for comparison and calibration. Note the similarity of the valid and invalid events and their dissimilarity to the Dalitz events. The good agreement between the Dalitz events and the ideal χ^2 function confirms the accuracy of the expected values and standard deviations used in the analysis. Making a cut at $\chi^2 = 6$, we have $N = 12 - \frac{37}{3}$ or -0.33 ± 4.0 net events. From the Dalitz data we determine the efficiency of this cut to be $\epsilon_\chi = 0.54 \pm 0.05$ (compared to 0.577 from the χ^2 function). The final result for the branching ratio is insensitive to the location of the χ^2 cut. The observed number of 3γ and 2γ events are related to the relative π^0 monitor N_π by, respectively,

$$N = bN_\pi \epsilon_G \epsilon_c \epsilon_\chi, \quad N' = b' N_\pi' \epsilon_G' \epsilon_c',$$

where ϵ_c is the efficiency of various cuts. The branching ratio is then

$$b = (N/N_\pi)(N_\pi'/N')(\epsilon_c'/\epsilon_c)(\epsilon_G'/\epsilon_G) b'/\epsilon_\chi.$$

We have $N_\pi = (2.27 \pm 0.02) \times 10^9$ and from the 2γ data we determine $\epsilon_c'/\epsilon_c = 1.25 \pm 0.06$ and $N_\pi'/N' = 1.95 \pm 0.04$, so that $b = (-0.2 \pm 2.5) \times 10^{-7}$. Rounding b to its minimum value of zero, the 90% confidence limit[8] for $\pi^0 \to 3\gamma$ branching ratio is $b < 3.8 \times 10^{-7}$. The 14% systematic error has been included.

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Phase Transitions and Magnetic Monopole Production in the Very Early Universe

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In grand unified models, the abundance of superheavy magnetic monopoles in the universe can be suppressed if (1) the phase transition which creates the monopoles occurs after much supercooling; and (2) immediately after the phase transition, the effective monopole mass is large compared with the temperature. These requirements impose constraints on the history of the early universe. The Georgi-Glashow SU(5) group probably breaks to SU(4) ⊗ U(1) before it reaches SU(3) ⊗ SU(2) ⊗ U(1).

There has recently been much interest in grand unified theories (GUT's) of the electromagnetic, weak, and strong interactions.[1,2] These models, combined with classical gravity, attempt to describe all physics which occurs at energy scales well below the Planck mass,[3] $M_P = G^{-1/2} = 1.2 \times 10^{19}$

GeV, at which point gravitational interactions will become strong. They, in principle, allow one to extrapolate the history of the universe back to a temperature of say $T = 10^{17}$ GeV ($t \sim 10^{41}$ sec). With reasonable success, such extrapolations have been used to obtain crude theoretical estimates of the net baryon-number density of the universe.[4]

These models contain stable magnetic monopoles[5] with mass M_m which is typically (although not necessarily) of order 10^{16} GeV. Recently, Zeldovich and Khlopov[6] and also Preskill[7] have attempted to estimate the abundance of magnetic monopoles which exist today as a result of production in the very early universe. Both studies assumed that the symmetry breaking takes place through a single second-order (or weakly first-order) phase transition, and both concluded that the number of monopoles would be unacceptably large. Their argument would also be applicable if the symmetry breaking persisted at all temperatures. The problem is evaded only if the GUT contains a mechanism to suppress the initial production of these monopoles. In this paper we will discuss some of the issues involved in this suppression, and the constraints imposed on the history of the early universe.

In a general GUT, a simple gauge group G undergoes a hierarchy of spontaneous symmetry breaking into successive subgroups: $G \to H_n \to \cdots \to H_0$, where $H_1 = \mathrm{SU}(3) \otimes \mathrm{SU}(2) \otimes \mathrm{U}(1)$ (quantum chromodynamics $\otimes$ Weinberg-Salam theory) and $H_0 = \mathrm{SU}(3) \otimes \mathrm{U}_1^{\mathrm{EM}}$. By general topological arguments, such theories necessarily contain classically stable magnetic monopole configurations of the 't Hooft–Polyakov type. The monopoles are actually associated with the symmetry breaking $H_{k+1} \to H_k$, where H_k is the first group in the sequence with a U(1) factor. The mass M_m is then on the order of M_X/α, where M_X is a typical mass of the gauge mesons associated with this level of symmetry breaking.

The high-temperature behavior of spontaneously broken gauge theories has been studied in the literature.[8] These models typically have symmetric phases at high temperature, and undergo one or more phase transitions before reaching a low-temperature phase with symmetry H_0. As we will see later in the example of SU(5), the intermediate phases may have symmetries which are not part of the gauge hierarchy. In any case, there will be one phase transition at which the magnetic monopoles come into existence, and we will call this critical temperature T_c. If the theory is to successfully suppress monopole production, then we believe that this transition must be first order. It will then proceed through nucleation of bubbles of the new phase. Bubbles exceeding a critical size will start to expand.

We shall consider two mechanisms by which monopoles might be produced during the course of this phase transition:

(i) *Bubble coalescence.*—The orientation of the Higgs field inside one bubble will have no correlation with that of another bubble not in contact.[9] When the bubbles coalesce to fill the space, it will be impossible for the uncorrelated Higgs fields to align uniformly. One expects to find topological knots, and these knots are the monopoles.[10]

Naively, we expect the number of monopoles so produced to be comparable to the number of bubbles, to within a few orders of magnitude.

(ii) *Bubble expansion.*—As a bubble expands, we expect that the interior will contain a density n_m of monopoles which is at least as high as thermal equilibrium. (Note that fewer monopoles would correspond to a *higher* degree of order in the Higgs fields, which seems unlikely.) Thus,

$$n_m \sim [M_m(T)T]^{3/2} \exp[-M_m(T)/T], \tag{1}$$

where $M_m(T)$ is the effective monopole mass computed using the Higgs expectation value at temperature T.

We shall ignore other mechanisms, such as (iii) monopole production from energy released in bubble-wall collisions, and (iv) conversion of monopoles from the previous phase (see below).

Mechanism (i) depends critically on λ, the probability per unit volume per unit time that a critical size bubble will nucleate. One can define $\lambda(T) \equiv T^4 f(T)$, where $f(T)$ is then dimensionless. We will now show that monopole suppression requires a very small value for $f(T)$.

The early universe can be described by a Robertson-Walker metric[11] with zero curvature: $d\tau^2 = dt^2 - R^2(t) d\vec{x}^2$. If bubbles expand at speed v, then the fraction of space which remains in the old phase at time t is given by

$$p(t) = \exp\left[-\frac{4\pi}{3}\int_0^t dt_1 R^3(t_1)\lambda(t_1)\left(\int_{t_1}^t dt_2 \frac{v}{R(t_2)}\right)^3\right] . \tag{2}$$

The number of bubbles per volume which have nucleated by time t is then given by

$$n_b(t) = R^{-3}(t)\int_0^t dt_1 R^3(t_1)\lambda(t_1)p(t_1)\,. \qquad (3)$$

To find qualitative answers, one can assume $f(T) = \bar{f}\theta(T_c - T)$, where $\bar{f}$ is a constant. We also assume for simplicity that $v = 1$ and that the universe evolves through the entire time period as if it were dominated by radiation with a fixed number of degrees of freedom. Then RT = const, and $T = (M_p/2\gamma t)^{1/2}$, where $\gamma^2 = (4\pi^3/45)[N_b + (7/8)N_f]$; N_b and N_f are the numbers of effectively massless bosonic and fermionic spin degrees of freedom, respectively. One then finds that for large times $n_b \to cT^3\bar{f}^{3/4}$, where $c = (3/\pi)^{1/4}\Gamma(5/4) \approx 0.90$. The ratio of bubbles to photons is then given by $3.68\bar{f}^{3/4}$. For $M_m \sim 10^{16}$ GeV, one requires[12] $n_m/n_\gamma < 10^{-24}$. If $n_m \sim n_b$, then we find $\bar{f} < 10^{-32}$. We also find that the temperature T^* at which half of the volume has entered the new phase is given by

$$\frac{1}{T^*} = \frac{1}{T_c} + \frac{0.90\gamma}{M_p \bar{f}^{1/4}}\,. \qquad (4)$$

Taking $\bar{f} \sim 10^{-32}$ and $\gamma = 20$, one finds that $T^* < 10^{10}$ GeV for all values of T_c.

If $T^* \ll T_c$ (such supercooling seems quite likely), then the evolution of the universe has an interesting "heat spike." A typical region will cool to about T^*, at which point the phase transition will take place and the latent heat will be released. The temperature will then rise to some $T_r \lesssim T_c$. It is this T_r which should be used on the right-hand side of Eq. (1). Furthermore, the number of photons will be increased by a factor of $(T_r/T^*)^3$, further suppressing the monopole/photon ratio. If one takes $T_r = 10^{13}$ GeV, one finds that the earlier bounds are replaced by $\bar{f} < 10^{-26}$ and $T^* < 2\times 10^{11}$ GeV.

The calculation of $\lambda(T)$ remains an important topic for future investigation. This is the finite-temperature generalization of the work of Coleman and Callan[13] on the fate of the false vacuum.

To illustrate the ideas discussed above, we will now examine in detail the simplest GUT: The SU(5) model of Georgi and Glashow.[1] For our purposes, the fields of interest are the gauge fields and the adjoint representation Higgs field, which is denoted by a Hermitian traceless matrix[14] $\Phi = \sum \varphi_i \lambda^i/\sqrt{2}$. The Langrangian contains a Higgs potential[15]

$$V_0[\Phi] = -\tfrac{1}{2}\mu^2\,\mathrm{Tr}\Phi^2 + \tfrac{1}{4}a(\mathrm{Tr}\Phi^2)^2 + \tfrac{1}{2}b\,\mathrm{Tr}\Phi^4 + \tfrac{1}{3}c\,\mathrm{Tr}\Phi^3. \qquad (5)$$

The low-temperature phase is determined by the minimum of this potential. There are three possible forms for this minimum[16]:

(I) $\Phi_0 = v\,\mathrm{diag}(1,1,1,-3/2,-3/2)$, where $v = \{c + [c^2 + 8(15a+7b)\mu^2]^{1/2}\}/2(15a+7b)$. The unbroken group is $SU(3)\otimes SU(2)\otimes U(1)$.

(II) $\Phi_0 \sim \mathrm{diag}(1,1,1,1,-4)$. The unbroken group is $SU_4\otimes U_1$.

(III) $\Phi_0 = 0$.

The phase structure can be described in terms of the two dimensionless variables $\eta \equiv a/b$ and $\xi \equiv -\mu^2 b/c^2$. Positivity of the quartic terms requires that $\eta > -7/15$, and one must take $b > 0$ to allow for the existence of phase I. The phase diagram is shown in Fig. 1.

The renormalized parameters are chosen to give phase I at zero temperature. Twelve of the 24 gauge particles will acquire masses given by $M_X^2 = (25\pi/2)\alpha v^2$. The lightest monopole[17] has magnetic charge $2\pi/e$ (Dirac quantization with respect to the electron), and in the Bogomol'nyi limit[18] its mass is given by $M_m = M_X/\alpha$. One expects $M_X \approx 10^{14}$ GeV; hence $M_m \approx 10^{16}$ GeV.

At $T \gg M_X$, one can evaluate the finite-temperature effective potential $V_{\mathrm{eff}}[\Phi, T]$ with the methods of Ref. (8). It is given approximately by the same form as Eq. (5), except that $-\mu^2$ is replaced by $-\mu_{\mathrm{eff}}^2 = -\mu^2 + \sigma T^2$, where $\sigma = (130a + 94b + 75g^2)/60$. From Fig. 1, one can see that for $T > T_c' \approx \mu/\sigma^{1/2} \sim 10^{14}$ GeV, the system will be in Phase III. One also notes that if $\eta > -\frac{2}{5}$, the system goes through the intermediate phase II. The II–I phase transition will occur at T_c, which can also be calculated (but not very reliably) from Fig. 1. In contrast to T_c', T_c can be made as low as one wants by choosing the parameters

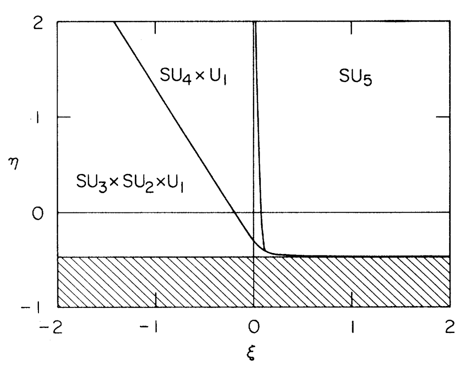

FIG. 1. Phase diagram for the SU(5) adjoint Higgs system. The crosshatched region is not allowed. The triple point occurs at $\eta = -\frac{2}{5}$, $\xi = \frac{1}{9}$. The $SU(3)\otimes SU(2)\otimes U(1)/SU(4)\otimes U(1)$ borderline approaches the asymptotic straight line $\xi = -0.610\eta - 0.206 + O(1/\eta)$.

very near the I-II borderline in Fig. 1. However, the natural scale is $T_c \lesssim M_X$.

Since $T_c \lesssim M_X$, the approximations used in the above analysis are somewhat dubious. It is therefore reassuring to note that the existence of phase II can also be inferred from a low-temperature approximation. One notes that if

$$-(10\eta+7) < 8\xi < 9/(10\eta+13), \tag{6}$$

then phase II is metastable (positive values of mass^2) at temperature $T=0$. For 10^2 GeV $<< T << M_X$, one can calulate the free-energy density $\mathscr{A}$ (which is just the negative of the pressure) of each phase with use of the massless ideal-quantum-gas approximation. Thus,

$$\mathscr{A}(T) = V_{\min} - (\pi^2/90)T^4[N_b + (7/8)N_f], \tag{7}$$

where N_b and N_f are the number of effectively massless physical spin degrees of freedom, bosonic and fermionic, respectively. One has $N_b^{\mathrm{II}} = N_b^{\mathrm{I}} + 8$, $N_f^{\mathrm{II}} = N_f^{\mathrm{I}}$, and so the critical temperature for the I-II transition is given by

$$T_c = [(90/8\pi^2)(V_{\mathrm{II}} - V_{\mathrm{I}})]^{1/4}. \tag{8}$$

The II-I phase transition will be first order, since $\Phi_0(\mathrm{II})$ cannot be continuously deformed to $\Phi_0(\mathrm{I})$ without passing through some other phase.

Note that monopoles exist in phase II, but they are topologically unrelated to those of phase I [the two U(1) factors are different]. There may be some probability of conversion when a bubble wall crosses a phase-II monopole [mechanism (iv) above], but we will assume that it is negligible.

Thus, our expectations for the very early universe in the SU(5) model can be summarized as follows: The phase remains symmetric down to $\sim 10^{14}$ GeV, at which point the symmetry breaks to SU(4)$\otimes$U(1). The transition point T_c to the SU(3)$\otimes$SU(2)$\otimes$U(1) phase would naturally lie in the 10^{13}–10^{14}-GeV range, but it could be arbitrarily low. A sufficient barrier against nucleation is needed in the model to suppress monopole production. In this case, the universe will supercool at least to $\sim 10^{11}$ GeV before the phase transition actually occurs. The latent heat will then warm the universe back up to near T_c. (In the ideal-gas approximation, it is warmed to $0.40T_c$.) These estimates suggest that mechanism (ii) of monopole production will be strongly suppressed.

It is clear that the (non-) observational bound on the monopole density imposes constraints on GUT's and on the early history of the universe. Our scenario requires a modification of the present understanding of baryon generation.[4] Also, the expansion and collision of bubbles after supercooling generate inhomogeneities which are perhaps related to galaxy formation. Details of the effects of phase transitions in the early universe will be discussed elsewhere.

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Measurement of the Cross Section for $\nu_\mu + e^- \to \nu_\mu + e^-$

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A total of ~ 249 000 neutrino interactions were observed at Fermilab in a high-angular-resolution electromagnetic shower detector, with ~ 0.947×10^{19} protons of 350-GeV energy incident upon the production target. Based on a data sample of 0.71×10^{19} protons, 46 electrons were observed with $\theta_e \leq 10$ mrad. Of these 46 events, 34 are attributed to the process $\nu_\mu + e^- \to \nu_\mu + e^-$, and 12 are attributed to background processes. This leads to the following results: $\sigma = (1.40 \pm 0.30)\times 10^{-42} E_\nu\ \mathrm{cm}^2$ and $\sin^2\theta_W = 0.25^{+0.07}_{-0.05}$.

Since the discovery in 1973 of weak neutral-current interactions,[1] efforts have been made to determine the nature of the weak-coupling constants[2] and to compare them with the predictions of the gauge theories of Weinberg and Salam.[3] To date, the most accurate measurements have involved hadronic currents as well as leptonic currents, and have been in overall good agreement with the Weinberg-Salam (WS), and the Glashow-Iliopoulos-Maiani (GIM) theory.[4] Measurements of purely leptonic processes, such as

$$\nu_\mu + e^- \to \nu_\mu + e^- , \tag{1}$$

$$\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^- , \tag{2}$$

$$\nu_e + e^- \to \nu_e + e^- , \tag{3}$$

$$\bar{\nu}_e + e^- \to \bar{\nu}_e + e^- , \tag{4}$$

have been limited by statistics, resulting in less conclusive evidence for agreement with the WS

