Calendars with Olympiad and Eclipse Prediction on the Antikythera Mechanism

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Previous research on the Antikythera Mechanism established a highly complex ancient Greek geared mechanism with front and back output dials[1-6] (Figure 1). The upper back dial is a 19-year calendar, based on the *Metonic* cycle, arranged as a five-turn spiral[1,6,7]. The lower back dial is a Saros eclipse-prediction dial, arranged as a four-turn spiral of 223 lunar months, with *glyphs* indicating eclipse predictions[6]. Here we add new and surprising findings concerning these *Back Dials*. Though no month names on the Metonic calendar were previously known, we have now identified all twelve months, which are unexpectedly of Corinthian origin. The Corinthian colonies of northwestern Greece or Syracuse in Sicily are leading contenders—the latter suggesting a heritage going back to Archimedes. Calendars with *excluded days* to regulate month lengths, described in a first century BC source[8], have hitherto been dismissed as implausible[9,10]. We demonstrate their existence in the Antikythera calendar and in the process establish why the Metonic Dial has five turns. The upper subsidiary dial is not a 76-year Callippic Dial as previously thought[7], but follows the four-year cycle of the *Olympiad* and its associated *Panhellenic Games*. Newly identified index letters at the bottom of each glyph on the Saros Dial show that a previous reconstruction

needs modification[6]. We explore models for generating the unusual glyph distribution and show how the eclipse times appear to be contradictory. We explain the four turns of the Saros Dial in terms of the *Full Moon Cycle* and the lower subsidiary *Exeligmos Dial* as indicating a necessary correction to the eclipse times in the glyphs. The new results reveal an unforeseen provenance and additional sophistication in the Antikythera Mechanism. The Metonic Calendar, Olympiad Dial and eclipse prediction link the cycles of human institutions with the celestial cycles embedded in the Mechanism's gearwork: a microcosm of the temporal harmonization of human and divine order in the Classical world.

This extraordinary astronomical mechanism from about 100 BC employed bronze gears to make calculations based on cycles of the Solar System[1]. Its corroded remains were recovered in 1901 by Greek sponge divers, and are now split into eighty-two fragments—seven larger fragments (A–G) and seventy-five smaller fragments (1-75)[6].

Scientific data, gathered in 2005, included still photography, digital surface imaging[11] and, crucially for this study, microfocus X-ray computed tomography (CT)[6,12].

The main upper back dial is now established as a *Metonic Calendar*[1,6,7] (Box 1).

The calendar dial bears hidden inscriptions, only viewable using X-ray CT. We have now identified all twelve months of this calendar (Figure 1), providing conclusive evidence of the regulation of a Greek civil calendar by a Metonic cycle, and clues to the instrument's origin. While the Babylonian calendar followed a Metonic cycle from about 500 BC, it has commonly been assumed that the intercalary months of the numerous lunisolar calendars of the Greek cities were determined arbitrarily—Metonic and Callippic cycles (Box 1) only being used by astronomers[13]. The month names on the Metonic spiral, however, belong to a regional calendar unassociated with technical astronomy, suggesting that it was common for Greek civil calendars to follow the

Metonic cycle by about 100 BC. The inscriptions show that not only the names and order of the months were regulated, but also which years had thirteen months, which month was repeated in these years, and which months had 29 or 30 days. The rules are similar to those given by the first century BC writer Geminos[8], whose accuracy has hitherto been in doubt[9,10]. Years are numbered 1 to 19, and intercalary months are spread as evenly as possible over the cycle, such that each year begins with the first new moon following solstice or equinox[14]. In a Metonic cycle 110 of the 235 months must have 29 days (Box 1). The divisibility of both 110 and 235 by 5 explains the five turns of the spiral: months on the same *radius* across all five turns are equal in length. The numbers on the inside of each 29-day radius indicate which day in these months is skipped (Figure). The skipped days are spread uniformly at intervals of 64 or 65 days across successive Metonic periods, improving on Geminos' scheme.

The month names and order in Greek regional calendars vary widely[15]. The months on the Mechanism belong to one of the Dorian family of calendars, with practically a complete match (eleven or twelve names) with Illyria and Epirus in northwestern Greece and with Corcyra (Corfu)—all Corinthian colonies. The calendars of Corinth and its other important colonial foundation, Syracuse, are poorly documented. Seven of the Mechanism's months, however, coincide in both name *and sequence* with the calendar of Tauromenion in Sicily, which was probably originated by settlers from Syracuse in the fourth century BC. The Mechanism's calendar is thus from Corinth or one of its colonies. Moreover, the estimated date of the Mechanism falls after the Roman devastation of Corinth (146 BC) and Epirus (171-168 BC). Syracuse's candidacy suggests a possible mechanical tradition going back to Archimedes (died 212 BC), who invented a planetarium described by Cicero[16] (first century BC) and wrote a lost book on astronomical mechanisms[17].

The subsidiary dial (Figure 1) inside the Metonic spiral was formerly believed to be a 76-year *Callippic Dial*[7] (Box 1). We have now established from its inscriptions that it displays the 4-year *Olympiad cycle*—a suggestion made previously for the main upper back dial[18]. The four sectors are inscribed anticlockwise with each sector containing a year number and two of the following panhellenic games: the ‘crown’ games of Isthmia, Olympia, Nemea and Pythia and two lesser games: Naa (at Dodona) and a second game not yet deciphered[19,20]. As biennial games, Isthmia and Nemea occur twice. The Olympiads were a common framework for chronology, with years normally beginning in midsummer. But here the year must start between early autumn and early spring because the Isthmian Games are in the years preceding their usual positions in the cycle (Figure 3). Several month names favour a start following the autumnal equinox. The design of the lunisolar calendar means that its year starts can vary by as much as a month relative to the solar year. The small (c. 8° - one month) offset of the dial ensured that the next Olympiad year would never start before the current year's games were over.

The *Olympiad Dial* must be turned from the existing gearing[6] at a rate of ¼ turns per year. Underneath the Olympiad Dial are the remains of an isolated gear with 60 teeth. Engaging this with a single additional gear with 57 teeth on the shaft of the Metonic pointer provides the correct anticlockwise rotation. Sizing this gear, with tooth pitch equal to the 60-tooth gear, gives a gear radius exactly as required by the interaxial distance: strong supporting evidence both for the Olympiad Dial and this mechanical arrangement. The *“76 years”* inscription (Figure 1) and other factors favour a Callippic Dial as a second subsidiary, symmetrical with the Olympiad Dial—though loss of evidence means confirmation is unlikely. Might a fourth subsidiary, symmetric with the Exeligmos Dial (Figure 1) complete the dial system? An existing shaft here does not penetrate the back plate and does not appear to rotate at any meaningful rate. So this seems doubtful.

We have increased the number of identified eclipse glyphs[6] (Figure 4) from 16 to 18. The decoding of these glyphs is extended here with the observation of $\mathrm{N}\backslash^{\mathrm{Y}}$ abbreviating ΝΥΚΤΟΣ (night) for solar eclipses and *index letters* at the bottom of each glyph in *alphabetical order*. These mean that the Saros Dial starts at the top (as initially suggested[4]) with index letter ‘A’ rather than at the bottom (as in a later model[6]). With any other dial start, extrapolation of the index letters back to the first glyph would force them to begin in the middle of the alphabet. The alphabetic index letters also constrain the number of glyphs: a previous reconstruction has too many to be correct[6]. If the 18 glyphs are aligned with lunar and solar eclipses in the last four centuries BC[21], they give a perfect match for 100 start dates, suggesting an excellent prediction scheme. However, we do not believe that the glyphs were based primarily on observations: they contained an estimated 65 eclipses and extensive observations over many decades would miss a high proportion of these (Box 2). The glyphs appear to have been generated by a scheme of eclipse possibilities, similar to those from Babylonian astronomy that exhibit an 8-7-8-7-8-pattern[22,23] (Box 2), which can be generated from a simple arithmetical model of *nodal elongation* at syzygy[24]. However, these schemes have 5- or 6-month gaps between all predictions, whereas the index letters imply that the Antikythera scheme has some longer gaps. We consider kinematic models for glyph generation, defined by different *nodal elongation criteria* and computable with similar technology to the Mechanism or an arithmetic method. With suitable criteria, these models generate all the Antikythera glyphs and no glyphs which are observably absent—whether the model uses *mean months* or *first anomaly months* (incorporating lunar and solar anomalies). However, none of these models exactly match the index letters. Because of parallax, the likelihood of a solar eclipse depends not only on a syzygy’s nodal elongation, but also on whether it occurs North or South of the ecliptic, as was recognized in antiquity[10]. Introducing this asymmetry produces models (both using *mean* and *first anomaly months*) that exactly match all 18 definite glyphs with a single index letter error, caused by the only instance where the models generate two adjacent lunar glyphs—something which never featured in Babylonian schemes. Discarding the second of these gives a perfect match. The *first anomaly* model only works assuming a lower lunar eccentricity than that of the Mechanism, so the mean month model seems more likely. These kinematic models provide a persuasive explanation of the glyph sequence.

Matching the glyph times with actual eclipse times over the last four centuries BC has not discovered close correlations, suggesting they were not accurate. Six out of the eight definite glyph times that include ‘$\mathrm{H}\backslash^{\mathrm{M}}$’ (‘ΗΜΕΡΑΣ’, ‘of the day’) (Figure 4) conform to a model that calculates glyph times from mean lunar months—but the other two do not. Introducing the first lunar and solar anomalies into the analysis of the glyph times should reveal a periodic cycle of corrections following the *Full Moon Cycle* (Box 2). But the glyph times do not conform to this pattern. We conclude that the process of generation of glyph times was not sound and may remain obscure.

We have discovered why the Saros Dial is a four-turn spiral: each quarter-turn of the dial covers a *Full Moon Cycle* (Box 2). So the apparent diameter of the Moon, which mediates the duration and type of an eclipse, is indicated by the angle of the pointer within each quarter turn of the dial.

Each Saros series eclipse occurs about 8 hours later in the day (Box 2). After three Saros cycles (the *Exeligmos*) the eclipse is at nearly the same time of day. The Exeligmos Dial is divided into three sectors, with no inscription in one sector and the numbers 8 and 16 in successive sectors (Figure 1). We here conclude that these numbers tell the user how many hours to add to the glyph time to get the time of the predicted eclipse.

The inscriptions reveal that the Antikythera Mechanism was not simply an instrument of abstract science but exhibited astronomical phenomena in relation to Greek social institutions. It is totally unexpected that it was made for use in northwestern Greece or Sicily, rather than Rhodes as is often suggested. The Metonic calendar, the Olympiad Dial and the Saros eclipse prediction scheme add new insights into the sophisticated functions of this outstanding landmark in the history of technology.

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Figure 1 | The Back Dials

Text in red is traced from X-ray CT; text in blue is reconstructed.

The **Metonic Dial** is the main upper dial: a 19-year calendar with 235 months round a five-turn spiral. The newly identified Corinthian months, written over two or three lines in each cell, are:

1.	ΦΟΙΝΙΚΑΙΟΣ	7.	ΑΡΤΕΜΙΣΙΟΣ
2.	ΚΡΑΝΕΙΟΣ	8.	ΨΥΔΡΕΥΣ
3.	ΛΑΝΟΤΡΟΠΙΟΣ	9.	ΓΑΜΕΙΛΙΟΣ
4.	ΜΑΧΑΝΕΥΣ	10.	ΑΓΡΙΑΝΙΟΣ
5.	ΔΩΔΕΚΑΤΕΥΣ	11.	ΠΑΝΑΜΟΣ
6.	ΕΥΚΛΕΙΟΣ	12.	ΑΠΕΛΛΑΙΟΣ

The numbers *A (1), E (5), Θ (9), ΙΓ (13)…* around the inside of the spiral specify the excluded days to be skipped in each of the five 29-day months on the same radius.

The **Olympiad Dial** (Figure 3) is the upper right subsidiary dial, which is identified here for the first time. It is a four-year dial, representing the cycle of the *Panhellenic Games*, a central part of ancient Greek culture and a common basis for chronology.

The Callippic Dial is the hypothetical upper left subsidiary dial, which follows a 76-year cycle, indicted on the Back Door inscriptions (Figure 2).

The **Saros Dial** is the main lower dial: an 18-year 223-lunar month scale over a four-turn spiral, for predicting eclipses. Predictions are shown in the relevant months as ***glyphs*** (Figure 4), which indicate lunar and solar eclipses and their predicted times of day. This new reconstruction has 51 glyphs, specifying 38 lunar and 27 solar eclipses. The glyph times are incomplete since their generation remains obscure. The divisions on the inside of the dial at the cardinal points indicate the start of a new *Full Moon Cycle* (Box 2).

The **Exeligmos Dial** is the lower subsidiary dial: a 54-year *Triple Saros* dial, whose function is now understood. The first sector is blank (representing 0) and the following are labelled with numbers H (8) and Iς (16). The dial pointer indicates which number must be added to the glyph times in hours to get the eclipse times.

Figure 2 | The ‘Instruction Manual’

Previously identified inscriptions[1,6] reveal remnants of an *‘Instruction Manual’*, describing the Mechanism's cycles, dials and functions, as seen in two examples from the Back Door.

a, *Polynomial Texture Mapping* of Fragment 19 shows fine surface detail, with text about 2 mm high. Highlighted are *‘76 years, 19 years’* for the Callippic and Metonic cycles (Box 1); and *‘223’*, for the Saros cycle (Box 2).

b, X-ray CT of Fragment E reveals more text about 2 mm high. Highlighted are *‘on the spiral subdivisions 235’*, confirming the Metonic Dial (Box 1); and *‘excluded days 2…’*, the final *‘K’* presumably standing for the number 20—part of the 22 excluded days round each of the five turns of the Metonic calendar—though *‘B’* that would complete *‘KB’* (22) remains speculative.

Figure 3 | Deciphering the Metonic & Olympiad Dials

a, CT of Fragment B, showing the Metonic Dial. The scales are 7 mm wide and the text 1.7 mm high. b, Text in red was traced from the CT—just enough being deciphered to discover all the month names. Text in blue is reconstructed. The months here are ΜΑΧΑΝΕΥΣ and ΔΩΔΕΚΑΤΕΥΣ. c, An X-ray CT slice through Fragment B, showing the Olympiad Dial. LΔ and NEMEA can be seen faintly on the left-hand side. d, The four sectors of the Olympiad Dial are labelled LA, LB, LΓ and LΔ—Years 1, 2, 3 and 4. Outside are the Panhellenic Games: Year 1: ΙΣΘΜΙΑ, ΟΛΥΜΠΙΑ; Year 2: NEMEA, NAA; Year 3: ΙΣΘΜΙΑ, ΠΥΘΙΑ; and Year 4: ΝΕΜΕΑ, Undeciphered text. To the right of the dial are the numbers (6) and IA (11) for the *excluded days*.

Labels to go beside images from top to bottom: Glyph 20, Glyph 25, Glyph 26, Glyph 131, Glyph 178

Figure 4 | The Glyphs

A selection of the 18 known eclipse prediction glyphs. Most of the glyph symbols were previously decoded. Σ = ΣΕΛΗΝΗ (Moon); Η = ΗΛΙΟΣ (Sun); $H\backslash^{M}$ = ΗΜΕΡΑΣ (of the day); $\omega\backslash^{\rho}$ = ωρα (hour) and the text that follows is the eclipse time in hours. Here we add $N\backslash^{Y}$ = ΝΥΚΤΟΣ (of the night) and the identification of *index letters* at the bottom of each glyph in *alphabetical order*. In the consecutive Glyphs 20, 25 and 26 the index letters E, Z and H can be seen. (Z is always written here as an I with long serifs.) The index letters have profound consequences for the design of the glyph sequence.

Box 1 | Metonic & Callippic Calendars

From ancient times, astronomers have distinguished a number of different orbital periods of the Moon. The *sidereal month* is the period of the Moon in its passage from a particular star back to the same star (27.32 days); the *synodic (or lunar) month* is the period from one phase of the Moon back to the same phase (29.53 days).

Ancient calendars were based either on the synodic lunar cycle, the solar cycle or both. Twelve lunar months is about 11 days short of a year, so calendars on this basis do not remain synchronized with the seasons. Attempts to rectify this meant finding integer periods of years, which are also integer numbers of lunar months. One of the most accurate of these is the 19-year cycle of 235 lunar months, attributed to Meton of Athens in the 5th Century BC. The *Metonic Cycle* is one of the two basic cycles that underlie nearly all the known gearing of the Antikythera Mechanism.

The length of the mean lunar month (29.53 days) can be well approximated in a 235-month calendar by making 125 of these *full* 30-day months and 110 *hollow* 29-day months. 19 12-month years would make 228 months, so 7 years need to contain a thirteenth *intercalary* month in order to make up the 235 months of the 19-year period. Based on these ideas, artificial lunisolar calendars can be devised that as far as possible evenly distribute both the hollow months and the 13-month years over the 19-year period. 125 30-day months and 110 29-day months add up to 6,940 days, which is the period of the Metonic calendar.

In the 4th Century BC, Callippos pointed out that the 6,940 days of the Metonic Calendar implies a year length that is $^{1}/_{76}$ days longer than the length of a year, taken as $365^{1}/_{4}$ days. So he proposed an improvement on the Metonic Calendar, which is based on a 76-year period, consisting of four 19-year Metonic sub-periods minus a day. So the Callippic Calendar has (4 x 6,940) – 1 = 27,759 days.

Box 2 | Saros, Exeligmos and Full Moon Cycles

The Moon's orbit is inclined to the Earth's orbit, so Sun, Moon and Earth do not always line up well enough every month for an eclipse. The two points where the Moon's orbit meets the plane of the Earth's orbit (the ecliptic) are the *lunar nodes*. The passage of the Moon from a node back to the same node is the *draconitic month* (27.21 days). The Moon's speed of motion varies through the *anomalistic month* (27.55 days) because the Moon's orbit is elliptical.

The term *syzygy* is a collective noun for *New Moon* and *Full Moon*. Eclipses occur when a syzygy is sufficiently close to a node. Lunar eclipses occur at Full Moon and are visible over almost half the globe, whereas solar eclipses occur at New Moon and have limited geographical visibility. Eclipse prediction cycles rely on the coincidence of an integer number of synodic months with an integer number of draconitic months. Known from at least the 6th Century BC, the *Saros Cycle* is an eclipse prediction cycle of 223 lunar months—just over 18 years—that predicts repeating eclipses over many centuries. The Saros period is also close to an integer number of anomalistic months, which means that repeating eclipses have similar characteristics. In numerical terms the *Saros Cycle* is: *223 synodic months = 242 draconitic months = 239 anomalistic months*. The *Saros Cycle* is the second of the two astronomical cycles that are the basis for nearly all the known gearing in the Mechanism. It is a period of about 6,585$^{1}/_{3}$ days. The $^{1}/_{3}$ day means that the repeat eclipse after 223 lunar months is shifted by about 8 hours in time and, for solar eclipses (where geographical visibility is limited) by about 120° in longitude. The *Exeligmos Cycle* ('Turn of the Wheel') is a *Triple Saros Cycle*, which restores the repeat eclipse to the same time and longitude.

The *Eclipse Year* is the period the Sun takes (as seen from the Earth) to complete an orbit relative to one of the Moon's nodes (11.74 lunar months). It determines when eclipses can occur. It can be seen as the *beat period* of the synodic and draconitic months. The Saros Cycle implies that it has 242 – 223 = 19 cycles per Saros Period. *Eclipse seasons* occur twice as frequently since the Moon has two nodes—making 38 eclipse seasons in every Saros period with an average interval of 5.87 months.

Babylonian astronomers produced eclipse prediction schemes with 38 eclipse possibilities in the Saros period with an 8-7-8-7-8-pattern: 8 eclipses with 6-month gaps are followed by a 5-month gap and then 7 eclipses with 6-month gaps and so on. This pattern can be repeated many times to produce a scheme for predicting all eclipse possibilities over a long timescale.

The *Full Moon Cycle* is the cycle of changes in diameter of the Full Moon, which depends on how close the Moon is to the Earth in its elliptic orbit. It is the period the Sun takes (as seen from the Earth) to complete an orbit relative to the Moon's perigee (13.94 lunar months). It can be seen as the *beat period* of the synodic and anomalistic months. The Saros Cycle implies that it has 239 – 223 = 16 cycles per Saros Period.