Eratosthenes and the Mystery of the Stades

Newlyn Walkup
University of Missouri - Kansas City
nqwcm8@umkc.edu
1717 N. Ridgeview Rd.
Apt. 300
Olathe, KS 66061

Supervising Professor: Dr. Richard Delaware

In the third century BCE, the brilliant librarian Eratosthenes of Cyrene (276-195 BCE) devised an ingenious method by which to measure the circumference of the Earth. Using geometry and the Sun, Eratosthenes accomplished the impossible. Although his original works have long since been lost, the legendary story has been retold for over two thousand years. Like all legends it has become difficult to sort the fact from the fiction. Some scholars claim that Eratosthenes' approximated the size of the Earth to within 2% of its actual value; while others believe that the accuracy of his measurement is greatly exaggerated. The key to this ancient riddle is the not-so-standard ancient unit of length – the *stade*. There is a great deal of uncertainty as to the actual length of the stade Eratosthenes used. It is also uncertain whether he made the measurements used in the calculation, or if he relied on the information of others. Perhaps the most puzzling question is why Eratosthenes inexplicably added 2000 stades to his original figure for the Earth's circumference. The mystery is one that drives scholars even today.

Eratosthenes was a man of great distinction among scholars in the ancient world. He was a good friend of the famous Greek scholar Archimedes of Syracuse (287-212 BCE). In fact one of Archimedes greatest works, *The Method*, was dedicated to Eratosthenes [12, p.104].

> *I sent you on a former occasion some of the theorems discovered by me, merely by writing out the enunciations and inviting you to discover the proofs, which at the moment I did not give.* […] *The proofs then of these theorems I have now sent to you. Seeing moreover in you, as I say, an earnest student, a man of considerable eminence in philosophy, and an admirer* […] [13, pp.12-13]

Because Eratosthenes was highly knowledgeable in all branches of study, yet he was not the "*Alpha*" (the greatest) in any one branch, his peers gave him the nickname "*Beta*" [12, p.104]. Eratosthenes received the equivalent of a college education in Athens and then went to the Egyptian city of Alexandria [1, p.388]. Attracting scholars and students from all over the ancient world, the great library in Alexandria became the center of scholastic achievement. It is written that the library contained over 500,000 scrolls [15, p.59]. Around 235 BCE, Eratosthenes was appointed head librarian of the library in Alexandria [1, p.388]. It was during this period that Eratosthenes would devise his method to approximate the circumference of the Earth.

The simplicity and elegance of Eratosthenes' measurement of the circumference of the Earth is an excellent example of ancient Greek ingenuity. While working at the library, he learned that on the first day of summer the Egyptian town of Syene cast no shadows [8, p.339]. This happens because at noon on the day of the summer solstice the Sun is positioned directly above the town of Syene, near the modern city of Aswan, Egypt [3, p.115]. In contrast, on that same day in Alexandria a staff, or *gnomon*, did cast a shadow [8, p.339]. With a few measurements, some assumptions, and a little geometry, Eratosthenes was ready to approximate the circumference of the Earth [8, p.339]. Eratosthenes' original account of this measurement does not survive, but his argument has been preserved in the writings of many other ancient scholars such as Cleomedes, Strabo, and Ptolemy [4, p.1].

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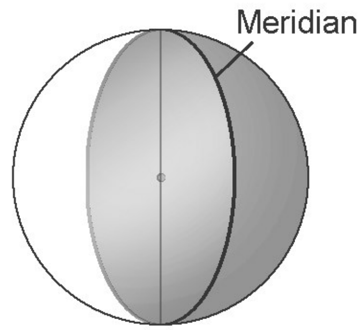
Eratosthenes makes five assumptions which he will use as hypotheses in his argument.

1. *That Alexandria and Syene lie on the same meridian* [11, p.109].
2. *That light rays from the Sun which strike the Earth are parallel* [11, p.109].
3. *That the distance between Alexandria and Syene is 5000 stades* [11, p.109].
4. *That the angle formed by the shadow and the staff in Alexandria at the summer solstice is equal to* $\frac{1}{50}$ *th of a circle* [11, p.109].
5. *That the Earth is a sphere.*

Let us examine each of Eratosthenes' assumptions along with their respective justifications.

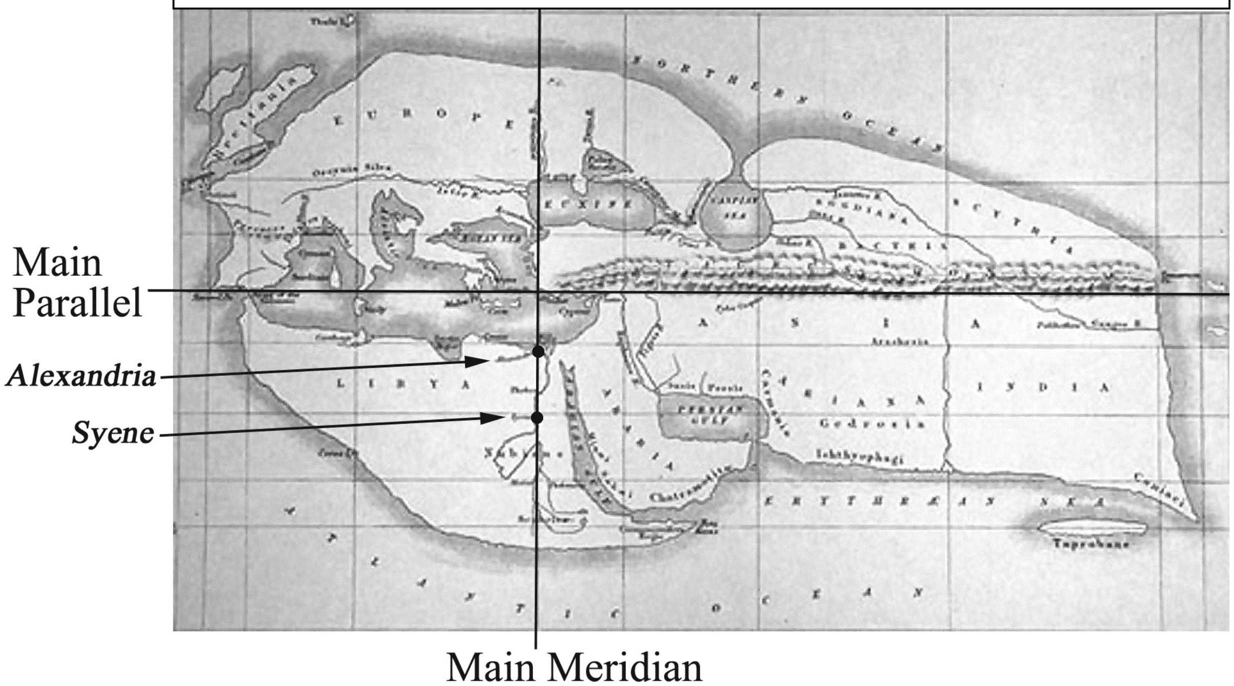
1. *That Alexandria and Syene lie on the same meridian* [11, p.109].

A *meridian* is an imaginary circle on the Earth's surface which passes through both the north and south poles [17, p.209]. The plane of any meridian bisects the Earth. If Alexandria and Syene both lie on the same meridian, Eratosthenes is guaranteed that two cities and the center of the Earth are all contained in the same plane. Now the geometrical argument takes place in two dimensions (the plane), rather than in three. How does Eratosthenes justify this assumption?



Before his famous calculation of the Earth's circumference, Eratosthenes attempted the earliest known scientific construction of a map based on mathematical geography. Using the tremendous amount of information available to him at the great library in Alexandria, Eratosthenes sought to correct the traditional Greek map of the world. He examined countless texts, compiling records of measured distances and comparing various accounts of similarities in flora, fauna, climate, astronomical observations, local peoples, etc. [1, p.389]. The map featured a main "parallel", running east to west through the city of Rhodes, and a main meridian, running north and south through Rhodes [4, p.63]. Using these two perpendicular main lines, Eratosthenes divided the map into rectangular regions he called *seals*, which could then be used to geometrically calculate any distance in the known world – hence the term "mathematical geography" [2, p.128]. The main meridian in this map runs directly through several cities, including Alexandria and Syene [1, p.389]. So Eratosthenes' first assumption is based on a map which he constructed using the wealth of knowledge available at the library of Alexandria.

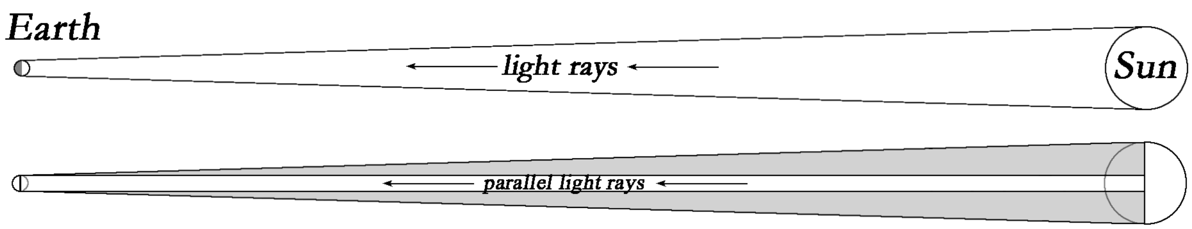
A 19th century reproduction of Eratosthenes' map of the world [19].



Eratosthenes' map of the world appeared in his work entitled *Geography*, which was long regarded as the highest authority on geography in the ancient world [1, p.389].

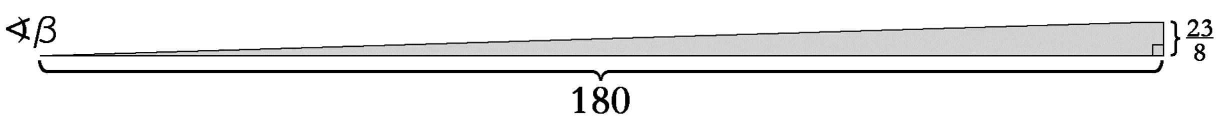
2. *That light rays from the Sun which strike the Earth are parallel* [11, p.109].

In fact, this assumption is incorrect. Sunrays striking the Earth are not parallel. How did Eratosthenes justify such a claim? Just years earlier a man named Aristarchus of Samos (310-230 BCE) produced a work entitled *On the Sizes and Distances of the Sun and Moon*. This masterpiece of ancient astronomy contains an elaborate geometric proof which asserts that the distance from the Earth to the Sun is approximately equal to 180 Earth diameters. Furthermore, he reasoned that the Sun's diameter is approximately $6\frac{3}{4}$ times that of the Earth. Actually, the Sun is almost 1200 Earth diameters from the Earth, and the Sun's diameter is around 109 times the Earth's, but the idea is the same – the Sun is much larger than the Earth, and light rays from the Sun travel a great distance to the Earth [8, pp.350-352].



The shaded regions represent the difference between the assumed parallel sunrays and actual nonparallel sunrays. Using Aristarchus' measurements and some modern mathematics, we can judge the significance of this difference. Consider one of the shaded regions.

Notice that the shaded region is a right triangle. Angle β, at the farthest vertex of the triangle, is the angular difference between the actual sunrays and the assumed parallel sunrays. Using the information provided by Aristarchus, angle β can be approximated. According to Aristarchus, the distance to the Sun is equal to 180 Earth diameters. So the length of the shaded triangle is 180 Earth diameters. Aristarchus also tells us that the Sun's diameter is equal to $6\frac{3}{4}$ Earth diameters. Subtracting one Earth diameter from the center of the Sun's diameter gives us $5\frac{3}{4}$. Dividing by 2, we find that each shaded triangle has a height of $\frac{1}{2}(5\frac{3}{4}) = \frac{1}{2}(\frac{23}{4}) = \frac{23}{8}$ Earth diameters.



Using modern trigonometry, we get

$$\operatorname{Tan}\beta = \frac{\frac{23}{8}\text{ Earth diameters}}{180\text{ Earth diameters}}$$

$$\beta = \operatorname{Tan}^{-1}\left(\frac{23}{1440}\right)$$

$$\beta \approx .915^{\circ}$$

At the time of Aristarchus and Eratosthenes, the instruments used to make angular measurements were so crude that an error of less than a degree was negligible [4, p.57]. Of course, Aristarchus and Eratosthenes did not have the benefit of our modern trigonometry, but using the Euclidean geometry available to them they were able to recognize that the small angular difference was relatively insignificant [5, p.154]. With this idea in mind, Eratosthenes is justified in making the assumption that sunrays striking the Earth are parallel.

3. *That the distance between Alexandria and Syene is 5000 stades* [11, p.109].

There is little doubt that Eratosthenes got this figure directly from his earlier map of the known world [4, p.62]. How he initially obtained this value is a controversial question which may never be answered, but it is doubtful that Eratosthenes actually measured the distance himself [5, p.154]. The writings of Strabo the Geographer (ca. 20 BCE) suggest that the lands along the Nile were measured every year.

> *In Egypt, the Nile passes in a straight line from the little cataract above* ***Syene*** *and Elephantine, at the boundary of Egypt and Ethiopia, to the sea. The country was divided into nomes* [provinces]*, which were subdivided into sections* […] *There was need of this accurate and minute division because of the continuous confusion of the boundaries caused by the Nile at the time of its increases* [flooding]*, since the Nile takes away and adds soil, and changes conformations of land, and in general hides from view the signs by which one's own land is distinguished from that of another. Of necessity, therefore, the lands must be re-measured again and again. And here it was, they say, that the science of land-measuring originated* [5, p.152].

The Nile flooded regularly every year, changing the landscape around it, so every year there were disputes between landowners over property lines. Thus, the lands around the Nile had to be re-surveyed each year after the flood. Long distances were measured by professional distance walkers, called *bematists*, who walked at a very regular pace and counted each step. Shorter distances were measured with lengths of knotted rope by men called *harpedonaptai*, which means "rope stretchers" [4, pp.56-58]. Knowing that Alexandria and Syene are both located on the Nile, Eratosthenes could have calculated the distance by compiling the yearly measurements of the land between the two cities [5, p.153]. The fact that 5000 stades is a round number might suggest that it was the traditionally accepted figure for the distance between the cities, established well before the time of Eratosthenes [6, p.411]. Whatever his reasoning, as the foremost authority on geography in the ancient world, Eratosthenes is justified in assuming that the distance between Alexandria and Syene is 5000 stades.

4. *That the angle formed by the shadow and the staff in Alexandria at the summer solstice is equal to* $\frac{1}{50}$ *th of a circle* [6, p.411].

This assumption states that the angle formed by the shadow cast by the staff in Alexandria is "$\frac{1}{50}$ th of a circle", meaning $\frac{360^{\circ}}{50} = 7\frac{1}{5}^{\circ} = 7^{\circ}12'$. Although it was used by Babylonian civilizations as early as the fifth century BCE, division of a circle into the familiar 360° was not introduced to Greek science until the second century BCE by Hipparchus of Rhodes (190-120 BCE) [2, p.149]. The system of angle measure used by Eratosthenes divided the circle into 60 parts, each called a *hexacontade*. As will be seen, this system provides one of the most compelling arguments as to the source of the additional 2000 stades. There is no way to know if Eratosthenes made this measurement himself, but many scholars argue that he probably did measure this angle using a hemispherical sundial, known as a *scaphe*, which was the best astronomical instrument of the day [5, pp.153-154]. Then based on his own observation, Eratosthenes is justified in assuming that the angle formed by the shadow in Alexandria is $\frac{1}{50}$ th of a circle.

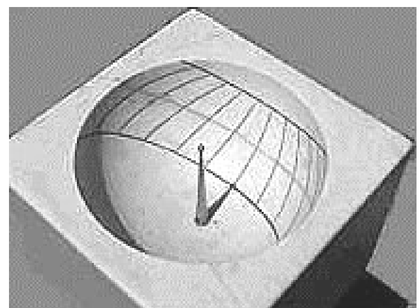
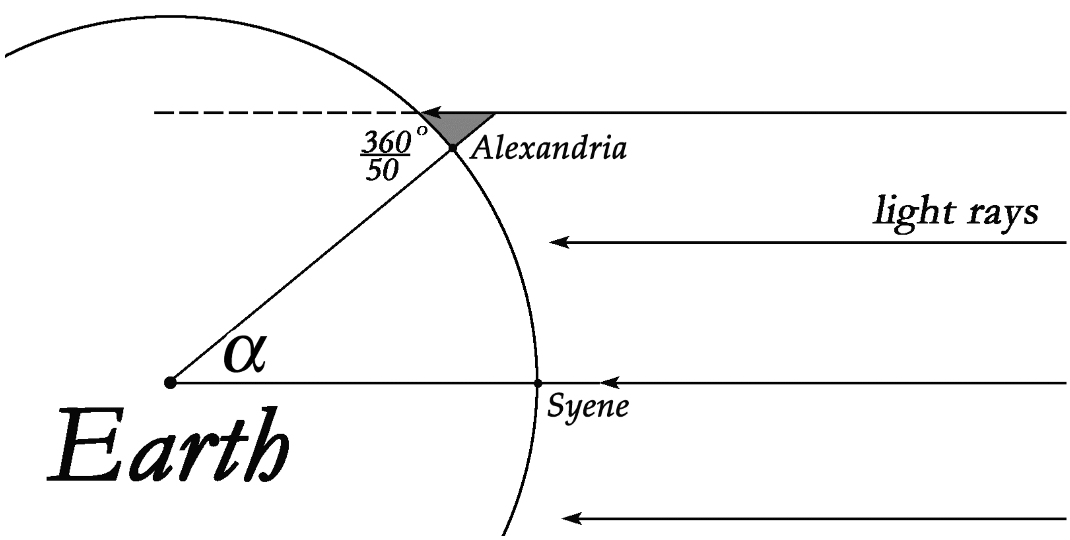


Illustration of a scaphe [16].

5. *That the Earth is a sphere.*

At a time when not everyone believed so, Eratosthenes clearly had no doubt that the Earth is a sphere [3, p.116]. Many early Greek philosophers held that the Earth was a disc or a cylinder. The idea of a spherical Earth was first suggested by Pythagoras of Samos (580-500 BCE), about 300 years before Eratosthenes [20, p.1]. The spherical Earth model did not begin to gain widespread acceptance among scholars until the well reasoned arguments of Aristotle (384-322 BCE), only about 100 years before Eratosthenes [4, p.59]. Around the time of Eratosthenes, the globe was becoming a popular model of the Earth as many Greek scholars came to accept the idea of a spherical Earth [20, pp.2]. As it was the prevailing belief in the scientific community at the time, Eratosthenes is justified in making the implicit assumption that the Earth is a sphere.

Eratosthenes uses these five main assumptions as hypotheses for his famous geometric approximation of the Earth's circumference. His approximation would not be surpassed for centuries to come. The method devised by Eratosthenes is the basis for the complex "astrogeodetic method" which is used to measure the Earth today [5, p.153]. His elegant geometric argument, illustrated below, is sound and simple.



Claim: The circumference of the Earth is approximately 250,000 stades.

Proof:

- Given that 1. That Alexandria and Syene lie on the same meridian.

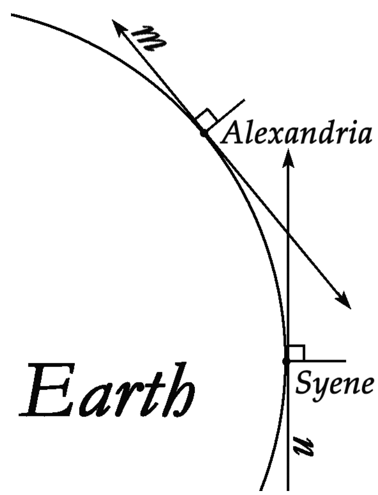
  2. That light rays from the Sun which strike the Earth are parallel.

  3. That the distance between Alexandria and Syene is 5000 stades.

  4. That the angle formed by the shadow and the staff in Alexandria at the summer solstice is equal to $\frac{1}{50}$ th of a circle.

  5. That the Earth is a sphere.

- By hypothesis, since Alexandria and Syene lie on the same meridian, the staffs and the center of the Earth all lie in the same plane.



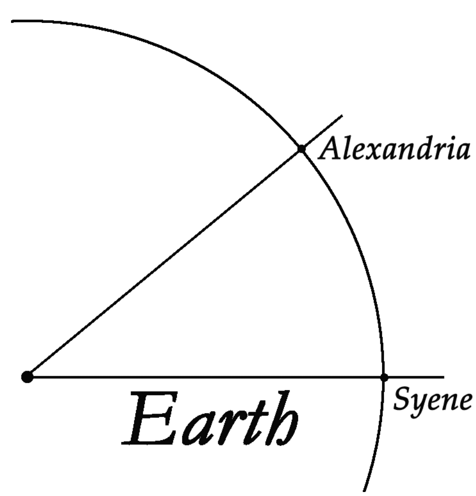
- By construction, the staff in Alexandria is perpendicular to the ground, so in the plane of the meridian it is orthogonal to the cross-sectional circle of the Earth.

- By definition of orthogonal, the staff in Alexandria is perpendicular to a line *m* which is tangent to the Earth at the base of the staff.

- Likewise, the staff in Syene is perpendicular to a line *n* tangent to the Earth at the staff's base.

- Euclid III-19: *If a straight line touches a circle, and from the point of contact a straight line is drawn at right angles to the tangent, the center of the circle will be on the straight line so drawn* [10, p.45].

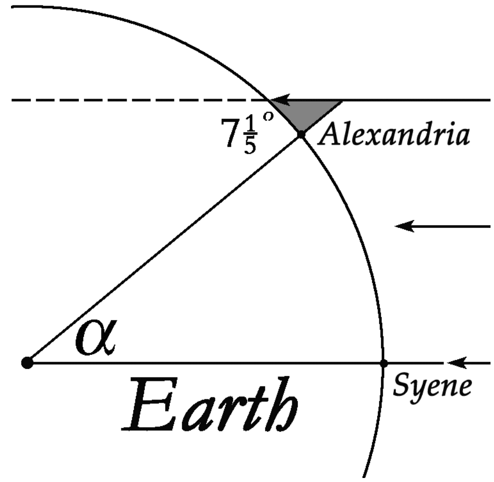
- By Euclid III-19, since the staffs are perpendicular to tangents *m* and *n*, if the staffs are extended toward the Earth, their lines intersect at the center of the Earth.



- By hypothesis, the light rays striking the Earth are parallel.

- Since the staff in Syene casts no shadow, no angle is formed by the intersection of the light rays and the staff, thus the line of the staff is parallel to the light rays.

- Euclid I-29: *A straight line falling on parallel straight lines makes the alternate angles equal to one another* […] [9, p.311].



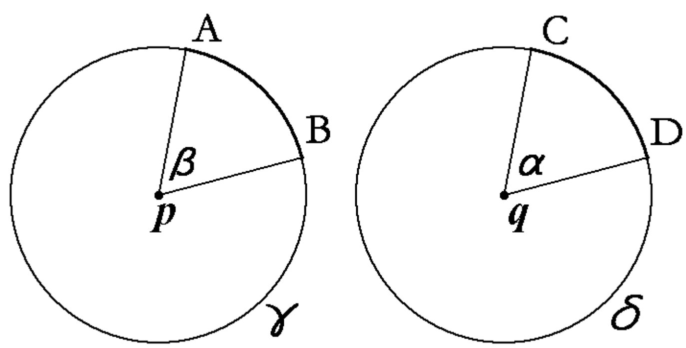
- Let the angle at the center of the Earth be called angle α.

- By hypothesis, the angle formed by the shadow in Alexandria is equal to $\frac{1}{50}$th of a circle. So the measure of this angle is $\frac{360^\circ}{50} = 7\frac{1}{5}^\circ$.

- By Euclid I-29, since the angle in Alexandria and angle α are alternate interior angles, the measure of angle α is also $\frac{360^\circ}{50} = 7\frac{1}{5}^\circ$.

- Euclid III-27: *In equal circles, angles standing on equal circumferences equal one another* […] [10, p.58].

    - Some explanation will help to reveal how Euclid III-27 is used in this argument.

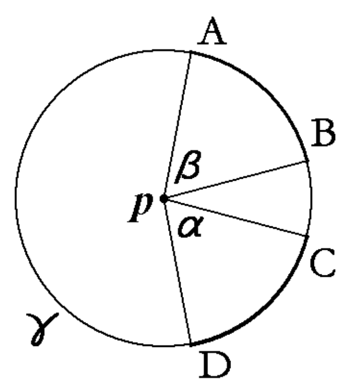


    - Given two equal circles γ and δ, with centers p and q respectively.

    - If arc AB $\cong$ arc CD, then angle β $\cong$ angle α.

    - Since every circle is equal to itself, by Euclid's 4th common notion, we can apply this proposition to a single circle.

    - Given circle γ, with center p.



    - If arc AB $\cong$ arc CD, then angle β $\cong$ angle α.

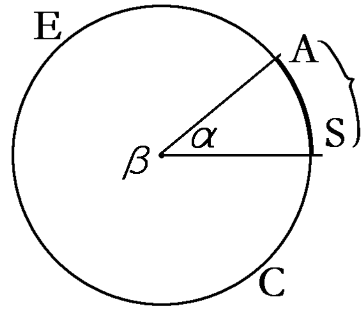
    - As real number values, these can be put into ratio form.

$$\frac{\text{arc CD}}{\text{arc AB}} = \frac{\text{angle}\alpha}{\text{angle}\beta}$$

    - Using this ratio form, Eratosthenes will now use three known values to solve for the unknown fourth value – the circumference of the Earth.

- Let the arc of the Earth between Alexandria and Syene be called arc AS, and let the full circumference of the Earth be called arc EC.

- Let the angle at the center of the Earth be called angle α, and let the full 360° of the circle be called angle β.



- By Euclid III-27, we have $\dfrac{\text{arc EC}}{\text{arc AS}} = \dfrac{\text{angle}\beta}{\text{angle}\alpha}$.

- By hypothesis, the length of arc AS is 5000 stades, and angle α is equal to $\frac{1}{50}$ th of a circle.

- Since angle β is the angle measure of a complete circle, angle β = 1 circle.

- Substituting these real number values into the previous ratio, we get

$$\frac{\text{arc EC}}{\text{arc AS}} = \frac{\text{angle}\beta}{\text{angle}\alpha}$$

$$\frac{\text{arc EC}}{5000\text{ stades}} = \frac{1}{\frac{1}{50}}$$

$$\text{arc EC} = \frac{5000\text{ stades}}{\frac{1}{50}}$$

$$\text{arc EC} = 250{,}000\text{ stades.}$$

- Therefore, since the length of arc EC is equal to the circumference of the Earth, we get that the circumference of the Earth is approximately 250,000 stades. ■

After seeing Eratosthenes' brilliant argument that the Earth's circumference is 250,000 stades, one naturally asks, " What is the length of a stade?" Unfortunately, this question has no simple answer. Without an International Bureau of Standards to ensure consistency of weights and measures throughout the ancient world, it is very likely that measures such as the stade varied slightly from region to region [2, p.46]. Scholars disagree greatly on the extent to which the stade may have varied in the ancient world. Scholar of Greek antiquity Carl Friedrich

Lehmann-Haupt claims the existence of at least six different stades [2, p.43]. To the contrary, astronomer and historian Dennis Rawlins makes the following claim.

> *That 1 stade = 185 meters (almost exactly 1/10 nautical mile) is well established. Nonetheless, some scholars are unwilling to believe that Eratosthenes'* C_E [approximation of the Earth's circumference] *could be so far in error as 17%* […] [18, p.211].

While the assertions of these two men represent the opposing extremes in this debate, there is an array of theories which lie somewhere in between. A common approach to this mystery is to examine the stade's relationship to other ancient units of length.

Book two in *The Histories* by the ancient historian Herodotus (480-425 BCE) tells us that 1 stade is equal to 600 Greek feet. Like the stade, the Greek foot exhibits some regional variation. However, all instances of the Greek foot appear to conform roughly to one of three basic lengths. To distinguish between these variations, scholar of Greek architecture Burkhardt Wesenberg refers to them as the "Attic" (from Asia Minor and southern Italy), the "Doric" (from Greece and Sicily), and the "Ionic" (used throughout the Greek civilization). Each of these variations of the Greek foot, when multiplied by 600, yields a stade length that corresponds closely to one of the six claimed by the previously mentioned scholar Lehmann-Haupt [7, pp.359-360]. Such correspondence lends to the argument that there was more than one stade used in the ancient world, and furthermore, that one of these stades may have been used by Eratosthenes.

The 185 meter stade, as claimed by Rawlins earlier, is the most commonly accepted value for the length of the stade used by Eratosthenes in his measurements of the Earth. This is so because a great number of authors from the first century CE onward make reference to the fact that 1 Roman mile is equal to 8 stades. History tells us that the Roman mile is equal to 5000 Roman feet, each of which is just short of the familiar English foot. The exact difference between the Roman foot and the English foot is uncertain, but if 1 Roman foot is taken to be approximately 11.65 English inches, then one Roman mile is approximately equal to 1479 meters. Taking $\frac{1}{8}$ of this Roman mile gives the length of 1 stade as approximately 184.8 meters. Again, this length corresponds to one of Lehmann-Haupt's six stades. He refers to this most frequently accepted stade as the "Italian" stade [2, pp.42-44].

By examining the relationship between the stade, the Greek foot, and the Roman mile, four distinct stade lengths are obtained. Using the names provided by the previously introduced scholars Wesenberg and Lehmann-Haupt, each of the four stades is listed in ascending order along with the corresponding Greek foot.

Greek Foot	Modern Equivalent Foot Length	Corresponding Stade	Modern Equivalent Stade Length
Attic	.2941 meters	Olympic	176.4 meters
		Italian	184.8 meters
Doric	.3269 meters	Babylonian-Persian	196.1 meters
Ionic	.3487 meters	Phoenician-Egyptian	209.2 meters

Using these four stades, modern approximations of Eratosthenes' 250,000 stades can be obtained. Below, the modern equivalent of 250,000 stades is given for each type of stade. Also given is the percent difference from the modern accepted value for the equatorial circumference of the Earth, which is approximately 40,075 kilometers [21].

Type of Stade	Modern Equivalent Stade Length	Stade × 250,000	Percent Difference from Modern Circumference
Olympic	176.4 meters	44,100 kilometers	+10.0%
Italian	184.8 meters	46,200 kilometers	+15.3%
Babylonian-Persian	196.1 meters	49,020 kilometers	+22.3%
Phoenician-Egyptian	209.2 meters	52,300 kilometers	+30.5%

Most of what is known about Eratosthenes' geometric argument comes from the writings of Cleomedes, in the first century BCE. Eratosthenes' argument, as described by Cleomedes, gives the circumference of the Earth as approximately 250,000 stades. However, most other ancient authors give 252,000 stades as Eratosthenes' value for the circumference of the Earth. Strabo's *Geography*, written in the late first century BCE, cites Hipparchus as the source of this figure. Another reference to this length appears in a letter written by Heron of Alexandria (ca. 75 CE) in the second century CE [4, pp.60-63].

> *The perimeter* [circumference] *of the Earth is 252,000 stades, as Eratosthenes, who investigated this question more accurately than others, has shown in the book he wrote "On the Measurement of the Earth"* [4, p.63].

In light of the many textual references stating 252,000 stades as the circumference given by Eratosthenes, many scholars believe that the addition of 2000 stades was a correction made by Eratosthenes shortly after his original calculation.

What could be the reason for Eratosthenes' correction? There are many theories as to why this correction may have been made. Let use examine three prevailing theories.

- Adding 40 stades to the original 5000 stades between Alexandria and Syene produces a final result of 252,000 stades, but it is unlikely that this was the correction made by Eratosthenes. As was mentioned earlier, the stretch of land between Alexandria and Syene was measured every year, decade after decade. It is doubtful that one year the measurement would be increased by a full 40 stades (over 7 kilometers).
- Similarly, changing the angle formed by the shadow in Alexandria from the original $7\frac{1}{5}^{\circ}$ to $7\frac{1}{7}^{\circ}$, a decrease of $\frac{2}{35}^{\circ}$, gives a final result of 252,000 stades. This is also unlikely. The best piece of astronomical equipment available at the time was the scaphe, which is basically a sundial [5, p.153]. It is doubtful that even the most precise scaphe was precise enough distinguish between $7\frac{1}{5}^{\circ}$ and $7\frac{1}{7}^{\circ}$ [6, p.412].
- It may be that the correction was not due to an improved measurement, but instead to simplify future calculations involving the result. Some scholars believe that

Eratosthenes added 2000 stades simply to make the final figure divisible by 60. Recalling that Eratosthenes divided the circle into 60 parts called hexacontades, dividing 250,000 stades by 60 results in approximately 4166.7 stades per hexacontade, whereas dividing 252,000 by 60 results in a round 4200 stades per hexacontade [5, p.154]. This reason seems far more likely. Today, altering measurements in order to obtain a simple result is considered highly unscientific, but in the ancient world practicality often took priority over accuracy [2, p.45].

Having established that Eratosthenes probably gave 252,000 stades as his best approximation of the Earth's circumference, and given four stade lengths which might represent reasonable approximations of the stade used by Eratosthenes, it is now possible to obtain some modern equivalents to Eratosthenes' approximation. Below is a table listing four approximations of the Earth's circumference by Eratosthenes' method, using each of the four previously mentioned types of stade.

Type of Stade	Equivalent Modern Length	Stade × 252,000	Percent Difference from Modern Circumference
Olympic	176.4 meters	44,450 kilometers	+10.9%
Italian	184.8 meters	46,560 kilometers	+16.2% (Rawlins' 17%)
Babylonian-Persian	196.1 meters	49,410 kilometers	+23.3%
Phoenician-Egyptian	209.2 meters	52,710 kilometers	+31.5%

How do these results reflect upon the accuracy of Eratosthenes' measurement of the Earth's circumference? By today's standards, these error percentages may seem high. However, for the ancient Greeks, the approximation is remarkably close.

While some modern scholars cling to theories which seem to indicate that Eratosthenes' approximation was highly accurate, others admire his approximation solely on the soundness of his reasoning and elegance of his argument. Mathematician Irene Fischer, having worked on modern measurements of the Earth, writes with great admiration of Eratosthenes' method.

> […] *the great thing for us about Eratosthenes' achievement was the method, the introduction of painstaking measurements instead of speculations, and not a specific number for the size of the Earth. It would not be fair to compare the ancient measuring precision, as advanced and sufficient as it may have been for that time, with modern precision in triangulation, astronomy, and satellite geodesy* [5, p.159].

While it is true that ancient scientists lacked the sophisticated scientific equipment necessary to make precise measurements, it is also necessary to realize that they did not place the same emphasis on precision that we do today. Therefore, assuming that a figure given by an ancient scientist is the most accurate measurement available at that time is not a safe assumption. Nor is it safe to assume that the ancient scientist holds in mind the same rigorously scientific ideals that scientists do today. Scholar of ancient astronomy D.R. Dicks comments on the futility of trying to determine the accuracy of ancient scientific works.

> *The Greek mentality cannot be judged correctly from the standpoint of the modern scientist, and any attempt to force a spurious accuracy on to ancient measurements and translate them into mathematically exact modern equivalents is bound to have misleading results* [2, pp.43-45].

Thus the specifics of Eratosthenes' measurement may elude us simply because Eratosthenes did not have specifics in mind when he conducted this calculation.

Eratosthenes' approximation of the Earth's circumference is a beautiful mathematical argument, regardless of the accuracy of its result. The modern length equivalent to the stade used by Eratosthenes may never be known, just as the reason for the addition of 2000 stades may never be discovered. Nonetheless, Eratosthenes helped to lay the foundation for science based on mathematics and empirical observation rather than on mere philosophical speculation. Most importantly, he demonstrated the awesome power of mathematics as a tool to model our world.

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