HYPOTHETICO-DEDUCTIVISM, CONTENT, AND THE NATURAL AXIOMATIZATION OF THEORIES*

KEN GEMES[†‡]

Department of Philosophy
Yale University

In Gemes (1990) I examined certain formal versions of hypothetico-deductivism (H-D) showing that they have the unacceptable consequence that "Abe is a white raven" confirms "All ravens are black"! In Gemes (1992) I developed a new notion of content that could save H-D from this bizarre consequence. In this paper, I argue that more traditional formulations of H-D also need recourse to this new notion of content. I present a new account of the vexing notion of the natural axiomatization of a theory. The notion is used to construct a form of H-D that allows for the type of selective confirmation without which Glymour (1980a,b) claims H-D is hopeless.

1. Hypothetico-Deductivism and Glymour's Tacking Problem.

Hypothetico-deductivism (H-D) in its simplest form is the claim that

H-D1 e (directly) confirms T iff T is consistent and T entails e.

Note, H-D is typically presented as an account of the confirmation of single hypotheses and/or as an account of the confirmation of theories. For our purposes a theory is any set of statements closed under the relation of deductive consequence. Typically a theory is specified by a finite axiom set, that is, a finite set of statements such that every member of the theory is a consequence of that set. Here and after, we use expressions such as T and $(T\&b)$ ambiguously: in the first case, to denote a single hypothesis T or a theory T; in the second, to denote the conjunction of a single hypothesis T and a statement b or the union set of theory T and statement (or theory) b.

A less ambitious and, perhaps, more plausible form of H-D claims that for (contingent) e to confirm (consistent) T, it is sufficient, but not nec-

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‡Send reprint requests to the author, Department of Philosophy, Yale University, P.O. Box 3650 Yale Station, New Haven, CT 06520, USA.

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essary, that T entails e. The problems affecting H-D dealt with in this paper are equally problems for even this modest form of H-D.

More complex forms of H-D make mention of T together with certain relevant background information and/or initial conditions:

H-D2 e (directly) confirms T relative to background evidence b iff $(T\&b)$ is consistent, $(T\&b)\vdash e$, and $b\nvdash e$.

C. Glymour has objected that hypothetico-deductive theory confirmation is too permissive in that where e hypothetico-deductively confirms T it will equally confirm $(T\&h)$ for any arbitrary h (consistent with T). Glymour calls this the tacking problem; where e hypothetico-deductively confirms T, it will also confirm T with arbitrary h tacked on (provided h is consistent with T). Thus suppose we have the following background evidence Pa. Then consider the following pair of theories, the first containing one and the second two axioms:

T1 **A1**: $(x)(Px \rightarrow Qx)$
T2 **A1**: $(x)(Px \rightarrow Qx)$
A2: $(x)(Fx)$.

According to **H-D2** the observational evidence Qa confirms both **T1** and **T2** relative to the background evidence Pa. Intuitively, and *pace* **H-D2**, what we want to say is that while this observational evidence confirms **T1**'s sole axiom **A1**, it does not confirm **T2** in toto. In particular, it does not confirm **T2**'s axiom $(x)(Fx)$. The problem, as Glymour points out, is that H-D, or, at least, classic expressions of H-D such as **H-D1** and **H-D2**, do not allow for such selective confirmation. They do not seem to allow for the possibility that evidence might be favorably relevant to only some axioms of a theory while being irrelevant to other axioms.

Attempts to construct more refined versions of H-D that allow for such selective confirmation have so far yielded little success. Let us see why.

In the above case it is tempting to say that the evidence Qa does not confirm **T2**'s axiom $(x)(Fx)$ because that axiom was not needed in the derivation of Qa from the background evidence Pa. The implicit suggestion here is that e only confirms those axioms of T needed in the derivation of e from the conjunction of T with the background evidence b. Yet consider the following alternative axiomatization of **T2**:

T2$^+$ **A1$^+$**: $(x)(Fx)$
A2$^+$: $(x)(Fx) \rightarrow (x)(Px \rightarrow Qx)$.

We call **T2$^+$** an alternative axiomatization, or reaxiomatization, of **T2** since, though **T2$^+$** and **T2** contain different axioms, they yield the same logical consequences. Where S and S' are sets of sentences sharing the same class of logical consequences, we say they are different axiomatizations/formulations of the same theory.

In the above case, **T2**$^+$'s axiom $(x)(Fx)$ is needed in the derivation of Qa from Pa. But surely Qa does not confirm **T2**$^+$'s axioms $(x)(Fx)$ relative to the background evidence Pa.

At this point, it is tempting to claim that what is wrong with **T2**$^+$ is that it can obviously be reaxiomatized in such a way that its first axiom $(x)(Fx)$ plays no part in the derivation of Qa from Pa. In particular **T2** is itself such a reaxiomatization. Can we then conclude that where T, or T together with the background information b, entails e, e only hypothetico-deductively confirms those axioms of T such that they, or some logical equivalents, are needed in the derivation of e under every reaxiomatization of T? But this would rule out Qa confirming **T1**'s axiom **A1**, that is, $(x)(Px \rightarrow Qx)$, relative to the background evidence Pa. To see this consider the following reaxiomatization of **T1**:

T1$^+$ **A1**$^+$: $(x)(Px \rightarrow Qx)$
A2$^+$: $Pa \rightarrow Qa$.

Using **T1**$^+$'s axiom **A2**$^+$ we may derive Qa from Pa without recourse to **T1**$^+$'s axiom **A1**$^+$ or to any logical equivalent thereof.

We might object to **T1**$^+$ on the grounds that it contains a redundant axiom, namely $Pa \rightarrow Qa$. But consider yet another formulation of **T1**:

T1* **A1***: $(x)(x \neq a \rightarrow (Px \rightarrow Qx))$
A2*: $(x)(x = a \rightarrow (Px \rightarrow Qx))$.

Using **T1***'s **A2*** we can derive Qa from Pa without recourse to (any logical equivalent of) **T1**'s sole axiom $(x)(Px \rightarrow Qx)$.

While **T1***, unlike **T1**$^+$, contains no redundant axioms, it, like **T1**$^+$, seems a thoroughly *unnatural* axiomatization of its logical equivalent **T1**. By the same token, **T2**$^+$ is a thoroughly unnatural axiomatization of **T2**. Here then we have a suggestive solution to the tacking problem: Where T together with background evidence b entails e, e hypothetico-deductively confirms axiom A of T iff for any natural axiomatization of T, A (or at least some logical equivalent) is needed in the derivation of e from b. The problem is that no one has yet to come up with an adequate analysis of the notion of the natural axiomatization of a theory.

One intuitive condition on a natural axiomatization is that it should contain no redundant axioms. This suffices to rule out **T1**$^+$ as a natural axiomatization of **T1**. However it does not rule out **T1***. Similarly, it does not rule out **T2**$^+$ as a natural axiomatization of **T2**. In other words, as good as the suggestion to rule out redundant axioms is, it does not take us very far.

I believe that at this point what is needed before we give a precise specification of the notion of a natural axiomatization is a radical reconception of what counts as part of the content of a theory. Consider again the two axioms of theory **T2**$^+$:

A1$^+$: $(x)(Fx)$

and

A2$^+$: $(x)(Fx) \rightarrow (x)(Px \rightarrow Qx)$.

This second axiom is equivalent to

A2*: $\sim(x)(Fx) \vee (x)(Px \rightarrow Qx)$.

Now traditionally every contingent consequence of a sentence/theory is counted as part of its content—for example, with respect to the content of sentences (see Carnap 1935, 56 and Popper [1959] 1972, 120). Yet if **A2*** counts as part of the content of **T2**$^+$, then on the evidence of $\sim Fa$ we would have to say that part of the content of **T2**$^+$ has been conclusively confirmed. Indeed, since **T2**$^+$ is equivalent to **T2**, and hence they share the same consequences, on the evidence of $\sim Fa$ we would have to say that part of **T2**—recall **T2** has only the two axioms,

A1: $(x)(Fx)$

and

A2: $(x)(Px \rightarrow Qx)$—

has been conclusively confirmed! This is plainly absurd. This is the kind of partial confirmation any theory can do without. The wff $\sim(x)(Fx) \vee (x)(Px \rightarrow Qx)$ may be a consequence of **T2** but it surely should not count as part if its content. Similarly, $(Fa \vee \sim Fb)$ may be a consequence of $(Fa \& Fb)$ but it should not count as part of its content, for if $(Fa \vee \sim Fb)$ counts as part of the content of $(Fa \& Fb)$ then on the evidence of $\sim Fb$ we would have to say that part of $(Fa \& Fb)$ has been conclusively confirmed! The lesson here is that we should reject the notion that every contingent consequence of a sentence (or theory) is part of its content.

In Gemes (1992) I give other reasons for rejecting the traditional notion of content and advance a new notion in its place. While this is not the appropriate place to rehearse my new account of content we can introduce a simple enough surrogate. Let α be a variable for well-formed formulas (wffs) of the language in question. Let β be a variable for wffs and sets of wffs of the language in question. A set of wffs β is contingent iff there is some contradiction μ such that $\beta \nvdash \mu$ and some ϕ such that $\phi \in \beta$ and $\{/\} \nvdash \phi$. Then, presuming the notion of atomic wff is defined for the language in question, we define content as follows, using "$\alpha < \beta$" to abbreviate "α is a content part of β":

D1 $\alpha < \beta =_{\text{df.}}$ α and β are contingent, $\beta \vdash \alpha$, and there is no σ such that $\beta \vdash \sigma$, σ is stronger than α, and every atomic wff that occurs in σ occurs in α.[1]

We say σ is stronger than α where $\sigma \vdash \alpha$ and $\alpha \nvdash \sigma$. Under this notion of content Fa is part of the content of $(Fa \& Fb)$ but $(Fa \vee {\sim}Fb)$ is not. Formula $(Fa \vee {\sim}Fb)$ is not a part of $(Fa \& Fb)$ because Fa is a consequence of $(Fa \& Fb)$ that is stronger than $(Fa \vee {\sim}Fb)$ and every atomic wff that occurs in Fa occurs in $(Fa \vee {\sim}Fb)$. Similarly, $(x)(Fx)$ is a content part of $(x)(Fx \& Gx)$ but $(x)(Fx \vee Hx)$ is not. Formula $(x)(Fx \vee Hx)$ is not a content part of $(x)(Fx \& Gx)$ because $(x)(Fx)$ is a consequence of $(x)(Fx \& Gx)$ which is stronger than $(x)(Fx \vee Hx)$ and contains only atomic wffs that occur in $(x)(Fx \vee Hx)$.

Applying this new concept of content to the theory **T2** gets the result that ${\sim}(x)(Fx) \vee (x)(Px \rightarrow Qx)$ is not a content part of **T2** since $(x)(Px \rightarrow Qx)$ is a consequence of the axioms of **T2** and is stronger than ${\sim}(x)(Fx) \vee (x)(Px \rightarrow Qx)$ yet contains only atomic wffs occurring in ${\sim}(x)(Fx) \vee (x)(Px \rightarrow Qx)$. Indeed, since our new definition of content has the intuitively appropriate property that logical equivalent hypotheses/theories have the same content parts, it follows that **T2**$^{+}$'s axiom **A2**$^{+}$, $(x)(Fx) \rightarrow (x)(Px \rightarrow Qx)$ is not part of the content of **T2**$^{+}$. Now how can an axiom of a theory not be part of that selfsame theory? Let us recall **T2**$^{+}$. It has two axioms

A1^{+}: $(x)(Fx)$

and

A2^{+}: $(x)(Fx) \rightarrow (x)(Px \rightarrow Qx)$.

The problem we started with before we began this digression about content was that of specifying in what way **T2**$^{+}$ is not a natural axiomatization of its equivalent **T2**. Recall that **T2** has the two axioms

A1: $(x)(Fx)$

and

[1]Note, for ease of comprehension, **D1** does not have the desirable property that the content part relationship is closed under logical equivalence. A more adequate, but more difficult to comprehend, version having this property runs as follows:

D1′ $\alpha < \beta =_{\text{df.}}$ (i) α and β are contingent; (ii) α is a consequence of β; and (iii) for some μ, μ is logically equivalent to α and there is no σ such that σ is stronger than μ, σ is a consequence of β and every atomic wff that occurs in σ occurs in μ.

For a full specification of this new notion of content for a number of languages, both propositional and quantificational, see Gemes (1992).

A2: $(x)(Px \rightarrow Qx)$.

I suggest that here we have the answer in hand: It is precisely because **T2**$^+$ contains axioms that are not part of the content of **T2** (or **T2**$^+$) that **T2**$^+$ is not a natural axiomatization of **T2** (or **T2**$^+$). Inversely, it is because all of **T2**'s axioms are content parts of **T2** and **T2**$^+$ that it represents a natural axiomatization of the theory captured by those two different axiom sets.

Similarly, we can now see why **T1*** does not count as a natural axiomatization of **T1**, or, for that matter, of **T1*** itself. The reason is that **T1***'s axiom $(x)(x \neq a \rightarrow (Px \rightarrow Qx))$ is not a content part of **T1** or **T1***. The wff $(x)(x \neq a \rightarrow (Px \rightarrow Qx))$ is not a content part of **T1** or **T1*** because $(x)(Px \rightarrow Qx)$ is a consequence of both **T1** and **T1*** that is stronger than $(x)(x \neq a \rightarrow (Px \rightarrow Qx))$, yet contains only atomic wffs occurring in $(x)(x \neq a \rightarrow (Px \rightarrow Qx))$.

Combining our demand that natural axiomatizations only contain as axioms content parts of the relevant theory with our previous demand that natural axiomatizations include no redundant axioms, we arrive at the following first attempt at an analysis of the notion of a natural axiomatization: Where T and T' are variables over sets of wffs

D2* T' is a natural axiomatization of T iff T' is a finite set of wffs such that T' is logically equivalent to T and every member of T' is a content part of T' and T' contains no redundant axioms.

In fact **D2*** does not hit the mark. The problem here is that **D2***'s condition that T' contains no redundant axioms is too weak. While it serves to rule out, say, $Pa \rightarrow Qa$ as an axiom in any natural axiomatization that entails $(x)(Px \rightarrow Qx)$, it does not prevent $Pa \rightarrow Qa$ from occurring as a part of some axiom of such a theory. Thus according to **D2***,

T2$^!$ **A1**$^!$: $(x)(Px \rightarrow Qx)$
A2$^!$: $(Pa \rightarrow Qa)$ & $(x)(Fx)$

is a natural axiomatization of

T2 **A1**: $(x)(Px \rightarrow Qx)$
A2: $(x)(Fx)$.

The problem here is that while **T2**$^!$ contains no redundant axiom, its axiom **A2**$^!$ contains a subformula, namely $Pa \rightarrow Qa$, which, in the context of **T2**$^!$, is redundant. This problem can be solved by demanding that no axiom of a natural axiomatization duplicate content contained in the other axioms.[2]

[2]Conversations with Ruth Barcan Marcus lead me to conclude that one might eliminate this problem without reference to the notion of content.

Here then is an improved account of the notion of a natural axiomatization, again letting T and T' be variables over sets of wffs:

D2 T' is a natural axiomatization of T iff (i) T' is a finite set of wffs such that T' is logically equivalent to T, (ii) every member of T' is a content part of T', and (iii) no content part of any member of T' is entailed by the set of the remaining members of T'.[3]

Now, at last, we are in a position to give a version of H-D that allows for the kind of selective confirmation advocated by Glymour. Here is our first attempt:

H-D3 e hypothetico-deductively confirms axiom A of theory T relative to background evidence b iff $(T\&b)\vdash e$ and there is no natural axiomatization $n(T)$ of T such that for some subset s of the axioms of $n(T)$, $(s\&b)\vdash e$ and A is not a content part of $(s\&b)$.

H-D3 yields the desired result that evidence statement Qa confirms **T2**'s axiom $(x)(Px\rightarrow Qx)$ relative to background evidence Pa. Now let us recall **T2**'s reformulated version:

T2$^+$ **A1$^+$**: $(x)(Fx)$
A2$^+$: $(x)(Fx) \rightarrow (x)(Px\rightarrow Qx)$.

Here **H-D3** yields the desired result that relative to background evidence Pa, Qa does not confirm **T2$^+$**'s axiom $(x)(Fx)$. This follows from **H-D3** since **T2** is a natural axiomatization of **T2$^+$** such that Qa can be derived from Pa and **T2**'s axiom $(x)(Px \rightarrow Qx)$ and that axiom does not contain $(x)(Fx)$ as a content part.

H-D3, unlike **H-D1** and **H-D2**, avoids Glymour's tacking problem because it, unlike **H-D1** and **H-D2**, does not allow that simple entailment of e by T—or $(T\&b)$—is sufficient for confirmation of T by e. It allows that only those content parts of T that play a role in the derivation of e

[3]Note, on this analysis where T' is a natural axiomatization of T there will typically be a natural axiomatization of T that contains just one axiom, namely, the conjunction of all the members of T'. This result has no bearing on our task of constructing a notion of natural axiomatization that may be used to give a formal account of hypothetico-deductive confirmation. However, if desirable, one could avoid this result by adding the following condition: (*iv*) *there is no T'' such that T'' meets conditions* (*i*)–(*iii*) *and T'' has more members than T'*. This modification would oblige us to make some minor modifications in the formal definitions of H-D given below. Such a condition, in maximizing the number of separate axioms in any natural axiomatization, might prove useful to Friedman's (1974) project of clarifying the notion of theoretical reduction. The basic idea would be that natural axiomatization T reduces natural axiomatization T' where every content part of T' is a content part of T and T has less axioms then T'.

Note, also, that **D2** does not prevent what might be called orthographic redundancy from occurring within an axiom. For instance, **D2** allows that **T2**'s axiom $(x)(Fx)$ could be replaced by $(x)(Fx\&Fa)$.

can be confirmed by e. In so doing it provides for the type of selective confirmation without which H-D would, in the words of Glymour, be hopeless.

We will now see that while **H-D3** avoids Glymour's tacking problem and provides for selective confirmation, it shares another tacking problem with **H-D1** and **H-D2**.

2. Tacking the Other Way. While any logical entailment is susceptible to tacking on the left through the use of conjunction, it is equally susceptible to tacking on the right through the use of addition. That is, where $T \vdash e$, it follows that $(T\&h) \vdash e$, and equally, where $T \vdash e$, it follows that $T \vdash (T \vee h)$. This suggests that if traditional forms of H-D, such as **H-D1** and **H-D2**, suffer from the problem caused by tacking (by conjunction) an arbitrary h onto a theory/hypothesis T, they will also suffer a problem when arbitrary h is tacked (by addition) onto the entailed statement e. Well, let us see.

The basic idea behind H-D is that one confirms a theory by finding that some consequence of the theory is true. Typically, one has in mind a consequence whose truth can be ascertained by observational means. Now suppose T is some arbitrary theory/hypothesis and h is some observational claim that is in an intuitive sense independent of T. For instance, suppose T is a theory of celestial mechanics including Kepler's laws, e is some observational consequence of T concerning the orbit of some planet, and h is the statement that Sydney has a harbor bridge. We presume that T itself has no implications for the question of whether Sydney in fact has a harbor bridge. From observation I know that h is true. Now from h I infer that $(e \vee h)$ is true. But $(e \vee h)$ is a consequence of T, so, if H-D as represented by **H-D1** and **H-D2** is to be believed, I now have confirming evidence for T. Surely something has gone seriously wrong! While my observing that Sydney has a harbor bridge, and hence ascertaining through observation that T's consequence $(e \vee h)$ is true, does not violate the law of **H-D1** or **H-D2**, it totally violates the spirit of hypothetico-deductive confirmation. The tacking of arbitrary h onto e to form the disjunction $(e \vee h)$ is as much a problem as the tacking of arbitrary h onto T to form the conjunction $(T\&h)$. Expressions of H-D such as **H-D1** and **H-D2** are not only too permissive in their accounts of what gets confirmed by arbitrary evidence e, they are also too permissive in their account of what confirms arbitrary theory/hypothesis T.

Now this new tacking problem poses difficulties for **H-D3** as well as **H-D1** and **H-D2**. This should hardly surprise us since **H-D3** was designed principally to solve Glymour's old tacking problem. Let us consider a particular case.

Let $(x)(Rx \rightarrow Bx)$ stand for the famous raven hypothesis and Hs for the claim that Sydney has a harbor bridge. Now consider the following one-axiom theory:

T3 A1: $(x)(Rx \rightarrow Bx)$.

Now suppose we have just observed the Sydney Harbor Bridge and solely on this observational basis we conclude that statement E, namely, $Hs \vee (x)(Rx \rightarrow Bx)$, is true. Now relative to any tautological evidence t, E confirms **T3**'s axiom **A1** according to **H-D3**. So on the basis of our observation of the Sydney Harbor Bridge, we have confirmed that all ravens are black!

What has gone wrong? Clearly the original proponents of H-D intended to confirm statements like **T3**'s **A1** by plugging in some background information—say, the claim that a is a raven, that is Ra—drawing out the consequence—in this case that a is black, that is Ba—and then observing that consequence to be true. However, the drawing out of such consequences of **T3**'s **A1** as $Hs \vee (x)(Rx \rightarrow Bx)$ was clearly not the kind of consequence they had in mind. Now note, while $Hs \vee (x)(Rx \rightarrow Bx)$ is a consequence of the conjunction of $(x)(Rx \rightarrow Bx)$ and the background evidence Ra, it is not a content part of that conjunction. On the other hand, Ba is a content part of that conjunction. Here then we have the makings of the solution to our new tacking problem: H-D should not be cast in terms of confirming a hypothesis/theory through confirming its consequences, but rather it should be cast in terms of confirming a theory through confirming its content parts. In particular, H-D should demand that the confirming evidence e be a content part of the theory (or the conjunction of the theory and background evidence) in question.[4] Here then is a first

[4]I first briefly mentioned this tacking problem and its solution at the end of Gemes (1990). Grimes (1990) notes the same problem and proposes a different, though not totally unrelated, solution. To this end, Grimes introduced a notion of narrow consequence, which, he claims, "seems better suited for expressing a relation of preserving content" (ibid., 520) than does the standard notion of consequence. However, Grimes's notion of narrow consequence is substantially different from the notion of content proposed above. His basic idea is that α is a narrow consequence of β iff β and $\sim\alpha$ are each consistent and for some Boolean disjunctive normal form $(d_1 \vee \ldots \vee d_n)$, α is logically equivalent to $(d_1 \vee \ldots \vee d_n)$ and for some d_i, $1 \leq i \leq n$, $\beta \vdash d_i$. This notion of narrow consequence has several drawbacks. First, it has the consequence that for any disjunction $(\alpha \vee \beta)$, where there is no nonempty set S such that each member σ of S is an atomic wff or negated atomic wff such that $\alpha \vdash \sigma$ and $\beta \vdash \sigma$, $(\alpha \vee \beta)$ has no narrow consequences. Thus $(p \vee q)$ is not a narrow consequence of itself. Furthermore, while p is a narrow consequence of $(p \& q)$ and r is a narrow consequence of $(r \& s)$, $(p \vee r)$ is not a narrow consequence of $(p \& q) \vee (r \& s)$. Second, it has the consequence that where $(\beta \& \alpha)$ and $\sim(\beta \vee \sim\alpha)$ are each consistent, $(\beta \vee \sim\alpha)$ is a narrow consequence of $(\alpha \& \beta)$. So, where H-D is defined in terms of Grimes's notion of narrow consequence, while we get the desirable result that $(Fa \vee Gb)$ does not hypothetico-deductively confirm $(x)(Fx)$ while Fa does, we also get the undesirable results that $(Fa \vee \sim Fb)$ hypothetico-deductively confirms $(x)(Fx)$ and $(Fa \vee Ga)$ does not hypothetico-deductively confirm $(x)(Fx \vee Gx)$.

attempt at a new version of H-D, which, since it allows for selective confirmation, we call selective hypothetico-deductivism, or **S-H-D***:

S-H-D* *e* hypothetico-deductively confirms axiom *A* of theory *T* relative to background evidence *b* iff *e* is a content part of $(T\&b)$, and there is no natural axiomatization $n(T)$ of *T* such that for some subset *s* of the axioms of $n(T)$, *e* is a content part of $(s\&b)$ and *A* is not a content part of $(s\&b)$.[5]

S-H-D* provides an account of H-D that, unlike **H-D1** and **H-D2**, is not open to Glymour's tacking-by-conjunction or our tacking-by-disjunction problems. However, it does not answer Glymour's other major criticism of H-D.

Glymour (1980a) points out that versions of H-D such as **H-D2** have the unfortunate consequence that for any *T* and true *e*, so long as $\sim T \nvdash e$, there is some true *b* (namely, $T \rightarrow e$) such that *e* confirms *T* relative to the background evidence *b*. Consider a concrete example. Theory **T3** contains one axiom being the famous raven hypothesis $(x)(Rx \rightarrow Bx)$ and *e* is the true evidence statement *Hs* which claims that Sydney has a harbor bridge. Then, according to **H-D2**, *Hs* confirms **T3**'s sole axiom relative to the true background theory $(x)(Rx \rightarrow Bx) \rightarrow Hs$. This is also a consequence of **S-H-D***. This type of consequence may be easily avoided by stipulating that where *e* confirms axiom *A* of theory *T* relative to background theory *b*, *b* itself must be a content part of the conjunction of *T* and *b*. In our case this condition is not met since *Hs* is a consequence of $(x)(Rx \rightarrow Bx)$ & *Hs* that is stronger than $(x)(Rx \rightarrow Bx) \rightarrow Hs$ and contains only atomic formulas that occur in $(x)(Rx \rightarrow Bx) \rightarrow Hs$. Furthermore, this condition is well within the spirit of H-D. The intuitive notion of H-D is to test some theoretical statement, typically a law statement, by combining it with initial conditions that are, in some sense, independent from the law in question. Demanding that the background evidence *b*, our stand-in for initial conditions, be a content part of the conjunction of *T* and *b* goes some way to ensuring this type of independence.

Here then is our final version of selective hypothetico-deductive confirmation:

S-H-D *e* hypothetico-deductively confirms axiom *A* of theory *T* relative to background evidence *b* iff *e* and *b* are content parts of $(T\&b)$, and there is no natural axiomatization $n(T)$ of *T* such that for some subset *s* of the axioms of $n(T)$, *e* is a content part of $(s\&b)$ and *A* is not a content part of $(s\&b)$.

[5] Since it follows from *e*'s being a content part of $(T\&b)$ that $(T\&b) \vdash e$, the clause $(T\&b) \vdash b$ common to **H-D2** and **H-D3** is omitted in **S-H-D***.

It is worth noting that this form of H-D does not fall prey to other charges Glymour brings against traditional expressions of H-D. Recall the following traditional version of hypothetico-deductive confirmation:

H-D2 e (directly) confirms T relative to background evidence b iff (T&b) is consistent, $(T\&b)\vdash e$, and $b\nvdash e$.

Against such versions of hypothetico-deductive confirmation, Glymour raises the objection that they have the unpalatable consequence that for any e and b, where b' is a consequence of b, e cannot confirm b' (relative to background evidence b). This follows from **H-D2** since where b' is a consequence of b either $(b'\&b)\nvdash e$ or $b\vdash e$. **S-H-D** does not fall prey to this criticism since it contains no direct analogue of **H-D2**'s condition that b not entail e. The purpose of **H-D2**'s condition that b not entail e is met in **S-H-D** by the demand that the content of axiom A plays an essential role in the derivation of e.

S-H-D, rather than **H-D1** or **H-D2**, is the best formal representation of the spirit that animates H-D. This is not to say that I would endorse **S-H-D**. Being a thoroughgoing inductivist I believe H-D to be misguided in spirit. In its heart H-D harbors an unhealthy sympathy for inductive skepticism. This is a theme I will return to soon.

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