many as 200 studs.

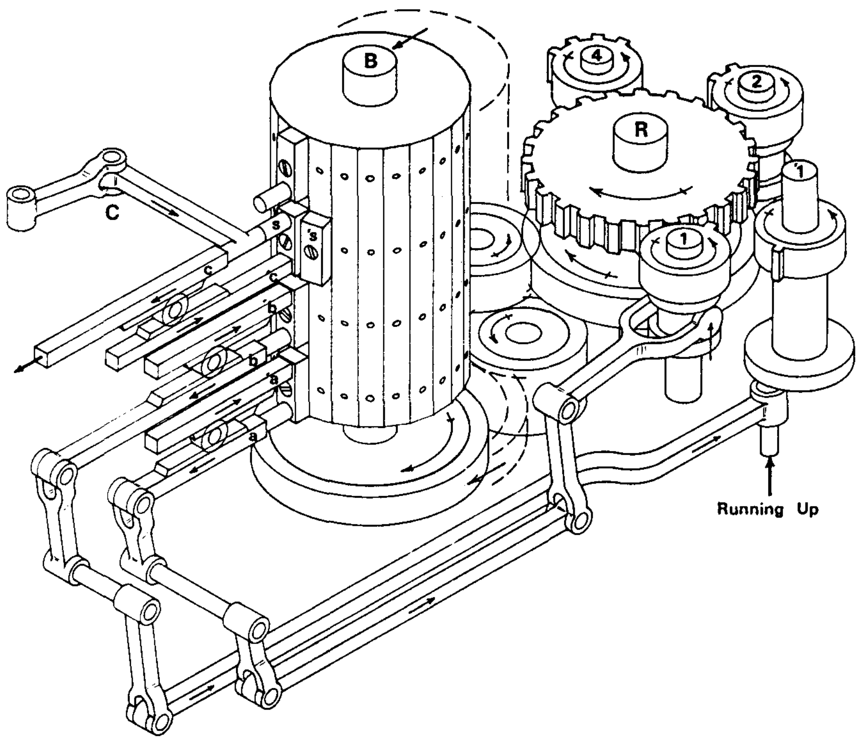

Figure 2.11. A barrel mechanism used in the Analytical Engine. The barrel may determine the sequencing between its own verticals by both the unconditional and conditional mechanisms shown here.

Some of the levers selected by the barrel control its rotation from one vertical to another. Some of these act unconditionally. Others establish a path by which a conditional event in the Analytical Engine, such as a carry propagation from the most significant digit position of a number (a running up), can move the barrel to another vertical. Thus, each vertical can determine which vertical will succeed it and which conditional events are to effect the choice. Interestingly, Babbage's arrangements provide only for what today we call "relative addressing"—i.e., a vertical can specify how far to go to the next vertical that is to act but cannot specify its absolute location. Babbage had no concept equivalent to the modern idea of the address of a word. A vertical may specify a return to a previous vertical and in this manner what we now call "loops" are provided.

Babbage's use of barrels was much more elaborate than this. There were, in general, several barrels—Figure 2.8 shows three but some designs had as many as seven. If these all turn together, the effect is nothing more than dividing up an inconveniently large barrel into a group of smaller ones. But in the case of addition and subtraction the barrels controlling the registers *I*, *A*, and *"A* and associated mechanisms acted independently of one another, responding in part to local conditional events, yet cooperating together to implement the string of addition and subtraction operations. The whole arrangement is enormously sophisticated yet finely judged to best exploit the capabilities of the calculating mechanisms.

The barrels specify in detail how multiplication, division, addition, subtraction, and other arithmetic operations, are to be carried out. The user of the Analytical Engine would regard these arithmetic operations as basic and specify a calculation in terms of them as elementary functions. We have, therefore, a hierarchical arrangement of the control.

For the higher level of control, Babbage, in 1836, adopted the punched cards developed by Jacquard for pattern-weaving looms and used extensively since 1810 (Figure 2.12). A card is pressed against the ends of control levers so that the pattern of holes in the card determines which levers act. The action is entirely analogous to the studs comprising a vertical on a barrel. The Jacquard cards are strung together by narrow ribbons so that they comprise, in essence, a paper tape. It is possible, by mechanisms similar to those used to rotate the barrels, to move forward or backwards through a string of the Jacquard cards. In effect the string of Jacquard cards is equivalent to

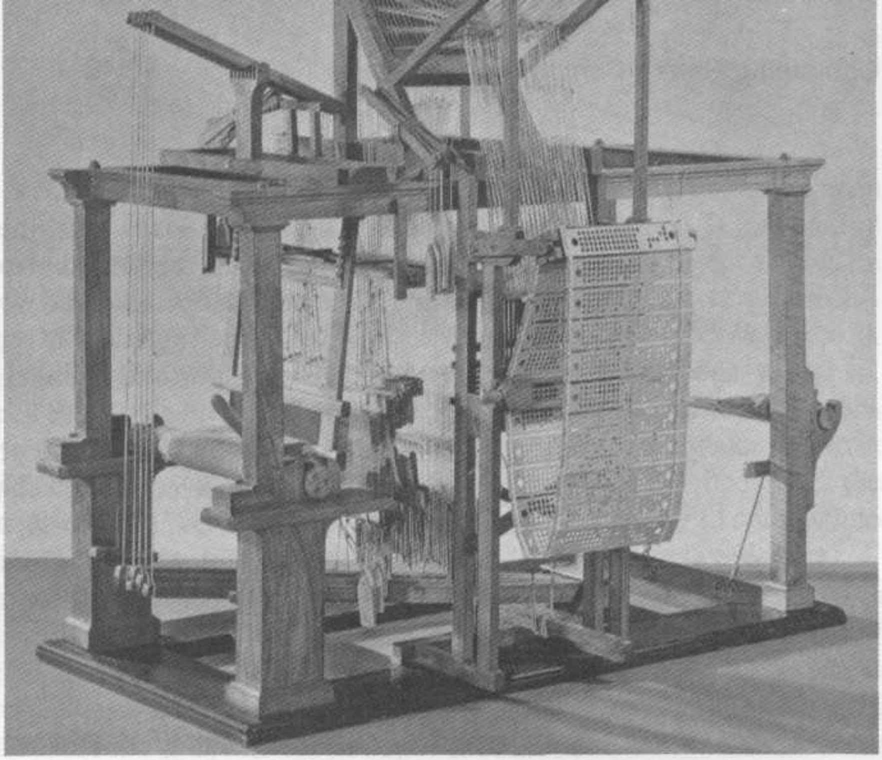

Figure 2.12. A Jacquard pattern-weaving loom controlled by punched cards. Babbage adopted and generalized this mechanism for the user-level specification of calculations for the Analytical Engine. Courtesy Science Museum, London.

a barrel with an indefinitely large number of verticals. The adoption of the cards represents less a conceptual breakthrough than a pragmatic improvement on the earlier use of a super barrel to specify the steps of a calculation. Babbage made little of this development and its importance has been considerably over-romanticized by the analogy with modern uses of punched cards.

Programming the Analytical Engine

Although Babbage's mechanical technology is vastly different from modern electronics, it is relatively easy to find analogies that make his organization of the calculating units and storage and their control by the barrels and Jacquard cards familiar to modern computer users. It is only when we come to examine the facilities available for programming the Analytical Engine that Babbage's designs begin to look strange to modern eyes.

Two strings of Jacquard cards were needed to specify a calculation to be performed by the Analytical Engine. One string, the "operation cards," specified the arithmetic operations to be performed. The second string, the "variable cards," specified the axes in the store that contained the operands and were to receive the results. These two strings cannot be regarded as separate parts of a single instruction, as are the operation and operand fields of an instruction in an electronic digital computer, because the operation and variable cards were intended to move and loop independently of one another under the direction of separate control mechanisms.

Babbage seems to have been led to separate the operation and variable cards on largely philosophical grounds stemming from his belief in the need to distinguish symbols for operation from those for quantity in mathematical notations. These views were probably reinforced when he considered the cards necessary for calculations such as the solution of simultaneous equations. There the pattern of operations required for carrying out row reductions is very simple and a straightforward loop of operation cards is readily found. No such simple loop structure exists for the variable cards, which can only specify single axes in the store. The loop structures that we now recognize concern rows of the matrix of coefficients of the equations and similar concepts related to the structuring of the data. As Babbage did not have the concept of a variable address in the store, neither was the Analytical Engine able to calculate the location of an operand in the store; there was no way in which the user programs could exploit this higher level structure in the data.

In reality, we know little of Babbage's programming ideas. There is nothing in the surviving papers in which this aspect of the machine is thoroughly discussed, e.g., nothing corresponding to a specification of a user instruction set. This is the more remarkable for it is the only aspect of the design that is discussed at length in a contemporary paper. In 1840, Babbage visited Turin in Italy and gave a series of seminars on the Analytical Engine. An account of these, by Menabrea, was translated into English by Ada Lovelace, who appended extensive notes prepared under Babbage's close guidance. These deal with the familiar modern ideas of flow of control in programs, particularly the formulation of simple loops and nested loops controlled by counters. However, the paper and notes carefully and deliberately skirt around any discussion of details of the means by which these are to be implemented.

Ada Lovelace has sometimes been acclaimed as the "world's first programmer" on the strength of her authorship of the notes to the Menabrea paper. This romantically appealing image is without foundation. All but one of the programs cited in her notes had been prepared by Babbage from three to seven years earlier. The exception was prepared by Babbage for her, although she did detect a "bug" in it. Not only is there no evidence that Ada Lovelace ever prepared a program for the Analytical Engine but her correspondence with Babbage shows that she did not have the knowledge to do so. Babbage seems to have deliberately employed independent persons to convey knowledge of the Analytical Engine to the wider public in exactly the same manner as, a decade earlier, he had used the well-known popularizer of science Dionysius Lardner to convey into print a detailed account of the purpose of the Difference Engine.

The conclusion seems inescapable that Babbage did not have a firm command of the issues raised by the user-level programming of the Analytical Engine. It would be quite wrong to infer that Babbage did not understand programming per se. The microprogramming of the barrels for multiplication and division show command of the basic branching and looping ideas and his skills in the microprogramming of addition and subtraction show complete virtuosity. It was from this base that Babbage explored the ideas of user-level programming. The issues of data structuring simply did not arise at the microprogramming level. There is some evidence to suggest that Babbage's ideas were moving in the directions now familiar in connection with the control mechanisms for loop counting in user-level programs. Had an Analytical Engine ever been brought to working order, there can be no doubt that Babbage's programming ideas would have been developed greatly.

Babbage realized that the Analytical Engine was a universal calculating machine in the sense that, given sufficient time, it could carry out any possible arithmetic calculation. The argument, clearly presented in simple terms in his autobiography, is based on three observations. First, arithmetic operations on numbers of more than forty digits can always be carried through by breaking them into 40-digit segments, so the limited number of digits on any store axis is no fundamental limit. Second, calculations can be specified by strings of operation and variable cards of unlimited extent, so there is no limitation to the size or complexity of programs. Third, numbers from the store can be punched onto number cards and later read back, and this provides a backing store of unlimited extent to overcome the limited number of axes in the store. This sophisticated argument has a very twentieth-century flavor. Babbage was not aware that there might be uncomputable numbers, a concept that derives from the brilliant work of Alan Turing in the 1930s.

Babbage's Later Calculating Engines

Work on the design of the Analytical Engine ended in 1847. At that time Babbage turned to the design of a Difference Engine No. 2, exploiting the improved and simplified arithmetic mechanisms developed for the Analytical Engine. The logical design was the same as for the earlier Difference Engine, but he employed simpler mechanisms for storing and adding numbers and carry propagation. The printing mechanism was simplified so that a whole number was impressed on a printing plate as a single action rather than in a digit-by-digit manner. A conventional print copy, using inked rollers, was made simultaneously. The control was arranged by a single barrel in a very straightforward manner. The design and a complete set of drawings was prepared by mid-1848. These Babbage offered to the British government, apparently to satisfy a commitment he felt existed in consequence of the failure of the project to build the first Difference Engine. The government showed no interest in the new design.

Babbage appears to have done no more work on calculating engines until the Scheutz Difference Engine (described in the next section) was brought to London in 1855. To the surprise of some, Babbage became an active and vigorous promoter of the Scheutzes and their machine.

Inspired, perhaps, by the Scheutzes' success, Babbage returned to design work on the Analytical Engine in about 1856 or 1857, when he was 65 years old. In this new phase of work Babbage was actively interested in building an Analytical Engine with his own resources. The logical design was somewhat simplified but, most importantly, far simpler and cheaper methods were proposed to implement the basic mechanisms. Babbage first experimented with sheet metal stamping and pressing for making gear wheels and similar parts. Later, he adopted pressure die casting for making parts—a newly invented technique that did not see extensive commercial use until the end of the nineteenth century. Babbage built many experimental models of mechanisms using these new techniques, and, at the time of his death in 1871, a model of a simple mill and printing mechanism was near completion. (Figure 2.13)

This last work of Babbage is poorly understood because of the disorganized and chaotic nature of the materials that remain. The impression is unavoidable that in this later work Babbage had lost the fine touch of genius exhibited in his earlier work, although his various

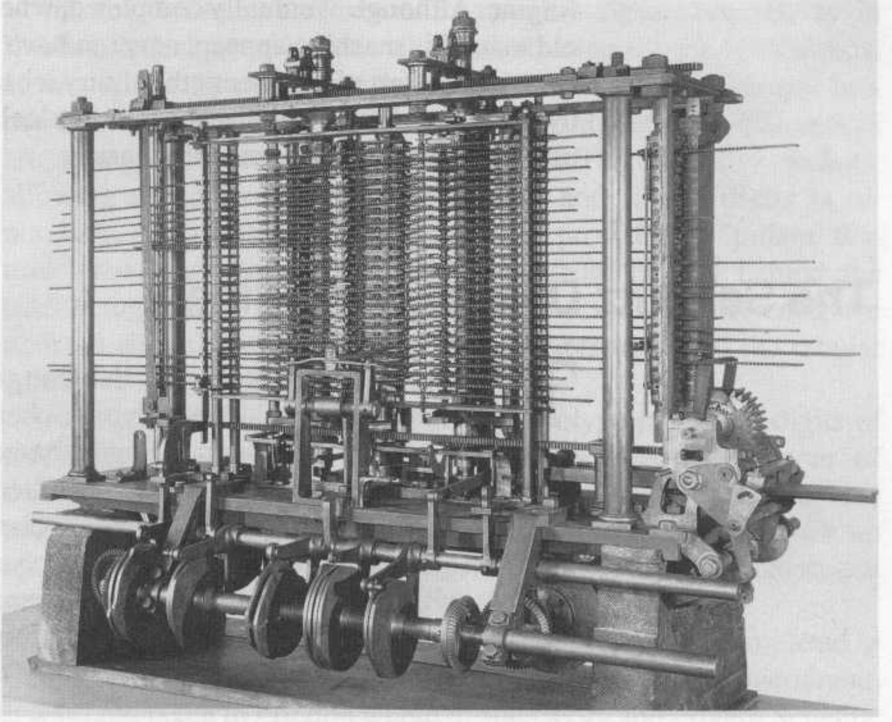

Figure 2.13. A model of the mill of the Analytical Engine that was under construction at the time of Babbage's death. The horizontal racks communicate numbers between the two number axes in the center and to the printing mechanism at the right. An anticipating carriage mechanism is located between the number axes. The calculating mechanism employs pressure die-cast metal components. Courtesy Science Museum, London.

experimental models still show much evidence of an ingenious and enquiring mind. In fairness, we must note that most of this later work was carried on when Babbage was between seventy and eighty years old.

Babbage's calculating machines and related materials were inherited by his youngest son, Major-General Henry P. Babbage, who had shown a strong interest in his father's work. Henry Babbage decided not to continue with the design of an Analytic Engine but instead to develop a manually operated machine for addition, subtraction, multiplication, and division (a four-function calculator), incorporating the mechanisms planned for the mill of the Analytical

Engine. Although eventually completed, when Henry was himself an old man, this machine appears never to have worked reliably. In any case, by the start of the twentieth century it had been rendered archaic by other developments of mechanical calculating machines, so that now it stands only as a scientific curio.

The Scheutz Difference Engine

The first successful automatic calculating machine was developed in Sweden in the 1840s by Georg Scheutz and his son Edvard. Their machine was a Difference Engine based directly on Babbage's design, which they learned about when George Scheutz translated Lardner's article into Swedish. In that Lardner's article contains only the most general descriptions of the mechanism of the Difference Engine (without drawings of the mechanisms) it is a small surprise that the Scheutz machine (Figure 2.14) looks very different from Babbage's (Figure 2.4).

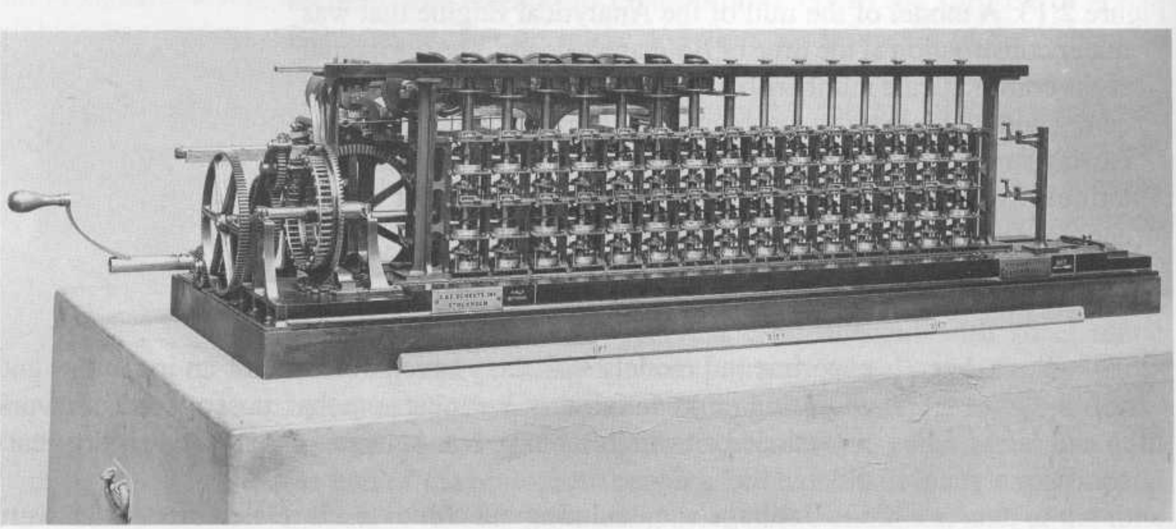

Figure 2.14. The copy of the Scheutz Difference Engine built for the General Register Office, London. The pillar at the right moves across the front of the figure wheels to effect the carriage propagation. The printing mechanism is behind the calculating wheels at the left. Courtesy Science Museum, London.

In the Scheutz Difference Engine the figure wheels, as in Babbage's design, are horizontal wheels rotating about a vertical axis. However, the figure wheels for the tabular value are arranged in a row on a horizontal shelf, while the first- and higher-order differences are arranged on successive shelves below. There are five shelves, allowing up to fourth-order differences, and fifteen digits in all numbers. A sequential carry propagation is provided by "pillars" that travel the length of the mechanism in front of and behind the calculating wheels. As in Babbage's machine, the odd differences are updated simultaneously, then the even differences (and the tabular value) are updated together.

The printing mechanism punches stereotype plates, all digits of a number being impressed simultaneously after the manner of Babbage's later designs. The Scheutz Difference Engine prepares only a single column of tabular values. The page layout is made up manually from these strips. There is no attempt at an automatic mechanism for this purpose.

Construction of the Scheutz Difference Engine was completed in October 1853 with some assistance from the Swedish government. It was later taken to London where it, and Georg and Edvard Scheutz, were championed by Babbage. The machine was exhibited in the Paris Exhibition of 1855, awarded a gold medal, and widely acclaimed. The original machine was sold to the Dudley Observatory in Albany, New York, in 1856 to be used in preparing astronomical tables. A copy of the machine was made by Bryan Donkin & Co. for the General Register Office in England about 1858.

Three sets of tables were published that had been calculated, at least in part, by the Scheutz Difference Engine. A set of *Specimen Tables*, including a table of five-figure logarithms of the integers from 1 to 1000, was prepared on the original machine in 1856. That same year, a set of *Mountain Barometer Tables*, for assessing heights on mountains or depths in mines from simultaneous observations of barometric pressure and temperature, were prepared by the English copy of the Scheutz Difference Engine. The major production was the preparation of the *English Life Tables*, published in 1864, at the General Register Office.

Neither model of the Scheutz Difference Engine was found to be very satisfactory and the use of both was quickly abandoned. A number of factors contributed to this failure. The calculations were slow, largely because of the awkward carry propagation mechanism used. The machine depended on friction alone to keep the figure wheels in their correct position when a number was stored—there were no spring detents or other mechanisms equivalent to Babbage's lockings to retain the figure wheels in place. Without such provision it seems the figure wheels easily became displaced from their correct position and the calculation spoiled. Although this could be detected by examining the last value printed it was no doubt a source of considerable annoyance. The printing mechanism seems also to have been unreliable and errors of that sort would have been difficult to detect. In general, both machines were found to be delicate instruments that required considerable skill to manipulate and hence were ill-suited to routine use.

Later Difference and Analytical Engines

That the Scheutz Difference Engine possessed faults is scarcely surprising in view of its being the first completed machine of its type. It is regrettable that more experience had not been gained with the original machine before the English copy was made, when the opportunity might have been taken to eliminate the difficulties. Production of a reliable difference engine required the investment of new, inventive effort to build on the Babbage and Scheutz achievements. However, there were only two more developments in that direction in the nineteenth century.

In Sweden, Martin Wiberg had built a difference engine by 1860 and used it to prepare a set of interest tables for publication. The Wiberg Difference Engine was both smaller and simpler than the Scheutz, though it possessed the same arithmetic capability. The machine appears to have worked reliably and was used in the preparation of logarithmic and trigonometric tables that appeared in 1875.

In America, George Grant developed a small model of a difference engine in 1871 and exhibited a complete machine in 1876. But this machine soon faded into obscurity and appears not to have been put to any practical use.

In the twentieth century the use of difference engines in table making again received some prominence. In this case, however, the construction of special purpose machines was not attempted but ways were found to adapt the general purpose calculating machines then on the market to this special purpose. The best known work is that of

L. J. Comrie at the British Nautical Almanac Office in the 1920s and 1930s, using multiple register accounting machines manufactured by Burroughs and National Cash Register.

If little effort was made to develop difference engines, it is scarcely surprising that nothing substantial followed in the tradition of Babbage's Analytical Engine.

In Ireland an analytical engine was designed by Percy Ludgate about 1905. Initially this work was independent of Babbage's but later Ludgate came to know and be influenced by Babbage's ideas. The design, which was purely mechanical, contains some striking features. The mechanisms in which numbers were stored were physically transported from the memory when the number was read. A pseudologarithmic representation of digits was used to simplify both multiplication and division operations. A most interesting feature was the abandonment of Babbage's separate operation and variable cards and the adoption of control by a paper tape in which each instruction comprised an operation code and four address fields. Very little information on Ludgate's design has survived, and there is no evidence that he ever attempted to construct the machine.

Very interesting designs of analytical engines were made in Spain in the 1910s and 1920s by Leonardo Torres y Quevedo. Torres was a well-known engineer who vigorously exploited the new electromagnetic technologies in the development of control mechanisms. He is particularly well-known for two fully automatic chess playing automata for the ending of king and rook against a king. In 1920 Torres constructed an electromagnetic calculating machine that was driven by operands typed on a typewriter and delivered its results using the same device. Torres's ideas for an analytical engine were sufficiently well developed that there is no doubt that a successful machine could have been built in the 1920s had the need for such a machine been pressing.

The Importance of Babbage's Calculating Engines

In the designs of the Difference Engine and the Analytical Engine Babbage made the first major intellectual contributions towards the development of automatic digital computing machines although his ideas were not realized until over a century later. Two major questions remain about Babbage's work. Why were his machines not successfully constructed? And what influence did his ideas have on the subsequent development of automatic computers?

The present evidence suggests, quite strongly, that both the Difference Engine and the Analytical Engine could have been built successfully with the mechanical technology at Babbage's disposal. The calculating part of the Difference Engine came close to completion and the portion in the Science Museum, London, works superbly. The failure of that project seems traceable in part to Babbage's relationship with the British government over the funding of the project but especially to Babbage's relationship with the engineer Clement. The large physical scale of the machine and, particularly, the very high degree of precision attained in the manufacture of its parts and the concomitant expense seem to have been the root causes of the failure to bring it to completion.

The Difference Engine has a direct line of descendants through the Scheutz to the Wiberg and Grant difference engines. That these were not extensively used or developed, despite the apparent complete success of the Wiberg machine, indicates that the entire idea was not well judged. The sub-tabulation task, though laborious, was not the dominant mathematical task in the preparation of tables nor, with adequate organization and management, was it of overwhelming practical importance. Babbage's argument for the accuracy in typesetting made possible by machines (later strongly held by Howard Aiken) was not widely accepted, and Babbage's own logarithm tables are proof of the accuracy that could be obtained by manual techniques. When machine sub-tabulation was adopted by Comrie, it was in the context of a large-scale mechanization of table making in which the balance of effort in the whole project was not much changed.

Although the Analytical Engine could have been built, Babbage chose, for most of his life, not to attempt to do so. This is a natural response to his experiences with the Difference Engine and the enormous intellectual appeal of the questions raised by the Analytical Engine. Of great regret is the fact that Babbage never published a detailed account of any of his many ideas and mechanisms. The Menabrea-Lovelace paper deliberately concentrates on the mathematical principles embodied in the machine and completely avoids describing their mechanization.

Without a detailed description of the Analytical Engine its influence on later developments was quite limited. Certainly the idea of an automatic calculating machine was well-known in English and, to a lesser extent, American scientific circles and closely associated with Babbage's name. But only the most limited technical guidance was provided for later designers, who in effect worked independently of Babbage. The fruits of Babbage's considerable genius were therefore effectively wasted as far as practical influence is concerned. Only in the tapes of the Turing machine, and the idea of mechanization of computation used there, is there any strong echo of Babbage's ideas. Turing's place in the English intellectual tradition makes such a line of influence plausible if unproven.

In the practical field of making automatic calculating machinery it is even possible that Babbage's influence was counterproductive. What point was there in attempting to make an automatic machine when a man of Babbage's acknowledged genius had failed? Indeed, it is difficult to understand why machines were not built using electromagnetic technology early in the twentieth century. Torres's designs showed that it was certainly feasible to do so by 1914, and Stibitz's designs could have been implemented decades before they were.

Notes

1. The trick is to add 5 to the initial value of the tabular function in the most significant digit position beyond those to be printed (i.e., 1/2 in the least significant digit) and thereafter to simply truncate all values to be printed.

2. The difference table shown at the right of Figure 7 was not produced by the Difference Engine, and the row and column headings would have required further runs through the machine to insert them.

3. The use of feedback here is very similar to that employed in differential analyzers and analog computers (Chapter 5). What Babbage proposed is effectively a form of digital differential analyzer.

Further Reading

Babbage, H. P. *Babbage's Calculating Engines.* Los Angeles and Cambridge, Mass.: Tomash Publishers and MIT Press, 1982. The best edition of the contemporary writings of Babbage and others concerning his machines, collected and published by his son after Babbage's death.

Bromley, A. G. "Charles Babbage's Analytical Engine, 1838." *Annals of the History of Computing* 4(July 1982):196-217.

———. "The Evolution of Babbage's Calculating Engines." *Annals of the History of Computing* 9(1987):113-136. These two works by Bromley describe the design of the Analytical Engine in more detail.

Hyman, A. *Charles Babbage: Pioneer of the Computer.* Oxford: Oxford University Press, 1982. Babbage has become, in the last two decades, something of a cult figure and has generated considerable literature, much of it unreliable and unsubstantiated by careful examination of the primary sources. So far, the only trustworthy biography of Babbage is that of Hyman.

Lindgren, M. *Glory and Failure.* Vol. 9, Linkoping Studies in Arts and Science. Linkoping University Press, 1987. Reprinted by MIT Press, 1989.

Merzbach, U. C. *Georg Scheutz and the First Printing Calculator.* Washington, D.C.: Smithsonian Institution Press, 1977. The Scheutz Difference Engine and its successors are described in the Lindgren and Merzbach publications.

Randell, B. *The Origins of Digital Computers.* 3d ed. New York: Springer-Verlag, 1982. This reprint of selected papers discusses the machines of Ludgate and Torres, as well as of Babbage.

Stein, D. *Ada: A Life and a Legacy.* Cambridge, Mass.: MIT Press, 1985. This work assesses the role of Ada Lovelace.