Decoding the Antikythera Mechanism: Investigation of an Ancient Astronomical Calculator

2 Supplementary Notes

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Supplementary Information Guide

There are three Supplementary Information sections:

Supplementary Notes 1 (Fragments) giving a key to fragment identification for Figure 1 of the main text and the dimensions of the fragments.

Supplementary Notes 2 (Glyphs & Inscriptions) giving details of the script of the characters, their dating and the Greek text and its provisional translation from (a) the Front Door inscriptions, (b) the Back Door inscriptions and (c) the Back Plate inscriptions near the Lower Back Dial.

Supplementary Notes 3 (Gears) giving a table to compare gear nomenclature and the gear tooth count estimates with previous estimates and to tabulate measured radii. Some notes are given on the individual gears and on the tooth count estimation procedure, including the effects of uncertainty in determining the centres of the gears. The gear train ratios are explained on the basis of simple Babylonian period relations. The equivalence of the epicyclic gearing and pin-and-slot mechanism to Hipparchos' theory of the moon is proved.

Supplementary Notes 1 (Fragments)

Key to Figure 1 of the main text:

Fragment	Area of maximum section [cm²]	Weight [g]	Thickness of discernible layers (mm)
A	224.209	369.1	
B	66.692	99.4	
C	65.767	63.8	
D	15.491	15.0	
E	12.623	22.1	
F	50.197	86.2	
G	68.757	31.7	7.6 (6 layers)
1	39.189	62.5	
2	16.018	15.3	
3	14.154	23.5	
4	12.195	9.6	
5	8.041	6.2	
6	7.166	10.9	
7	5.846	7.0	
8	5.383	3.2	2 (1 layer)
9	3.512	1.7	3.2 (3 layers)
10	2.296	1.2	
11	1.262	0.7	
12	1.878	0.6	
13	1.062	02	
14	1.091	0.2	
15	0.733	0.1	
16	0.629	0.3	
17	0.658	0.2	
18	0.438	0.1	
19	12.822	5.2	1.58 (1 layer)
20	5.920	2.2	1.24(1 layer), 1.0 (1 layer)
21	5.651	2.0	1.0 (1 layer)
22	9.547	2.7	1.6 (1 layer)
23	7.570	5.8	6.9 (6 layers)
24	2.153	0.5	1.0 (1 layer)
25	1.945	0.6	1.0 (1 layer)
26	2.951	1.1	2.6 (1 layer)
27	2.873	1.5	5.3 (5 layers) 1 mm (1layer)
28	3.379	1.1	2.8 (2 layers)
29	3.402	1.0	2.1 (1 layer)
30	1.385	0.3	1.5 (1 layer)
31	9.414	15.8	
32	8.585	14.9	
33	2.170	1.1	
34	0.286	>0.1	
35	0.222	0.1	
36	0.180	0.1	
37	2.027	0.7	1.6 (1 layer)
38	1.575	0.5	1.5 (1 layer)
39	1.376	0.4	
40	1.026	0.3	
41	1.228	0.5	1.7 (1 layer)
42	0.724	0.2	
43	1.079	0.3	
44	0.954	0.4	

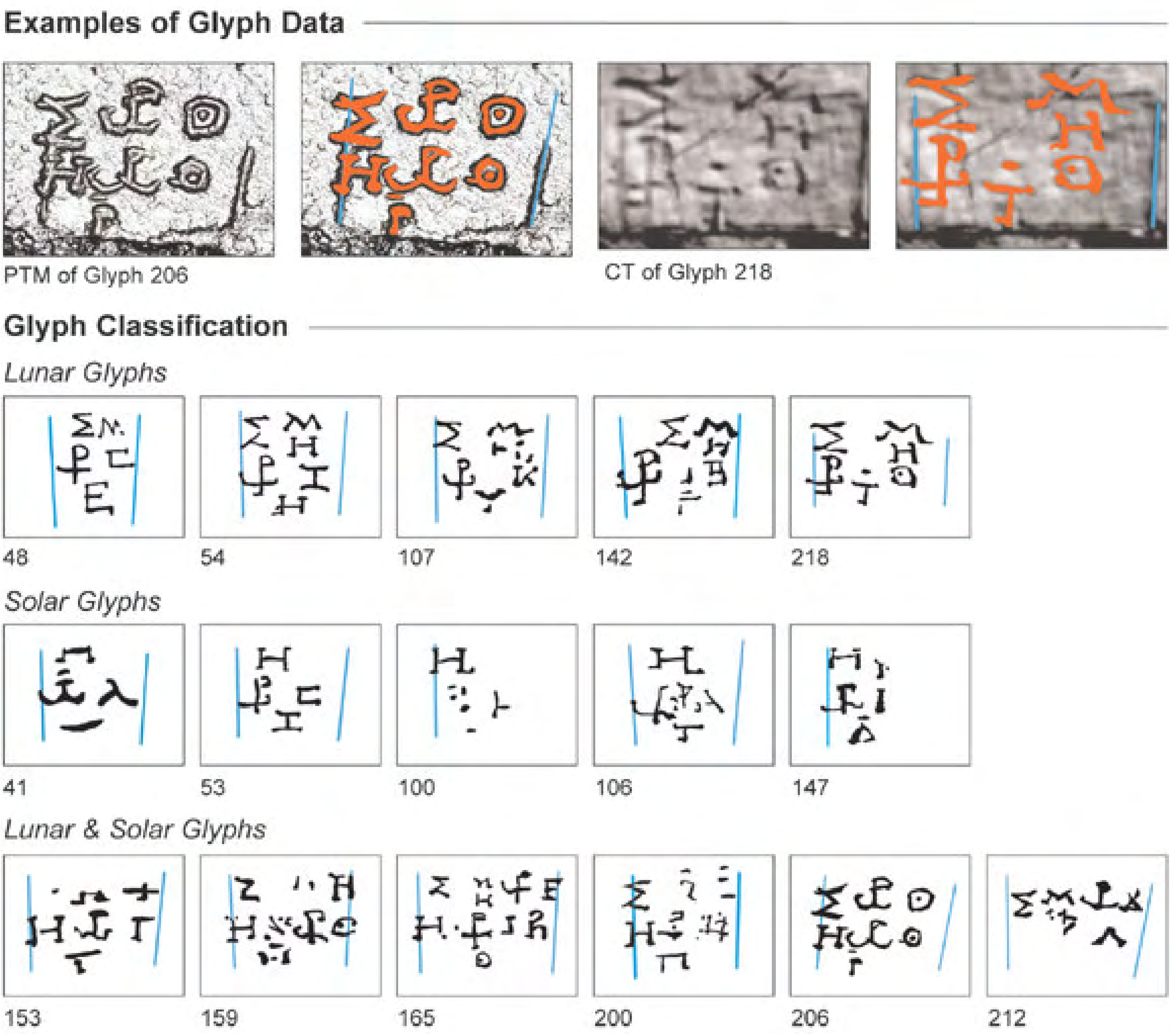
45	1.660	0.6	1.5 (1 layer)
46	0.592	0.2	
47	0.911	0.3	
48	0.395	0.1	
49	0.489	0.1	
50	0.322	0.1	
51	1.108	0.2	1.5 (1 layer)
52	0.781	0.3	1.9 (1 layer), 1.2 (1 layer)
53	0.849	0.3	2.1 (1 layer)
54	0.651	0.2	1.7 (1 layer)
55	0.881	0.2	1.0 (1 layer), 1.0 (1 layer)
56	0.497	0.2	
57	0.346	0.1	
58	0.565	0.2	
59	0.285	0.1	
60	0.604	0.1	1 (1 layer)
61	0.456	0.1	
62	0.357	0.1	
63	0.334	0.1	
64	0.237	>0.1	
65	0.266	>0.1	
66	0.208	0.1	
67	0.528	0.2	
68	0.208	0.1	
69	0.187	>0.1	
70	0.238	>0.1	
71	0.270	0.1	
72	0.270	0.1	
73	0.485	0.1	
74	0.201	0.1	
75	0.146	0.1	

For the dimensions in column 2 we have used digital scans of photographs, taken for us by Costas Xenikakis. The surface area in column 2 is the surface area of the largest section of each fragment (horizontal section, after positioning it down flat on a horizontal surface). Areas were estimated from prints of A4 images by square-counting using transparent-millimeter-graph-paper. Image distortion was checked to be small from horizontal and vertical scales photographed with the fragments. The errors in area measurement are estimated as certainly no more than 0.01 cm^2

The thickness of most of the metal sheets appears to be from 1 mm up to 2 mm, except for fragment 26, which is has a layer of 2.6 mm.

Supplementary Notes 2 (Glyphs and Inscriptions)

Glyphs



The 16 observed "Glyphs" from the Lower Back Dial. The lunar month number around the Saros dial is shown below each glyph. The data for the glyphs is transcribed directly from the PTMs in the rare cases when it is visible on the surface (e.g. Glyph 206), or from the CT when it is not (e.g. Glyph 218). Glyph 206 was noted by Price but not interpreted. Nearly all contain Σ (lunar eclipse, from ΣΕΛΗΝΗ, Moon) or H (solar eclipse, from ΗΛΙΟΣ, Sun). We classify the glyphs into lunar, solar and lunar & solar, making reasonable inferences where there is only partial information. In the period 400-1 BC there are 121 possible start dates where the month sequence of glyphs exactly match not only the eclipses but also eclipse type. Where there is a lunar & solar glyph, both types occur in the same month. The anchor-like symbol is probably the "omega-rho" denoting "hour" (hora) – probably indicating the predicted hour of the eclipse after sunrise or after sunset. The hour is indicated by a Greek letter used as a numeral, including Θ in its early form ⊙ for the number 9. The same symbol also appears in the Parapegma inscription. The eta with mu above it (e.g. in the right hand column of Glyph 218) may be the standard abbreviation of "day" (hemera) – possibly indicating that the (predicted lunar) eclipse was diurnal.

Inscriptions

Mirror image script found on some fragments is probably due to the accretion of fine silt against the original inscriptions, which became infused with bronze corrosion products and set in a hard matrix against the original. The style of writing is almost identical on the different fragments, except for the text near the Lower Back Dial, whose variation could be due either to the smallness of the characters or to a different hand. Full details of variant readings and translations of all the inscriptions will be published in due course

				Size (mm)		Characters in Price (1974)			Characters by this work		
Fragment	**Position**	**Text**	**Type**	**A**	**I**	**OK**	**?**	**Total**	**OK**	**?**	**Total**
A-2	Back door plate	Astronomic	Mirror			97	24	121	185	8	193
A-2	Lower back dial	Misc.	Direct	1.2	2.1	46	17	63	51	20	71
B-1	Back door plate	Astro/Mech	Mirror	2.0	3.4	157	41	198	239	105	344
B-1	Upper back scales	Calendrical?		-	-	-	-	-	5	10	15
C-1	Parapegma	Calendrical	Direct	2.8	5.0	95	12	107	105	10	115
C-1	Front scales	Calendrical	Direct	-	-	23	0	23	23	0	23
C-2	Parapegma	Calendrical	Mirror	2.7	6.0	13	0	13	16	0	16
E-1	Back door plate	Mechanical	Mirror	2.0	3.6	-	-	-	109	8	117
D	Gear (internal)	-	-	-	-	-	-	-	6	0	6
E-2	Lower back dial	Misc.	Direct	1.3	2.3	-	-	-	10	7	17
F	Lower back dial	Misc.	Direct	1.6	2.7	-	-	-	77	10	87
G-1	Front door plate	Astronomic	Direct	1.9	2.5	153	27	180	785	147	932
19-1	Back door plate	Astro/Mech	Direct	2.3	3.5	117	10	127	124	1	125
20-1	Parapegma	Calendrical	Direct	2.6	-	6	0	6	5	0	5
21-1	Front door plate?	Astronomic	Mirror	1.9	2.5	45	10	55	39	16	55
22-1	Parapegma	Calendrical	Direct	2.4	5.0	21	0	21	24	8	32
24-1	Lower back dial	Glyph									
25-1	?	-	Direct			-	-	-	6	1	7

In addition to these, there are visible traces of inscriptions on fragments 23-2, 26-1, 28-1, 29-1, 37-1 to 44-1, 51-2, 53-2, 61-1 and 67-2. A classification of fragments with visible inscriptions could also be made based on the colour surface texture and colour, which, together with text size and type, will help in assembling texts from disparate minor fragments. The "?" indicates a doubtful transcription. Fragment D has the letters "*ME*" at three different places on a gear wheel. More isolated characters will become available as reconstruction from the CT scans of smaller fragments is completed

Totals: Price 923, this work 2160. (A portion of 19-1 is counted twice in B-1, and some minor inscription is not included in the table above)

Data from the PTM has proved to be invaluable in inspecting the surface of the fragments and the inscriptions. The CT, whose primary aim was to collect information about the internal structure of the Mechanism, has allowed the discovery of unknown characters within fragments A, B, C, D, E, F and G, and within some of the smaller fragments. The case of Fragment G is exemplary: Price (1974) notes that its inscription is "almost illegible', reading only 180 characters. The CT images, viewed at various angles, enable us to read 932 characters. The inscription on the fragment F (newly discovered and identified by M. Zafeiropoulou in 2005) has characters whose height is often less than 1.6 mm, totally invisible because they are covered by sea accretions.
We propose two reconstitutions: the text from the back door plate, where part of the gap in Price (1974) is completed with text from Fragment E. Based on the internal structure of the fragment, showing portions of the scales, we are able to establish where the first line from fragment E joins with line 28 from fragment B1 and where the last line from fragment E joins with line 34 belonging to the fragment A2 (line 30 in Price). Similar results were produced with the text near the lower back dial, at the right side of the Mechanism. We are able to join characters from fragments A, E and F. We also believe that some characters in smaller

fragments may join with the big and intriguing text from fragment G.

The surviving part of the Front Door Plate probably comes from the middle of the original plate and we unfortunately lack the beginning and the end of phrases, and, because of this, possible planet names that would greatly aid interpretation.

The mechanical terms of fragment E (trunnions, pointers and gears) are common in Heron's "Dioptra".

The frequency of the (Parapegma) key letters in the Zodiac signs on the Front Dial suggest that the 24 letters of the greek alphabet might have been used twice here.

The figure shows an example of part of the inscription from the Back Door Plate on fragment 19, enhanced by the PTM technique.

According to Dr. Haralambos Kritzas (Director Emeritus of the Epigraphic Museum, Athens) the style of the writing could date the inscriptions to the second half of the 2^{nd} Century BC and the beginning of the 1^{st} Century BC, with an uncertainty of about one generation (50 years). Dates around 150 BC to 100 BC are a plausible range.

We give here a few examples of the **epigraphic clues to the dating**, but detailed analysis will be published elsewhere:
Π pi has unequal legs - second half of 2^{nd} century BC
Σ sigma has the two lines not horizontal but at an angle - second half of 2^{nd} century BC, beginning of 1^{st} century BC
M mu has the two lines not vertical but at an angle - second half of 2^{nd} century BC. There is one M with vertical lines
Y upsilon has the vertical line short - second half of 2^{nd} century BC
A alpha - just post Alexander
Z zeta is written like I with long horizontal lines - 2^{nd} century BC
Ω omega and not like ω - 2^{nd} century BC
B beta unequal upper circle, compared with the lower circle - old
O omicron very small - old
Θ theta has a short line in the middle, in one case a dot - 2^{nd} century BC
Φ phi is arc like - old
Ξ xi middle line short - old

Greek Text of Front Door Inscription

Mainly from fragment G. Red indicates dubious characters

1
2 Ο
3 Ο Δ Ε Η Ο Σ
4 Ο Σ Α Π Ο Σ Τ Η Μ Α
5 Δ   Ο Ν Ε Ξ Α Ρ Χ Η Σ Α
6 Ε   Σ Π Ο Μ Ε Ν Α Ο Δ Ε
7 Ο Υ   Ο Ε Σ Π Ο   Α Π Ο Κ Α Τ Α Σ Τ Α Σ
8 Π   Α Σ Τ Ο Υ Δ Α Π Ο Κ   Ο Τ Α Σ Ν   Ν Α Τ Α Ι Σ Ο
9 Λ Ο Π Ρ Ο Σ Τ Ο Ν Η Λ Ι Ο Ν Σ Ο   Π Ο Δ Ε
10 [Α Ξ Ι   Α Λ Ι Σ Α Σ Κ [Α Ι] Π Ρ Ο Σ Α Γ Ε Ι Ν Ε Π [Ι] Σ Η Σ Τ Ο Ν Η Λ [Ι Ο Ν]
11 [Π Ρ] Ο Σ Α Γ Ε Τ Α Ι Ε Π Ι Τ Ο Ν [Η Λ] Ι Ο Ν Ε Λ Α Σ Σ Ο Ν Α Σ Τ Η Ρ Ι Γ Μ Ο Ι Σ Τ Υ Γ Χ Α Ν Η Α Π Ο Σ
12 [Π Ρ] Ο Σ Α Γ Ε Ι Π Ρ Ο Σ Τ Ο Ν [Η] Λ Ι Ο Ν Ε Ω Σ   Η Σ Ε Ι Ο Ν Κ Α Ι Σ Υ Ν Ο Δ Ο Ν Α
13 Ε Π Ι Τ Ο Μ Ε Γ Ι Σ Τ Ο Ν Ε Π Ο Μ Ε Ν Ο   Ο Ε Ν Α Λ Λ Α Ι Σ Η Μ Ε Ρ Α Ι [Σ]
14 [Σ Τ Η Ρ Ι Γ] Μ Ο Ν [Ω Σ] Ο Π Ρ Ο Η Γ [Ο]Υ Μ Ε Ν Ο Σ   Α Π Ο Σ Τ   Α Λ Θ
15 Α Σ   Ε Ν Η Μ Ε Ρ Α [Ν] Π Ο Ι Ε Ι Π [Ρ Ο]Ε Ν Ο Σ Ε Ι Σ Τ Η [Ν]Κ Ε Π
16 [Δ Ι] Α Σ Τ Η Μ Α Τ Ο Σ Π [Ρ]Ο Σ Α   [Ο] Π Ρ Ο Σ Τ
17 Κ Η Σ   Ε Ι Ο Σ Χ [Ε]Π Ι Τ Ε Μ   Χ Α
18 Ο Σ Ι Η Τ Ο Ν Η Λ Ι Ο Ν Η Α Φ Ρ Ο [Δ Ι Τ Η]   Η Ν Ο Ι Σ Ν
19 Σ Τ Η Π Ρ Ο Σ Α Γ Ε Ι Π Α Σ Ι Ν Γ Ω Ν Ι Α Ι Σ
20 Ο Ι   [Η Μ Ε Ρ Α Σ] Π Ρ Ο Σ Α Γ Ε Ι Τ Η Ν   Π Ι Π Ε Τ Α   Ν Α Κ Α   Σ Α   Ε
21 Ε Σ Π   Η Μ Ε Ρ Α Σ Κ Α Ι Υ Π Ο Λ Ε Ι Π Ε Τ Α Ι Μ Ε Χ Ρ Ι Τ Η Σ Ε Ω Ι Α Σ Σ Τ
22 Η [Μ Ε Ρ] Α Ι   Τ Μ   Η Μ Ε Ρ Α Σ Σ Ο Ν Ο Η Ο Ν Ο Τ Ο Υ   Λ   Α Κ Μ Η
23 Α Ν Ι Σ Α Τ Ο Ν Ε Π Ι Σ Ο Ν Ε Χ Ω Ν Σ Τ Η Ρ Ι Γ Μ Ο Ν Ε Π Ι Σ Χ Ω Ν Α Π Ο Τ Ο Υ Η Λ Ι Ο Υ Ν   Ε Σ Λ Ι
24 Λ Ε Σ   Α Ι Π Ι Σ Ξ Ε   Η Λ [Ι Λ] Κ Η Ν Π Α Ρ Α Τ Ε Ι Ν Ε Τ [Η Ν] Α Π Ο Σ Τ Α Σ Ι Ν   Ε Σ Α Π Ο
25 Ρ Ε Σ   Α   Σ Ξ Ε Η Λ Ι Α Κ Η Ν Ε Π Ε Τ Ε Ι Ν Ε Ν Τ Ε Σ Σ Α Ρ Α Κ / Ε Ν Α Ε Υ Δ Ο Μ Ο Ν Κ Α
26 Ν Ε Ξ Η Μ Ε Ρ Α Σ Η   Γ Ε Ν [Ε] Σ Ε Ω Σ Ε Ν Ε Ν   Ε Χ   Ε Ω Α Ν Κ Α Τ Α Λ Ο
27 Ε Α   Ε Ι Ν   Π   Δ Ι Α Σ Τ Α Σ Ι Ν   Σ   Η Μ Ε Ρ Α Ι Σ Μ Ε Ν Μ Ε Γ Α Λ Α Ι Σ
28 Α Κ Α Ι Δ Ω   Δ Ε Κ Α   Τ Η Μ Ο Ρ Ι Ο Σ Κ Α   Α Φ Α Ι Λ Ε Τ Η Ν Υ Π Ο Λ Ο Ι Π Ο [Ν]
29 Α Μ Ο Ν Ε Ξ   Α Π Ο Τ Ο Υ Ε Σ Π Ε Ρ Ι Ν Ο Υ Ε   Ε Χ   Σ   Κ Α Ι Υ Π Ο Λ Ο Ι Π Ο Ν
30 Α Ν Τ   Μ   Η Μ   Ε Ν Χ Ρ Ο Ν Ω   Α Π Α Ν Α Ι Ο Ν   Η Μ Ε Ρ Α Ι Σ Τ Ο
31 Ε Α   Σ Ε Π Α Γ Ε Ι Σ   Ρ Λ Θ   Ε Π Ι Τ Ο Ν Η Λ Ι Ο Ν Τ Ο Ν Σ Τ Η Ρ Ι Γ Μ Ο Ν   [Ε]
32 Ο   Ο Ν Ο Μ Α   Η Μ Ε Ρ Α Σ Λ Α Π Ρ Ο Η Γ Ε Ι Τ Α Ι Η Μ Ε Ρ Α Ι Σ Τ Ο
33 Ν Α Ν Α Τ Ο Λ Η Σ Ε Ι Ν Α Ι Ο Η Λ Ι Ο Σ Μ Η Μ Ε Ρ Α Σ Π Α Λ   Ο Ν
34 Α Ν   Α Η Μ Ε Ρ Α Ν Γ Ι Ν Ε Τ Α Ι Η Π Ε Ρ Α Ι   Ε Ι Σ
35 [Η Μ] Ε Ρ Α Σ Σ Ε Ι Σ Α   Η Μ Ε Ρ Α Ι Α Π Ο   Ο Ε
36 Ρ Α Σ   Ι Ν Ε   Μ
37 Τ Η

Greek Text of Back Door Inscription

Black and blue letters are believed to be good, red and orange are dubious. Black and red are from fragments A and B, blue and orange from fragment E.
The second column in line numbering in Price (1974: reference 1 of main paper)

1 1 Τ Α Υ Τ Η Ν Δ
2 2 Δ   Ι Δ Υ Π Ο Λ Λ
3 3 Υ Π Ο Δ Ε Τ Ο Ν   Τ
4 4 Α Τ
5 5 Ε
6 6
7 7 Ο
8 8 Ι Ρ Μ Ο Σ
9 9 Α Κ Ρ Ο Υ Δ
10 10 Μ Ε Ν Ο
11 11 Μ
12 12 Ο Λ   Ν
13 13 Υ Π Ο Λ Α
14 14 Ο Υ   Δ   Σ Φ Α Ι Ρ Ι Ο Ν Φ Ε Ρ Ε
15 15 Π Ρ Ο Ε Χ Ο Ν Α Υ Τ Ο Υ Γ Ν Ω Μ Ο Ν Ι Ο Ν Σ
16 16 Φ Ε Ρ Ε Ι Ω Ν Η Μ Ε Ν Ε Χ Ο Μ Ε Ν
17 17 Τ Ο Σ Τ Ο Δ Ε Δ Ι Α Υ Τ Ο Υ Φ Ε Ρ Ο Μ Ε Ν
18 18 Τ Η Σ Α Φ Ρ Ο Δ Ι Τ Η   Ε Ρ Ο Υ
19 19 Τ Ο Υ   Σ Ψ Ο Ρ Ο Υ Ι Ε   Ε Ρ Ε Τ Α Ν
20 20 Γ Ν Ω Μ Ω   Κ Ε Ι Τ Α Ι Χ Ρ Υ Σ Ο Υ Ν Σ Φ Α Ι Ρ Ι Ο Ν
21 21 Η Λ Ι   Α Κ Τ Ι Ν   Υ Π Ε Ρ Δ Ε Τ Ο Ν Η Λ Ι Ο Ν Ε Σ Τ Ι Ν Κ Υ
22 22 Υ Λ Ρ Ε Σ   Λ Υ Ρ Ο Ε Ν Τ Ο Τ Ο Δ Ε Δ Ι Α Π Ο Ρ Ε
23 23 Ε Θ Ο Ν Ο   Σ Τ Ο Δ Ε Δ Ι Α Π Ο Ρ Ε Υ Ο Μ Ε Ν Ο Υ
24 24 Ι Ν Ο Ν   Ο Υ Κ Υ Κ Λ Ο Σ Τ Ο Δ Ε Σ Φ Α Ι Ρ Ι Ο Ν Φ
25 25 Μ   Ε   Τ Ο Υ Κ Ο Σ Μ Ο Υ Κ Ε Ι Τ Α Ι Σ Φ
26 26 Μ Ε Ν   Σ Τ Ο Ι Χ Ε Ι Α Π Α Ρ Α Κ Α   Ν
27 27 Α Υ Τ Α Τ Α Ι Σ Α Σ Π Ι Δ
28 28 Λ   Ο Τ Ω Ν Δ Ι Α Ω   Σ Τ Ω Ν Μ Ε Ν
29 - Ν Ο Μ Η Τ Η Ι Ε Λ Ι Κ Ι Τ Μ Η Μ Α Τ Α Σ Λ Ε
30 - Τ Α Ι Σ Κ Α Ι Ε Ξ Α Ι Ρ Ε Σ Ι Μ Ο Ι Η Μ Ε Ρ Α Ι Κ

31 - Χ Ο Ν Σ Τ Η Μ Α Τ Ι Α Δ Υ Ο Π Ε Ρ Ι Τ Υ Μ Π Α Ν
32 - Π Ρ Ο Ε Τ Ρ Η Μ Ε Ν Α Σ Τ Η Μ Α Τ Ι Α Τ Η Μ
33 - Α Τ Ω Ν Τ Ρ Η Μ Α Τ Ω Ν Δ Ι Ε Λ Κ Ε Σ Θ Α Ι
34 30 Ο Μ Ο Ι Α Ω Σ Τ Ο Ι Σ
35 31 Φ Υ Ε Σ Π Ο Ι Η
36 32 Κ Α Ι Σ Υ Μ Φ Υ
37 33 Τ   Π Α
38 34
39 35 Ε   Ο Υ
40 36 Ε Ν Λ Χ   Π Α Ν   Ε
41 37 Μ Η Ν Ο Θ Ε Ν Ε Ξ Η Λ
42 38 Τ Η Σ Π Ρ Ω Τ Η Σ Χ Ω Ρ Α Σ
43 39 Μ Ο Ν Ι Α Δ Υ Ο   Ω Ν   Τ Α Α Κ Ρ Α Φ Ε
44 40 Τ Ε Σ Σ Α Ρ Α Δ Η Λ Ο Ι Δ Ο Μ Ε Ν Τ
45 41 Σ Α Ι   Ν Τ Η Σ   Ο C L   Ι Θ L   Τ Ο Υ
46 42 Ο Σ Ε Ι   Ε Ι Σ Α   Σ Κ Γ   Σ Υ Ν Τ Ε Σ
47 43 Ι Ο Ν Τ   Ο Σ Δ Ι Α Ι Ρ Ε Θ Η   Η   Ο Λ Η
48 44 Δ Ο Ι Ε   Ε Γ Λ Ε Ι Π Τ Ι Κ Ο Ι Σ
49 45 Ι Ο Μ Ο   Τ Ο Ι Σ   Ε Π Ι Τ Η Σ Ε
50 46 Χ Ρ Ο Ν Φ Ε Ρ Ε   Τ Α
51 47 Π Ι Ν   Ε Ν Τ

Greek Text of Back Plate inscription, near the Lower Back Dial

Black and blue letters are believed to be good, red and orange are dubious. Black and red are from fragments A and F, blue and orange from the other side of fragment E.

1 Ι Π Ο
2 Ι Κ Ο Λ Ι Τ
3 Ι Ν Ο Ν
4 Α Π Ο Χ Ο   Ο
5 Δ Ε Κ Α Τ   Ξ Ω   Ν Τ Α Ν
6 Λ Ι Β Α   Ν   Χ Ι Π
7 Λ   Μ Α
8 Ν
9
10 Π Ρ Ο
11 Ι Σ Τ Ω Ν Τ Α Σ
12 Κ Α Τ Α Λ Η
13 Π Ρ Ο Σ Α Π Η Λ
14 Ω Τ Η Ν Ω
15 Λ Η Ν Τ Ω
16 Χ Ρ Ω Ν Ι Α
17 Π Υ   Ο
18 Ι Ο
/ /
19
20 Φ
21 Ρ Ι Ι Σ
22 Τ Α Α Δ
23 Ν Ο Τ Ο Ν
24 Κ Α Ι Α Ρ Η Σ
25 Σ Η Φ Α Ρ Ο Σ
26 Λ Ε Ν Τ Η Ν Κ
27 Σ Α Φ Υ Λ Α Ξ Α Σ
28 Ι Λ Α Μ Ε Λ Α Ν
29 Χ 2   Π Κ Ζ Φ
30 Α Π Ο Ν Ο Τ Ο Υ Π Ε Ρ Ι
31 Ι Σ Π Α Ν Ι Α Σ Δ Ε Κ Α
32 Δ Υ Σ Α Ν
33
34

Table 1. Provisional Translation of the Front Door Inscriptions

1. ---

2. ---

3. ---

4. space (*or* distance) between

5. from the beginning

6. ---

7. --- restore (*or* which has been restored)

8. ---

9. towards the Sun <370>

10 .equal and brings the Sun upon to the equal

11. brought upon the Sun the minor stationary point <s *or* 200> then occurs distance

12. brings towards the Sun up to --- and conjunction

13. on to the maximum following within other days

14. [stationa]ry point as the previous one 39

15. day, makes before one to the

16. interval brings upon to the

17. ---

18. <Venus> <approach> the Sun

19. brings upon every <angle> (*verb could be* coincide)

20. brings upon [days]

21. days and remains until the eastern (*eastern = adjective in the sense of dawn*)

22. 34<0?> days 270 days ---

23. the stationary point which is at equal distance, is at a distance from the Sun

24. 265 of the Sun, extend the distance

25. 265 of the Sun, has extended four and one seventh

26. 8 days --- of the origin --- dawn

27. interval (*or* separation, length, distance; *greek*: diastasin) <2??> large days

28. twelfth part of the circle (*greek*: dodecatemorios) --- subtract the remaining (*genre is feminine*)

29. <six> from the evening --- and the remaining

30. in time --- <370> days

31. brings on <139> the Sun the stationary point

32. days 31 is leading <37?> days

33. of the rising is the Sun 40 days

34. day is becoming the <completion>

35. 205 days <equal> days from

36: ---

37: ---

Conventions used:

--- : either unreadable, or non-translatable string

< > : enclosing either dubious characters or one amongst many reading choices (e.g. either number or beginning of word)

[ ]   : enclosing restored sections

?   : uncertain character

( )   : alternative translation, indication of greek word translated or (if in *italics*) comment from reader-translator.

Table 2. Provisional Translation of the Back Door Inscriptions

1. this
2. ---
3. and under the
4. ---
5. ---
6. ---
7. ---
8. ---
9. (of the) extremity
10. ---
11. ---
12. ---
13. ---
14. [and is carrying] little [golden] sphere
15. the pointer that protrudes from it
16. carries, of which the next one
17. which is carried through (*or* the other carried by it)
18. of Venus
19. ---
20. on the [extremity of] the pointer stands a little golden sphere (golden *or* goldish)
21. the ray [towards the] Sun and above, the Sun is ---
22. --- when it moves through (through its orbit; *greek*: diaporevomenon)
23. --- and the moving through (*same meaning as in line 22*)
24. --- circle and the little sphere
25. stands --- the [sphere] of the world (*world in greek*:cosmos)
26. --- elements ---
27. ---
28. ---
29. the spiral divided in 235 sectors
30. and days to be excluded 2? (*twenty to twenty-nine; "excluded" means "taken out of the calendar"*)
31. --- two trunnions (*greek*: stematia) around gear (*greek*: tympanon)
32. --- perforated trunnions (*possibly pre-perforated*)
33. through the perforations to be pulled (*haul*)
34. the same manner as
35. ---
36. ---
37. ---
38. ---

39. ---
40. ---
41. from where it came out of
42. the first position
43. two pointers, whose ends carry
44. four, the one indicates
45. the 76 years, 19 years of the
46. 223 coming together
47. so that the whole will be divided
48. (of the) ecliptic
49. similar to those on the
50. carries
51. ---

Conventions used: *see Table 1*

No translation of the Back Plate Inscriptions, near the Lower Back Dial is attempted, as the text is rather incomplete. Work is in progress.

Supplementary Notes 3 (Gears)

The first publication that comprehensively estimated the tooth counts of the gears was Price's *Gears from the Greeks* (1974 - reference 1 in the Main Text). The original counts were done for Price by Charalambos and Emily Karakalos (based on conventional film x-radiography). These counts were then adapted by Price to suit his model. M.T. Wright has subsequently re-counted the teeth on digitised X-rays (with some limited use of tomography) undertaken by Bromley and Wright in 1990-1994. Our estimates are based on our CT data.

Gear	Gear Price	Gear Wright	Average outer radius to gear tips mm	Inner radius from best-fit circle ±0.5 mm	Outer radius from best-fit circle ±0.5 mm	Karakalos tooth count	Price tooth count	Wright tooth count	Wright limits	Our best fit tooth count	Our limits
a1	A	A	13.6±0.2				45(48)	48	44-52	48	Definite
b0		B6								*20*	
b1	B1	B1	64.9±1.1	63.8	65.0	223-226	225	223	216-231	223	223/224
b2	B2	B2	15.5±0.2	14.9	15.7	64-66	64	64		64	64-66
	B3	B3				32	32				
b3	B4	B4	8.6±0.2	8.2	9.3	32	32	32		32	Definite
b4		B6						*24*			
c1	C1	C1	10.3±0.3	9.4	10.3	38	38	38		38	38/39
c2	C2	C2	11.3±0.4	10.5	11.0	48	48	48	47-48	47/48	47-49
d1	D1	D1	5.6±0.3	5.1	5.8	[24]	24	24		24	Definite
d2	D2	D2	31.6±0.2	30.6	31.7	128	127	127		127	Definite
e1	E1	E6	9.4±0.3	8.6	9.7	32?	32	32		32	Definite
e2	(E2i)	E7	7.8±0.2	7.1	7.8	32?	32	32		32	Definite
e3	E4	E4	52.6±0.3	51.5	52.4	222	222	223	218-228	220-225	217-235
e4	E3	E3	50.2±0.3	49.1	49.9	192	192	191	188-192	187-191	180-192
e5	(E2ii)	E8	13.4±0.2	12.2	13.1	(32?)		51	50-52	52	50-52
e6	E5	E5	13.9±0.2	12.9	13.9	50-52	48	53	51-55	50	49/50
f1	F1	F1	14.0±0.2	13.6	14.6	54	48	54	53-54	53	53/54
f2	F2	F2	8.3±0.3	7.4	8.2	30	30	30		30	Definite
g1	G2	G2	14.2±0.3	13.4	14.4	54/55	60	55	54-55	54	54-56
g2	G1	G1	4.9±0.1	4.1	4.9	20	20	20	20	20	Definite
h1	H1	H1	14.0±0.1	13.0	13.7	60-62	60	60	57-64	60-64	60-64
h2	H2	H2	3.9±0.2	3.0	3.8	16	15	15		15	Definite
i1	I	I	13.4±0.3	12.6	13.2	60	60	60	59-60	60	59-62
k1	(K1)	K3	13.5±0.3	12.6	13.3	(32)		49	48-50	49/50	48-51
k2	K2	K2	14.0±0.2	13.1	14.0	48 or 51	48	49	48-50	50	48-52
l1	L1	L1	9.1±0.2	8.3	9.0	36+	36	38	37-38	38	Definite
l2	L2	L2	13.1±0.4	12.5	13.3	52	54	53		53	Definite
m1	M1	M1	24.5±0.5	23.6	24.7	96+	96	96	95-98	96/97	96-99
m2	M2	M2	4.4±0.3	3.7	4.0	14	16	15		15	Definite
m3											
n1		N1						*53*			
n2		N2						*15*			
o1		O	13.3±0.1	12.2	12.8			60	57-62	60*	57-61
p1		P1						*60*			
p2		P2						*12*			
q1		Q	5.3±0.2					24		20	Definite
r1	N	Δ1	16.4±0.2	15.9	16.9	63	64	63		63	Definite
		Δ2						65			

*Strong preference

(In the table: Price, Karakalos and Wright data taken from Wright, M.T., Bull.Sci.Instr.Soc. 85, 2-7, 2005 – reference 4 in the Main Text)

Columns 3-5 give measured radii from CT data. Column 3 is the mean of the radii to the tooth tips from the assumed centres. Columns 4 and 5 gives the radii of "best fit" circles to the pits between the teeth (inner radius) and to the tooth tips (outer radius), with an estimated error of order or less than 0.5 mm. Gears in italics are hypothetical.

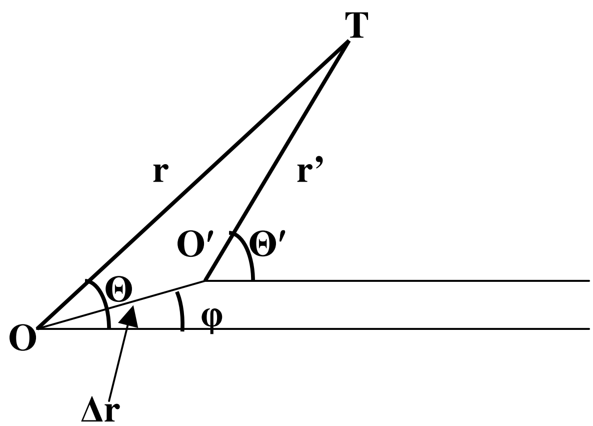
Tooth Counting Method

The angular data Θ_i of tooth tips from the centre is compared with a model $a + n\Delta\Theta$. The "goodness-of-fit" is the parameter $\varepsilon = \sum_n [\Theta_n - (a + n\Delta\Theta)]^2$ where Θ_n is chosen as the model point closest to the data point. The shift parameter a is fixed to minimise ε by requiring that $\sum_i \Theta_i = \sum_i (a + i\Delta\Theta)$. We then investigate peaks in $1/\varepsilon$ as a function of $\Delta\Theta = 360/T$ where T is the implied total number of teeth on the gear, i.e. we seek to minimise ε. If the errors in the angular data are Gaussian, this should give a "maximum likelihood" estimate of the true total tooth count.

The counts are sensitive to the positioning of the assumed centre of the gear. This can be investigated by transformation from a measured set of data (see below), and we endeavour to find the "best" fit – i.e. strongest peak in $1/\varepsilon$ - for reasonable variation of the centre position. In some cases a unique tooth count gives a very clear isolated peak, in other cases a range of peaks implies a range of possible total tooth counts, with adopted values implied (subjectively) from the relative peak heights.

Gear Count Analysis: Moving the Centre

Consider a measured tooth tip T at angle Θ from a given centre O, with a measured distance r from the centre to the tooth tip. Now move the centre to O′ by a distance Δr in direction ϕ. The new angle of T from O′ is Θ′ where:



In triangle OO′T:

angle OO′T = 180 - (Θ′ - ϕ)

angle OTO′ = 180 – (Θ - ϕ) – [180 - (Θ′- ϕ)] = Θ′ - Θ

By sine rule:

$$\frac{\mathbf{r}}{\sin[180 - (\Theta' - \phi)]} = \frac{\Delta\mathbf{r}}{\sin[\Theta' - \Theta]}$$

so $\sin[\Theta' - \Theta] = \frac{\Delta\mathbf{r}}{\mathbf{r}} \sin(\Theta' - \phi)$ Eqn(1) and hence $\sin\Theta'\cos\Theta - \cos\Theta'\sin\Theta = \frac{\Delta\mathbf{r}}{\mathbf{r}}\left[\sin\Theta'\cos\phi - \cos\Theta'\sin\phi\right]$ which re-arranges to:

$$\sin\Theta'\left[\cos\Theta - \frac{\Delta\mathbf{r}}{\mathbf{r}}\cos\phi\right] = \cos\Theta'\left[\sin\Theta - \frac{\Delta\mathbf{r}}{\mathbf{r}}\sin\phi\right]$$

Giving finally (or by simple geometry, dropping a perpendicular from T) :

$$\Theta' = \arctan\left[\frac{\sin\Theta - \frac{\Delta\mathbf{r}}{\mathbf{r}}\sin\phi}{\cos\Theta - \frac{\Delta\mathbf{r}}{\mathbf{r}}\cos\phi}\right]$$

This is the formula used in the analysis of the effects of searching for a "best" centre. A useful approximation can be made for *small* Δr:

From Eqn 1 above: $\sin[\Theta' - \Theta] = \frac{\Delta\mathbf{r}}{\mathbf{r}}\sin(\Theta' - \phi)$

Suppose $\Theta' - \Theta = \Delta\Theta$ and is small, then

$$\sin[\Theta' - \Theta] = \frac{\Delta\mathbf{r}}{\mathbf{r}}\sin(\Theta + \Delta\Theta - \phi) = \frac{\Delta\mathbf{r}}{\mathbf{r}}\left[\sin(\Theta - \phi)\cos\Delta\Theta - \cos(\Theta - \phi)\sin\Delta\Theta\right]$$

And as $\sin\Delta\Theta \approx \Delta\Theta;\quad \cos\Delta\Theta \approx 1 - \Delta\Theta/2$

$$\sin[\Theta' - \Theta] = \frac{\Delta\mathbf{r}}{\mathbf{r}}\left[\sin(\Theta - \phi)(1 - \frac{\Delta\Theta}{2})\cos\varepsilon - \cos(\Theta - \phi)\Delta\Theta\right]$$

So to first order if both $\Delta\mathbf{r}/\mathbf{r}$ and $\Delta\Theta$ are small:

$$\sin[\Theta' - \Theta] \approx \Delta\Theta \approx \frac{\Delta\mathbf{r}}{\mathbf{r}}\sin[\Theta - \phi]$$

So $\Theta' \approx \Theta + \frac{\Delta\mathbf{r}}{\mathbf{r}}\sin[\Theta - \phi]$

By differentiating this, consider the deduced tooth separation with primed and unprimed centres

$$\Delta\Theta' \approx \Delta\Theta\left\{1 + \frac{\Delta\mathbf{r}}{\mathbf{r}}\cos[\Theta - \phi]\right\}$$

The deduced tooth counts would be $n' = 2\pi/\Delta\Theta'$; $n = 2\pi/\Delta\Theta$, so

Averaging $\cos[\Theta - \phi]$ over a whole circle of Θ implies $\Delta\Theta' = \Delta\Theta$, but *over a half circle* (i.e. semi-circular segment) between $\phi - \pi/2$ and $\phi + \pi/2$ (for maximum effect of centre shift)

$$\frac{n}{n'} = \frac{\int_{-\pi/2}^{\pi/2}\left[1 + \frac{\Delta\mathbf{r}}{\mathbf{r}}\cos(\Theta - \phi)\right]d\Theta}{\int_{-\pi/2}^{\pi/2} d\Theta} = \frac{\pi - 2\frac{\Delta\mathbf{r}}{\mathbf{r}}}{\pi} \quad \text{or} \quad \frac{n}{n'} \approx 1 - \frac{2}{\pi}\frac{\Delta\mathbf{r}}{\mathbf{r}} \quad \text{or} \quad \Delta n \approx n\frac{2}{\pi}\frac{\Delta\mathbf{r}}{\mathbf{r}}$$

As examples, consider the following gears:

e3: radius 52.3 mm; nominal n around 220-230; so a centre shift of 1mm can change the teeth count for a semicircle by 3
h1: radius 14 mm; nominal n of 60-64; so a centre shift of 1mm can change the teeth count for a semicircle by 3.
If the segment is less than a semicircle, then the effect can be even stronger, since the average value of $\cos(\Theta - \phi)$ will increase a little.

◄

Notes on Individual Gear Measurements

On Centres:
We estimate that the assumed centre of e3 is within 0.3 mm of the correct centre, both on the x-axis and the y-axis. This implies that the maximum error is about 0.4 mm in terms of distance. Similarly, h1 is within 0.2 mm on both x and y, hence about 0.3 mm in terms of distance.

These errors apply to the original hubs. However the teeth are often clearly distorted and moved relative to the hubs. So, for example, there might be a whole sector that has moved by several millimetres relative to the hub even though the stated estimate for the hub itself is within 0.2 mm. However, a best fit circle through the tips may not get a good centre because some of the tips are often corroded to the point of virtual non-existence whereas others look virtually perfect.

For nearly all the gears, the assumed centre is based on finding evidence of a hub with some sort of geometry (either a circle or a square).

a1
This is the contrate input gear. Since the tooth tips are variable worn, a CT slice has been taken through the tooth pits and the angles of the pits measured rather than the tips.
b1
This is the main drive wheel. The teeth are very damaged and mostly non-existent. The centre is easy to identify. The reason for the builder's choice of 223 or 224 teeth is not known.
b2
There is only one sector of teeth. It is a fairly complete gear, though some of the tips are hard to identify. There are two clear central round hubs. They are nearly coaxial and the centre is based on a compromise between the two.
b3
It is a complete gear, though some teeth are corroded.
c1
The centre is almost impossible to identify accurately. We used a second CT slice that shows it better than the slice for the gear tips. The centre cannot really be identified with a best-fit circle through the tips since they are so damaged. Identification of the tips is also very difficult in some cases.
c2
Many of the teeth are hard to identify and the centre is also uncertain.
d1
Gear is damaged. Several of the teeth are hard to identify. The centre is clear in one of the CT slices.
e1/e2
Both these gears have all their teeth present, though quite damaged in some instances. The angular information should give a fairly good idea of the original variation of angle, but the radial information would not give a good estimate for the original variation of radius. This is because corrosion has removed most of the gear tips in some cases. So the variation of radius mostly represents corrosion differences at the tips, though their angular position has not significantly changed. Even the angular data is not that reliable due to difficulties identifying exactly the right angle (point of the tip) due to corrosion.
The hub at e2 (based on our interpretation): This is square with a circle inside it. We believe that the square hub is attached to the pipe that links e2 and e5 and is the way that e2 is attached to the pipe so that it is secure and does not rotate. The circle inside this square hub is the inner shaft that carries the output from e6 to e1 (and thence to b3 and the Front Dial).

e3/e4

The centre has been determined in the same way as e5 and e6 by looking at the pentagon on e5 and the central arbour and pipe on Axis E. Inevitably, it is a bit of a compromise but believed to be quite good.

f1

The centre is hard to determine accurately since it looks blurred. We also tried with features in a different CT slice - the bearing in the main plate. This gives a roughly consistent centre. But it could be a few millimetres in any direction.

Wright finds "a run of 21 teeth". We find a clear run of 24. Wright also sees "...two points on a projecting tongue of metal on the opposite side...". We think we see these, but that they are not teeth because they are not at the right radius from the centre (wherever the centre actually is). So we have not included them.

From images of Axis F: The axis can be seen as a hole just above the centre and just to the right. Gear f2 can be seen as a row of six teeth. We think that a single tooth of f1 can be seen below f2 and slightly to the right (and maybe even three more teeth). It is likely that this is what Wright identified as another couple of teeth. In any case, the CT shows that this part has been displaced from its correct position and should be discounted as data.

f2

All the teeth are present, though some are very difficult to see and cannot all be seen in a single CT slice.

g1

All the data points are good. The square hub at the centre is fairly clear. There is a part of the gear that has broken off, which contains some extra damaged teeth. None of these have been used as data since they are very clearly in the wrong place.

g2

Nearly all the tips are present, but the central square hub is not clear. It was centred using the hub in a parallel CT slice

h1

Much of this gear is damaged or missing. The hub is hard to locate and a different CT slice has been used to determine the best guess for the centre.

h2

All the teeth are identifiable, though some are a bit faint. The centre is difficult to locate exactly.

l1

The data is just one sector.

l2

There is a prominent sector of teeth. These teeth are well-formed and regular. A second sector is much more doubtful. The gear is severely cracked and the tips are very worn. There is a clear central hub and the centre is based on this.

o1

About one third of the teeth are present and in good condition. The margins of the teeth have the “characteristic” higher density and it looks as if a strip of teeth might be about to tear off as in the gear r1 in Fragment D. The centre was found from a round arbour in a parallel CT slice.

q1

This is the small contrate gear in the Moon Phase Mechanism in Fragment C.

r1

This is the gear in Fragment D. It is certain that there is only one gear in Fragment D. The outside strip is only present where the gears attached to the main body are rudimentary and conversely, where they are not rudimentary, there is no outside strip. The inside of the torn-off strip of gears matches, tooth for tooth, the shape of the outside of the remaining teeth attached to the body of the gear - where the inside remnant is pointed, the outside one is also; where it is rounded, the outside is also etc. It is clear also in the cross-section that the hub of the gear has split where a pin went through the arbour.

The Gear Trains

The existing gear trains appear to be based almost completely (except for the amplitude of the lunar anomaly) on the following Babylonian period relationships:

"Metonic"

19 year = 235 synodic months = 254 sidereal months

"Saros"

223 synodic months = 239 anomalistic months

After a Saros cycle the Moon has returned to its same position relative to the nodes – the points where the Moon's orbit crosses the plane of the Sun-Earth orbit, and in between has come close enough to a node alignment with the Sun for 38 eclipse possibilities for both sun and moon[29].

When a gear with p teeth engages with a gear of q teeth, then the second gear rotates at a rate of –p/q times the first. In the following we have suppressed the minus signs for simplicity.

The Main Drive and the Date Pointer

The Mechanism was driven through crown gear a1, presumably operated by a hand-turned shaft, which drives the large four-spoked gear wheel b1. The outer shaft of b, on which b1 is fixed, probably turned a date marker on the Front Dial, which could also indicate the approximate position of the Sun in the Zodiac. One revolution of b1 represents a year. b2 is fixed to b1 and hence the rest of the Mechanism.

The Metonic Gearing

The outer shaft of b, indicating the date/mean Sun position rotates once per year. The Metonic dial has 5 full turns in 19 years. So required ratio is:

$$\frac{5}{19}$$

The gearing b2-l1+l2-m1+m2-*n1* has the ratio:

$$\frac{64}{38}\times\frac{53}{96}\times\frac{15}{53} = \frac{2\times 32}{2\times 19}\times\frac{53}{3\times 32}\times\frac{3\times 5}{53} = \frac{5}{19} \quad \text{as required.}$$

One turn on the "Callippic " dial of four Metonic cycles of five turns each of the axis *n* is generated by *n2-p1+p2*-01 with ratio:

$$\frac{15}{60}\times\frac{12}{60} = \frac{1}{4}\times\frac{1}{5} \quad \text{as required.}$$

The Saros Gearing

The Saros dial has four full turns in 223 synodic months. There are 235 synodic months in 19 years, so the required ratio is

$$\frac{4}{223}\times\frac{235}{19}$$

The gearing b2-l1+l2-m1+*m3*-e3+e4-f1+f2-g1 has the ratio:

$$\frac{64}{38}\times\frac{53}{96}\times\frac{27}{223}\times\frac{188}{53}\times\frac{30}{54}$$
$$=\frac{2\times 32}{2\times 19}\times\frac{53}{3\times 32}\times\frac{27}{223}\times\frac{4\times 47}{53}\times\frac{2\times 3\times 5}{2\times 27}=\frac{1}{19}\times\frac{4\times 47}{1}\times\frac{1}{223}\times\frac{5}{1}$$
$$=\frac{4}{223}\times\frac{235}{19}$$

as required.

One turn on the "Exeligmos" dial is three Saros cycles of four turns each of axis g and is generated by g2-h1+h2-i1 with ratio

$$\frac{20}{60}\times\frac{15}{60}=\frac{1}{3}\times\frac{1}{4}\quad \text{as required.}$$

The Sidereal Month

The Moon is carried around the Sun with the Earth, so in 19 years there are 19 extra rotations of the Moon relative to the Zodiac in addition to the 235 "synodic" rotations, the origin of the period relation 254 sidereal months = 235 synodic months.

Gear e2, and also (we believe) the outer shaft of axis e with e5 attached, rotates once every sidereal month. This requires the ratio:

$$\frac{235}{19}\times\frac{254}{235}=\frac{254}{19}$$

The gearing b2-c1+c2-d1+d2-e2 has the ratio:

$$\frac{64}{38}\times\frac{48}{24}\times\frac{127}{32}=\frac{2\times 32}{2\times 19}\times\frac{2\times 24}{24}\times\frac{127}{32}=\frac{2\times 127}{19}=\frac{254}{19}\quad \text{as required.}$$

The Lunar Anomaly

Hipparchos developed two equivalent lunar theories based on the idea that the moon exhibits a simple periodic anomaly. In the first eccentric theory, the Moon rotates at the rate of the mean *sidereal month* about an eccentre that in turn rotates about the Earth at the rate of the *anomalistic month*. In the second theory, the Moon rotates on an epicycle at the rate of the *anomalistic month* relative to a deferent circle that rotates at the rate of the *sidereal month*. Apollonius of Perga (c. 240-190 BC) had already shown[30,31] that these are equivalent using (in today's language) a simple vector diagram and the commutativity of vector addition. The theory introduces a harmonic variation into the Moon's motion that has the period of the *anomalistic month*.

The lunar display is again driven from b2. The train b2-c1+c2-d1+d2-e2 results in e2 turning with the period of the sidereal month (i.e. position of the Moon relative to the Zodiac). The subsequent gears in the train introduce no further multiplication or division, but introduce a quasi-sinusoidal variation in the Moon's motion *at the period of the anomalistic month* – i.e. modelling the "first anomaly". The sequence starts with an outer shaft, which is free to turn within e3, connecting e2 to e5. The train is then e5-k1+k2-e6+e1-b3 and through to the lunar pointer and phase mechanism on the Front Dial. The link e6+e1 is via the inner shaft of e. A pin-and-slot device on gears k1 and k2, clearly seen in the CT, provides the variation. This device was originally identified by Wright (reference 5 of main text), although he rejected its use as a lunar mechanism. The purpose of mounting the pin-and-slot mechanism on the gear e3 is to change the period of variation from sidereal month, which would occur if k1 and k2 were on fixed axes, to the anomalistic month by carrying the gears at a rate that is the *difference* between the rates of the sidereal and anomalistic months – i.e. at the rate of rotation of the Moon's apogee. We show that this models Hipparchos' lunar theory.

All rotations will be measured in rotations per year with clockwise rotations on the Front Dial being positive. Negative rotations on the Back Dials are clockwise. All the dials of the Antikythera Mechanism run clockwise. Let ω_{Syn} be the rotation of the synodic lunar cycle; ω_{Si} the rotation of the sidereal lunar cycle; ω_a the rotation of the Moon's return to an apse (i.e. the rotational speed corresponding to the anomalistic month); ω_n the rotation of the line of apses (apogee and perigee) of the Moon's orbit.

From the Metonic and Saros relationships, we get:

$$\omega_{Syn} = \frac{235}{19} \approx 12.368$$

$$\omega_{Si} = \frac{254}{19} \approx 13.368$$

$$\omega_a = \frac{239}{223} \times \omega_{Syn} = \frac{239}{223} \times \frac{235}{19} \approx 13.256$$

$$\omega_n = \omega_{Si} - \omega_a = \frac{254}{19} - \frac{239}{223} \times \frac{235}{19} = \frac{1}{19 \times 223}\{254 \times 223 - 239 \times 235\} = \frac{56642 - 56165}{19 \times 223} = \frac{477}{19 \times 223}$$
$$\approx 0.1126$$

The gearing from the main drive wheel to e3 is b2-l1+l2-m1+*m3*-e3 and has the ratio:

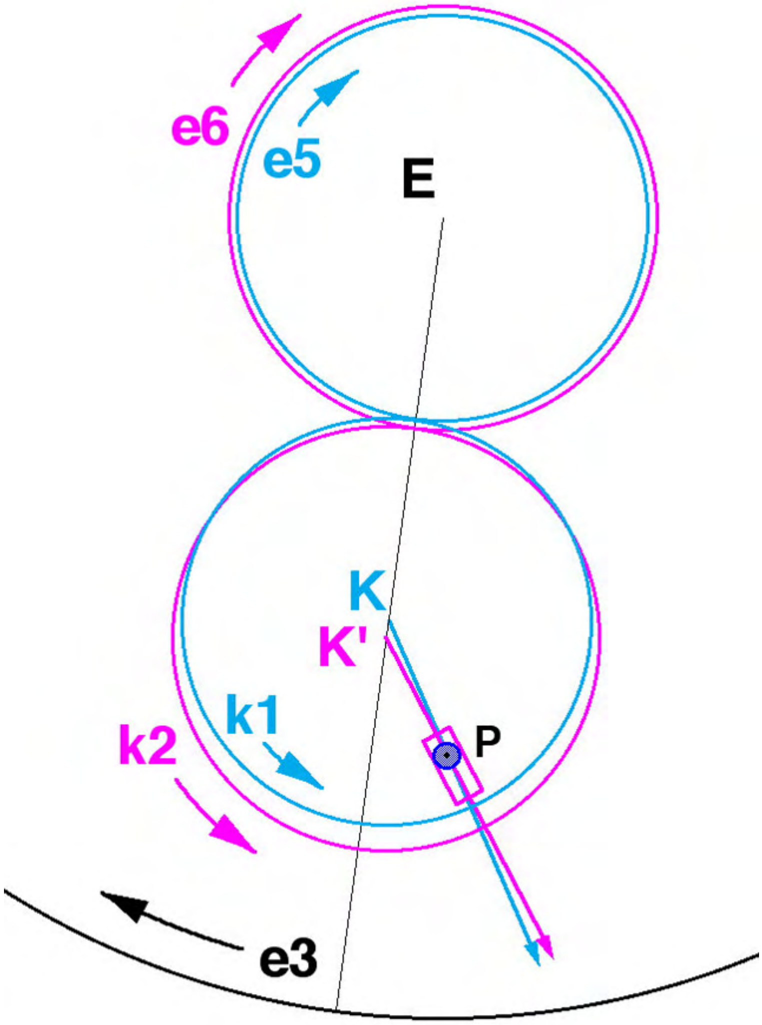
$$-\frac{64}{38} \times -\frac{53}{96} \times -\frac{27}{223} \quad = -\frac{2 \times 32}{2 \times 19} \times \frac{53}{3 \times 32} \times \frac{3 \times 9}{223} = -\frac{53 \times 9}{19 \times 223} = -\frac{477}{19 \times 223}$$

So e3 rotates at the rate of rotation of the line of apses (angular speed ω_n). This is how the prime factor 53 arises in the tooth counts. Here we restore the minus signs to be certain of getting the sense of rotation of the gears correct. The minus sign here means that e3 rotates clockwise if viewed from the back of the Mechanism.

In what follows, it is essential to distinguish absolute and relative rotations in the epicyclic system. In order to calculate the rotations on the epicyclic system, we need to look at the rotations relative to e3 since the gears on e3 are on axes that are fixed relative to e3. We can therefore use the basic properties of fixed-axis gearing to calculate the rotations relative to e3.

As discussed above, in absolute terms, e5 rotates at the rate of $-\omega_{Si}$ and e3 rotates at the rate of $-\omega_n$. So, relative to e3, e5 rotates at the rate of $-\omega_{Si} - (-\omega_n) = -\omega_{Si} + (\omega_{Si} - \omega_a) = -\omega_a$. Since e5 and k1 both have 50 teeth, relative to e3, k1 rotates at the rate of ω_a. This is the critical factor that ensures that the anomaly introduced by the pin-and-slot mechanism has the period of the anomalistic month, as required by Hipparchos' lunar theory.

Pin-and-Slot Mechanism



In the diagram, e3, e5 and e6 rotate about E, k1 rotates about K and k2 rotates about K′. The pin-and-slot mechanism on k1/k2 introduces a small quasi-sinusoidal variation in k2's rotation rate. As k1 rotates, the pin on its face engages with the slot on k2. k2 rotates about K′ and is forced to rotate by the pin-and-slot arrangement. The difference between the blue and magenta arrows shows the magnitude of the variation introduced. The period of rotation of k2 relative to e3 is the same as k1—in other words the anomalistic month.

Let A(x) be a function (the "anomaly function") that is the difference between the rotation of k2 and that of k1 after x rotations. This has the correct geometric form for Hipparchos' eccentric lunar theory and we demonstrate that it acquires the correct period by means of its eccentric placement.

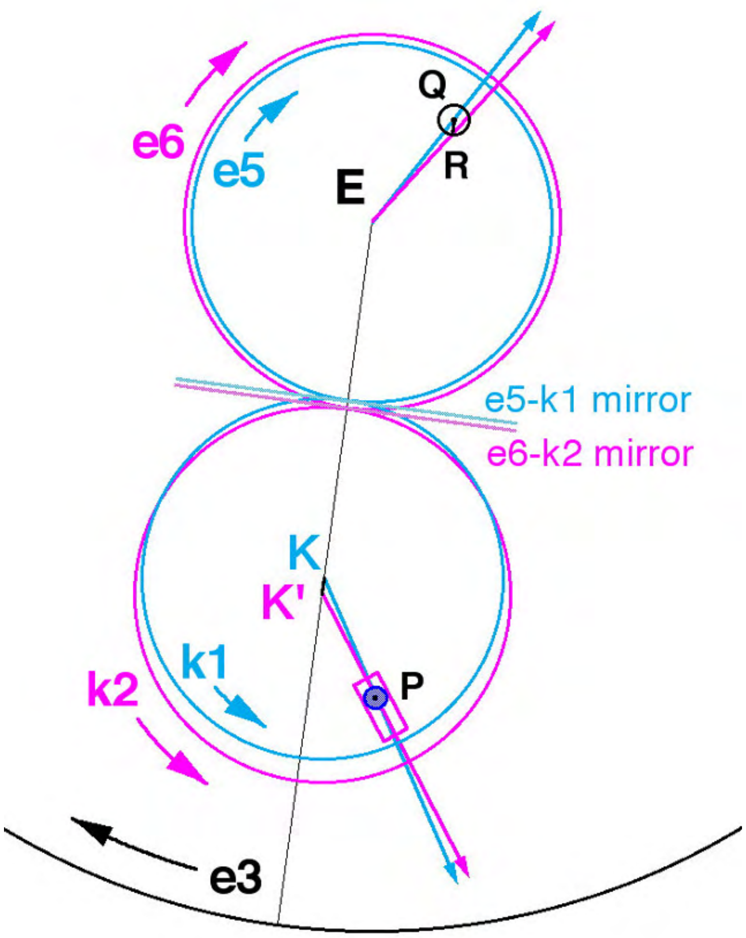
We assume that the origin of x is set so that A(0) = A(0.5) = A(1) = 0. Since rotations are no longer constant when the pin-and-slot mechanism takes effect, we need to introduce a time parameter (expressed in years). Recall that rotations are measured in rotations per year. The rotation of k1 relative to e3 at time t is $\omega_a t$. The rotation of k2 relative to e3 at time t is then given by: $\omega_a t + A(\omega_a t)$. Since k2 and e6 have the same number of teeth, relative to e3, they rotate at the same rate in opposite directions. So the rotation of e6 relative to e3 at time t is: $-\omega_a t - A(\omega_a t)$

Returning now to absolute rotations, the absolute rotation of e6 is its rotation relative to e3 plus the absolute rotation of e3. In other words:

Rotation of e6 at time t = $-\omega_a t - A(\omega_a t) + (-\omega_n t) = -\omega_a t - A(\omega_a t) + (\omega_a - \omega_{Si})t = -(\omega_{Si} t + A(\omega_a t))$

So e6 rotates at the rate of the mean sidereal month plus an eccentric anomaly that has the period of the anomalistic month. This is Hipparchos' first lunar theory. In the Mechanism, e6 is linked by a shaft to e1 that engages with b3 (that has the same tooth count as e1) and thence to the lunar indicators on the front dial.

Geometric Proof



We now also give a geometric proof using elementary methods, and so in principle accessible in ancient Greece, establishing that the pin-and-slot mechanism is equivalent to Hipparchos' epicyclic lunar theory.

In the diagram, Q is a point obtained by reflecting the centre of pin P in a rotating mirror defined by the line that is tangent to both pitch circles of e5 and k1. This is referred to as the "e5-k1 mirror" and Q as the "mirror pin". From Q construct a line segment QR (shown in black) that is parallel to and of the same length as the line segment KK'. The line ER will be our output.

First we establish that the mirror pin Q rotates as if fixed to wheel e5. As established previously, e5 rotates at the rate of $-\omega_{Si}$, the epicyclic table e3 rotates at the rate $-\omega_n$; and the rate of rotation of e5 relative to e3 is $-\omega_a$. In addition, P is fixed to k1 and so rotates at the rate of ω_a relative to e3. Therefore its mirror Q rotates at the rate of $-\omega_a$ relative to e3, which is the same rate as the rotation of e5 relative to e3. Also, EQ = KP, so Q moves on a circle centred at E. Thus Q is fixed to e5 and rotates at the rate ω_{Si}.

In order to show that the mechanism satisfies Hipparchos' lunar theory we want to show that the rotation of R about Q relative to e5 is ω_a and that R appears fixed to e6.
QR is defined to be parallel to KK′. So in absolute terms it rotates at the rate of - ω_n (the rate of rotation of e3). But Q rotates about E at the absolute rate ω_{Si}. So, relative to e5, QR rotates at the rate $-\omega_n \omega_{Si}$ $\omega_{Si}\omega_n = \omega_a$.

It remains to show that R is fixed to e6. The triangle EQR is congruent to the mirror image of the triangle PKK′. This is because QR is defined to be equal to KK′ and EQ is equal to KP. Also QR is parallel to KK′, so angle K′KP is equal to angle RQE. Hence the angle between the blue and magenta arrows at Axis E is the same as the angle between the blue and magenta arrows at Axes K and K′ (in the other direction). In other words, ER mirrors K′P and so R appears to be fixed to e6. In fact it is not difficult to see that ER is the mirror of K′P in the e6-k2 mirror.

This establishes that the pin-and-slot mechanism models Hipparchos' epicyclic lunar theory — subject only to the correct eccentricity of axis K relative to axis K′.

An Alternative Period for the Lunar Anomaly?

The period relation 223 synodic months = 239 anomalistic months is not surprising for the period of construction. But if we try to associate the conception of the mechanism with Hipparchos it might be wondered why the Mechanism does not use the relation 251 synodic months = 269 anomalistic months that he is believed to have preferred (e.g. G.J. Toomer's Note 10 on page 176 of his edition of Ptolemy's Almagest, Princeton University Press, 1998 – reference 15 in the main text). The period of rotation of the apsides given by the 223/239 relation is 8.8826 years, and by the 251/269 relation is 8.8479 years, which is certainly closer to the modern value 8.8504 years. If Hipparchos was involved, then presumably it was the possibility of the combination of the gearing with the Saros train that appealed and which also avoided a large (and perhaps difficult to accommodate) additional prime 251 gear.

Further References

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