ASTRONOMICAL MAGNITUDES

Why Can We See the Moon and Planets in Daylight?

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Several different types of "magnitude" are used in astronomy. Most commonly, <u>apparent</u> magnitudes are used to describe the brightness of stellar and solar system objects while <u>absolute</u> magnitudes are required to assess intrinsic brightness. A method is described to evaluate extended object versus sky brightness to calculate contrast, which explains why we can see Venus in daylight sky. There are specific procedures for calculating absolute magnitudes of asteroids, comets, and meteors. Factors such as color, albedo, and phase angles must, at times, be considered in their evaluation. The article will cover this topic with example calculations that can provide a firmer understanding of astronomical magnitudes.

1. SOME HISTORY

Greek astronomers were thought to be the first to classify stars by their brightness. Around 129 B. C. Hipparchus ranked the 1,080 stars in his catalogue in a simple way; the brightest were termed "first magnitude," the next brightest were "second magnitude" through to the "sixth magnitude." It was a rough estimation of star brightness. Our knowledge of Hipparchus is via Ptolemy, who, in the 2nd century A. D., utilized a similar stellar brightness scheme in his *Almagest.* Of the 1, 028 stars in his catalogue, 156 were given the descriptions of a little more or a little less than the integer values, but his precision was off by ± 0.6 magnitudes. The Persian astronomer Abu'l-Husayn al-Sufi re-estimated the *Almagest's* magnitude values in the tenth century. Tycho Brahe (1546-1601) added to the list of magnitudes, but with no precision improvement.

The introduction of the telescope allowed astronomers to see fainter stars. Galileo (1564-1642) estimated he could see magnitude +12 stars. However, it wasn't until William Herschel (1738-1822) that magnitude accuracy reached ± 0.2 magnitudes. He felt that brightness was solely based on stellar distance, and (unsuccessfully) attempted to delineate the Milky Way based on the distribution of stars of different magnitudes. To standardize magnitudes, Norman Pogson (1829-1891) proposed that a brightness ratio of 100:1 be equivalent to five magnitudes. In other words, a difference of one magnitude implied a brightness difference of 2.512. This Pogson ratio is now the accepted magnitude standard. Today, the eyeball has been supplanted by CCD photometry for very accurate magnitude estimation.

2. BASIC RADIATION PHYSICS

As a body gets hotter, its peak radiated wavelength gets smaller. Wien's Displacement Law illustrates this as follows:

$$\lambda_{peak} = \frac{0.002898}{T} \qquad (1)$$

Where λ_{peak} is in meters and T is in degrees Kelvin.

Note the visible range in Figure 1. The Sun's intensity peaks in the visible range while hotter stars peak in the ultraviolet, versus cooler stars in the infrared. The eyeball judges brightness via radiation in the visible spectrum, but total radiation is the area under each temperature curve in the diagram, most of which cannot be seen.
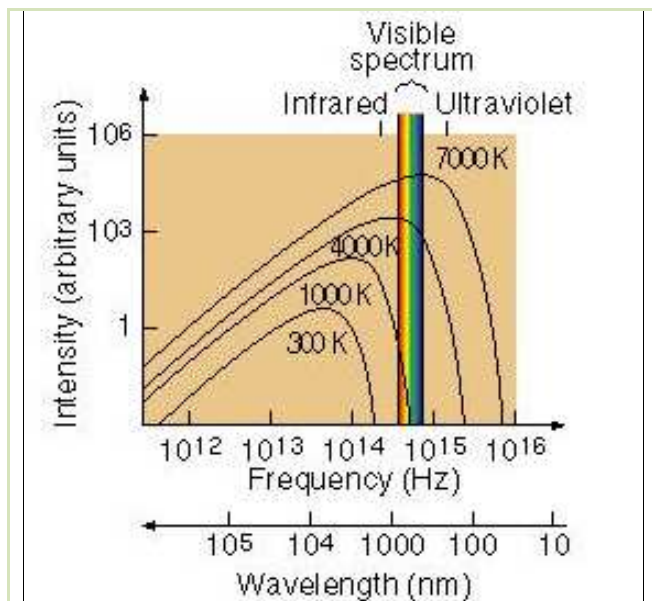

Figure 1. Wein's Displacement Law
http://feps.as.arizona.edu/outreach/bbwein.html

An important radiation energy/temperature relationship is the Stefan-Boltzmann Law:

$$F = \sigma T^4 \qquad (2)$$

Where F = flux, watts/m^2, σ = Stefan-Boltzmann constant, 5.67 x 10^{-8} watts/m^2/T^4, T in Kelvin.

The total luminosity from an object would be:

$$L = 4\pi R^2 \sigma T^4 \qquad (3)$$

Where L is luminosity in watts, and R is the object's radius. Radiation output depends on a body's temperature and size. The perceived radiation also varies inversely as the square of the distance from the object.
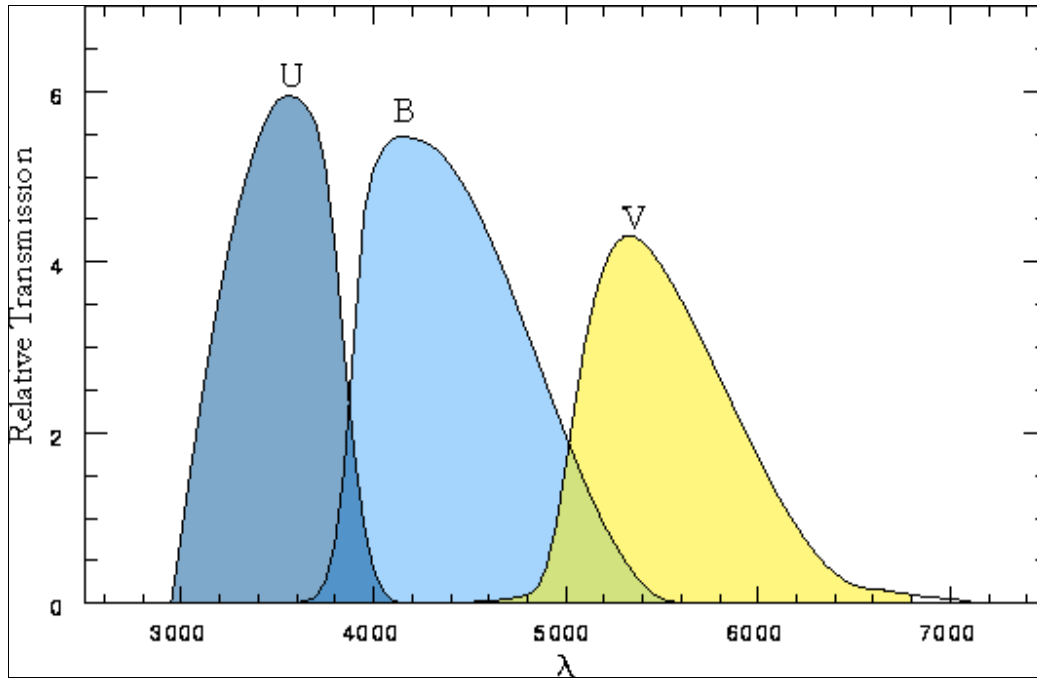
2.1 STAR COLOR

Since the radiation spectrum of stars depends on temperature, astronomers have devised a method of determining their temperature via filters. Figure 2 illustrates the wavelength response of standard U (ultraviolet), B (blue) and V (visual) filters. Magnitude measurements through such filters are used to create a color index (CI) for a star. Examples are U-B or B-V color indices with B-V the most used. The Sun's B-V index is 0.63. The color index is positive for stars redder than a white star (A0 V stars on the HD diagram) and negative for stars bluer than a white star. Equation 4 is an approximate equation to calculate temperature of stars similar to the Sun.

$$T \approx \frac{8540}{B - V + 0.865} \qquad (4)$$

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FIGURE 2. UBV Bandpass, Angstroms

3. ASTRONOMICAL MAGNITUDES

Astronomical magnitudes are dimensionless values. They are the ratio of a star's radiation, to that of some standard star. The standard star used is Vega. Thus, Vega has U = V = B = 0 values by definition so Vega has zero color indices. Bolometric magnitudes take into account all the radiation of a star, not just those in the visible range. Since the atmosphere blocks some wavelengths, bolometric magnitudes can only be easily obtained from space-borne instruments, like the Hipparcos device and will not be discussed in this article.

3.1. APPARENT MAGNITUDE

We define apparent magnitude as a logarithmic luminosity ratio of a body to some standard:

$$m = -2.5 \log(L / L_0) \tag{5}$$

Where m is the apparent magnitude, L is the luminosity of a body as determined through a V filter, and L_0 is the standard's luminosity, the luminosity of Vega. Vega has been assigned an m = 0 although its actual magnitude is 0.03. It follows that

$$m_1 - m_2 = -2.5 \log(L_1 / L_2) \tag{6}$$

Equation 5 can be rewritten as:

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$$L = L_0 x 10^{-m/2.5}$$

(7)

Figure 3 illustrates the apparent magnitude relationship to luminosity.

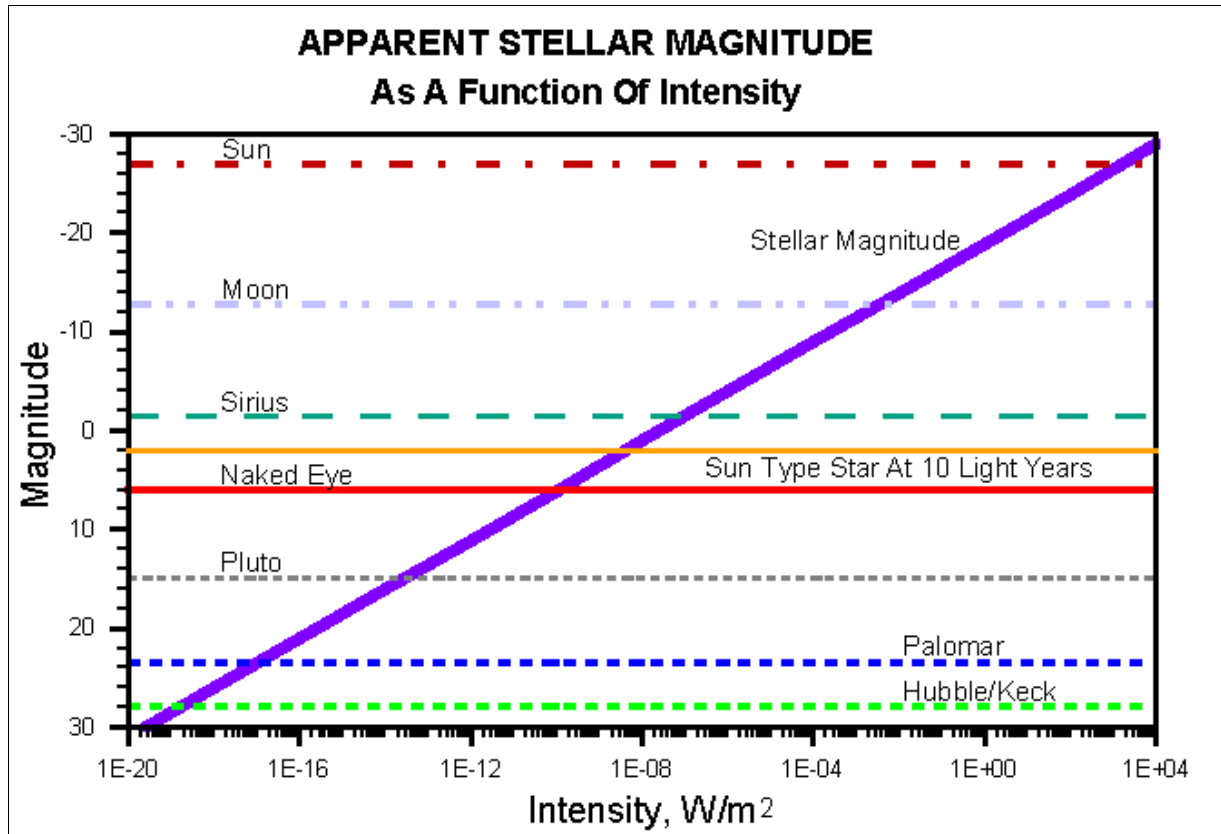**FIGURE 3. Luminosity versus Apparent Magnitude**

http://www.coseti.org/9006-023.htm

EXAMPLE 1. *The Sun is about 480,000 times more luminous than the full Moon. What is the difference in their apparent magnitude?*

L_{moon} =1, L_{Sun} = 480,000 $\Delta m = -2.5 \log(\frac{L_{Sun}}{L_{moon}}) = -2.5 \log(\frac{480,000}{1}) = -14.2$ magnitude.

The Sun's apparent magnitude is -26.8, while the full moon's is -12.6.

EXAMPLE 2. *The individual apparent magnitudes of two binary stars are +2 and +4. What is the combined apparent magnitude of the binary system?*

It's obvious we cannot simply add or subtract the two magnitudes. We must revert to their original luminosities using equation 7, which are additive, then convert the sum to magnitude using Equation 5.

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$$\frac{L_{binary}}{L_0} = \frac{L_{+2}}{L_0} + \frac{L_{+4}}{L_0} = 10^{-4/2.5} + 10^{-2/2.5} = 0.1836 \text{ from Equation 7}$$

$$m_{binary} = -2.5\log(\frac{L_{binary}}{L_0}) = -2.5\log(0.1836) = 1.840 \text{ magnitude}$$

EXAMPLE 3. *Sirius is 8.6 light years distant, with an apparent magnitude of -1.5. What would the apparent magnitude of the Sun be at Sirius' distance?*

There are 63,241 AU in a ly so Sirius is 543,873 AU distant. The Sun would be $1/(543,873)^2$ fainter at Sirius' distance. At 8.6 ly, the Sun's magnitude would change by

$$\Delta m_{Sun} = -2.5\log(\frac{1}{(543,873)^2}) = 28.7 \text{ magnitudes fainter}$$

So the Sun's apparent magnitude would be -26.8 + 28.7 = +1.9 at Sirius' distance, a star slightly brighter than Polaris.

EXAMPLE 4. *At opposition, Mars apparent magnitude m_1 was -1.6 and solar distance r_1 was 1.55 AU. At opposition seven years later, r_2 was 1.64 AU. Find Mars apparent magnitude, m_2.*

At opposition, the Mars-Earth distance $d = r - 1$. Also, perceived luminosity varies inversely as r^2 and d^2. Using equation 6:

$$m_1 - m_2 = -2.5\log(\frac{L_1}{L_2}) = -2.5\log(\frac{d_2^2 r_2^2}{d_1^2 r_1^2}) = -5\log(\frac{r_2(r_2-1)}{r_1(r_1-1)})$$

$$m_2 = -1.6 + 5\log(\frac{1.64^2 - 1.64}{1.55^2 - 1.55}) = -1.15$$

The apparent magnitude at Mars' second opposition was -1.15.

3.1.1. MAGNITUDE per ARCSECOND^2

Astronomers sometimes find use for magnitudes based on an object's arcseconds^2 area, rather than its entire area. This is especially appropriate for extended objects like the Sun, Moon, nearby solar system objects, nebula, galaxies and even the sky itself. The *Observer's Handbook* has a chapter (Magnification and Contrast in Deep Sky Observing) stating:

> … the visibility of an object is a function of its apparent size and contrast, where contrast is the ratio of an object's surface brightness (neglecting the contribution of the background sky) to that of the background sky (p. 50).

Determining an object's contrast requires evaluations of magnitude per arcsecond^2 for the object and the sky background.

EXAMPLE 5. *At opposition, Jupiter has an apparent visual magnitude of -2.5, with a diameter of 49.8 arcseconds. Find Jupiter's magnitude/arcsecond^2.*

Jupiter's area = $\pi/4(49.8)^2 = 1,948$ arcseconds^2.

$$L_{Jupiter}/L_0 = 10^{-(-2.5/2.5)} = 10 \quad \text{and} \quad m_{a\sec^2} = -2.5\log(L_{Jupiter}/L_0) = -2.5\log(10/1948) = 5.72$$

The *Observer's Handbook* has a useful approximation to calculate S, magnitude/arcsecond^2:

$$S = m + 2.5\log(2827 + ab) \tag{8}$$

Where a and b are the dimensions of an elliptical object in *arcminutes*. Using equation (8) yields a Jupiter magnitude/arcsecond^2 of 6.13, about 7% over the exact value.
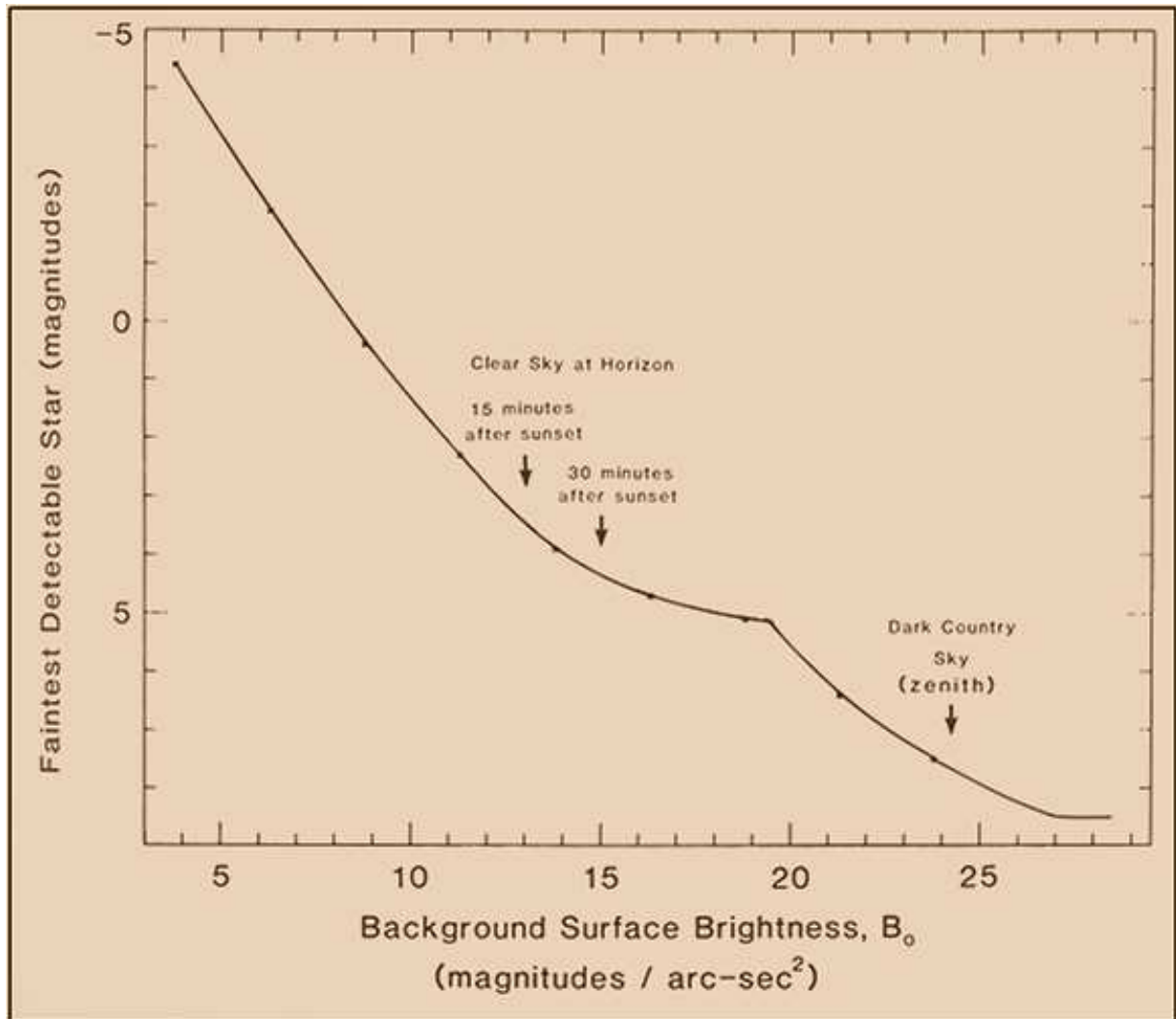

FIGURE 4. Star Visibility versus Sky Brightness

Clark, R. N., 1990, *Visual Astronomy of the Deep Sky,* Sky Publishing Corp. p 16

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Figure 4 illustrates the visibility of stars on sky background brightness based on magnitude/arcsecond^2, for a trained observer with dark adapted vision. It shows why bright planets may be naked-eye visible during daylight, especially if one knows where to look. Normal daylight skies have magnitude/arcsecond^2 of about 4.0. When the Sun is 5^0 above the horizon, the zenith sky has a magnitude/arcsecond^2 of about 6.5. Jupiter's 5.72 magnitude/arcsecond^2 versus daylight sky's 6.5 magnitude/arcsecond^2 has sufficient contrast to be visible just before sunset or just after sunrise. The full Moon's size and 3.6 magnitude/arcsecond^2 makes for an easy daylight sighting. At greatest elongation, Venus' magnitude/arcsecond^2 is 1.9. The large contrast with the daylight sky makes Venus a relatively easy naked-eye object if one is aware of its location.

3.2 ABSOLUTE MAGNITUDE

Since stellar brightness depends on distance, astronomers utilize magnitudes at a standard distance, 10 parsecs (32.62 ly), to compare stellar intrinsic luminosity. We already determined in Example 3 that if the Sun was at Sirius's distance, it would appear 28.7 magnitudes fainter. Absolute magnitudes are used in the Hertzsprung-Russell diagram as illustrated in Figure 5. The curved diagonal area on the diagram contains main sequence stars, those that are "burning" hydrogen. The absolute magnitude of the Sun is +4.8. The right ordinate illustrates luminosity versus that of the Sun.
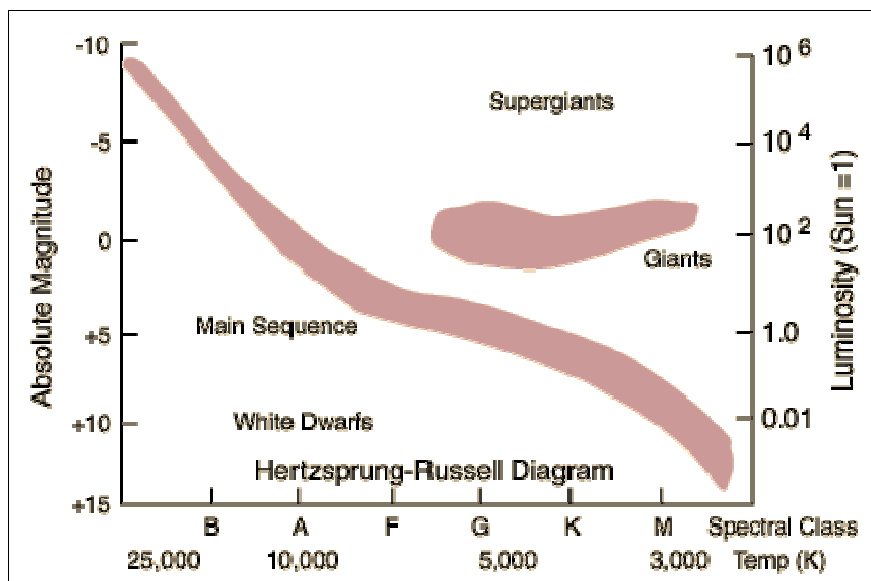

FIGURE 5. Hertzsprung-Russell Diagram

http://hyperphysics.phy-astr.gsu.edu/Hbase/Astro/herrus.html

We'll relate three quantities: apparent magnitude, m; absolute magnitude, M; and stellar distance in parsecs, d. (1 parsec = 3.26 light years.)

$$m - M = 5 \log (d) - 5 \qquad\qquad (9)$$

Equation 9 can be rewritten as follows:

$$d = 10^{\frac{m-M+5}{5}}$$ (10)

The expression (m – M) is called the distance modulus. The distance modulus is negative if a star is closer than 10 parsecs.

EXAMPLE 6. *Find the absolute magnitude of the Sun with m = -26.8.*

We need the Sun's distance to be in parsecs. There are 206,265 AU in a parsec, so d = 1/206,265. Using Equation 9,

M = m – 5 log (d) + 5 = -26.8 – 5 log (1/206,265) + 5 = 4.77

EXAMPLE 7. *A star's m = 8 and M = -2. Find the star's distance.*

Notice that the distance modulus is 8 – (-2) = 10. This indicates the star is more distant than 10 parsecs. Using Equation 10,

$$d = 10^{\frac{m-M+5}{5}} = 10^{\frac{8-(-2)+5}{5}} = 10^3 = 1,000 \text{ parsecs.}$$

3.3. ASTEROID ABSOLUTE MAGNITUDES

An asteroid's absolute magnitude, H is the visual magnitude an observer would record if the asteroid was located 1 astronomical unit (AU) away and 1 AU from the Sun at a zero phase angle. Of course, this definition would place the observer at the center of the Sun, so H has to be calculated. At zero phase angle, the Sun-asteroid-Earth angle, an asteroid's apparent magnitude m, and absolute magnitude H are related as follows:

$$m = H + 5\log \Delta + 5\log r$$ (11)

Where Δ and r are distances in AU from the Earth and Sun, respectively.

The IAU has defined a plutoid as a "dwarf planet" with a semi-major axis greater than Neptune. It is usually impossible to optically determine if distant KBOs are in hydrostatic equilibrium (round). The IAU assumes an object is a plutoid if its absolute magnitude, H, is +1 or brighter. Using equation 11, a body at 40 AU must have an apparent magnitude of at least +17 to be considered a "dwarf planet" and a plutoid. The plutoid, Makemake, has an H of -0.48 and at opposition, an m of +16.7.

A sun-lit body's magnitude depends on its albedo, the ratio of reflected to incoming luminosity. In addition, its phase angle can have a major effect on magnitude due to what is called the opposition effect. This is evident in comparing lunar magnitudes during phase

changes. We'll not cover the complex semi-empirical formula relating an asteroid's apparent magnitude m, and its absolute magnitude H, at varying phase angles.

Astronomers determine asteroid sizes based on H magnitudes and assumptions of albedo. Table 1 shows asteroid sizes (km) with different magnitude and albedo assumptions:

H	Albedo 0.50	Albedo 0.25	Albedo 0.05
-2.0	4,700	6,700	14,900
0	1,900	2,600	5,900
+2.0	750	1,050	2,400
+4.0	300	420	940
+6.0	120	170	370
+8.0	45	65	150
+10.0	19	25	60
+12.0	7	11	24
+14.0	3	4	9

TABLE 1. Asteroid Sizes (km)

**http://cfa-www.harvard.edu/iau/lists/Sizes.html**

3.4. COMET MAGNITUDES

Comet brightness varies inversely as Δ^2 (Earth distance) and inversely as r^n (r = heliocentric distance). The photometric exponent n typically has a value of about 4. The relationship between a comet's apparent magnitude m, and absolute magnitude H_{10} would be:

$$m = H_{10} + 5\log \Delta + 10 \log r \qquad (12)$$

As we all know, predicting cometary brightness is not for the faint-hearted. It is difficult to forecast magnitude changes for a new comet approaching the Sun. Comet Kohoutek (1973) was unusually bright when discovered near Jupiter's orbital distance and there was speculation that it could be "the comet of the century." However, it was only a disappointing magnitude +4 in the night sky after perihelion.

3.5. METEOR MAGNITUDES

Apparent meteor magnitudes are usually assessed by comparison to nearby stars. Absolute magnitude for meteors is defined as the magnitude of a meteor if it was at the zenith at a height of 100 km. Figure 6 Is an illustration of how a meteor's distance d is affected by increasing zenith angle z:
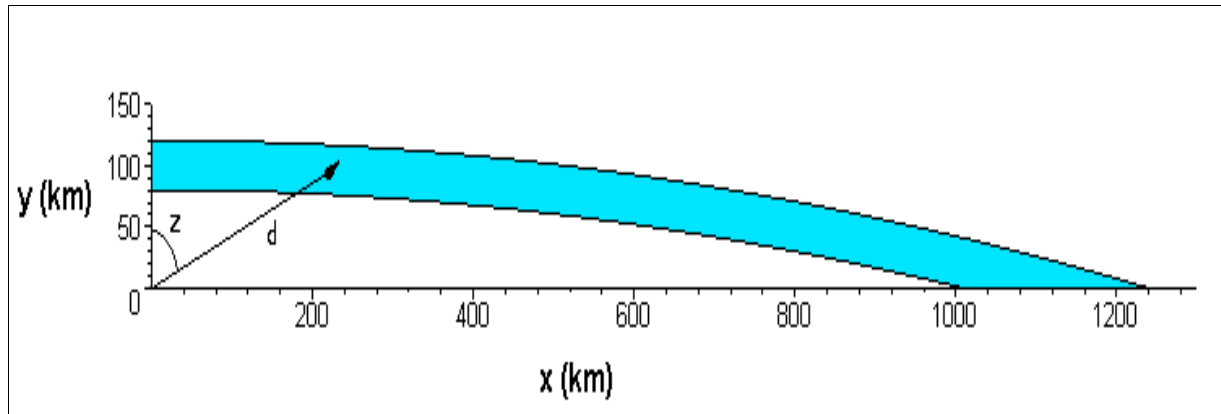
FIGURE 6. Meteor Distance (d) Relationship to Zenith Angle (z)

http://www.amsmeteors.org/richardson/distance.html

Assuming a meteor height of h and a zenith angle z, the distance d to the meteor is:

$$d = \sqrt{4.07x10^7 \cos^2(z) + 1.28x10^4 h + h^2} - 6.38x10^3 \cos(z) \qquad (13)$$

Where d and h are in km.

A relatively accurate "flat earth" solution for d is

$$d = h/\cos(z) \qquad (14)$$

EXAMPLE 8. *A magnitude +2 meteor is sighted with a zenith angle of 60^0. Assume we already know its height h, is 120 km. Find the absolute magnitude of the meteor.*

Using Equation 13, with $z = 60^0$ and h = 120 km yields distance d = 234 km. It's obvious that if the meteor were at the zenith and a height of 100 km, it would be brighter. The luminosity increase would be

$$[\frac{\sqrt{234^2 + 120^2}}{100}]^2 = 6.92$$

So the absolute magnitude of the meteor would be

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$$M = +2 - 2.5 \log 6.92 = -0.1$$

An additional factor that should be considered is that of atmospheric extinction. At a zenith angle of 60^0, atmospheric extinction is about 0.28 magnitudes more than at the zenith (see AAI Tutorials "Atmospheric Extinction and Refraction"), so the actual absolute magnitude of the meteor would be -0.10 – 0.28 = -0.38.

4. REFERENCES

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