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THE CHINESE-UIGHUR CALENDAR IN ṬŪSĪ'S ZĪJ-I ĪLKHĀNĪ

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1. *Introduction*

In the ninth century the Uighurs, a Turkic people, established themselves in the oases of Gansu, the Turfan Depression, and the Tianshan region. There they settled down among, and merged with, the local Indo-European-speaking populace until they were absorbed into the empire of Chinggis Khan. Being relatively close to regions of ancient high civilization, the Uighurs had been exposed to and had adopted cultural elements of all varieties from China, India, and Iran. These acquisitions tended to make them the intellectuals of the nascent Mongol imperial bureaucracy (*Allsen*, pp. 245–248. Here and in the sequel, references in italics are to the bibliography at the end of the paper).

The Uighurs used a calendar which was essentially Chinese, but it also employed a period relation which had first appeared in Babylonian astronomy. This calendar was adopted by the ruling class in all parts of the Mongol empire. In Iran and Central Asia it was introduced by Hülegü Khan (d. 1265), grandson of Chinggis and founder of the Ilkhanid dynasty. In Iran it was used systematically for about a century (*Melville*, p. 83), and vestiges of it remain to this day, for a Tehran astrological almanac of 1995 (*Nujūmī*) lists the animals of the duodecimal cycle as alternative year names. The years so designated are not Chinese, however, but follow the ancient Iranian tradition, and commence with the vernal equinox.

Some information about the Uighur calendar may be found in the published literature, e.g. *Ideler*, *Kennedy*, *Mercier*, and *Sédillot*, but all such studies are either incomplete or difficult of access. The object of this paper is to describe the Uighur calendar to the extent that, given a date in the Iranian Yazdigird calendar, a reader may convert it into the equivalent Uighur date. This is the only transformation described by our principal source. However, a DOS computer program (available at cost) has been developed by the first author. This enables the user, given

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a date in the Chinese-Uighur, Muslim, Yazdigird, Seleucid, Julian, or various other calendars, to convert it immediately into all the others.

There are tables and rules for conversions into the Uighur calendar in a number of *zīj*es, astronomical handbooks, most of them written in Persian, although a few are in Arabic. But for the most part their information seems to have been obtained from the Zīj-i Īlkhānī, which is therefore our principal source. This was the work of Naṣīr al-Dīn al-Ṭūsī (d. 1274), director of the famous observatory at Maragha, the Ilkhans' first capital, in northwestern Iran. Among the astronomers assembled to staff the institution a Chinese is reported, one Fu Meng-chi or Fu Mu-zhai (*Boyle*, p. 253). It is probable that he was responsible for the Chinese material in our source.

Many manuscript copies of the Īlkhānī Zīj are extant, of which we have used two. The first is British Museum Ms. Or. 7464, copied only three years after the death of the author. References to it are preceded by a BM, and give folio and line numbers, separated by a colon. The second, also very early, is Paris Bibliothèque Nationale Ms. Ancien Fonds 163. It is referred to as BN.

Some information missing from the Īlkhānī Zīj has been supplied from the Zīj-i Khāqānī, written by Jamshīd Ghiyāth al-Dīn al-Kāshī (d. 1429), director of the Samarqand observatory of the astronomer-prince, Ulugh Beg. References to it, preceded by KA, are from the India Office copy, 430 (Ethé 2232).

It is not difficult to sketch the leading characteristics of the Uighur calendar. It is built upon three astronomical periods: (1) the solar year, (2) the mean lunation, and (3) the anomalistic month. All three are known Chinese parameters. The beginning of each mean lunar year is defined as the mean new moon just preceding the second of the standard twenty-four equal divisions of the solar year. This insures that the year maintains its place among the seasons. It also implies that some years contain a thirteenth, leap, month. A rule defines where the leap month falls in the interior of the year, usually not at the end. But the calendar is based on true, not mean, new moons, so that it is necessary to calculate an equation for the moon and an equation for the sun. Here a procedure is used which occurs only in very few (unofficial) Chinese calendar systems. For new moons in the interior of any year the Chinese anomalistic month is approximated by the Babylonian period relation which puts nine anomalistic months in 248 days. Moreover, the lunar and solar equations are calculated by means of parabolic schemes.

By and large, it appears that the people who constructed the Uighur calendar as described in the Īlkhānī Zīj were operating in Central Asia, but within the Chinese astronomical tradition, and in Chinese. They did not base their work upon a single Chinese calendrical system, rather making an eclectic choice from elements of several. However, since they used the era for the creation of the universe of the Revised Da Ming calendar of the Jin Dynasty, which was adopted by the Mongols after their conquest of northern China in 1215, it seems plausible that this was one of their main sources.

Sections 3, 5, 10, and 11 below give tables listing the names of such things as the duodecimal animal cycle, the sexagesimal cycle of days and years, solar year divisions, and month names. Most of these Chinese and Turkish names are standard and well known. However, for the benefit of scholars wishing to identify the same names when encountered in Persian or Arabic manuscripts, it has been thought well to repeat the tables here, giving their transcriptions into Perso-Arabic characters as found in our sources.

Sections 12 through 17 define and describe the concepts used in the construction of the calendar, such as the luni-solar year, lunar and solar equations, determination of the true new moon, and the identification of leap months. All this is applied in Section 18, which, illustrated by a running worked example, explains how to convert a given date from the Yazdigird calender into the equivalent Chinese-Uighur date.

Sections 19 and 20 present a description and recomputation of a table in the Īlkhānī Zīj for converting Hijri dates of the century commencing in 599 H (1202 A.D.) into Chinese-Uighur equivalents.

In the course of the explanations in the Īlkhānī Zīj, eighteen technical terms are given, presumably transcriptions from the Chinese. A few of these have been recognized as standard in the Chinese calendary lore, but over half have not. All are listed in an appendix at the end of the paper, in alphabetical order of their transcriptions into Latin characters. From this it is hoped that Sinologists will be able to identify the original Chinese of all these terms, thence be in a position to draw inferences concerning the sources of the calendar.

The authors express gratitude to several scholars who have been helpful, but who, of course, bear no responsibility for any errors. Professor Nathan Sivin gave us much information concerning Chinese calendars. For the tables the late Professor Joseph Fletcher supplied Uighur words and transcriptions of the Chinese, plus crucial encouragement. Professor Thomas T. Allsen has straightened us out in such varied topics as Mongol and Uighur history and the transcription of Chinese. A preliminary translation of the Īlkhānī Zīj by Professor Javad Hamadani Zadeh has been immensely useful. The research on the Uighur calendar by the first author was supported by the Alexander von Humboldt Foundation and the Japan Society for the Promotion of Science.

2. *Symbols, Technical Terms, and Parameters*

an_c a Chinese anomalistic month, 27.5546 days.

an_b a Babylonian anomalistic month, 27.5556 ($\approx 248/9$) days.

Aram the first (lunar) month of the year. (This word, like the names of other periods, will also be used to designate the instant when the period begins.)

d days.

division a twenty-fourth of a Chinese solar year, 15.21848333... days.

f *fen*, a ten-thousandth of a day. In the Persian texts *fen*s are expressed in sexagesimals. They are decimal fractions, and we will frequently thus display them, e.g. $365^{\mathrm{d}}\,0{,}40{,}36^{\mathrm{f}} = 365.2436^{\mathrm{d}}$. As customary, sexagesimal digits are separated from each other by commas, and a semicolon is the "sexagesimal point".

int (x) the integer part of the positive number x.

Li-chun the first division of the solar year.

m a mean lunation, 29.5306^{d}.

madkhal (pl. *madākhil*, English *feria*) the Arabic and Persian name for a number denoting the initial week day of a month or year: Sunday 1, Monday 2, Tuesday 3, …, Saturday 7.

rem (x/y) the residue remaining after dropping from a positive x all possible multiples of y.

Shun Ay the intercalary month.

Y a given Yazdigird year.

y a Chinese solar year, 365.2436^{d}.

Yu-shui the second division of the solar year.

	Chinese			Turkish		Mongol	Persian
	transcr.	BM	BN	transcr.	Text		
1 rat	子 zi	زه zh	ژه žh	küskü	کسکو kskū	qulugana	mouse
2 ox	丑 chou	جیو jyū	چیو čyū	ud	اوط āūṭ	hükär	cow
3 tiger	寅 yin	یِم yim	یَم yam	bars	بارس bārs	bars	panther
4 hare	卯 mao	ماوُ māū	ماوو māūū	tavïšγan	طاوشقن ṭāvšqn	taulai	
5 dragon	辰 chen	جن jn	چن čn	lū	لو/لوی lū/lūy	lu	crocodile
6 snake	巳 si	صز ṣz	ضر ḍr	yïlan	ییلان yīlān	mogai	
7 horse	午 wu	ووُ vū	وو vū	yund	یوند yūnd	morin	
8 sheep	未 wei	وی vī		qoy	قوی qūy	qonin	
9 monkey	申 shen	شن šn	سن sn	bičin	بیجین/بیچن bījīn/bīčn	bichin	
10 cock	酉 you	یوو yūū	پوُوُ pūū	taquq	داقوق/دقوق dāqūq/dqūq	takiya	hen
11 dog	戌 xu	سو sū	سیو syū	ït	ایت āīt	noqai	
12 pig	亥 hai	خائی khāʾī	خایی khāīī	tonguz	طونقوز/طنغوز ṭūnqūz/ṭnghūz	gaqai	

The elements of the duodecimal animal cycle (see page 116)

3. *The Duodecimal Animal Cycle and the Chinese Decimal and Sexagesimal Cycles*

Days and years are given names based on a set of twelve so-called earthly branches, represented by animals, as shown in the table on the preceding page (BM f. 17v:13–15; BN f. 5v:9–11; *Needham*, pp. 396, 403, 405). The Chinese words we give are not the names of the animals themselves, but those of the branches they represent. Variants in the transcriptions of the Chinese names as between BM and BN are sufficiently numerous that they are given in separate columns. Various Persian historical works from the Ilkhan period use the Mongol forms of the animal names in Chinese-Uighur dates. Furthermore, some Persian texts name different animals than the Chinese. Both sets of variants are shown in separate columns. Note that in the Hijri-Uighur conversion table given in Section 19 below, the animal month names are Turkish. The transliteration conventions used in this article are explained in the Appendix.

An ordered set of ten Chinese words makes up a decimal cycle, the heavenly stems. It is (BM f. 4v:3; BN f. 6r:14,15; *Needham*, p. 396):

	Chinese	Text		Chinese	Text
1	甲 jia	كا kā	6	己 ji	كى kī
2	乙 yi	بي bī	7	庚 geng	كن kn
3	丙 bing	پين pīn	8	辛 xin	سن sn
4	丁 ding	تين tīn	9	壬 ren	ژم žm
5	戊 wu	وو vū	10	癸 gui	كوى kūī

The elements of the animal and decimal cycles are combined in ordered couples to produce a sexagesimal cycle used for naming days and years. This cycle is displayed in the table below where, instead of repeating the names of the component elements, they are represented by their numbers, the decimal component first, the duodecimal second (BM f. 4v:8–13):

1	2	3	4	5	6	7	8	9	10
1,1	2,2	3,3	4,4	5,5	6,6	7,7	8,8	9,9	10,10
11	12	13	14	15	16	17	18	19	20
1,11	2,12	3,1	4,2	5,3	6,4	7,5	8,6	9,7	10,8
21	22	23	24	25	26	27	28	29	30
1,9	2,10	3,11	4,12	5,1	6,2	7,3	8,4	9,5	10,6
31	32	33	34	35	36	37	38	39	40
1,7	2,8	3,9	4,10	5,11	6,12	7,1	8,2	9,3	10,4
41	42	43	44	45	46	47	48	49	50
1,5	2,6	3,7	4,8	5,9	6,10	7,11	8,12	9,1	10,2
51	52	53	54	55	56	57	58	59	60
1,3	2,4	3,5	4,6	5,7	6,8	7,9	8,10	9,11	10,12

Thus the name of the first element, (1,1), of the sexagesimal cycle is *jia-zi*; that of the twenty-fourth element, (4,12), *ding-hai*, and so on.

4. *Divisions of the Civil Day (nychthemeron)*

In general, fractions of a day are expressed using ten-thousandths, which are called *fen*s (Chinese 分, transcribed into Persian as فنگ *fng*, pl. فنگها *fnghā*, BM f. 4v:17,18). Each day is divided into twelve *chāgh* (Turkish), two of our hours, hence a chāgh is 10,000/12 = 833.33... = 13,53;20 fens. Each chāgh is in turn divided into eight *kih* (Chinese 刻 *ke*, Mercier, p. 50), so that a kih is 833.33.../8 = 104.1666... = 1,44;10 fens. Note that in Chinese astronomy a *ke* was usually taken as a hundredth of a day, i.e. slightly less than the Uighur value. The first chāgh of any day straddles midnight, so that the second chāgh begins an hour after midnight, and so on. However, the civil day commences at midnight (BM f. 17v:16–20; BN f. 5v:12–19).

5. *The Solar Year and Its Divisions*

The mean solar year is taken as $365.2436^{d} = 365^{d}\,0,40,36^{f}$, a frequent value in Chinese calendrical systems from the eleventh to the thirteenth centuries. Since the true solar year is never defined, there is no need to employ the adjective *mean* with the solar year and its subdivisions.

Each year is divided into twenty-four equal parts which, following the Persian texts, will be called *division*s (قسم *qism*, pl. اقسام *aqsām*).

They correspond to the 24 *qi* (氣) in Chinese astronomy. The length of each division is

$$365.2436^{d}/24 = 15.21848333...^{d} = 15^{d}\,0,36,24;50^{f}.$$

The Chinese names of the individual divisions are tabulated below with the Persian transcriptions next to each one (BM f. 5r:8–13, BN f. 6v:12-17, *Needham*, p. 405). Entries for the table in both manuscripts are corrupt, and we have not hesitated to add dots where they seemed called for by the Chinese original:

	Chinese		Text	
1	立春	Li-chun	ليجن	lījn
2	雨水	Yu-shui	ووشى	vūšī
3	驚蟄	Jing-zhe	كنجه	knjh
4	春分	Chun-fen	شونفوند	šūnfūnd
5	清明	Qing-ming	سنك مينك	snk mīnk
6	穀雨	Gu-yu	كووو	kūvū
7	立夏	Li-xia	ليحه	līḥh
8	小満	Xiao-man	سيومن	sīūmn
9	芒種	Mang-zhong	منجون	mnjūn
10	夏至	Xia-zhi	شاجر	šājr
11	小暑	Xiao-shu	سياوشو	syāūšū
12	大暑	Da-shu	دايشو	dāyšū
13	立秋	Li-qiu	ليجو	lījū
14	處暑	Chu-shu	جيوشو	jyūšū
15	白露	Bai-lu	پيلو	pīlū
16	秋分	Qiu-fen	شيوفن	šyūfn
17	寒露	Han-lu	حنلو	ḥnlū
18	霜降	Shuang-jiang	شون كون	šūnkūn
19	立冬	Li-dong	ليتون	lītūn
20	小雪	Xiao-xue	سياوسه	syāūsh
21	大雪	Da-xue	دايسه	dāīsh
22	冬至	Dong-zhi	دونچن	dūnčn
23	小寒	Xiao-han	سيوحن	sīūḥn
24	大寒	Da-han	دايحن	dāīḥn

As for the seasons, spring commences with the first division, summer with the seventh, and so on. Naṣīr al-Dīn remarks that the beginnings of "our" seasons are in the middle of "their" seasons. Thus the Chinese spring commences when the sun is about the middle of Aquarius, in agreement with Chinese common practice (cf. *Ginzel*, p. 468).

6. *The Age of the Universe* (BM f.5r:14–5v:6)

Large spans of time are measured in a unit of 10,000 years called a *wan* (Chinese 萬, Persian transcription ون *vn* or ون *vin*, BM f. 5v:1, BN 6v:23). It is stated that 8,863 wans have elapsed since the creation of the universe. This era is elsewhere only found in the Revised Da Ming calendar of the Jin Dynasty, which was adopted by the Mongols after their conquest of northern China in 1215 and was in official use till 1281 (*Boyle*, p. 251).

For shorter periods the unit is a set of three cycles of sixty years each, thus 180 in all. These are called

	Chinese		Text	
1.	上元	Shang-yuan	شانك ون	šānk vn
2.	中元	Zhong-yuan	جونك ون	jūnk vn
3.	下元	Xia-yuan	خا ون	khā vn

Within the cycles the years are named with the elements of the sexagesimal cycle as explained in Section 3.

Of the current wan, 9679 years had elapsed until the year of the accession of Chinggis Khan. So from the creation to the year of the accession was a total of 88,639,679 years, the accession being in the eightieth year, the last (sixtieth, *gui-hai*) year of a Zhong-yuan cycle. Several copies of the Īlkhānī Zīj in Iranian libraries, including the one in the Tehran University central library, add a clause to the effect that this year corresponded to 571 Y. This last assertion will be examined in Section 8 following the determination of the zīj's epoch.

7. *The Epoch of the Zīj and the Beginning of Any Solar Year*

The text states that the Uighur base year of the Īlkhānī Zīj is the first Chinese solar year of the Shang-yuan cycle following the accession of Chinggis Khan. An entry in a table (BM f. 7r:5) indicates further that this year began during Yazdigird year 633 (1264 A.D.), and that the value of the sexagesimal cycle of days at its beginning was $11^{\mathrm{d}}\,2,7,40^{\mathrm{f}} = 11.7660^{\mathrm{d}}$. In order to determine the year beginning, however, knowledge of the cycle of days is insufficient. What is needed is the time from the beginning of 633 Y to the beginning of Li-chun (the first division) of the next Chinese year. This we failed to find in the Īlkhānī Zīj.

However, on KA, f. 12v, there is a table with an entry stating that for 781 Y the Li-chun of the corresponding Chinese year was after the beginning of the Yazdigird year by 55.8188^{d}. Since the length of the Yazidigird year is 365 days, the difference between the two year lengths is $365.2436 - 365 = 0.2436$ days. Hence for any two successive Yazdigird years the time from the beginning of the earlier year to the Li-chun occurring during that year is 0.2436 days less than the corresponding time for the following year.

The epoch of the Īlkhānī Zīj falls in 633 Y. So, since $781 - 633 = 148$, the time from the beginning of that year to its Li-chun will be $148 \cdot 0.2436 = 36.0528$ days less than the corresponding time for 781 Y. Thus the epoch of the Īlkhānī Zīj is

$$55.8188 - 36.0528 = 19.7660^{\mathrm{d}} = 19^{\mathrm{d}}\,2,7,40^{\mathrm{f}}$$

after the beginning of 633 Y, 10 January 1264. As a partial check, notice that the fractional part of days in the above is identical with the fractional part of the sexagesimal cycle of days.

The epoch having been secured, for any subsequent Uighur year, the time from the beginning of Y, the Yazdigird year in which it starts, to its beginning (Li-chun) is DF in the fold-out figure after page 152. In order to evaluate this magnitude, recall that for each year after 633 Y, the time from the Yazdigird new year to the corresponding Uighur solar new year will increase by 0.2436 days. Hence

$$DF = 19.7660 + 0.2436 \cdot (Y - 633) \text{ days.}$$

8. *The Era of Chinggis Khan*

Between the year of accession of Chinggis (the last of a Zhong-yuan) and that of the epoch (the first of the next Shang-yuan) there are sixty-one years, the entire sixty of the intervening Xia-yuan, plus one. Hence as seen above, in receding from the epoch to the accession, the time between the respective Yazdigird year beginnings and the next Li-chun will decrease by $61 \cdot 0.2436 = 14.8596$ days. For the epoch year, 633 Y, the time from its beginning to the epoch Li-chun has been shown to be 19.7660 days. So, since $633 - 61 = 572$, the Li-chun of the accession occurs $19.7660 - 14.8596 = 4.9064$ days after the beginning of 572 Y. This implies that the accession took place during the Chinese solar year beginning on 5 Farvardīn 572 Y (29 January 1203), and that the 571 Y given in some copies of the Īlkhānī Zīj is wrong. The year 1203 is confirmed by *Taqizadeh*, p. 118, who inferred it from a work by the court astrologer of Shah Abbas I of Iran as marking Chinggis' decisive defeat of the Kerayits.

However, a different era for Chinggis is found in the *zīj* by Abū Muḥammad ʿAṭā ibn Aḥmad ibn Muḥammad ibn Khwāja Ghāzī al-Samarqandī al-Sanjufīnī (or Sanjafīnī, see *Franke*, p. 97). This work was written in 1366 for the Mongol viceroy of Tibet and is extant as Paris BN Ar. 6040. The epoch of the calendar which al-Samarqandī al-Sanjufīnī calls Khānī is the vernal equinox of 14 March 1207. It is not to be confused with the better known Khānī epoch of 13 March 1302, which celebrates the accession of Ghazan Khan (*Taqizadeh*, p. 118). The relationship of the *zīj* of al-Samarqandī al-Sanjufīnī with other Islamic works written either in Persia or by the Muslim astronomers who served the Yuan Dynasty in China, is still unclear. It can be seen, in any case, that the Chinese dates given by al-Samarqandī al-Sanjufīnī correspond to the epoch of the Uighur calendar used in the Īlkhānī Zīj.

9. *The Sexagesimal Cycle of Days*

It was noted in Section 7 above that the value of this cycle at the beginning of Li-chun of 633 Y was 11.7660^{d}. To find the value of the sexagesimal cycle of days at some subsequent time, calculate the number of days between the beginning of the Chinese solar year of 633 Y and the given time. To this add 11.7660 and find the remainder after dividing by 60.

For example, for Li-chun of the Chinese solar year beginning in 781 Y the value of the sexagesimal cycle of days is

$$\text{rem}\,[\{11.7660 + (781 - 633)y\}/60] = 7.8188 \text{ days} = 7^{d}\,2,16,28^{f},$$

which is the entry for this year at the head of the table in KA f. 12v.

However, the texts also give rules (BM, f. 11v:16–18; KA, f. 12r:15) indicating that the number attached to any particular day for the sexagesimal cycle is to be an integer. If the fens are less than half a night plus a day (three quarters of a nychthemeron) they are to be taken as one day, but if they are more than half a night plus a day, two days must be added to the complete (*tām*) days. Apparently, although to begin with, days are reckoned from midnight, for these purposes they are regarded as commencing from the previous evening. With respect to this new starting point any event will occur a quarter of a day later. So the value of the sexagesimal cycle found above must be increased by 0.25, and the result becomes $7.8188 + 0.25 = 8.0688$. The integer part of this, 8, signifies completed days. But the instant in question is in the next, the ninth, day. Hence for the first day of the Chinese solar year which begins in 781 Y, the value of the sexagesimal cycle is 9.

The same procedure for converting fractional results into integer days is used for the times of other events obtained in the calculation of Uighur dates. For instance, if d is the time of a true new moon expressed in completed days and fens, the current day of the true new moon is given by $\text{int}\,(d + 1.25)$, which increases the integer part of d by 2 if its fractional part is three quarters or more, otherwise by 1.

10. *The Cycle of Choices*

In addition to the sexagesimal, there is another cycle of days running independently, called the cycle of choices (Persian *ikhtiyārāt*). This has twelve elements, tabulated below (BM f. 12v:4,5, BN f. 10v:22,23):

	Text	Chinese		Text	Chinese
1	كِنْ kin	建 jian	7	پوو/پو pū/pūū	破 po
2	جيو/چو čū/jīū	除 chu	8	وِى viy	危 wei
3	مَن man	滿 man	9	جِن/چن čn/jin	成 cheng
4	پِن pin	平 ping	10	شيو šīū	收 shou
5	تِن tin	定 ding	11	خاىِ khāīy	開 kai
6	جه/چِه čih/jh	執 zhi	12	پِي pī	閉 bi

Each element is associated with a color and an attribute, these being shared as shown below (BM f. 12v:6–10, BN f. 11r:17–20):

Elements	Color			Attribute
	Transcr.	Chinese	Translation	
1, 3, 4, 10	خی khī	黑 hei	black	corruption
2, 5, 6, 8	خونگ khūng	黄 huang	yellow	inclined toward beneficence
7, 12	هون hūn	?	polluted dust	extremely corrupt
9, 11	په ph	白 bai	white	extremely beneficent

For the color associated with elements 7 and 12 neither BM nor BN have a translation into Persian. The entry is translated from KA f. 9v, in a table at the bottom of the folio, which has خاك آلود *khāk ālūd*.

In this cycle the element pertaining to a particular day depends upon its place in the solar (not the lunar) year to this extent, that if the cycle is about to recommence at the beginning of an odd-numbered division, the element for the preceding day is repeated, whereupon the cycle begins one day later.

There are complicated verbal rules, with worked examples, presumably defining how to find the element of the cycle which rules a given date in the lunar calendar. It seems hardly worthwhile to decipher these. As for the epoch element, the table on BM f. 13r gives 10 as the element which rules the Li-chun of 633 Y.

There is material on the cycle of choices in *Ginzel*, pp. 490–492; *Schram*, p. 280; and *Rachmati*, pp. 98–99.

11. *Mean Lunar Months*

The Uighur calendar being luni-solar, a mean month commences with each mean new moon. A mean lunation is taken as (BM f. 7v:9,10)

$$m = 29^{\mathrm{d}}\,1{,}28{,}26^{\mathrm{f}} = 29.5306 \text{ days.}$$

No historical inferences can be drawn from this value, since it could have been rounded off to fens from any common historical value, including the classical Babylonian $29;31,50,8,20^{\mathrm{d}}$ (29.530594 days), Ṭūsī's $29;31,50,8,9^{\mathrm{d}}$ (29.530593 days), and most Chinese values (all close to 29.53059 days).

The month names are tabulated below. Naṣīr al-Dīn does not give the Chinese names, which we have transcribed from the Khāqānī Zīj. For use in date conversions, the Yazdigird month names (which do not generally run parallel to the Uighur months) are also shown. Lists with variant Turkish months are given in *Ginzel*, pp. 499–503.

month	Turkish (BM f. 12r)	Chinese	Transcription of Chinese (KA f. 13v)	Yazdigird
1	Aram ay	正月 Zheng-yue	چنوه čnvh	Farvardīn
2	Ikindi ay	二月 Er-yue	ژژوه žžvh	Urdībihišt
3	Üčünč ay	三月 San-yue	ساموه sāmvh	Khurdād
4	Törtünč ay	四月 Si-yue	ضژوه ḍžvh	Tīr
5	Bešinč ay	五月 Wu-yue	اووه āūvh	Murdād
6	Altïnč ay	六月 Liu-yue	لووه lūvh	Shahrīvar
7	Yetinč ay	七月 Qi-yue	چيوه čīvh	Mihr
8	Sekizinč ay	八月 Ba-yue	باوه bāvh	Ābān
9	Toquzïnč ay	九月 Jiu-yue	کهوه khvh	Ādhar
10	Onunč ay	十月 Shi-yue	شيوه šīvh	Dai
11	Biryegirminč ay	十一月 Shi-yi-yue	شي ايوه šīāīvh	Bahman
12	Čaqšapat ay	十二月 Shi-er-yue	شروه šrvh	Isfandārmudh

The Turkish word *ay* is "moon". Aram, the name of the first month, is said to be from the Iranian word *ram*, "joyous", hence "the joyous month", the month of celebrations. The name of the twelfth month, Čaqšapat, is from Sanskrit *šikṣāpada*, meaning "prescriptions". Both names are evidence of Buddhist influence (*Bazin*, p. 294). All the rest of the names are Turkish ordinal numbers: second, third, …, eleventh, as are the Chinese.

Since $12 \cdot 29.5306 = 354.3672$ days, which is about eleven days less than a solar year, if the calendar is to keep its place among the seasons it is necessary to insert an intercalary (leap) lunation from time to time. The leap month is called in Uighur شون *Šūn* (or very rarely شيون *Šyūn*), which we will write as *Shun*. It is apparently a loan from Chinese 閏 *run* (*Bazin*, p. 158). Hence there are in fact two mean lunar year lengths, one of twelve, the other of thirteen lunations.

12. *Mean Lunar Years*

The beginning of the first mean lunar month, Aram, of any year is defined as the beginning of the last mean lunation preceding the Yu-shui (second division) of the corresponding solar year (BM f. 7v:2–4). The second month commences with the following lunation, and so on until the next year begins.

From the fourth entry in the top line of the table on BM f. 9v we read that for the epoch year 633 Y the time from the beginning of Aram to Yu-shui was 14.4676 days $= 14^{d}\,1,17,56^{f}$. This reading is confirmed by BN f. 9r. In Section 7 we found the time from the beginning of 633 Y to Li-chun, the beginning of the corresponding solar year, to be 19.7660 days. Increase this by the length of a division (Li-chun to Yu-shui, rounding the fractional fens) to obtain the time from the beginning of 633 Y to Yu-shui. Next, subtract the value from the table to obtain

$$(19.7660 + 15.2185) - 14.4676 = 20.5169 \text{ days} = 20^{\mathrm{d}}\,1{,}26{,}9^{\mathrm{f}}.$$

This is the time from the beginning of 633 Y to mean Aram, the beginning of the corresponding Chinese mean lunar year. The fractional part of this number is identical with the fractional part of the value of mean Aram in the sexagesimal cycle of days, which is found as the sixth entry in the same line of the above-mentioned table.

For any subsequent lunar year, say the one commencing during Yazdigird year Y, it suffices to determine the time EG on the figure after page 152, E being the last mean new moon preceding G, the Yu-shui of that year. For DF, the time of the solar new year (Li-chun) of that year can be calculated as explained in Section 7 above. Hence G, the following Yu-shui, is found by adding a division, a twenty-fourth of a solar year, to the time of F. From B, the new moon commencing the mean lunar epoch year, lunations of length m are then stepped off, the last one before G being E. Now AF and CG are $Y - 633$ Chinese solar years of length y, and $BG = BC + CG$. So

$$EG = \mathrm{rem}\,[BG/m] = \mathrm{rem}\,[\{14.4676 + (Y - 633)y\}/m],$$

whence the time from the beginning of Yazdigird year Y to the Chinese mean lunar new year occurring during that year is

$$DE = DF + FG - EG = 19.7660 + 0.2436(Y - 633) + y/24 - EG.$$

In these computations the texts use the auxiliary parameter $y - 12m = 10.8764$ days $= 10^{\mathrm{d}}\,2{,}26{,}4^{\mathrm{f}}$.

13. *The Anomalistic Month*

The Chinese realized that the motions of both the moon and the sun are subject to small periodic variations from the mean. Such a variation is known as an equation, and the true moon or sun is the algebraic sum of its mean and its equation. Since the Chinese calendar is based upon true rather than mean new moons, it must be possible to calculate the equations of both luminaries.

The Uighur calendar, like the Indian and the pre-Ptolemaic Greek models for the motion of the moon and most Chinese calendrical systems, incorporated only one lunar equation, with a period equal to the anomalistic month. We have not found explicit mention in the text of the value of this parameter, but at BM f. 9r:9–12 is a statement to the effect that $7^{\mathrm{d}}\,0{,}5{,}38^{\mathrm{f}}$ is the excess of a solar year over thirteen (anomalistic) lunar cycles. That is

$$7^{\mathrm{d}}\,0{,}5{,}38^{\mathrm{f}} = 7.0338 = 365.2436 - 13an_c,$$

where an_c is the length of an anomalistic month. So

$$an_c = (365.2436 - 7.0338)/13 = 27.5546 \text{ days}.$$

This value was used in various Chinese calendrical systems during the twelfth and thirteenth centuries.

As for the initial position of the anomalistic month, at BN f. 8v:16 the text says that at the beginning of the epoch year (but meaning at Yu-shui) the argument of the lunar anomaly was 23.2836 days = $23^{\mathrm{d}}\,0{,}47{,}16^{\mathrm{f}}$.

In the preceding section it was shown that the mean lunar new year preceded Yu-shui by 14.4676 days. Hence the anomaly at the beginning of mean Aram in 633 Y (the time from the beginning of the last anomalistic month preceding mean Aram to mean Aram itself, *HB* in the figure after page 152) was

$$23.2836 - 14.4676 = 8.8160 \text{ days} = 8^{\mathrm{d}}\,2{,}16{,}0^{\mathrm{f}}.$$

This is the next to the last entry in the first line of the table on BM f. 9v, thus confirming the derivation.

To determine the anomaly at mean Aram in any subsequent year, say Y of Yazdigird, again consider the figure, where it is seen that CG, the time from epoch Yu-shui to Yu-shui of year Y is $Y - 633$ Chinese solar years. This is the same as AF, the time between the Li-chuns of the same years. Now J is the beginning of the last anomalistic month before mean Aram of year Y. So JE is the required time, and it can be calculated as

$$\operatorname{rem}(HE/an_c) = \operatorname{rem}[\{23.2836 + (Y - 633)(y - 13an_c) - EG\}/an_c],$$

where the auxiliary parameter $y - 13an_c$, the number 7.0338 introduced above, is the increase in lunar anomaly from one solar new year to the next, and EG is the time from mean Aram to Yu-shui, calculated as in the preceding section.

For example, calculate the lunar anomaly at the mean Aram of 642 Y.

$$\begin{aligned} EG &= \operatorname{rem}[\{14.4676 + (642 - 633) \cdot 365.2436\}/29.5306] \\ &= 23.7634 \text{ days} = 23^{\mathrm{d}}\, 2,7,14^{\mathrm{f}} \end{aligned}$$

as given in the table on BM f. 9v in the fourth column of the line for 642 Y. Then

$$\begin{aligned} JE &= \operatorname{rem}[\{23.2836 + 9(y - 13an_c) - 23.7634\}/27.5546] \\ &= 7.7152 \text{ days} = 7^{\mathrm{d}}\, 1,59,12^{\mathrm{f}}. \end{aligned}$$

This is twenty fens more than the next to the last entry of the same line of the same table. The difference can be explained as follows: In the interior of a lunar year, the Īlkhānī Zīj prescribes the use of a different parameter for the anomalistic month, namely $an_b = 27.5556$ days. As will be seen in Section 14 below, this is in agreement with the method by which the lunar argument is calculated, which puts nine anomalistic months equal to 248 days ($248/9 = 27.5555...$). Thus the lunar anomaly is in fact found as

$$\operatorname{rem}[\{23.2836 + (Y - 633)(y - 13an_c) - EG\}/an_b].$$

In the above example, we now obtain

$$\begin{aligned} JE &= \operatorname{rem}[\{23.2836 + (642 - 633) \cdot 7.0338 - 23.7634\}/27.5556] \\ &= \operatorname{rem}[62.8244/27.5556] = 7.7132 \text{ days} = 7^{\mathrm{d}}\, 1,58,52^{\mathrm{f}} \end{aligned}$$

as given in the table.

14. *The Lunar Equation* (BM ff. 10r:17–10v:2, 11r)

The argument of the equation at any time is $n = \operatorname{int}(9a)$, where a is the lunar anomaly in days at that time. The equation, in fens, is defined as

$$eq_{\text{moon}} = \begin{cases} n(124 - n) & \text{for } 0 \le n < 124 \\ -(n - 124)(248 - n) & \text{for } 124 \le n \le 248. \end{cases}$$

As an example, note that at mean Aram of the epoch year 633 Y the value of the lunar anomaly has been shown to be 8.8160 days (Section

13), hence the argument of the lunar equation was $\mathrm{int}(9 \cdot 8.8160) = \mathrm{int}(79.3440) = 79$. Herefrom the lunar equation at that time can be computed as $79 \cdot (124 - 79) = 3555$ fen, to be added to the time of mean Aram.

Notice that in graphical terms the equation is a periodic wave function resembling a sine, but composed of joined congruent segments of a parabola, the segments opening downwards above the horizontal axis, and upwards below the axis. In terms of n the period of the function is 248, but in terms of the anomalistic argument, since the latter has been multiplied by nine, it is $248/9$.

This is a matter of considerable interest, for the period relation equating nine anomalistic months to 248 days is of ancient Babylonian origin and was subsequently often applied in Greek and Indian astronomy (see *Jones*), and also in the Qintian calendar of the late Zhou Dynasty (tenth century) in China (*Yabuuti*, p. 95). The anomalistic month it implies,

$$248/9 = 27.55555... \approx 27.5556 = 27^{\mathrm{d}}\,1{,}32{,}36^{\mathrm{f}}$$

appears explicitly many times in our text at, e.g., BM f. 9r:17.

In the text (BM f. 10r:19) the increment to the original argument for succeeding months within the year is

$$17^{\mathrm{d}}\,2{,}9{,}14^{\mathrm{f}} = 17.7754 \text{ days} = 9m - 248.$$

It is nine times the excess of a mean lunation over a Babylonian anomalistic month.

The maximum value of the equation is found by putting $n = 62$ in the definition above, which gives a result of 0.3844 days. It has been shown (in *Kennedy*, p. 441) that this can be converted approximately into ecliptic degrees by multiplying it by 12.19°, the Chinese-Uighur daily rate of mean elongation. The product, 4;41°, is not far from the medieval Indian maximum lunar equation of 4;56° or the Ptolemaic 5;0°.

The fact that the equation is expressed in (fractions of) days is another typically Chinese feature of the Uighur calendar. In Chinese astronomy the ecliptic was divided into approximately $365\frac{1}{4}$ *du* (度), each corresponding precisely to the solar motion during a single day.

15. *The Solar Equation* (BM ff. 9r:2–6, 10r:1–16, 10v)

The period of the solar equation is a solar year. The text states that to determine the argument of the solar equation at mean Aram, subtract the time from Aram to Yu-shui from a sixth of a solar year

$$y/6 \approx 60.8740 \text{ days} = 60^{\text{d}}\,2,25,40^{\text{f}}.$$

Evidently one subtracts because Aram precedes Yu-shui. To generalize, if the event in question is after Yu-shui, add the time from Yu-shui to the event to $y/6$ to obtain the solar argument. If a solar argument exceeds a year, subtract y from it.

The solar equation, in fens, is defined as

$$eq_{\text{sun}} = \begin{cases} \text{int}\,[(2/9) \cdot n(182 - n)] & \text{for } 0 \leq n < 182 \\ -\text{int}\,[(2/9) \cdot (n - 182)(364 - n)] & \text{for } 182 \leq n \leq 364, \end{cases}$$

where n is the integer part of the solar argument. Most of the operations being on integers for ease of hand computation, the period has been shortened from a year to 364 days. The equation corrections are in any case quite small compared to the values of the other variables involved.

For example, in the epoch year 633 Y the time from Aram to Yu-shui has been found to be 14.4676 days (Section 12), hence the argument of the solar equation at mean Aram was $\text{int}\,(60.8740 - 14.4676) = \text{int}\,(46.4064) = 46$ days. This leads to a solar equation of

$$\text{int}\,[(2/9) \cdot 46 \cdot (182 - 46)] = \text{int}\,(1390\tfrac{2}{9}) = 1390 \text{ fen},$$

to be added to the time of mean Aram.

To find the maximum value of the equation, put $n = 91$ to obtain 0.1840 days. This may be converted into degrees as indicated in the previous section to give a Chinese maximum solar equation of 2;15°, very close to the medieval Indian 2;14° and not far from the Ptolemaic 2;23°.

In the eighth-century Futian calendar, presumably compiled by an astronomer originating from the western part of China, a solar equation is found which is similar to the one underlying the Uighur calendar (*Nakayama*, p. 451). In the Futian calendar the solar equation, expressed in days of solar motion, is calculated according to the rule $eq_{\text{sun}} = 1/3300 \cdot n(182 - n)$. The difference between the two equations derives mainly from the fact that the Uighur theory expresses the equation in days of lunar elongation instead of solar motion. The Futian calendar was never officially adopted, but was widely known. Since it was the first Chinese calendar which consistently used ten-thousandths of a day (*fen*s), it seems possible that it was another important source of the compilers of the Uighur calendar besides the Revised Da Ming calendar of the Jin Dynasty (cf. Section 6).

16. *Leap Years, True Aram, and the Other Months*

An ordinary, twelve month, mean lunar year is almost eleven days shorter than a solar year, for

$$y - 12m = 365.2436 - 354.3672 = 10.8764 \text{ days.}$$

Hence, as time passes, the ends of successive ordinary years precede the next Yu-shui more and more until the interval exceeds a lunation. A thirteenth month is then added to produce a mean leap year of length $13m = 383.8978$ days.

There tend to be about half as many leap years as ordinary (non-leap) years. From the mean years true years are obtained by the addition of solar and lunar equations, but the equations are so small that the properties remarked above for mean years apply also to true years.

For any year, to the beginning of mean Aram add the compound equation, the algebraic sum of the solar and lunar equations for that time. The result is the time of the true new moon, hence the beginning of true Aram (cf. *Kennedy*, pp. 438–441).

It may happen that the mean new moon marking the beginning of the second mean lunar month occurs after Yu-shui by only a small amount, and that the compound equation turns out to be negative and in absolute value greater than this amount. In this case the true new moon will fall before Yu-shui, and it will become the true Aram, the first lunar month of the new year. The preceding month, mean Aram, becomes the last month of the preceding year. Conversely, the mean Aram may fall just before Yu-shui by a small amount. If the compound equation is positive and exceeds this amount, the true Aram will be the preceding month.

By the above rules, we can now compute the date of the Chinese-Uighur epoch used in the Īlkhānī Zīj. Adding the lunar and solar equations for mean Aram of the year 633 Y, which were determined in Sections 14 and 15, to the number of days from the beginning of that year to mean Aram (Section 12), we find that true Aram occurred

$$20.5169 + 0.3555 + 0.1390 = 21.0114 \text{ days} = 21^{\text{d}}\,1{,}54^{\text{f}}$$

after the beginning of the year. From the rounding rule presented in Section 9 it follows that the first day of the true lunar month and hence the epoch of the Īlkhānī Zīj was 22 Farvardīn 633 Y (31 January 1264).

As we have seen in Sections 13 and 14, in the interior of each year the Chinese anomalistic month is replaced by the Babylonian one,

$an_b = 27.5556$ days. Using this parameter to calculate the lunar argument, add to the time of the mean new moon following true Aram the compound equation for that instant. The sum is the true new moon marking the beginning of the second lunar month, and so on for the remaining months.

With regard to the number of months of the year, Kāshī claims (KA f. 12r:10) that any year for which the time from (mean) Aram to Yu-shui exceeds 18.6542^d ($= 13m - y$, the difference between a mean leap year and a solar year) will be a leap year. But because this rule is based on the mean year, it will occasionally give wrong results. The exact method is to calculate the time of the true moon which ends the assumed thirteenth month of the year in question. If this time is less than the time of the next Yu-shui, the year is leap, otherwise it is ordinary. Since this test involves the beginning of a different Uighur year, the lunar argument should be calculated anew as in Section 13, rather than applying the increment based on the Babylonian anomalistic month, which is only used in the interior of a year.

17. *Identification of the Leap Month (Shun Ay)*

In a leap year the leap month in general is not the last, the thirteenth, month. The texts state (BM f. 11v:26, KA f. 12r:11) that Shun Ay is that month which contains only one division beginning. To this Kāshī adds (KA f. 12r:12) that the single division beginning which characterizes the leap month will be odd-numbered. The validity of these rules, which are standard in Chinese calendrical systems, can be demonstrated as follows.

In any mean lunar leap year mean Aram occurs at least $13m - y = 18.6542^d$ before Yu-shui, since otherwise the next mean Aram would not fall before the next Yu-shui, and the mean lunar year would not be a leap year. It follows that both the first, odd, division beginning Li-chun and the second, even, division beginning Yu-shui fall within the first mean lunar month and also that the time between Yu-shui and the end of mean Aram is at most $m - 18.6542^d = 10.8764^d$ ($= y - 12m$). Two divisions are $2(y/24) - m = 0.9063666...$ days longer than a mean lunar month, so in each consecutive month the even division beginning will be 0.9063666... days closer to the end of the month. This implies that for the thirteenth month at the latest the even division beginning will pass the end of the corresponding mean lunar month, leaving in that month only a single, odd division beginning. By the above definition, that month is Shun Ay.

Since the solar and lunar equations are relatively small, the same argument holds for true lunar months. In fact, if we denote the times of the true new moons in a leap year by TrueNewMoon(i), $i = 1, 2, \ldots, 13$ and write $y/12$ for the length of two divisions, the leap month is given by the smallest value of i for which the inequality

$$\text{TrueNewMoon}(i + 1) < \text{Yu-shui} + (i - 1) \cdot (y/12)$$

is satisfied.

18. *Conversion of Yazdigird Dates: a Worked Example*

To find the Chinese-Uighur date corresponding to

17 Dai 642 Yazdigird (20 October 1273 A.D.),

first subtract 632 from the Yazdigird year. The result, $642 - 632 = 10$, is the number of the Chinese year, reckoning from the beginning of the Shang-yuan cycle, which commences during the given Yazdigird year. In general, if the difference exceeds sixty but is less than 120, the Chinese year will be in the Zhong-yuan. If it exceeds 120 but is less than 180 it is in the Xia-yuan. From there on the triple of yuans repeats itself.

Next, calculate d, days of the given Yazdigird year, by multiplying the elapsed months by thirty and adding the day number of the current month. Include any of the five epagomenal days which may be necessary, recalling that if the date is in the Old Style (generally prior to 375 Y), they are inserted after Ābān, the eighth month, otherwise at the end of the year. In our example we obtain $d = 9 \cdot 30 + 17 = 287$, taking the date to be in the New Style.

Now find the number of days from the beginning of the given Yazdigird year to the Chinese solar new year, Li-chun, occurring in that year. As explained in Section 7, this, *DF* in the fold-out figure after page 152, is

$$19.7660 + 0.2436(Y - 633),$$

so for year 642 Y we obtain $DF = 19.7660 + 0.2436 \cdot 9 = 21.9584^{d}$. Hereafter all such intervals, measured from the beginning of year Y, will be called *Yazdigird days*.

Calculate next the Yazdigird days of the beginning of true Aram for the given year. Using the expression at the end of Section 12 we find for the time from mean Aram to Yu-shui in 642 Y

$$\begin{aligned} EG &= \operatorname{rem}[\{14.4676 + (9 \cdot 365.2436)\}/29.5306] \\ &= \operatorname{rem}[3301.6600/29.5306] = 23.7634. \end{aligned}$$

According to another expression at the end of Section 12, DE, the Yazdigird days of mean Aram, can now be calculated as $DF + FG - EG$, which, for 642 Y, yields $21.9584 + 15.2185 - 23.7634 = 13.4135^{\mathrm{d}}$.

From this true Aram must be found, which entails calculating the lunar and solar equations. For mean Aram of year 642 Y, the lunar anomaly has already been calculated, at the end of Section 13, as 7.7132^{d}. So the corresponding argument will be $\operatorname{int}(9 \cdot 7.7132) = 69$, and the resultant lunar equation $69 \cdot (124 - 69) = 3795^{\mathrm{f}}$. By the rule of Section 15, the solar argument is a sixth of a solar year less the time from mean Aram to Yu-shui, which, in our example, gives $60.8740 - 23.7634 = 37.1106^{\mathrm{d}}$. Then the solar equation is found as

$$\operatorname{int}[(2/9) \cdot 37 \cdot (182 - 37)] = 1192^{\mathrm{f}}.$$

Addition of the equations to mean Aram gives the Yazdigird days of true Aram, the beginning of the true lunar year, as $13.4135 + 0.3795 + 0.1192 = 13.9122^{\mathrm{d}}$. Note that, according to the rule for rounding fractional results presented in Section 9, this implies that the Uighur new year falls on 15 Farvardīn.

In case the Yazdigird days of true Aram exceed d, the given Yazdigird date will fall in the Chinese year which commenced during the preceding Yazdigird year. The entire operation should then be repeated, working now with the previous year. It will almost always suffice to use mean Aram in making this test; only when the difference between d and the Yazdigird days of mean Aram is very small could the equations make a difference in the result.

In our example, the year remains 10 Shang-yuan, and it is now necessary to determine whether it is ordinary or leap. By Kāshī's criterion of Section 16 it will be the latter, for the time from mean Aram to Yu-shui, 23.7634^{d}, is larger than 18.6542^{d} ($= 13m - y$). Since the maximum compound equation amounts to 0.5684^{d} (cf. Sections 14 and 15), it is obvious that in this case the equations cannot affect the result. In general, a rigorous test can be performed by calculating a true next Aram under the assumption that the year in fact has thirteen months. If this hypothesized true next Aram occurs before the next Yu-shui, the year is indeed a leap year.

The year 10 Shang-yuan having been shown to be a leap year, the rule at the end of Section 17 is now applied to locate the leap month. Without putting down any of the details for the first six months, suffice it to say that all of them retain an even-numbered division-beginning in their respective interiors. For $i = 7$, however, the inequality given in Section 17 is satisfied:

$$\begin{aligned}\text{TrueNewMoon}(8) &= 219.5842\\ &< 219.7987 = 37.1769 + 182.6218 = \text{Yu-shui} + 6(y/12).\end{aligned}$$

So Month 7 is designated Shun; Month 8 is called Yetinč (seventh), and so on through the thirteenth.

It remains only to find the month and day within the Uighur year, a procedure which is essentially the same for leap and ordinary years. The month is clearly the one which contains day d. As an approximation, find the number n of mean lunations contained in the interval from mean Aram to d, that is

$$n = \text{int}\,[(d - \text{mean Aram})/m].$$

Since d lies in the following month, its month number will be $n + 1$. In our example we obtain

$$n = \text{int}\,[(287 - 13.4135)/29.5306] = 9,$$

so d is presumably in the tenth month.

But the calendar uses true, not mean, months. So the presumed month number must be tested and the day number within the month determined by calculating the Yazdigird days of TrueNewMoon$(n+1)$. In our example we obtain TrueNewMoon$(10) = 278.8241^{\text{d}}$. Again using the expedient of Section 9 to shift from fractional to integer days, we conclude that the first day of the tenth Uighur month falls on the 280th day of the Yazdigird year, i.e. on 10 Dai 642. From this it can be seen at once that the date to be converted, 17 Dai 642, indeed falls in the tenth month and that its day number within that month is 8.

In the general case, if it turns out that the given Yazdigird date falls before the beginning of month $n + 1$, the actual month number will be n and the above determination should be carried out on the basis of TrueNewMoon(n). If, on the other hand, the day number becomes 30, TrueNewMoon$(n + 2)$ must be calculated as well, since month $n + 1$

could have 29 days, in which case the desired date would be the first day of month $n + 2$.

To complete the worked example, we note that the Uighur month name of the desired date is Toquzïnč (ninth), for this is a leap year, and we have seen above that the leap month is inserted after the sixth month, pushing the remaining months forward. Thus the full Uighur date corresponding to 17 Dai 642 Yazdigird is:

Day 8 of Toquzïnč Ay of the year 10 Shang-yuan,
Year of the Cock.

19. *The Table for Conversion from the Hijri Calendar*
(BM ff. 13v–16r; BN ff. 11v–13r)

In the Īlkhānī Zīj the section on the Uighur calendar ends with an extensive table for converting from Hijri dates to Chinese-Uighur. This table is preceded by a short explanation of how it is to be used (BM f. 13r:1–20), but there is no indication of how it was computed. A recomputation of the complete table appears in Section 20 below.

The first column of the table gives the arguments in Hijri years 599, 600, 601, . . ., 704 H. Thus it runs through the century commencing in 1202 A.D. (571 Y). The remaining columns are each headed by a Hijri month name in order: Muḥarram, Ṣafar, etc. In the interior of the table each cell determined by a year and a month has four numerical entries. They are: **(1)** In the upper right corner of the cell a number indicating the *madkhal*, the initial week day of that Hijri month. **(2)** The number in the lower right corner of the cell gives the initial day of the Chinese month determined by the same lunation. **(3)** The middle lower number is that of the Chinese lunar month in the current Chinese year, not including the leap month, hence never more than twelve. **(4)** The number in the lower left hand corner is the number of days in that particular Chinese month. The middle lower number is written in the decimal place value numeral forms the Arabs took over from the Indians. The other three are in the Arabic alphabetical *abjad* numerals customarily used for sexagesimals.

In addition to the four numbers, some cells have a word written in them. This is either Shun, if it is a leap month, or the Turkish name of the year in the duodecimal animal cycle, if it is the first month of a year. There is no way to tell the complete year name in the sexagesimal cycle, nor the place in the cycle of yuans.

The first page of the Hijri conversion table in the Īlkhānī Zīj
(Or 7464, folio 13v, reproduced by permission of The British Library)

The use of the table is straightforward and simple. Convert, for instance,

15 Ṣafar 610 H

into Chinese-Uighur. In the cell determined by the Ṣafar column and the 610 line, the Hijri *madkhal* is 6 (Friday), the Chinese is 5 (Thursday), and the month number is 6. This says that the Chinese month starts a day before the Hijri month, and that it is the sixth month, hence our date will be the sixteenth day of the sixth month. The given date is just two weeks beyond the beginning of Ṣafar, so it also is a Friday. So the Chinese-Uighur date is

16 Altïnč Ay, year of the Cock, a Friday.

From the week days of the Hijri month given in the table we conclude that its base is the astronomical (not the civil, or popular) mean Hijri calendar, the variant of the thirty year cycle which puts an intercalary day after the fifteenth (rather than the sixteenth) year.

20. *The Recomputed Conversion Table*

A complete recomputation of the Hijri conversion table in the Īlkhānī Zīj can be found on pages 139 to 146. Its format is exactly that of the texts, except that, because of the opposite directions of the writing employed, it has been mirrored. All the variants between entries in the recomputation and the two manuscripts are listed in the critical apparatus immediately following the table.

In general, corresponding entries in the recomputated table and the manuscripts agree. In fact, of the 1267 ($= 105 \times 12 + 7$) beginnings of Uighur months given in the table, the recomputation of only forty-one (3 %) yields a different result. In thirty cases the recomputed beginnings are one day later than the texts, in eleven cases they are one day earlier. Furthermore, of the thirty-eight leap months indicated in the table, the position within the Uighur year of thirteen (34 %) is different. In twelve cases the recomputed position of the leap month is one month later than that in the table, in the remaining case it is one month earlier.

Note that both an error in the beginning of an Uighur month and an error in the position of a leap month lead to more than one difference between text and recomputation. In the case of an erroneous beginning of a month, the *madkhal* as well as the lengths of the preceding and current month will be different. If the position of a leap month is in error, the month numbers of (at least) two consecutive months will be different. Consequently, errors of these two types can easily be distinguished from individual scribal errors. In the two copies of the Īlkhānī Zīj that we have used, frequent scribal errors are confusion of 6 (Arabic و *wāw*) and 7 (Arabic ز *zā'*) and a displacement of (parts of) rows and columns. For instance, as can be checked with the critical apparatus, in both manuscripts most of the Chinese *madkhal*s and month lengths for the year 620 Hijri were displaced one month towards the end of the year. Furthermore, in BM the Chinese month lengths corresponding to Muḥarram of the Hijri years 606 to 612 (at the end of the page), as well as the Uighur month numbers, including "Shun", corresponding to Muḥarram of years 655 to 672 (again at the end of the page) were all displaced one line upwards.

Some patterns can be noted in the differences between the original table and the recomputation. Firstly, the number of differences in the beginnings of Uighur months increases drastically towards the end of the table, whereas the first part shows hardly any differences at all. Secondly, the differing recomputed beginnings of months are in most cases one day later than those in the original table, in only a few cases one day earlier. Thirdly, except for one, all differing recomputed positions of leap months are one month later than those in the texts.

We have not been able to find a possible reason for these patterns. Any changes in the underlying parameters increase the number of differences between table and recomputation rather than decreasing it. We investigated, without succes, the presence of correlations between the solar, lunar and compound equations on one side and the errors in the resulting true new moons on the other. One thing which may be noted is that for most of the differing beginnings of months the recomputed true new moons have a decimal fraction very close to the critical 0.75, at which the day of the true new moon changes (cf. Section 9). Therefore, the differences in these beginnings of months could have arisen from relatively small errors made by the author of the table in the complicated calculation of the true new moons. As far as the positions of the leap months in the Īlkhānī table are concerned, all except two agree with the contemporary official Chinese calendars of the Jin and Yuan dynasties. It seems possible that, in order not to deviate too strongly, the leap months in the Uighur calendar were inserted on the basis of the official Chinese calendar rather than by the method of calculation presented in the Īlkhānī Zīj.

Hijri year	Muḥarram	Ṣafar	Rabīʿ I	Rabīʿ II	Jumādā I	Jumādā II	Rajab	Shaʿbān	Ramaḍān	Shawwāl	Dhū al-qaʿda	Dhū al-ḥijja
599	5 4 9 30	7 6 10 30	1 1 11 29	3 2 12 30	4 *Shun* 4 30	6 *Pig* 6 1 29	7 7 2 30	2 2 3 29	3 3 4 29	5 4 5 30	6 6 6 29	1 7 7 29
600	3 1 8 30	5 3 9 30	6 5 10 29	1 6 11 30	2 1 12 30	4 *Rat* 3 1 30	5 5 2 29	7 6 3 30	1 1 4 29	3 2 5 29	4 3 6 30	6 5 7 29
601	7 6 8 29	2 7 9 30	3 2 10 29	5 3 11 30	6 5 12 30	1 *Ox* 7 1 30	2 2 2 29	4 3 3 30	5 5 4 30	7 7 5 29	1 1 6 29	3 2 7 30
602	4 4 8 29	6 *Shun* 5 29	7 6 9 30	2 1 10 29	3 2 11 30	5 4 12 30	6 *Tiger* 6 1 29	1 7 2 30	2 2 3 30	4 4 4 29	5 5 5 30	7 7 6 29
603	2 1 7 30	4 3 8 29	5 4 9 29	7 5 10 30	1 7 11 29	3 1 12 30	4 *Hare* 3 1 30	6 5 2 29	7 6 3 30	2 1 4 30	3 3 5 29	5 4 6 30
604	6 6 7 29	1 7 8 30	2 2 9 29	4 3 10 29	5 4 11 30	7 6 12 29	1 *Drag.* 7 1 30	3 2 2 29	4 3 3 30	6 5 4 30	7 7 5 29	2 *Shun* 1 30
605	3 3 6 29	5 4 7 30	6 6 8 30	1 1 9 29	2 2 10 30	4 4 11 29	5 5 12 29	7 *Snake* 6 1 30	1 1 2 29	3 2 3 30	4 4 4 29	6 5 5 30
606	1 7 6 29	3 1 7 30	4 3 8 30	6 5 9 29	7 6 10 30	2 1 11 30	3 3 12 29	5 *Horse* 4 1 30	6 6 2 29	1 7 3 29	2 1 4 30	4 3 5 29
607	5 4 6 30	7 6 7 29	1 7 8 30	3 2 9 30	4 4 10 29	6 5 11 30	7 7 12 30	2 *Sheep* 2 1 29	3 3 2 30	5 *Shun* 5 29	6 6 3 29	1 7 4 30
608	3 2 5 29	5 3 6 29	6 4 7 30	1 6 8 30	2 1 9 29	4 2 10 30	5 4 11 30	7 6 12 30	1 *Monk.* 1 1 29	3 2 2 30	4 4 3 29	6 5 4 29
609	7 6 5 30	2 1 6 29	3 2 7 29	5 3 8 30	6 5 9 29	1 6 10 30	2 1 11 30	4 3 12 30	5 *Cock* 5 1 29	7 6 2 30	1 1 3 30	3 3 4 29
610	4 4 5 29	6 5 6 30	7 7 7 29	2 1 8 29	3 2 9 30	5 4 10 29	6 *Shun* 5 30	1 7 11 30	2 2 12 30	4 *Dog* 4 1 29	5 5 2 30	7 7 3 29
611	2 1 4 30	4 3 5 29	5 4 6 30	7 6 7 29	1 7 8 29	3 1 9 30	4 3 10 29	6 4 11 30	7 6 12 30	2 *Pig* 1 1 29	3 2 2 30	5 4 3 30

Hijri year	Muḥarram	Ṣafar	Rabīᶜ I	Rabīᶜ II	Jumādā I	Jumādā II	Rajab	Shaᶜbān	Ramaḍān	Shawwāl	Dhū al-qaᶜda	Dhū al-ḥijja
612	6 6 4 29	1 7 5 30	2 2 6 29	4 3 7 30	5 5 8 29	7 6 9 29	1 7 10 30	3 2 11 29	4 3 12 30	6 *Rat* 5 1 29	7 6 2 30	2 1 3 30
613	3 3 4 29	5 4 5 30	6 6 6 29	1 7 7 30	2 *Shun* 2 30	4 4 8 29	5 5 9 29	7 6 10 30	1 1 11 29	3 2 12 30	4 *Ox* 4 1 29	6 5 2 30
614	1 7 3 29	3 1 4 30	4 3 5 30	6 5 6 29	7 6 7 30	2 1 8 29	3 2 9 30	5 4 10 30	6 6 11 29	1 7 12 29	2 *Tiger* 1 1 30	4 3 2 29
615	5 4 3 30	7 6 4 29	1 7 5 30	3 2 6 29	4 3 7 30	6 5 8 30	7 7 9 29	2 1 10 30	3 3 11 30	5 5 12 29	6 *Hare* 6 1 30	1 1 2 29
616	3 2 3 29	5 *Shun* 3 30	6 5 4 29	1 6 5 29	2 7 6 30	4 2 7 30	5 4 8 29	7 5 9 30	1 7 10 30	3 2 11 30	4 4 12 29	6 *Drag.* 5 1 30
617	7 7 2 29	2 1 3 29	3 2 4 30	5 4 5 29	6 5 6 29	1 6 7 30	2 1 8 29	4 2 9 30	5 4 10 30	7 6 11 30	1 1 12 29	3 *Snake* 2 1 30
618	4 4 2 30	6 6 3 29	7 7 4 29	2 1 5 30	3 3 6 29	5 4 7 29	6 5 8 30	1 7 9 29	2 1 10 30	4 3 11 30	5 5 12 30	7 *Shun* 7 29
619	2 *Horse* 1 1 30	4 3 2 29	5 4 3 30	7 6 4 29	1 7 5 30	3 2 6 29	4 3 7 29	6 4 8 30	7 6 9 29	2 7 10 30	3 2 11 30	5 4 12 29
620	6 *Sheep* 5 1 30	1 7 2 30	2 2 3 29	4 3 4 30	5 5 5 29	7 6 6 30	1 1 7 29	3 2 8 29	4 3 9 30	6 5 10 29	7 6 11 30	2 1 12 29
621	3 *Monk.* 2 1 30	5 4 2 30	6 6 3 29	1 7 4 30	2 2 5 30	4 4 6 29	5 5 7 30	7 7 8 29	1 *Shun* 1 29	3 2 9 30	4 4 10 29	6 5 11 30
622	1 7 12 29	3 *Cock* 1 1 30	4 3 2 29	6 4 3 30	7 6 4 30	2 1 5 29	3 2 6 30	5 4 7 29	6 5 8 30	1 7 9 29	2 1 10 30	4 3 11 29
623	5 4 12 30	7 *Dog* 6 1 29	1 7 2 30	3 2 3 29	4 3 4 30	6 5 5 29	7 6 6 30	2 1 7 30	3 3 8 29	5 4 9 30	6 6 10 29	1 7 11 30
624	2 2 12 29	4 *Pig* 3 1 30	5 5 2 29	7 6 3 30	1 1 4 29	3 2 5 30	4 *Shun* 4 29	6 5 6 30	7 7 7 29	2 1 8 30	3 3 9 30	5 5 10 29

Hijri year	Muḥarram	Ṣafar	Rabīᶜ I	Rabīᶜ II	Jumādā I	Jumādā II	Rajab	Shaᶜbān	Ramaḍān	Shawwāl	Dhū al-qaᶜda	Dhū al-ḥijja
625	7 6 11 30	2 1 12 30	3 *Rat* 3 1 29	5 4 2 29	6 5 3 30	1 7 4 29	2 1 5 29	4 2 6 30	5 4 7 29	7 5 8 30	1 7 9 30	3 2 10 30
626	4 4 11 29	6 5 12 30	7 *Ox* 7 1 30	2 2 2 29	3 3 3 29	5 4 4 30	6 6 5 29	1 7 6 29	2 1 7 30	4 3 8 29	5 4 9 30	7 6 10 30
627	2 1 11 30	4 3 12 29	5 *Tiger* 4 1 30	7 6 2 30	1 *Shun* 1 29	3 2 3 29	4 3 4 30	6 5 5 29	7 6 6 29	2 7 7 30	3 2 8 29	5 3 9 30
628	6 5 10 29	1 6 11 30	2 1 12 30	4 *Hare* 3 1 30	5 5 2 29	7 6 3 30	1 1 4 29	3 2 5 30	4 4 6 29	6 5 7 29	7 6 8 30	2 1 9 29
629	3 2 10 30	5 4 11 29	6 5 12 30	1 *Drag.* 7 1 30	2 2 2 30	4 4 3 29	5 5 4 30	7 7 5 29	1 1 6 30	3 3 7 29	4 4 8 29	6 5 9 30
630	1 7 10 29	3 *Shun* 1 30	4 3 11 29	6 4 12 30	7 *Snake* 6 1 30	2 1 2 29	3 2 3 30	5 4 4 30	6 6 5 29	1 7 6 29	2 1 7 30	4 3 8 29
631	5 4 9 30	7 6 10 29	1 7 11 30	3 2 12 29	4 *Horse* 3 1 30	6 5 2 29	7 6 3 30	2 1 4 30	3 3 5 29	5 4 6 30	6 6 7 29	1 7 8 30
632	2 2 9 29	4 3 10 30	5 5 11 29	7 6 12 30	1 *Sheep* 1 1 29	3 2 2 30	4 4 3 29	6 5 4 30	7 7 5 29	2 1 6 30	3 3 7 30	5 *Shun* 5 29
633	7 6 8 30	2 1 9 29	3 2 10 30	5 4 11 29	6 5 12 30	1 *Monk.* 7 1 29	2 1 2 30	4 3 3 29	5 4 4 29	7 5 5 30	1 7 6 30	3 2 7 29
634	4 3 8 30	6 5 9 30	7 7 10 29	2 1 11 30	3 3 12 29	5 *Cock* 4 1 30	6 6 2 29	1 7 3 30	2 2 4 29	4 3 5 29	5 4 6 30	7 6 7 29
635	1 7 8 30	3 2 9 30	4 4 10 29	6 5 11 30	7 7 12 30	2 *Dog* 2 1 29	3 3 2 30	5 5 3 29	6 *Shun* 6 30	1 1 4 29	2 2 5 29	4 3 6 30
636	6 5 7 29	1 6 8 30	2 1 9 29	4 2 10 30	5 4 11 30	7 6 12 30	1 *Pig* 1 1 29	3 2 2 30	4 4 3 29	6 5 4 30	7 7 5 29	2 1 6 29
637	3 2 7 30	5 4 8 29	6 5 9 30	1 7 10 29	2 1 11 30	4 3 12 30	5 *Rat* 5 1 30	7 7 2 29	1 1 3 30	3 3 4 29	4 4 5 30	6 6 6 29

Hijri year	Muḥarram	Ṣafar	Rabīʿ I	Rabīʿ II	Jumādā I	Jumādā II	Rajab	Shaʿbān	Ramaḍān	Shawwāl	Dhū al-qaʿda	Dhū al-ḥijja
638	1 7 7 29	3 1 8 30	4 3 9 29	6 4 10 30	7 6 11 29	2 7 12 30	3 *Shun* 2 30	5 *Ox* 4 1 29	6 5 2 30	1 7 3 30	2 2 4 29	4 3 5 30
639	5 5 6 29	7 6 7 29	1 7 8 30	3 2 9 29	4 3 10 30	6 5 11 29	7 6 12 30	2 *Tiger* 1 1 29	3 2 2 30	5 4 3 30	6 6 4 29	1 7 5 30
640	2 2 6 29	4 3 7 30	5 5 8 29	7 6 9 30	1 1 10 29	3 2 11 30	4 4 12 29	6 *Hare* 5 1 30	7 7 2 29	2 1 3 30	3 3 4 29	5 4 5 30
641	7 6 6 30	2 1 7 29	3 2 8 30	5 4 9 29	6 *Shun* 5 30	1 7 10 29	2 1 11 30	4 3 12 29	5 *Drag.* 4 1 30	7 6 2 29	1 7 3 30	3 2 4 29
642	4 3 5 30	6 5 6 29	7 6 7 30	2 1 8 30	3 3 9 29	5 4 10 30	6 6 11 29	1 7 12 30	2 *Snake* 2 1 29	4 3 2 30	5 5 3 29	7 6 4 29
643	1 7 5 30	3 2 6 29	4 3 7 30	6 5 8 30	7 7 9 29	2 1 10 30	3 3 11 30	5 5 12 29	6 *Horse* 6 1 30	1 1 2 29	2 2 3 30	4 4 4 29
644	6 5 5 29	1 *Shun* 6 30	2 1 6 29	4 2 7 30	5 4 8 29	7 5 9 30	1 7 10 30	3 2 11 30	4 4 12 29	6 *Sheep* 5 1 30	7 7 2 29	2 1 3 30
645	3 3 4 29	5 4 5 29	6 5 6 30	1 7 7 29	2 1 8 30	4 3 9 29	5 4 10 30	7 6 11 30	1 1 12 30	3 *Monk.* 3 1 29	4 4 2 30	6 6 3 29
646	1 7 4 30	3 2 5 29	4 3 6 29	6 4 7 29	7 5 8 30	2 7 9 29	3 1 10 30	5 3 11 30	6 5 12 30	1 *Cock* 7 1 29	2 1 2 30	4 *Shun* 3 30
647	5 5 3 29	7 6 4 30	1 1 5 29	3 2 6 29	4 3 7 29	6 4 8 30	7 6 9 29	2 7 10 30	3 2 11 30	5 4 12 29	6 *Dog* 5 1 30	1 7 2 30
648	2 2 3 29	4 3 4 30	5 5 5 29	7 6 6 30	1 1 7 29	3 2 8 30	4 4 9 29	6 5 10 29	7 6 11 30	2 1 12 30	3 *Pig* 3 1 29	5 4 2 30
649	7 6 3 30	2 1 4 29	3 2 5 30	5 4 6 29	6 5 7 30	1 7 8 29	2 1 9 30	4 3 10 29	5 *Shun* 4 30	7 6 11 29	1 7 12 30	3 *Rat* 2 1 29
650	4 3 2 30	6 5 3 29	7 6 4 30	2 1 5 29	3 2 6 30	5 4 7 30	6 6 8 29	1 7 9 30	2 2 10 29	4 3 11 30	5 5 12 29	7 *Ox* 6 1 30

Hijri year	Muḥarram	Ṣafar	Rabīʿ I	Rabīʿ II	Jumādā I	Jumādā II	Rajab	Shaʿbān	Ramaḍān	Shawwāl	Dhū al-qaʿda	Dhū al-ḥijja
651	1 1 2 29	3 2 3 29	4 3 4 30	6 5 5 30	7 7 6 29	2 1 7 30	3 3 8 29	5 4 9 30	6 6 10 30	1 1 11 29	2 2 12 30	4 *Tiger* 4 1 29
652	6 5 2 30	1 7 3 29	2 1 4 29	4 2 5 30	5 4 6 29	7 *Shun* 5 30	1 7 7 30	3 2 8 29	4 3 9 30	6 5 10 30	7 7 11 29	2 1 12 30
653	3 *Hare* 3 1 29	5 4 2 30	6 6 3 29	1 7 4 29	2 1 5 30	4 3 6 29	5 4 7 30	7 6 8 29	1 7 9 30	3 2 10 30	4 4 11 30	6 6 12 29
654	7 *Drag.* 7 1 30	2 2 2 29	3 3 3 30	5 5 4 29	6 6 5 29	1 7 6 29	2 1 7 30	4 3 8 29	5 4 9 30	7 6 10 30	1 1 11 30	3 3 12 29
655	5 *Snake* 4 1 30	7 6 2 30	1 1 3 29	3 2 4 30	4 *Shun* 4 29	6 5 5 29	7 6 6 29	2 7 7 30	3 2 8 29	5 3 9 30	6 5 10 30	1 7 11 29
656	2 1 12 30	4 *Horse* 3 1 30	5 5 2 30	7 7 3 29	1 1 4 30	3 3 5 29	4 4 6 29	6 5 7 30	7 7 8 29	2 1 9 29	3 2 10 30	5 4 11 30
657	7 6 12 29	2 *Sheep* 7 1 30	3 2 2 30	5 4 3 29	6 5 4 30	1 7 5 29	2 1 6 30	4 3 7 29	5 4 8 30	7 6 9 29	1 7 10 29	3 1 11 30
658	4 3 12 29	6 *Shun* 4 30	7 *Monk.* 6 1 30	2 1 2 29	3 2 3 30	5 4 4 30	6 6 5 29	1 7 6 30	2 2 7 29	4 3 8 30	5 5 9 29	7 6 10 29
659	1 7 11 30	3 2 12 29	4 *Cock* 3 1 30	6 5 2 30	7 7 3 29	2 1 4 30	3 3 5 29	5 4 6 30	6 6 7 30	1 1 8 29	2 2 9 30	4 4 10 29
660	6 5 11 30	1 7 12 29	2 *Dog* 1 1 29	4 2 2 30	5 4 3 29	7 5 4 30	1 7 5 29	3 1 6 30	4 3 7 30	6 5 8 29	7 6 9 30	2 *Shun* 1 30
661	3 3 10 29	5 4 11 30	6 6 12 29	1 *Pig* 7 1 30	2 2 2 29	4 3 3 29	5 4 4 30	7 6 5 29	1 7 6 30	3 2 7 29	4 3 8 30	6 5 9 30
662	7 7 10 30	2 2 11 29	3 3 12 30	5 *Rat* 5 1 29	6 6 2 30	1 1 3 29	2 2 4 29	4 3 5 30	5 5 6 29	7 6 7 29	1 7 8 30	3 2 9 30
663	5 4 10 30	7 6 11 29	1 7 12 30	3 *Ox* 2 1 30	4 4 2 29	6 5 3 30	7 7 4 29	2 1 5 29	3 *Shun* 2 29	5 3 6 30	6 5 7 29	1 6 8 30

Hijri year	Muḥarram	Ṣafar	RabIᶜ I	RabIᶜ II	Jumādā I	Jumādā II	Rajab	Shaᶜbān	Ramaḍān	Shawwāl	Dhū al-qaᶜda	Dhū al-ḥijja
664	2 1 9 30	4 3 10 29	5 4 11 30	7 6 12 30	1 *Tiger* 1 1 30	3 3 2 29	4 4 3 30	6 6 4 29	7 7 5 29	2 1 6 29	3 2 7 30	5 4 8 29
665	6 5 9 30	1 7 10 29	2 1 11 30	4 3 12 30	5 *Hare* 5 1 30	7 7 2 29	1 1 3 30	3 3 4 29	4 4 5 30	6 6 6 29	7 7 7 30	2 2 8 29
666	4 3 9 29	6 4 10 30	7 6 11 29	2 7 12 30	3 *Drag.* 2 1 30	5 4 2 29	6 *Shun* 5 30	1 7 3 30	2 2 4 29	4 3 5 30	5 5 6 29	7 6 7 30
667	1 1 8 29	3 2 9 29	4 3 10 30	6 5 11 29	7 6 12 30	2 *Snake* 1 1 30	3 3 2 29	5 4 3 30	6 6 4 29	1 7 5 30	2 2 6 30	4 4 7 29
668	6 5 8 30	1 7 9 29	2 1 10 29	4 2 11 30	5 4 12 29	7 *Horse* 5 1 30	1 7 2 29	3 1 3 30	4 3 4 30	6 5 5 29	7 6 6 30	2 1 7 29
669	3 2 8 30	5 4 9 30	6 6 10 29	1 7 11 30	2 *Shun* 2 29	4 3 12 29	5 *Sheep* 4 1 30	7 6 2 29	1 7 3 30	3 2 4 29	4 3 5 30	6 5 6 29
670	7 6 7 30	2 1 8 30	3 3 9 29	5 4 10 30	6 6 11 30	1 1 12 29	2 *Monk.* 2 1 30	4 4 2 29	5 5 3 29	7 6 4 30	1 1 5 29	3 2 6 30
671	5 4 7 29	7 5 8 30	1 7 9 30	3 2 10 29	4 3 11 30	6 5 12 30	7 *Cock* 7 1 29	2 1 2 30	3 3 3 29	5 4 4 29	6 5 5 30	1 7 6 29
672	2 *Shun* 1 29	4 2 7 30	5 4 8 30	7 6 9 29	1 7 10 30	3 2 11 30	4 4 12 30	6 *Dog* 6 1 29	7 7 2 30	2 2 3 29	3 3 4 29	5 4 5 29
673	6 5 6 30	1 7 7 29	2 1 8 30	4 3 9 29	5 4 10 30	7 6 11 30	1 1 12 30	3 *Pig* 3 1 29	4 4 2 30	6 6 3 30	7 1 4 29	2 2 5 29
674	4 3 6 29	6 4 7 30	7 6 8 29	2 7 9 30	3 2 10 29	5 3 11 30	6 5 12 30	1 *Rat* 7 1 30	2 2 2 29	4 3 3 30	5 5 4 29	7 *Shun* 6 30
675	1 1 5 29	3 2 6 30	4 4 7 29	6 5 8 29	7 6 9 30	2 1 10 29	3 2 11 30	5 4 12 30	6 *Ox* 6 1 29	1 7 2 30	2 2 3 30	4 4 4 29
676	6 5 5 30	1 7 6 29	2 1 7 30	4 3 8 29	5 4 9 29	7 5 10 30	1 7 11 29	3 1 12 30	4 *Tiger* 3 1 29	6 4 2 30	7 6 3 30	2 1 4 29

Hijri year	Muḥarram	Ṣafar	Rabīᶜ I	Rabīᶜ II	Jumādā I	Jumādā II	Rajab	Shaᶜbān	Ramaḍān	Shawwāl	Dhū al-qaᶜda	Dhū al-ḥijja
677	3 2 5 30	5 4 6 29	6 5 7 30	1 7 8 29	2 1 9 30	4 3 10 29	5 4 11 30	7 6 12 29	1 *Shun* 7 30	3 *Hare* 2 1 29	4 3 2 30	6 5 3 29
678	7 6 4 30	2 1 5 30	3 3 6 29	5 4 7 30	6 6 8 29	1 7 9 30	2 2 10 30	4 4 11 29	5 5 12 29	7 *Drag.* 6 1 30	1 1 2 29	3 2 3 30
679	5 4 4 29	7 5 5 30	1 7 6 29	3 1 7 30	4 3 8 30	6 5 9 29	7 6 10 30	2 1 11 30	3 3 12 29	5 *Snake* 4 1 30	6 6 2 29	1 7 3 29
680	2 1 4 30	4 3 5 29	5 4 6 29	7 5 7 30	1 7 8 30	3 2 9 29	4 *Shun* 3 30	6 5 10 30	7 7 11 30	2 2 12 29	3 *Horse* 3 1 30	5 5 2 29
681	6 6 3 29	1 7 4 30	2 2 5 29	4 3 6 29	5 4 7 30	7 6 8 29	1 7 9 30	3 2 10 30	4 4 11 30	6 6 12 29	7 *Sheep* 7 1 30	2 2 2 30
682	4 4 3 29	6 5 4 29	7 6 5 29	2 7 6 30	3 2 7 29	5 3 8 30	6 5 9 29	1 6 10 30	2 1 11 30	4 3 12 30	5 *Monk.* 5 1 29	7 6 2 30
683	1 1 3 29	3 2 4 30	4 4 5 29	6 *Shun* 5 30	7 7 6 29	2 1 7 29	3 2 8 30	5 4 9 29	6 5 10 30	1 7 11 30	2 2 12 29	4 *Cock* 3 1 30
684	5 5 2 30	7 7 3 29	1 1 4 30	3 3 5 29	4 4 6 30	6 6 7 29	7 7 8 29	2 1 9 30	3 3 10 29	5 4 11 30	6 6 12 29	1 *Dog* 7 1 30
685	3 2 2 30	5 4 3 29	6 5 4 30	1 7 5 30	2 2 6 29	4 3 7 29	5 4 8 30	7 6 9 29	1 7 10 30	3 2 11 29	4 3 12 30	6 *Pig* 5 1 29
686	7 6 2 30	2 *Shun* 1 29	3 2 3 30	5 4 4 30	6 6 5 29	1 7 6 30	2 2 7 29	4 3 8 30	5 5 9 29	7 6 10 30	1 1 11 29	3 2 12 30
687	5 *Rat* 4 1 29	7 5 2 30	1 7 3 29	3 1 4 30	4 3 5 29	6 4 6 30	7 6 7 30	2 1 8 29	3 2 9 30	5 4 10 29	6 5 11 30	1 7 12 29
688	2 *Ox* 1 1 30	4 3 2 29	5 4 3 30	7 6 4 29	1 7 5 30	3 2 6 29	4 3 7 30	6 5 8 29	7 6 9 30	2 1 10 30	3 3 11 29	5 *Shun* 4 30
689	6 6 12 30	1 *Tiger* 1 1 29	2 2 2 29	4 3 3 30	5 5 4 29	7 6 5 29	1 7 6 30	3 2 7 29	4 3 8 30	6 5 9 30	7 7 10 30	2 2 11 29
690	4 3 12 30	6 *Hare* 5 1 30	7 7 2 29	2 1 3 29	3 2 4 30	5 4 5 29	6 5 6 29	1 6 7 30	2 1 8 29	4 2 9 30	5 4 10 30	7 6 11 30

Hijri year	Muḥarram	Ṣafar	Rabīᶜ I	Rabīᶜ II	Jumādā I	Jumādā II	Rajab	Shaᶜbān	Ramaḍān	Shawwāl	Dhū al-qaᶜda	Dhū al-ḥijja
691	1 1 12 29	3 *Drag.* 2 1 30	4 4 2 29	6 5 3 30	7 7 4 29	2 1 5 30	3 3 6 29	5 *Shun* 4 29	6 5 7 30	1 7 8 29	2 1 9 30	4 3 10 29
692	5 4 11 30	7 6 12 30	1 *Snake* 1 1 30	3 3 2 29	4 4 3 30	6 6 4 29	7 7 5 30	2 2 6 29	3 3 7 29	5 4 8 30	6 6 9 29	1 7 10 30
693	3 2 11 29	5 3 12 30	6 *Horse* 5 1 30	1 7 2 30	2 2 3 29	4 3 4 30	5 5 5 29	7 6 6 29	1 7 7 30	3 2 8 29	4 3 9 30	6 5 10 29
694	7 6 11 30	2 1 12 29	3 *Sheep* 2 1 30	5 4 2 30	6 6 3 29	1 7 4 30	2 *Shun* 2 29	4 3 5 30	5 5 6 29	7 6 7 30	1 1 8 29	3 2 9 30
695	4 4 10 29	6 5 11 30	7 7 12 29	2 *Monk.* 1 1 30	3 3 2 29	5 4 3 30	6 6 4 30	1 1 5 29	2 2 6 30	4 4 7 29	5 5 8 30	7 7 9 29
696	2 1 10 30	4 3 11 29	5 4 12 30	7 *Cock* 6 1 29	1 7 2 30	3 2 3 29	4 3 4 30	6 5 5 29	7 6 6 30	2 1 7 29	3 2 8 30	5 4 9 30
697	6 6 10 29	1 7 11 30	2 2 12 29	4 *Shun* 3 30	5 *Dog* 5 1 29	7 6 2 30	1 1 3 29	3 2 4 29	4 3 5 30	6 5 6 30	7 7 7 29	2 1 8 30
698	4 3 9 30	6 5 10 29	7 6 11 30	2 1 12 29	3 *Pig* 2 1 30	5 4 2 29	6 5 3 30	1 7 4 29	2 1 5 29	4 2 6 30	5 4 7 29	7 5 8 30
699	1 7 9 30	3 2 10 29	4 3 11 30	6 5 12 30	7 *Rat* 7 1 29	2 1 2 30	3 3 3 29	5 4 4 30	6 6 5 29	1 7 6 29	2 1 7 30	4 3 8 29
700	5 *Shun* 4 30	7 6 9 29	1 7 10 30	3 2 11 30	4 4 12 30	6 *Ox* 6 1 29	7 7 2 30	2 2 3 29	3 3 4 30	5 5 5 29	6 6 6 29	1 7 7 29
701	3 1 8 30	5 3 9 30	6 5 10 29	1 6 11 30	2 1 12 30	4 *Tiger* 3 1 30	5 5 2 29	7 6 3 30	1 1 4 29	3 2 5 30	4 4 6 29	6 5 7 29
702	7 6 8 29	2 7 9 30	3 2 10 29	5 3 11 30	6 5 12 30	1 *Hare* 7 1 30	2 2 2 29	4 3 3 30	5 5 4 30	7 7 5 29	1 1 6 29	3 *Shun* 2 30
703	4 4 7 29	6 5 8 30	7 7 9 29	2 1 10 29	3 2 11 30	5 4 12 30	6 *Drag.* 6 1 29	1 7 2 30	2 2 3 30	4 4 4 29	5 5 5 30	7 7 6 29
704	2 1 7 30	4 3 8 29	5 4 9 30	7 6 10 29	1 7 11 30	3 2 12 29	4 *Snake* 3 1	6 2	7 3	2 4	3 5	5 6

Critical Apparatus

The apparatus gives all the entries in the two manuscript texts, BM and BN, which are different from those of the recomputed table. The arguments consist of the Hijri year, in boldface, then the month number followed by a period. Next is any one or all of the four numbers (or Sh, for Shun) in each cell which is different from the recomputed value: **(1)** A number from one to seven preceded by an H is a Hijri *madkhal* which differs from the recomputed value. **(2)** A number preceded by a C is a Chinese *madkhal*. **(3)** A number from one to twelve preceded by no letter is the number of the Chinese month. In each of the three circumstances defined above the entry is followed by the recomputed value for the particular entry enclosed in parentheses. Finally, **(4)** a 29 or 30 is the number of days in the particular Chinese month if the text value is different from that recomputed. In this case there is no need for a number in parentheses, since if the variant is 29, the recomputed value must be 30, and conversely.

In all four cases, if the number (or Sh) is followed by an **M**, this indicates that the variant occurs in the BM copy, whereas the entry in BN is the same as the recomputed value. If the number is followed by an **N**, this implies that the variant is in BN and not in BM. If there is neither an **M** nor an **N**, this indicates that the entry which differs from the recomputed value is common to both copies.

599 1. H4**N**(5) 2. C7(6) **602** 7. C5**N**(6) **604** 11. Sh(5) 12. 5(Sh) **606** 1. 30**M** **607** 1. 29**M** **608** 1. 30**M** 3. 29**M** 8. 29 9. C7(1) 30 **609** 1. 29**M** 8. 29**N** **610** 1. 30**M** 6. Sh**M**(10) 7. 10**M** 11**N**(Sh) 8. 12**N**(11) 9. 13(*abjad*)**N**(12) Dog(-) **611** 1. 29**M** 5. C1(7) **612** 1. C5**N**(6) 7. 29 8. 30**N** **613** 3. 30 4. C1(7) 29 8. 29**N** **614** 11. 29**N** **615** 2. H2**M** 4**N**(7) **618** 7. 29**M** **619** 2. H3(4) 30 3. H4(5) C5(4) 29 4. H6(7) 5. H7(1) 6. H2(3) 7. H3(4) 30**M** 8. H5(6) 9. H6(7) 10. H1(2) 11. H2(3) 12. H4(5) **620** 2. C2**M**(7) C blank in **N** 3. C7(2) 30 4. C2(3) 29 5. C3(5) 30 6. C5(6) 29 7. C6(1) 30**N** 8. C1(2) 9. C2(3) 29 10. C3(5) 30 11. C5(6) 29 12. C6(1) 30 **625** 2. 29**N** **628** 10. 30**N** **630** 1. Sh(10) 2. 10(Sh) **631** 3. C6**M**(7) 29 4. C1(2) 30 5. 29 6. C4(5) 30 **632** 3. C4**M**(5) **634** 6. H4(5) **635** 9. 4(Sh) 10. Sh(4) **636** 7. 30**N** **639** 7. 29**N** **640** 7. 30**N** 9. H6(7) 12. 29 **641** 3. 29**N** 4. Sh(9) 5. 9(Sh) **642** 8. C blank in **N** **643** 5. C6(7) **644** 1. Sh(5) 2. 5(Sh) 3. 30**N** **646** 4. 30 5. C6(5) 29 **647** 5. 30 6. C5(4) 29 **648** 12. 29**N** **654** 5. H3(6) 6. 30 7. C2(1) 29 **655** 7. 30 8. C1(7) 29**N** 9. H4N(3) **656** 8. 29**M** 12. 29 **657** 1. C5(6) Sh**M**(12) 30 8. 30**M** **658** 1. 11**M** Sh**N**(12) 2. 12(Sh) **659** 4. 29 5. C6(7) 30 **660** 1. 10**M**(11) 2. C6(7) 4. 29**N** 6. H6**M**(7) 7. C6**N**(7) 8. 29**M** **661** 8. 30**M** **662** 8. 29**M** **663** 1. 9**M**(10) 8. 30**M** 9. 30 10. C4(3) 29 **664** 7. 29 8. C5(6) 30**N** **665** 8. 30**M** 11. 29 12. C1(2) 30 **666** 1. 8**M**(9) 6. Sh(2) 7. 2(Sh) 11. C6**N**(5) **668** 8. 29**M** 9. 29 10.

C4(5) 30 **669** 1. 7**M**(8) **670** 3. 30 4. C5(4) 29 8. 30**M** 12. 29 **671** 1. C3(4) Sh**M**(7) 30 8. 29**M** 12. C6(7) **672** 1. 6**M**(Sh) 12. 30 **673** 1. C6(5) 29 10. 29 11. C7(1) 30 **674** 8. 29 9. C1(2) 30 11. Sh(4) 12. 4(Sh) **675** 2. 29 3. C3(4) 30 **676** 7. 30**N** **677** 8. Sh**N**(12) 9. 12**N**(Sh) The preceding two places in the film of **M** are blank **678** 1. C5**N**(6) **679** 10. 29 11. C5(6) 30 **680** 1. 29 2. C2(3) 30 6. Sh(9) 7. 9(Sh) **682** 3. 30 4. C1(7) 29 **683** 4. 29 5. C6(7) 30 7. C3**N**(2) **684** 5. 29 6. C5(6) 30 11. H7(6) **686** 1. Sh(2) 2. 2(Sh) **687** 7. C5(6) **688** 5. 29 6. C1(2) 30 11. Sh(11) 12. 11(Sh) **689** 1. C7**M**(6) **690** 2. 29 3. C6(7) 30 5. 29 6. C3(4) 30 **691** 6. 29 7. C2(3) 30 12. 30 **692** 1. C5(4) 29 7. 29 8. C1(2) 30 **693** 4. 29 5. C1(2) 30 8. 30 9. C1(7) 29 **694** 4. 29 5. C5(6) 30 **695** 7. 29 8. C7(1) 30 **697** 6. 29 7. C7(1) 10. 29 11. C6(7) 30 **698** 3. C7(6) **699** 8. 29 9. C5(6) 30 **700** 9. 29 10. C4(5) 30 **701** 10. 29 11. C3(4) 30 **702** 9. 29 10. C6(7) 30 11. Sh(6) 12. 6(Sh) 29 **703** 1. C3(4) 30 4. 30 5. C3(2) 29 7. 30 8. C1(7) 9. C3(2) 29 10. 30 11. C6(5) 12. C1(7) **704** 1. C2(1) 2. C4(3) 3. C6(4) 4. C7(6) 5. C1(7) 6. C3(2) 7. C4(3). With month 7 of 704 Hijri a Chinese year begins. The *madkhal* of the new year is calculated, incorrectly. But from there on to the end of the Hijri year there are entries only for the *madkhal*s of the Hijri months and the numbers of the Chinese months. Apparently the main calculations for these months were never carried out.

Appendix: Chinese Technical Terms

For the benefit of persons who do not read Arabic characters the words have been transliterated into Latin letters, sometimes with diacritical marks. Most of these are the equivalents in common use. Note, however, that for Persian *che* چ and *zhe* ژ, *č* and *ž* are used respectively, and for Arabic *shīn* ش and *yā*ʾ ي, *š* and *ī* (or *y*). The letter *wāw* و is transcribed as *v* or *ū*. Vowel signs rarely appear in the texts, so the letters *a*, *i*, and *u* are rare in the transliterations. But on the few occasions when, say, an *a* is in a transliteration, this implies the text has a *fatḥa*, and so on. As between different manuscript copies there tend to be several variant writings for the same word. This is to be expected, since the probability of a scribe having any Chinese is very small. In particular a *che* چ in one copy may well be a *jīm* ج in another, and conversely. In the manuscripts, Persian *gāf* گ is always written as Arabic *kāf* ك. A copyist may forget to put dots over or under a letter, which will then be read differently. These inherent ambiguities must be borne in mind in attempting to reconstruct the Chinese original from a transcription.

bījūšā (BM f. 10r:23; BN f. 9v:19 بيجوشا) is the constant 124^d, half the period 248^d of the lunar anomaly (Section 14).

bījūtn (BM f. 10r:9 بيجوتن) the approximate half-period, 182^d, of the solar anomaly (Section 15). Compare the entry above.

jnjūn (BM f. 9r:18 جنجون) Chinese 轉終 *zhuan zhong* (Mercier, p. 51), the (Babylonian) anomalistic month, 27.5556^d (Section 14).

jūnčā (BM f. 9r:10 جو نجا, BN f. 8v:19 جو نجا, KA f. 11r:14 چو نجا) is the excess of a solar year over thirteen Chinese anomalistic months, 7.0338^d (Section 13).

jūnjūnkā (BM f. 9r:8 جو نجونكا) for any year is the lunar anomaly for that year, the time from the beginning of the last anomalistic month preceding Yu-shui to Yu-shui itself (Section 13).

jūnjūnšā (BM f. 10r:20 جو نجونشا) is nine times the excess of a mean lunation over a Babylonian anomalistic month, 17.7754^d (Section 14). *Mercier*, p. 51, gives the Chinese 轉終差 *zhuan zhong cha*. Compare the preceding entries.

kījā (BM f. 9r:5 كيجا) is the argument of the solar equation at mean Aram of any year (Section 15).

kījū (BM f. 5v:12 كيجو) according to KA f. 12v, in a column heading, this denotes the initial day of the first division of any solar year (Section 7).

knjh (BM f. 5r:1; BN f. 6v:5 كنجه) or *knčh* (KA f. 9v:11 كنچه), probably Chinese 中氣 *zhong-qi*, a division of one twenty-fourth of a solar year (Section 5).

nū (BM f. 10r:11,25 نو) Chinese 朒 *nü*, used for the slow phase of the moon. In our text it has the meaning "additive", said of the solar and lunar equation (Sections 14 and 15, cf. *tyāvū* below).

sījū (BN f. 6v:2 سيجو) or *sīnjūr* (BM f. 4v:16 سينجور) Chinese 歲實 *sui shi*, length of the solar year, taken as 365.2436^d, a frequent value in Chinese systems from the eleventh to the thirteenth centuries (Section 5).

sī yū (BM f. 6r:3 سی یو) Chinese 歲餘 *sui yu* is the excess of a solar year over 360, 5.2436^d. This parameter occurs with the preceding one (Section 5).

sūyjā (BM f. 7v:8 سويجا or BN f. 8v:1 سو نجا) the difference between the solar year and twelve mean lunations, 10.8764^d (Section 12). This parameter resembles the 歲閏 *sui run* mentioned in the Song dynastic history.

šūjh (BM f. 7v:10 شوجه), a mean lunation, 29.5306^d, found in most Chinese systems (Section 11). *Mercier*, p. 51, makes it 朔實 *shuo shi*.

šūnjn (BM f. 7v:4 شونجن) is the interval from mean Aram to Yu-shui for any particular year (Section 12).

tāyānk žjūn (BM f. 10v:1 تايانك ژجون) is the lunar equation, for which 太陽日中 *tai-yang ri-zhong* has been suggested (Section 14). *Mercier*, p. 50, reads *tānkāz*, Turkish for "equation".

tāyānk žkī (BM f. 10r:16 تايانك ژكى) is the solar equation (Section 15). 太陽 *tai yang* is "sun" in Chinese. Compare the entry above.

tyāvū (BM f. 10r:15 تياوُو) Chinese 朓 *tiao*, used for the fast phase of the moon. In our text it has the meaning "subtractive", said of the solar and lunar equation (Sections 14 and 15, cf. *nū* above).

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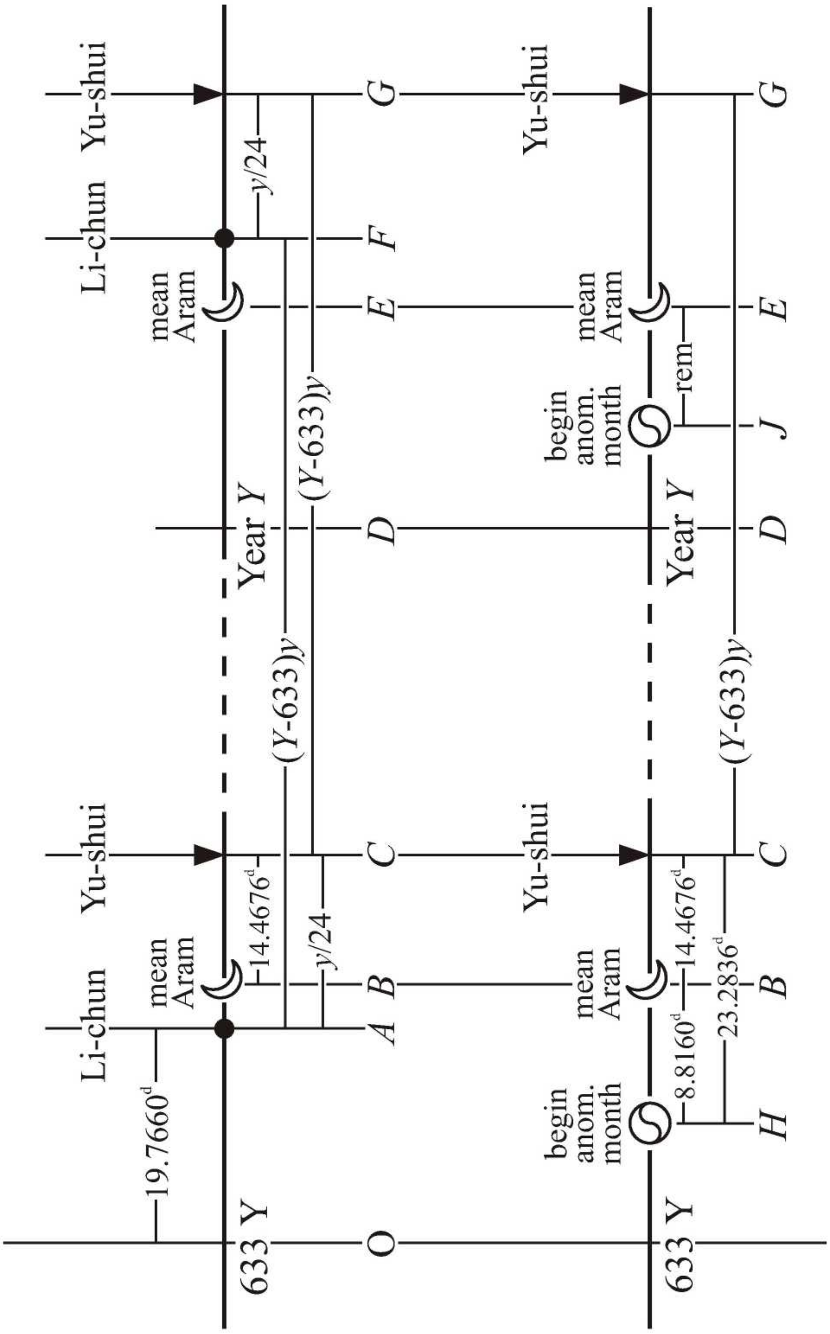
Li-chun
Yu-shui
mean Aram
Year Y
633 Y
begin anom. month
y/24
(Y-633)y
14.4676^d
19.7660^d
8.8160^d
23.2836^d
rem
O
A
B
C
D
E
F
G
H
J