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## OVERVIEW:

In this article, we will examine the possibility to enhance the divisibility check that is done by the Osculation technique in Vedic Mathematics. We will look how the technique works currently, and how this can be simplified. The new approach is presented as a concept, which can be expanded further.

## PRESENT APPROACH:

A brief explanation of the current Osculation procedure is discussed in this section. As an example, to check the divisibility of 4752993 by 7 , we first find the number with which we can Osculate 4752993. This number is also known as the Ekadhika. Ekadhika is the number "One more than the one before when the number ends in a 9 " or in a series of 9 s .

The Ekadhika of 7 is 5 , because to obtain a number ending in 9 for 7 , we should multiply 7 by 7 . We get 49 as a result, and the Ekadhika for 49 is 5 .We osculate a number by multiplying its last digit by the Osculator (or Ekadhika) and add the result to the previous figure. If a carry is generated in this operation, we multiply only the unit digit on the next round and add the next figure with the carry. For this example, the steps are provided below:

| 4 | 7 | 5 | 2 | 9 | 9 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 42 | 37 | 6 | 10 | 31 | 24 |  |

The list of steps is captured in the table below [Addition from the original number is marked in blue, Carry figure is marked in Red]

| Step | Description | Result |
| :--- | :--- | :--- |
| 1 | $5 * \mathbf{3}+\mathbf{9}$ | 24 |
| 2 | $5 * 4+2+\mathbf{9}$ | 31 |
| 3 | $5 * 1+3+\mathbf{2}$ | 10 |
| 4 | $5 * 0+1+\mathbf{5}$ | 6 |
| 5 | $5 * 6+\mathbf{7}$ | 37 |
| 6 | $5 * 7+3+4$ | 42 |

In this process, the final number 42 is divisible by 7 , so the original number 4752993 is also divisible by 7 . To those who are keen to understand this visually, refer to the following link below:


Video 1 - Basic Osculation Steps

## NEW MODIFICATIONS:

## 1. REDUCE THE NUMBER BEING OSCULATED

In the new modification we intend to reduce the number being osculated to its simplest level before running the process. It stems from the fact that reducing a number by its divisor or multiples of it, does not change its divisibility. So, in the case of number 7 as divisor, we reduce the dividend by any multiple of 7 from any of the figures, or the combination of figures. Let us see an example to reduce 4752993 by 7 below.

| Step | New Number |
| :---: | :---: |
| Replace all 7 by 0 and all 9 by 2 | 4052223 |
| Consider the number as 405222 3, and <br> reduce the highlighted portion by 7. The <br> divisibility does not change, and we leave only <br> the remainders back. | 0503013 |
| Consider the number as 50 30 13 and reduce |  |
| further |  |$\quad 010206$

We can just use 203 to do the Osculation process. The divisibility result would be the same as of Osculating 4752993 which is a considerable simplification to the original process. In some cases, you don't have to Osculate but can simply conclude if you know the final number is divisible by the divisor.

Note that there is no standard approach to do the reduction, which gives a greater flexibility. You can keep reducing till you reach a comfortable point of performing the Osculation. You can refer to the video below that explains the same reduction technique visually.


Video 2 -Reducing Osculated Number

## 2. CONSIDER ONLY THE REMAINDERS AT EACH STEP

The second modification stems from the fact that at each stage of the Osculation process, when we get an intermediate number, we can reduce that number from the dividend and consider only the remainder as the next number in the whole process. This reduces the Osculation we did earlier to

| Steps | $\mathbf{4}$ | $\mathbf{7}$ | $\mathbf{5}$ | $\mathbf{2}$ | $\mathbf{9}$ | $\mathbf{9}$ | $\mathbf{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Original | 42 | 37 | 6 | 10 | 31 | 24 |  |
| Modified | 14 or 0 | 2 | 6 | 3 | 3 | 3 |  |

This is much easier to handle and eliminates the need to manage carries in the Osculation process. A video is available below that explains this process;


Video 3-Consider Remainders at Each Step

## 3. USE BAR NUMBERS IN REDUCTION PROCESS

The Osculation process can be further simplified with the use of bar numbers to reduce the dividend. The use of bar numbers does not affect the divisibility equation. As an example, let us consider doing a divisibility check for 36323424 by 17 . We can approach the reduction process, two figures at a time as shown below: (Note: bar numbers are shown in bold red with underline)

| Step | New Dividend |
| :---: | :---: |
| Consider the number as 36323424 and <br> reduce each part by 17, | $020 \underline{2} 00 \underline{10} 0$ |
| Consider the new number as $20 \underline{2} 00 \underline{1} 0$ and <br> reduce | $03 \underline{2} 00 \underline{1} 0$ |

The Ekadhika for a divisor of 17 is 12 . We can now Osculate the new Dividend by 12. The steps are shown below. Note that we have used the concept of considering only remainders at each step below:

| 3 | $\underline{2}$ | 0 | 0 | $\underline{\mathbf{1}}$ | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 51 (or) 0 | 4 | 9 | 5 | $\underline{1}$ |  |

The last number 51 is divisible by 17 so, the number 36323424 is divisible by 17 . A video that explains the steps in this approach is available below:


Video 4-Reduce with Bar Numbers

## 4. USING BAR DIGITS AS EKADHIKA

This is another simplification step where instead of using bigger number as Ekadhika, we use bar digits. We can repeat the same example above, this time instead of using 12 to Osculate, we use 5 [bar 5, as $12-17=\underline{5}]$. The steps are shown below:

| 3 | $\underline{2}$ | 0 | 0 | $\underline{1}$ | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| -17 or 0 | 4 | $\underline{8}$ | 5 | $\underline{1}$ |  |

From the last step, we can infer that the number 36323424 is divisible by 17 . A video that explains how this Osculation is performed is available below:


Video 5 -Using Bar Digits as Ekadhika

## CONCLUSION

In summary, if we can reduce the number to be checked for divisibility to its lowest level using the divisor d-point circle, we can cut down significant number of steps in the Osculation process and speed up the entire process further. We can also use remainders at each step / bar digits to further refine the technique. From some of the examples above, the step to Osculate is optional as the prefilter layer itself is capable to confirm the divisibility check. These options are presented as a concept here with a scope for further expansion.

## REFERENCES:

- http://vedicmathsforum.org/viewtopic.php?f=6\&t=37
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- Vedic Mathematics Teacher's Manual, Intermediate Level, Kenneth R.Williams

