TPPmark2014

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Let $\mathbf{N} = \{0, 1, 2, 3, \cdots\}$ be the set of all natural numbers, $p \in \mathbf{N}$ and $q \in \mathbf{N}$. We denote $(p \,\mathbf{mod}\, q) = r$ if and only if there exist $k \in \mathbf{N}$ and $r \in \mathbf{N}$ such that $p = kq + r$ and $0 \leqq r < q$. Further, we denote $(q \,|\, p)$ if and only if $(p \,\mathbf{mod}\, q) = 0$. Prove the following questions:

(i) For any $a \in \mathbf{N}$, $(a^2 \,\mathbf{mod}\, 3) = 0$ or $(a^2 \,\mathbf{mod}\, 3) = 1$.

(ii) Let $a \in \mathbf{N}$, $b \in \mathbf{N}$ and $c \in \mathbf{N}$. If $a^2 + b^2 = 3c^2$ then $(3 \,|\, a)$, $(3 \,|\, b)$ and $(3 \,|\, c)$.

(iii) Let $a \in \mathbf{N}$, $b \in \mathbf{N}$ and $c \in \mathbf{N}$. If $a^2 + b^2 = 3c^2$ then $a = b = c = 0$.