ANARCHY AND ITS BREAKDOWN

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Abstract

Anarchy is not mere chaos, and can constitute a stable system. However, not all environments can sustain an anarchic order. Each contestant in the model here balances optimally between producing out of current resources versus fighting to seize or defend a resource base. Anarchy is viable only when there are strongly diminishing returns to fighting effort (the 'decisiveness parameter' m must be sufficiently low), else any contender with an initial advantage would always gain total control. Under Cournot conditions, assuming viability, as m rises fighting levels increase and achieved incomes decrease. Also, larger numbers of contenders N imply higher per-capita fighting efforts and lower per-capita incomes, even if the resource base expands proportionately with N. Under Stackelberg conditions all sides benefit from reduced levels of fighting, but followers do better than the leader -- a consideration that tends to stabilize anarchy by reducing the motivation to seek leadership.

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Anarchy is not chaos. It is, rather, a pattern of relationships that constitutes, at least potentially, a stable system. However, not all environments can sustain an anarchic order. Anarchy can break down, to be replaced by another structure of relationships.

Whereas law and other socially ordered constraints define a 'political economy', anarchy is a 'natural economy'.[1] The biological realm, with its varied and subtle social regularities,[2] illustrates how there can be order even under anarchy. As another instance, while all nation-states govern by some form of law, relations among these states remain anarchic. And yet, this international system also has its regularities and systematic analyzable patterns.[3]

Biologists recognize two main forms of competition in Nature. Fish in the open sea, living as they do mainly on resources that are fugitive in space and time, can only engage in what is termed 'scramble' competition. 'Interference' competition, in contrast, occurs in environments where organisms can capture and deny others access to valuable resources such as territories or mating partners.[4] The latter more closely approximates the most important forms of human rivalry and struggle.

Whenever it is possible to sequester resources, competitors have to divide their efforts between two main types of activities: (1) productive exploitation of the resources currently controlled, versus (2) seizing and defending a resource base. There are two corresponding technologies: a <u>technology of production</u> and a <u>technology of appropriation, conflict, and struggle</u>.[5] There are ways of tilling the land, and quite a different set of ways of capturing land and securing it against intruders.

The analysis of anarchy as a system is directly applicable, as already indicated, to lawless arenas such as the biological struggle for survival and the competition among nation-states for territory and influence. More generally, however, no civil order is ever perfect. Institutions of political economy can only attenuate, rather than completely eliminate, anarchic competition for control of resources.

In earlier papers I analyzed the choice between productive and conflictual activities in environments characterized by an element of jointness in production, for example the struggle between capital and labor within the firm.[6] When each side's productive inputs help generate income for all parties, their interests are not totally opposed and a degree of de facto cooperation typically emerges. Also, in those papers the underlying resource base on each side was assumed safe from capture, the issue being only how to divide up the jointly generated income. Here we are concerned with a starker environment, where the productive opportunities are entirely disjoint and the fighting is over the resources themselves.

The analytical key distinguishing the theory of conflict is recognition that (i) competitors are each trying to balance optimally between productive effort versus conflictual effort, and (ii) their separate decisions will jointly determine an equilibrium involving levels of production and extent of fighting activity on each side, together with a final distribution of product among the claimants. While there are precursors, notably Schelling [1960], Boulding [1962], and Tullock [1974], the modern type of analysis can be said to begin with Bush and Mayer [1974]. Recent contributions, apart from my own papers cited elsewhere, include Skogh and Stuart [1982], Wolfson [1985], Usher [1989], Garfinkel [1990], Grossman [1991] and Skaperdas [1992].[7]

Among the issues to be considered here are:

1. When is there a stable anarchic solution?: Under what conditions can unorganized contestants all retain a viable share of the social resources in equilibrium? Or put the other way, in what circumstances does the anarchic system 'break down' in favor of tyranny or some other form of social control?

2. Equilibrium allocations of effort: Assuming a stable anarchic equilibrium, what fractions of resources will be devoted to fighting? What levels of incomes will be attained?

3. Numbers: If the number of contenders N is exogenously given, how are the equilibrium fighting efforts and attained incomes levels of income affected as N changes? Alternatively, if N is endogenous, how many contenders can survive?

4. Technology and comparative advantage: How do the outcomes respond to parametric variations, one-sided or two-sided, in the technology of production or in the technology of struggle?

5. Strategic position: How do the outcomes respond to positional asymmetries, for example where one side is a Stackelberg leader?

For the simplest case of two competitors ($N = 2$), Section I below describes the conditions for a stable interior equilibrium under Nash-Cournot behavior, while Section II analyzes the optimizing decision and final outcomes. Section III generalizes to any number of contenders, where N may be either exogenous or endogenous. Section IV examines the consequences of asymmetrical capacities (absolute or comparative advantage in production or in conflict), while Section V covers positional asymmetries. Section VI concludes with a discussion of the results obtained.

I. CONDITIONS FOR EQUILIBRIUM IN THE STRUGGLE FOR RESOURCES (N = 2)

Each of two rival claimants aims solely to maximize own income. Neither benevolent nor malevolent preferences play a role, nor is there any taste for leisure or other non-income-generating activity.

At any moment of time each contender i will be dividing his or her current resource availability R_i between productive effort E_i (aimed at extracting income from resources or territory currently controlled) and fighting effort F_i (aimed at enlarging territory at the expense of competitors, or repelling them as they attempt to do the same):

(1-1) $$R_i = a_i E_i + b_i F_i$$

Here the a_i and b_i are unit conversion costs (assumed constant) of transforming resources into productive effort or into fighting effort, respectively. Making use of a military metaphor, b_i is a logistics cost coefficient that indicates the resource burden per fighting unit supported. Similarly a_i, the production cost coefficient, measures the resources expended to maintain a worker or machine in civilian production. In the decades preceding the American Civil War, inventions like the steamboat and railroad sharply reduced

a_i (since workers could be fed and machines built more cheaply) and also b_i (since supplies could more easily be delivered to fighting troops). In consequence, vastly larger armies were able to take the field in the Civil War as compared with the Revolutionary War or the War of 1812.

It will sometimes be more convenient to deal with the corresponding "intensities" e_i and f_i:

(1-2) $$e_i \equiv E_i/R_i \quad \text{and} \quad f_i \equiv F_i/R_i$$

In what follows, the e_i and f_i will be the crucial decision variables on each side, subject of course to:[8]

(1-3) $$a_i\, e_i + b_i\, f_i = 1$$

As a crucial simplification, I assume each side makes an optimal once-and-for-all fractional choice of e_i and f_i.

Symbolizing income to side i as Y_i, let the production function take the simple form:

(1-4) $$Y_i = E_i^h = (e_i R_i)^h \qquad \text{Production Function}$$

As pictured in Figure 1, there will be increasing, constant, or decreasing returns to productive effort depending upon whether the <u>productive scale parameter</u> h exceeds, equals, or falls short of unity .

[Figure 1]

The outcome of the struggle for resource control is measured by the <u>success fractions</u> p_1 and p_2, where of course $p_1 + p_2 = 1$. In equilibrium, the success ratio p_1/p_2 determines the division of aggregate resources R into the respective shares R_1 and R_2:

(1-5) $$R_1/R_2 = p_1/p_2 \qquad \text{Resource Equilibrium Condition}$$

What I will call the Contest Success Function (CSF) determines the success ratio p_1/p_2 as a function of the ratio of the fighting efforts F_1/F_2 and a <u>decisiveness parameter</u> $m > 0$:[9]

(1-6a) $p_1/p_2 = (F_1/F_2)^m$ Contest Success Function

Or equivalently:

(1-6b) $p_1 = F_1^m/(F_1^m + F_2^m)$ and $p_2 = F_2^m/(F_1^m + F_2^m)$

Figure 2 illustrates how (with F_2 held fixed) the success fraction p_1 responds to changes in fighting effort F_1. Evidently, the sensitivity of p_1 to F_1 grows as the decisiveness parameter m increases.[10]

[Figure 2]

Putting equations (1-5) and (1-6) together:

$$R_1/R_2 = (F_1/F_2)^m = (f_1R_1)^m/(f_2R_2)^m$$

which reduces to:

(1-7a) $f_1^m R_1^{m-1} = f_2^m R_2^{m-1}$

Or, equivalently:

(1-7b) $p_1/p_2 = (f_1/f_2)^{m/(1-m)}$ Equilibrium Success Ratio

Equations (1-7a) or (1-7b) describe the logically required steady-state relationships between the parties' chosen fighting intensities f_i and the equilibrium success ratio p_1/p_2 or resource ratio R_1/R_2. Figure 3 is a plot for different values of m. Note that, as $m \to 1$, the curve approaches a limiting step function such that $p_1/p_2 = 0$ when $f_1 < f_2$ and $p_1/p_2 = \infty$ when $f_1 > f_2$. It follows, therefore, that <u>for a stable steady-state equilibrium, the decisiveness parameter must lie in the range $0 < m \leq 1$</u>.

[Figure 3]

NUMERICAL EXAMPLE 1:

Suppose $m = 2/3$. Then (1-7a) and (1-7b) simplify to:

$$p_1/p_2 = R_1/R_2 = (f_1/f_2)^2$$

If the total resources available are $R = 100$ and the fighting intensities on each side have been chosen (not necessarily optimally) to be $f_1 = .1$ and $f_2 = .2$, respectively, then:

$$p_1/p_2 = (.1/.2)^2 = 1/4$$

implying that, in equilibrium, $R_1 = 20$ and $R_2 = 80$.

To illustrate that this is dynamically stable (and unique), start with any arbitrary initial distribution of resources -- say, $R_1 = R_2 = 50$. The parties having chosen $f_1 = .1$ and $f_2 = .2$, by equation (1-7b) the conflict outcome in the first period is $p_1/p_2 = (30/60)^{2/3} = .630$. Since this ratio diverges from $R_1/R_2 = 1$, we are not yet in equilibrium. The ratio $p_1/p_2 = .630$ implies $p_1 = .386$ and $p_2 = .614$; redistributing the initially equal resource endowments accordingly, at the end of the first period the revised resource distribution becomes $R_1 = .386x100 = 38.6$ and $R_2 = .614x100 = 61.4$. Repeating the process in successive periods, the equilibrium resource distribution $(R_1,R_2) = (20,80)$ is approached asymptotically.

What happens when the conditions for equilibrium are not met? Then, we can say, the anarchic system breaks down. The preceding discussion has brought out one source of breakdown: <u>dynamic instability</u>. Another possible source of breakdown is <u>non-viability</u>. Supposing that some minimum income y is required to sustain life, anarchy cannot be stable if the equilibrium of the dynamic process implies per-capita incomes $Y_i < y$. Summarizing:

•<u>RESULT #1</u>: The conditions for non-breakdown of a 2-party anarchic system are:

$$(1\text{-}8)\quad \begin{array}{ll} m \leq 1 & \text{Condition for dynamic stability} \\ Y_i \geq y \quad (i = 1,2) & \text{Condition for viability} \end{array}$$

II. OPTIMIZATION AND EQUILIBRIUM

Figure 3 does not illustrate the solution of the system, but only the relations that must hold in equilibrium among the dependent variables R_1 and R_2 and the decision variables f_1 and f_2. The actual solution involves optimizing behavior on each side. As in the traditional duopoly model of classical economic theory, there is a problem of defining the optimum when what is best for each side depends upon the other's action. Without further apology, at this point I will use the traditional Cournot assumption -- that contender i chooses an f_i on the assumption that the opponent's f_j will remain unchanged.[11] The individual chooses between e_i and f_i so as to maximize Y_i. A larger f_i captures more territory, but a larger e_i generates more income from the territory currently controlled.

The maximand for individual #1, given the opponent's f_2, is:

$$(2\text{-}1)\quad \text{Max}\quad Y_1 = (e_1R_1)^h = (e_1p_1R)^h = \{e_1Rf_1^M/(f_1^M+f_2^M)\}^h,$$

subject to $a_ie_1 + b_if_1 = 1$

and defining for compactness $M = m/(1-m)$

Forming the Lagrangian and solving, straightforward steps lead to his Reaction Curve RC_1.[12] A corresponding analysis of course leads the opponent to her Reaction Curve RC_2:

(2-2a) $f_1^M/f_2^M = M/(b_1 f_1) - (M + 1)$ Reaction Curve RC_1

(2-2b) $f_2^M/f_1^M = M/(b_2 f_2) - (M + 1)$ Reaction Curve RC_2

The Reaction Curves show how each side's fighting intensity f_i responds to the fighting intensity chosen by the opponent. RC_i, the Reaction Curve for player i, depends on the decisiveness parameter m and upon his or her own logistics cost coefficient b_i; it is independent of the productive scale parameter h, his or her own production cost coefficient a_i, and the opponent's cost coefficients. It is evident from the analytical form of the equations that the Reaction Curves both have positive slopes throughout, as illustrated in Figure 4. (This feature will prove important in interpreting the Stackelberg solutions later on.)

[Figure 4]

Equations (2-2a) and (2-2b) may be solved for f_1 and f_2, determining the equilibrium of the entire system. Unfortunately, there is no convenient general analytic solution. However, consider the special case of <u>symmetrical conflict</u>, i.e., where $b_1 = b_2 = b$. In that case it must be that $f_1 = f_2$ at equilibrium, so (2-2a) and (2-2b) reduce to:

(2-3) $f_1 = f_2 = M/[b(M + 2)] = m/[b(2 - m)]$

Symmetrical conflict equilibrium

Symmetrical solutions for $b = 1$ are illustrated by the intersections of the paired RC_1,RC_2 curves in Figure 4. If $m = 1/2$, the inner pair of curves apply and the solution is $f_1 = f_2 = .333$. At the larger decisiveness parameter $m = 2/3$, the intersection occurs at

$f_1 = f_2 = .5$. Fighting effort being more decisive, each side is forced to "try harder" -- to choose a higher fighting intensity and a correspondingly larger fighting effort F_i. The net result of is reduced incomes for both sides.

Furthermore, from the form of equation (2-3) it is evident that a <u>symmetrical</u> reduction in the logistics cost coefficients will have consequences entirely parallel to what happens when the decisiveness parameter m increases. Thus:

•<u>RESULT #2</u>: Assuming that the conditions for dynamic stability and viability both hold, in the symmetrical conflict situation (equal logistics cost coefficients $b_1 = b_2 = b$) larger values of the decisiveness parameter m imply higher equilibrium fighting intensities f_1 and f_2, and thus higher fighting effort levels F_i and F_2. And similarly, the lower is the common value b of the logistics cost coefficient, the greater will be the f_i and F_i.

Since $p_1 = p_2 = 1/2$ in the symmetrical conflict situation, direct substitutions lead to the equilibrium per-capita incomes:

$$(2\text{-}4) \qquad Y_i = (e_i p_i R)^h = \left[\frac{1 - m}{a_i(2 - m)} R\right]^h$$

provided of course that $m \le 1$ and $Y_i \ge y$. The form of equation (2-4) leads directly to:

•<u>RESULT #3</u>: In the symmetrical conflict situation, assuming once again that the conditions for dynamic stability and viability both hold, the incomes achieved (i) are independent of the level of the logistics cost coefficient b, (ii) increase as aggregate resource availability R or the productive scale parameter h rise, but (iii) decrease as the decisiveness parameter m increases. Also, (iv) for either player, income also decreases as his/her production cost coefficient a_i rises.

NUMERICAL EXAMPLE 2:

In the previous Numerical Example, with $R = 100$ and $m = 2/3$, the contenders were arbitrarily assumed to have chosen fighting intensities $f_1 = .1$ and $f_2 = .2$, leading to the equilibrium resource distribution $R_1 = 20$ and $R_2 = 80$. Suppose now that the parties choose optimally under the Nash-Cournot assumption, holding to the parameter values $R = 100$ and $m = 2/3$, and now explicitly assuming symmetrical logistics cost coefficients $b_1 = b_2 = b = 1$. From equation (2-3) the equilibrium choices are $f_1 = f_2 = .5$, implying an equal equilibrium resource division $R_1 = R_2 = 50$. From (2-4), and assuming production cost coefficients $a_1 = a_2 = 1$ and a productive scale parameter $h = 1$, the associated incomes are $Y_1 = Y_2 = 25$. (In contrast, had no conflict occurred -- i.e, if the choices had been $f_1 = f_2 = 0$ -- the per-capita incomes would have been 50 each.)

Apart from m, the decisiveness parameter, none of the parameters in (2-4) affect the Reaction Curve on either side or consequently the f_1, f_2 solution. However, since these other parameters do affect Y_i, the level of income achieved, they have a bearing upon whether or not the viability condition is met.

NUMERICAL EXAMPLE 3:

In Numerical Example 2 the equilibrium choices were $f_1 = f_2 =$

.5, implying an equal resource division $R_1 = R_2 = 50$ and associated incomes $Y_1 = Y_2 = 25$. Maintaining all the previous assumptions except for reducing the value of the productive scale parameter from $h = 1$ (constant returns) to $h = .95$ (diminishing returns), the per-capita incomes fall from $Y_i = 25$ to $Y_i = 19.5$. So if the viability threshold were, say, $y = 20$, for $h = .95$ anarchy would break down.

To summarize: Of the two conditions for non-breakdown of anarchy, the dynamic stability condition depends solely upon the decisiveness parameter, to wit, $m \leq 1$. Only if m is sufficiently low, i.e., if the success ratio is sufficiently insensitive to the fighting effort ratio F_1/F_2, can there be a stable interior equilibrium. In the special case of symmetrical conflict (equal logistics cost coefficients $b_1 = b_2 = b$), the equilibrium levels of the fighting intensities f_i increase as the decisiveness parameter m rises and as the logistics cost parameter b falls. But whether the viability condition $Y_i \geq y$ is met will depend also upon the aggregate resource availability R, the productive scale parameter h, and the party's productive cost coefficient a_i -- since all these enter also into the determination of the absolute income levels Y_i.

III. VARYING NUMBERS -- EXOGENOUS VERSUS ENDOGENOUS N

Exogenous N

Suppose now that there are now a fixed number of competitors N, where $N > 2$. Assume that the contest is a melee, a Hobbesian struggle of each against all, and that R, the aggregate of resources available, is constant. Using the Cournot solution concept again, each contender i chooses an f_i (fraction of resources devoted to fighting) on the assumption that every opponent j will be holding his or her f_j fixed. Generalizing equations (1-7a):

(3-1a) $f_1^m R_1^{m-1} = f_2^m R_2^{m-1} = \ldots = f_N^m R_N^{m-1}$

Or, equivalently:

(3-1b) $p_1:p_2:\ldots:p_N = (f_1:f_2:\ldots:f_N)^M$,

For dynamic stability, once again it is necessary to have $m \leq 1$. In addition, as before the viability condition $Y_i \geq y$ must also hold.

For simplicity, in this section conflict symmetry (equal logistics cost coefficients $b_1 = b_2$) and productive symmetry (equal production cost coefficients $a_1 = a_2$) are assumed to hold, and as a numerical convenience all these parameters can be set at unity. Then $f_1 = f_2 = \ldots = f_N$ at equilibrium. For any contender i, the maximand is:

(3-2) $\text{Max} \quad Y_i = e_i R_i = e_i p_i R = e_i R f_i^M/(f_1^M + f_2^M + \ldots + f_N^M)$,

subject to $e_i + f_i = 1$

Proceeding by straightforward steps, the Reaction Curve for competitor i, the analog of equation (2-2a), is:

(3-3) $f_i^M/(f_1^M + \ldots + f_N^M) = M/f_i - (M + 1)$ Reaction Curve RC_i

Using the fact that in symmetrical equilibrium all the f_i are equal, the solution for the fighting intensities is:

(3-4) $$f_1 = f_2 = \ldots = f_N = \frac{M}{M + 1 + 1/(N-1)} = \frac{m(N - 1)}{N - m}$$

It is evident that the fraction of resources devoted to fighting <u>rises</u> as the number of contestants increases. The equilibrium incomes are:

$$(3\text{-}5) \qquad Y_i = (e_i p_i R)^h = \left(\frac{1 - m}{N - m} R\right)^h$$

provided of course that $m \leq 1$ and $Y_i \geq y$

•<u>RESULT #4A</u>: Assuming productive and fighting symmetry (the production cost coefficients a_i are equal and so are the logistics cost coefficients b_i), and if aggregate resources remain fixed (that is, if $\Sigma_i R_i = R$, a constant), then as the number of contestants increases the average competitor is impoverished in two ways: <u>first</u>, because the equilibrium pro-rata resource share $p_i = 1/N$ must fall as N grows, and <u>second</u>, because the equilibrium f_i rises. That is, each contender has to waste proportionately more effort in fighting even to obtain this reduced share.[13]

Now consider a more friendly environment such that the aggregate resource base is not fixed but grows in proportion to the number of claimants. We can imagine that each entrant brings into the economy a resource quantum r, so that $R \equiv Nr$. Evidently, the expanding resource base would exactly cancel out the adverse effect upon per-capita incomes of the fall in $p_i = 1/N$. But the effect of the larger commitments to fighting efforts remains. Under this more optimistic assumption the equilibrium incomes are:

$$(3\text{-}6) \qquad Y_i = (e_i p_i R)^h = \left(\frac{1 - m}{N - m} Nr\right)^h$$

•<u>RESULT #4B</u>: Even if aggregate resource availability increases in proportion to numbers N, the average level of income still falls as N rises, owing to the higher equilibrium fighting intensities f_i.

Figure 5 illustrates how fighting intensity f_i rises with numbers N, and the implications of that fact for per-capita income $Y_i = (e_i p_i R)^h$ under the more and less favorable assumptions about the relation of aggregate resources to the number of contenders. (The parameter values for the diagram are as stated in Numerical Example 4 below.)

[Figure 5]

Once again, of course, these results are valid only if the stability condition ($m \leq 1$) and the viability condition ($Y_i \geq y$) both hold.

NUMERICAL EXAMPLE 4: In Numerical Example 2 with parameters $m = 2/3$ and $a_i = b_i = h = 1$, for $N = 2$ and aggregate resources $R = 100$ the equilibrium fighting intensities were $f_1 = f_2 = .5$ while the associated per-capita incomes were $Y_1 = Y_2 = 25$. For $N = 3$, with aggregate resources still fixed at 100, from (3-4) the equilibrium fighting efforts rise to $f_1 = f_2 = .571$, while from (3-5) the per-capita incomes fall to $Y_i = 14.3$, approximately. If on the other hand resources rise in proportion to numbers -- specifically here, if $R \equiv Nr$, where $r = 50$ -- equation (3-6) implies $Y_i = 21.4$, approximately. Thus, even when the resource base expands in proportion to N, there is a per-capita income loss owing to the larger optimal f_i.

Endogenous N

If the economy is not closed but is instead subject to immigration or emigration, the equilibrium number of competitors N will be determined by the viability limit -- a kind of "zero-profit" condition:

(3-7) $Y_i(N) = y$ Condition for equilibrium N

Once again, the actual viable population will depend upon whether aggregate resources R are fixed or alternatively grow in proportion to numbers N.

NUMERICAL EXAMPLE 5: With the same parameter values as in the previous Example, for the case where aggregate resources are fixed at $R = 100$ suppose the viability threshold is $y = 4$. It can be verified from (3-5) that the equilibrium incomes are $Y_i = 4$ at $N = 9$. So this fixed resource magnitude will support a population of $N = 9$ competitors.

If instead resources expand with population in such a way that $R \equiv 50N$, the situation is much more favorable. In such an environment, equation (3-7) indicates that $N = 9$ would be the equilibrium population for a viability threshold as high as $y = 18$.

Summarizing, larger N implies lower per-capita incomes Y_i even if each added player brings in a pro-rata increment of resources. The reason is that with larger N each contestant has to fight harder just to maintain his or her proportionate share. If N is endogenously determined, a "zero-profit" condition will establish the viable number of contestants.

IV. ASYMMETRICAL CAPACITIES: PRODUCTIVE ADVANTAGE vs. FIGHTING ADVANTAGE

The emphasis to this point has been upon symmetrical solutions. This section examines the implications of asymmetrical <u>capacities</u> as between the players. The section following will address asymmetries of position.

Asymmetrical conversion cost coefficients

One type of asymmetry has already been mentioned: differences between competitors in their abilities to convert resources into productive effort E_i or fighting effort F_i, as reflected in the magnitudes of a_i and b_i respectively. As previously noted, apart from the special case of equal logistics cost coefficients ($b_1 = b_2$) there is no convenient analytical solution for equations (2-2a) and (2-2b) -- that is, for the intersection of the Reaction Curves that determines the equilibrium of the system as a whole. Nevertheless, the implications of divergences in the conversion cost coefficients a_i and b_i can be illustrated by the numerical simulations pictured in Figures 6 and 7.

[Figures 6 and 7]

All the simulations maintain the parameter values of Numerical Example 2, apart from the asymmetries introduced by the indicated changes in a_1 (Figure 6) and b_1 (Figure 7). Without going over the details, we can summarize:

•RESULT #5: As a_1 rises relative to a_2, other things equal Y_1 must fall while Y_2 and all the other variables remain unaffected. Thus, an absolute disadvantage in production ($a_1 > b_1$) impacts only upon own income. But an absolute disadvantage in fighting ($b_1 > b_2$) affects the fighting intensities as well ($f_1 < f_2$), implying both lower own income Y_1 and higher opponent income Y_2.

Other asymmetries

Other types of asymmetries may also be important. In the production function equation (1-4), for example, it might be that the respective production processes do not reflect the same degree of returns to scale. Instead of a common exponent h there might be distinct exponents h_1 and h_2. Owing once again to the maintained assumption that the production processes on the two sides are entirely disjoint, any such changes will impact only upon the parties' own incomes.

On the fighting side, instead of a common decisiveness parameter m in equation (1-6a) there might be distinct exponents m_1 and m_2. If $m_1 > m_2$, the implication is that a higher degree of fighting effort on the part of contender #1 ($F_1 > F_2$) is <u>more</u> decisive with respect to the outcome than the reversed disparity ($F_2 > F_1$) would be. This might be a way of modelling a situation in which player 1 'takes the offensive' (aims for a decisive outcome) while player 2 'stands on the defensive' (hoping to reduce the chances of a decisive outcome).

V. POSITIONAL ASYMMETRIES: STACKELBERG SOLUTIONS

Just as capacities may differ in several ways, there are several kinds of <u>asymmetries of position</u>. Only the Stackelberg situation is considered here, in which one of the rivals is a "leader" who moves first in choosing his fighting intensity -- to which the opponent then responds with her optimal reaction.[14] Ability to move first is often an advantage, for example 'taking the high ground' in a military context. But the second-mover, being able to optimize in the light of the opponent's known choice, has an informational advantage. So it is not immediately clear whether, in the present context, a Stackelberg leader can be expected to come out ahead.

If contender #1 is the Stackelberg leader, he can choose his fighting intensity f_1 in the knowledge that the opponent's f_2 reply will be governed by her known Reaction Curve RC_2. To isolate the effect of positional asymmetries, in this section fully symmetrical capacities are assumed -- and specifically, all the conversion cost parameters are set at $a_i = b_i = 1$. Also, nothing is lost by setting the scale parameter $h = 1$. Then, player 1's analytical problem is:

$$(5\text{-}1) \qquad \text{Max} \quad Y_1 = e_1 p_1 R = R(1 - f_1) \quad \frac{f_1^M}{f_1^M + f_2^M}$$

$$\text{subject to } f_2^M/f_1^M = M/f_2 - (M + 1)$$

where the constraint is RC_2 as given by equation (2-2b).

The question is, given the opportunity to do so, would the Stackelberg leader choose an f_1 that diverges from his Cournot solution, and if so in which direction? The actual maximization is analytically somewhat intractable. But recall now that both Reaction Curves necessarily have positive slope throughout. Thus, if player 1 were to choose a larger-than-Cournot f_1, player 2 would respond with a somewhat larger f_2 -- implying a lower aggregate income for the two sides together. It has been shown by Esther Gal-Or [1985] that, <u>whenever the Reaction Curves are positively sloped, the relative advantage is always to the second-mover</u>. An example: in duopoly equilibrium, when quantity is the decision variable the Reaction Curves are negatively sloped and it is advantageous to move first; when price is the decision variable, the curves are positively sloped and the advantage is to the last-mover.

That the relative advantage is to the second-mover is confirmed by the numerical simulation pictured in Figure 8. With player #1 as leader, the diagram shows the respective fighting intensities f_1 and f_2 and the incomes Y_1 and Y_2, as a function of his choice of f_1 (using once again the quantitative assumptions of Numerical Example 2). The Cournot equilibrium is represented as before by $f_1 = f_2 = .5$ and $Y_1 = Y_2 = 25$. As can be seen, under the assumptions here the leader does best by choosing a somewhat lower $f_1 = .41$, approximately, his income rising slightly to about 25.7. The second-mover optimally responds by cutting back her fighting intensity only to about .466, reaping a considerably higher income of around 30.1.

[Figure 8]

•RESULT #6: In shifting from the Cournot to the Stackelberg equilibrium, both parties gain. But the follower does better than the leader!

VI. CONCLUDING REMARKS

It will be convenient to summarize by responding briefly to the specific questions raised in the Introduction.

1. When is there a stable anarchic solution? The conditions for non-breakdown of anarchy are:

(i) Dynamic stability: to preclude the situation in which an initially more powerful contender captures more and more resources and eventually eliminates all opposition, it must be that the sensitivity of the conflict outcome to force disparities must be sufficiently low.

Specifically, the decisiveness parameter m in the Contest Success Function (CSF) must lie in the range $m \leq 1$.

(ii) Viability: Per-capita incomes under anarchy must meet or exceed the minimum survivable income level ($Y_i \geq y$). The attained Y_i will depend upon many parameters including the aggregate resources R, the conversion cost coefficients a_i and b_i, and the returns-to-scale coefficient h.

2. Equilibrium allocations of effort: In the Cournot solution with $N = 2$ contestants, the equilibrium fighting intensities on each side are determined by the intersection of the parties' Reaction Curves RC_i. A closed solution was obtained for the symmetrical conflict case with equal logistics cost coefficients $b_1 = b_2 = b$. The crucial result is that, as the decisiveness coefficient m rises, each side is forced to fight harder -- leading to increased mutual loss of potentially achievable income. In the symmetrical conflict model, the incomes achieved Y_i are independent of the actual level of b but vary in the expected way with R, h, and the production cost coefficients a_i.

3. Numbers: The key result is that, as N grows, the equilibrium fighting intensities f_i rise. Consequently, if aggregate resources R are fixed, per-capita income Y_i falls for two reasons: **first**, because each party's pro-rata share $p_i = 1/N$ is less, but **second**, because f_i is rising. (A contestant has to fight harder just to retain the pro-rata share.) And even in a more generous environment where resources grow in proportion to N, there will still be lower per-capita incomes owing to f_i being greater. If the number of

contenders N is endogenous, the equilibrium N will be determined by the viability condition $Y_i \geq y$ -- that is, entry occurs up to the point of "zero profit."

4. Technology and comparative advantage: In the model here, an asymmetrical productive improvement in, say, contestant #1's production cost coefficient a_1 increases own income Y_1 but does not otherwise affect any of the results. Asymmetrical improvements on the conflict side, however -- say, a reduction in the logistics cost coefficient b_1 -- not only increases own income but also reduces the opponent's income Y_2.

5. Strategic position: As compared with Cournot equilibrium, the Stackelberg solution involves reduced fighting effort on both sides -- but the reduction is greater for the leader than the follower. So, although both sides end up absolutely better off as compared with the Cournot outcome, the leader is relatively worse off. This evidently tends to stabilize the anarchic system. Although all parties could gain by shifting to a Stackelberg leadership pattern, each participant is motivated to hold back and let the opponent take the lead.

As a general qualification, all the results here depend upon a particular way of modelling anarchy that omits many possibly important aspects of the anarchic situation. To mention only a few: (1) Full information was assumed throughout, so that factors like deception have been set aside.[15] (2) Apart from opportunity costs in the form of foregone production, fighting was assumed non-destructive.[16] (3) The effects of distance and other geographical factors were not been considered. (4) The steady-state assumption rules out issues involving timing, such as arms races, economic growth, or (on a smaller time-scale) signalling resolve through successive escalation. (5) In the postulated ratio form of the Contest Success Function, 'peace' -- i.e., where it is optimal on both sides to engage in zero fighting effort -- is impossible. If the outcome of conflict depended, possibly more reasonably, upon the <u>difference</u> between the fighting commitments, a peaceful equilibrium would be possible.[17]

The justification for these omissions is that one must begin somewhere. The model illustrates a method of analysis. In some particular context, it might be unacceptable to omit the element of collateral damage (qualification #2 above), for example. Still, that effect could be incorporated by means of an adjustment that nevertheless preserves the general analytical framework developed here.

As a final question, supposing that anarchy does break down, what happens next? Such a query moves outside the range of the model, but nevertheless there are some suggestive indications.

Anarchic breakdown, the analysis has indicated, could be due either to <u>dynamic instability</u> or to <u>non-viability</u>. In the former case, owing to the decisiveness parameter m being too great, whichever side has a sufficient initial advantage would capture more and more resources and ultimately become all-powerful. So any momentary advantage will lead to total extinction of opponents.[18] Or, it might be that m is not too high, but the incomes Y_i achieved under anarchy are too low for viability.

In either case, on the social level what is needed is to reduce the wastage of potential income due to excessive fighting. One way out might be a Hobbesian social contract in which parties facing extinction accept subordination as the price of survival. Or alternatively, a Lockian social contract that replaces anarchic defense of resources with secure property rights. Either solution promises to increase aggregate output via new modes of production and exchange that can take advantage of the combined efforts of the parties. Still, neither of these solutions is costless. In a Hobbesian world the struggle to achieve alpha rank also consumes resources, while Lockian property rights have to be defended against coercion and fraud. But even if there were a clear social profit to be gained, the questions of who might be motivated to bring about the transition from anarchy and how he or she could achieve it remain to be modelled.[19]

ENDNOTES

1.Natural economy and political economy are compared in Ghiselin [1978] and Hirshleifer [1978].

2.See, for example, Wilson [1975], Trivers [1985].

3.*Cf* Waltz [1954], Snyder and Diesing [1977], Bernholz [1985].

4.A convenient survey is McNaughton and Wolf [1973], Ch. 11-12.

5.On these two technologies see Hirshleifer [1991a].

6.Hirshleifer [1988, 1991b].

7.The somewhat parallel literature on rent-seeking, starting with Tullock [1967], differs in two main respects. First, rent-seeking is typically concerned only with a rather innocuous process of *bidding* for resource control, rather than fighting for it. Second, the rent-seeking literature typically postulates prizes that are exogenously given rather than determined endogenously through the interaction of production and struggle.

8.While e_i and f_i are necessarily non-negative, either or both may exceed unity (since E_i and F_i need not be scaled in the same units as R_i). In the special case where $a_i = b_i = 1$, however, e_i and f_i are each bounded by 0 and 1.

9.This form of the CSF, in which the success fractions are determined by the *ratio* of the fighting efforts, was proposed in Tullock [1980]. If instead the outcome depends upon the *difference* between the fighting efforts, the CSF takes the form of a 'logistic' curve (see Hirshleifer [1988]. Both forms are instances of the more general logit functions described in Dixit [1987]).

10.When $m \leq 1$, diminishing returns to fighting effort hold throughout. For $m > 1$, however, there are increasing returns to fighting effort over an initial range.

11.I assume that fixed one-time choices of f_1 and f_2 are made on each side. More generally, the decision for contender i would involve choosing an optimizing curve $f_i(R_i)$ as a function of $f_j(R_j)$. I do not address this more difficult problem.

12.For player #1 the first-order conditions, where λ is the Lagrangian multiplier, are:

$$\frac{\partial L}{\partial e_1} = h(e_1 p_1 R)^{h-1} \frac{R f_1^M}{f_1^M + f_2^M} - \lambda a_1 = 0$$

$$\frac{\partial L}{\partial f_1} = h(e_1 p_1 R)^{h-1} \frac{e_1 R M f_1^{M-1} f_2^M}{(f_1^M + f_2^M)^2} - \lambda b_1 = 0$$

Routine steps then lead to equation (2-2a), the Reaction Curve for player 1.

13.However, the wastage fractions f_i do not approach unity in the limit. Instead, as is evident from equation (3-4), as N grows indefinitely large we have: $\lim f_i = M/(M + 1) = m$.

14.The Stackelberg type of positional asymmetry may arguably be taking us outside the domain of anarchy, properly speaking. Definitely diverging from anarchy (and therefore not considered here) is 'hierarchial' asymmetry -- where, in order to influence the follower's behavior, the leader is able to issue a prior threat and/or promise while guaranteeing to execute it ex-post (Hirshleifer [1988]). The relation between Stackelberg and hierarchial positions has also been explored in unpublished work by Raymond E. Franck, Jr., and see also

Thompson and Faith [1981].

15.On this see, for example, Tullock [1974, Ch. 10] and Brams [1977].

16.In Becker [1983], for instance, incidental damage to the economy (called 'deadweight cost') plays a crucial role in limiting the extent of conflict.

17.As emphasized in Hirshleifer [1988] and Skaperdas [1992].

18.A rather drastic version of such an assumption appears in Niou and Ordeshook [1986], for example. In their model, if at any moment of time a state controls more than half the global aggregate of resources, it can always costlessly gobble up the remainder!

19.So far as I know, this question has been systematically addressed only in the biological literature. Specifically, biologists have analyzed the conditions leading to various forms of social structures, notably territoriality (anarchy with sequestered resources) or dominance (hierarchy). One general concern has been whether _species or group advantage_ (for example, in reducing the resource loss due to fighting) can of itself effectuate social change or, alternatively, whether such change only occur to the extent that individuals responding to their _private genetic advantages_ are motivated to bring it about. The modern consensus favors the latter interpretation -- the 'selfish gene' hypothesis (Dawkins [1976]). As an example of a more specific question, biologists have asked whether impoverishment of the environment, other things equal, tends to favor territorial (anarchic) or hierarchial systems. See, for example, Morse [1980, pp. 246-251] and especially Vehrencamp [1983].

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Figures

Figure 1
PRODUCTION FUNCTION -- RETURNS TO SCALE

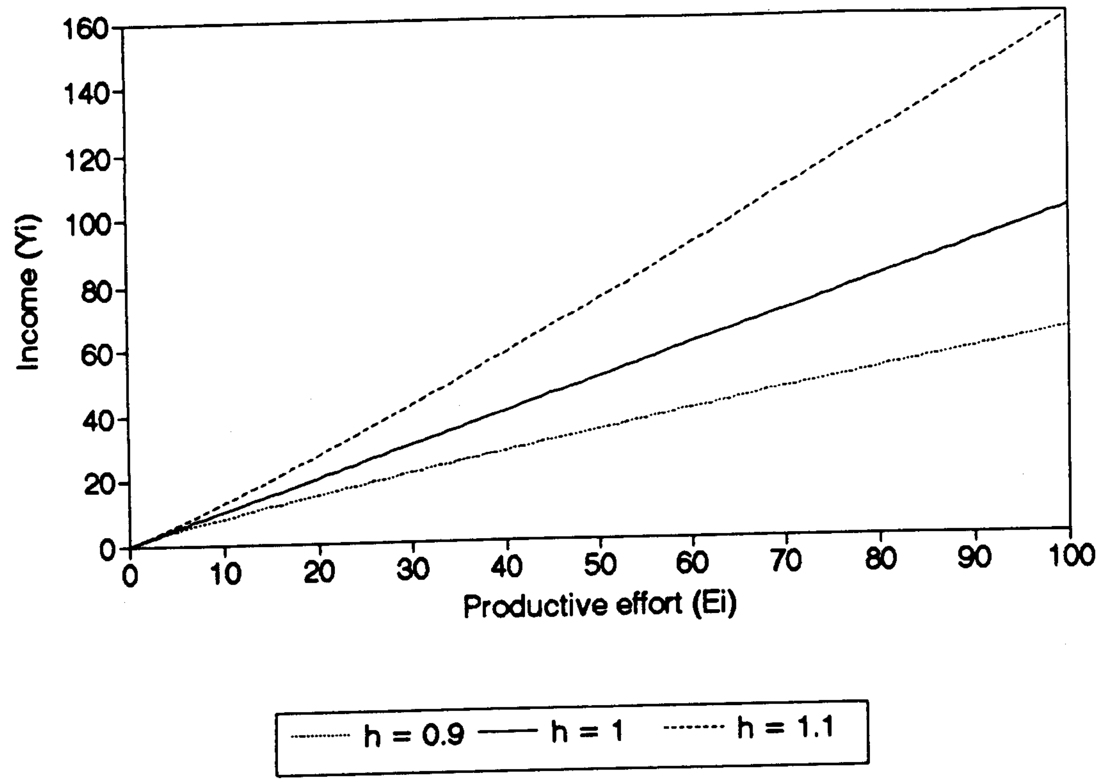

Figure 2
CONTEST SUCCESS FUNCTION (CSF)

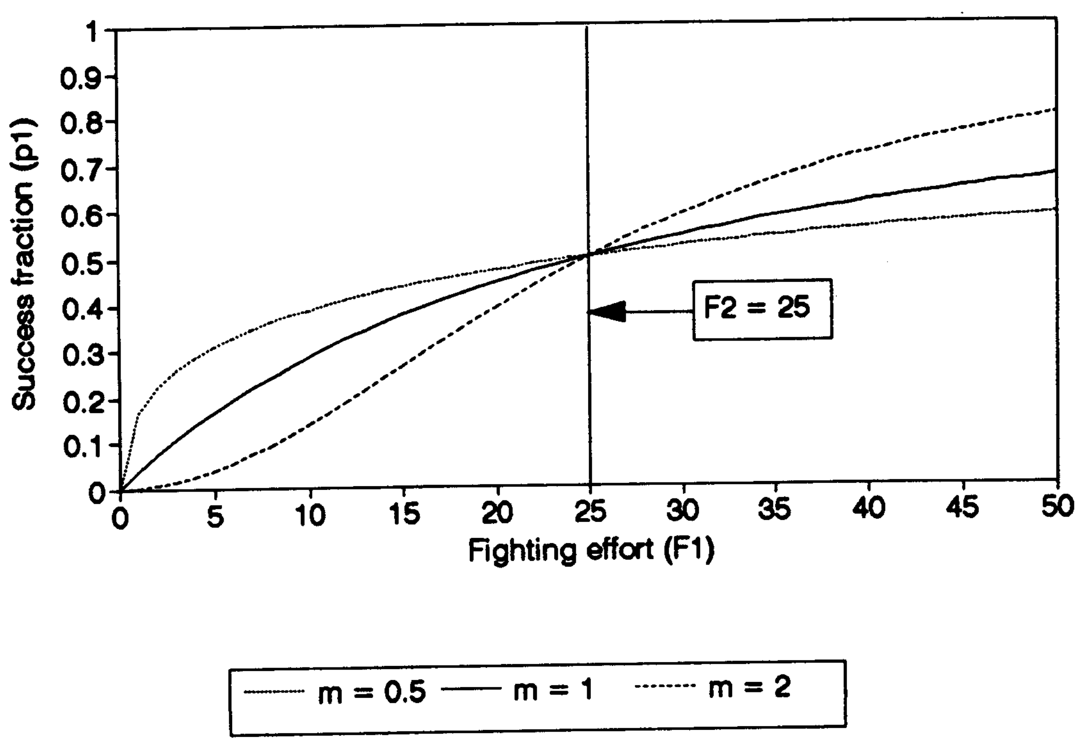

Figure 3
FIGHTING INTENSITIES AND SUCCESS RATIO

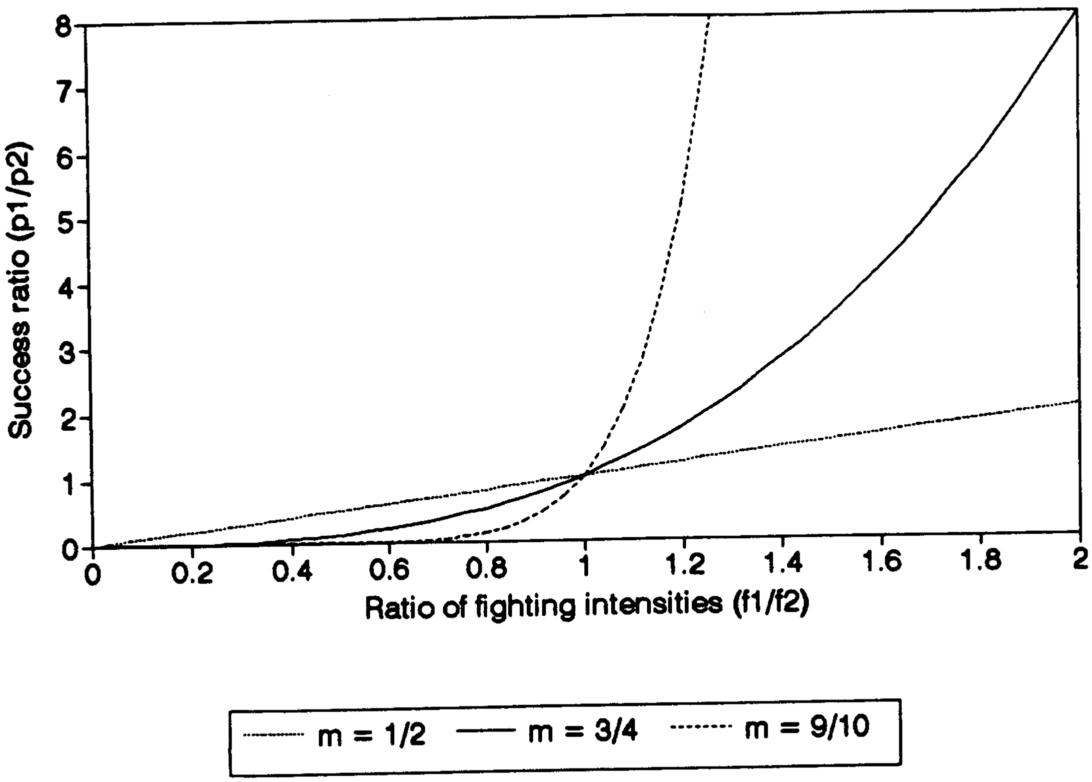

Figure 4
REACTION CURVES (m = 1/2 and m = 2/3)

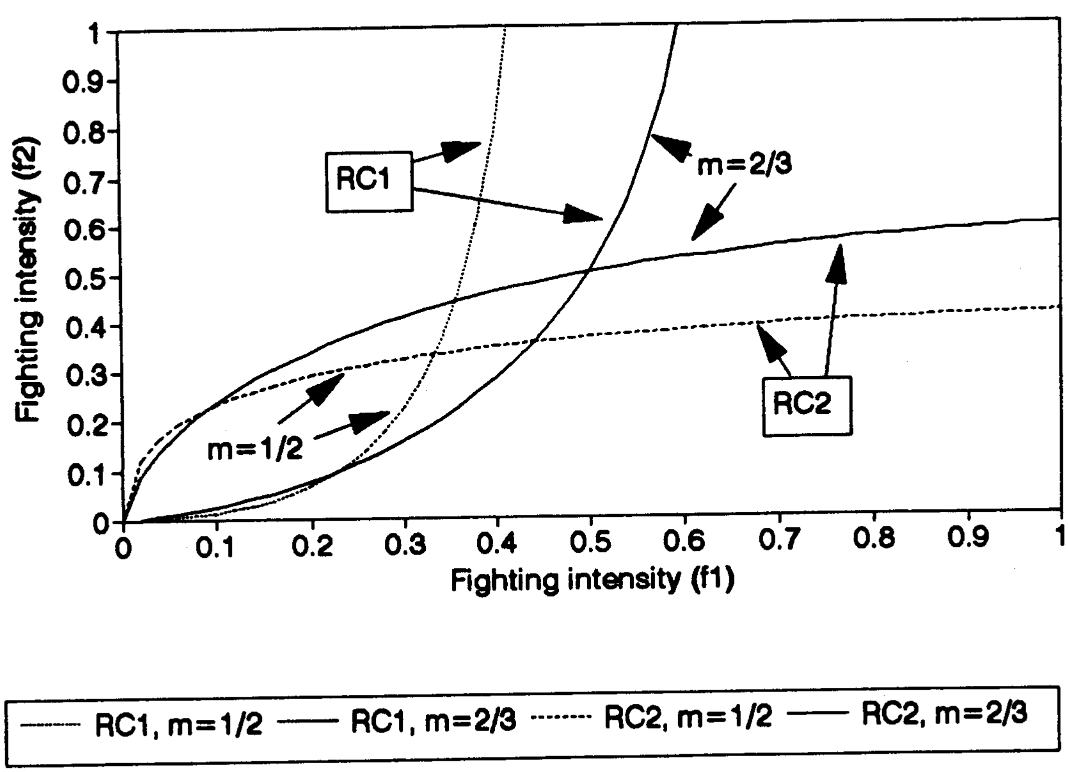

Figure 5
EFFECT OF RISING NUMBERS (N)

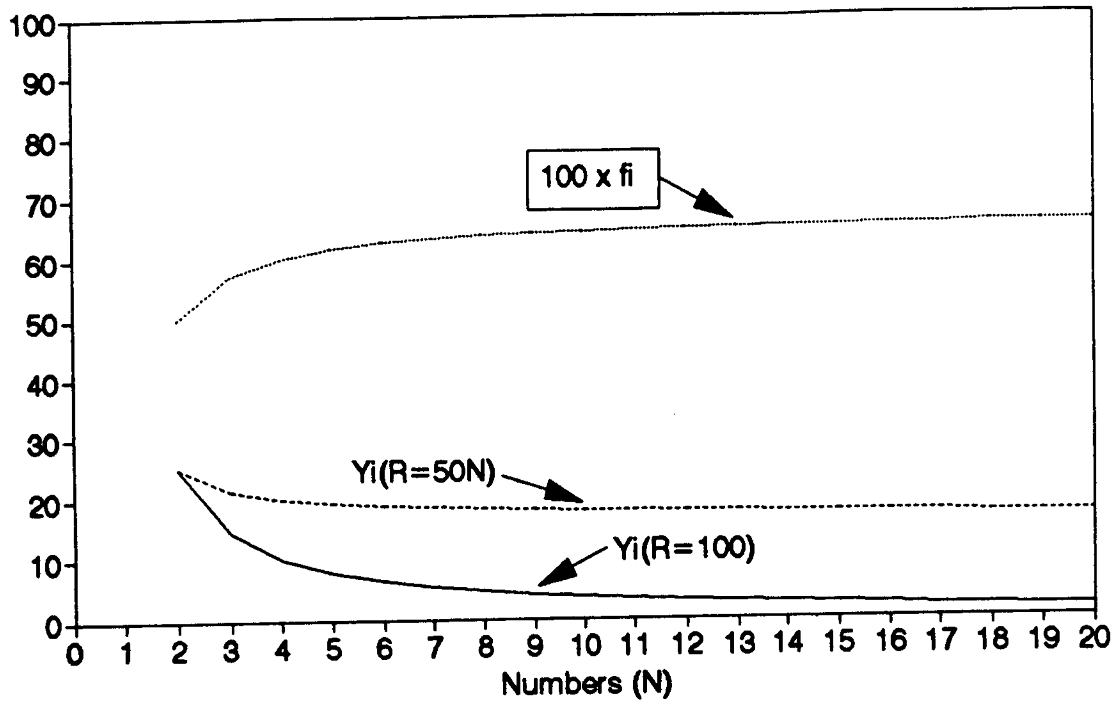

Figure 6
EFFECT OF PRODUCTION COST ASYMMETRY

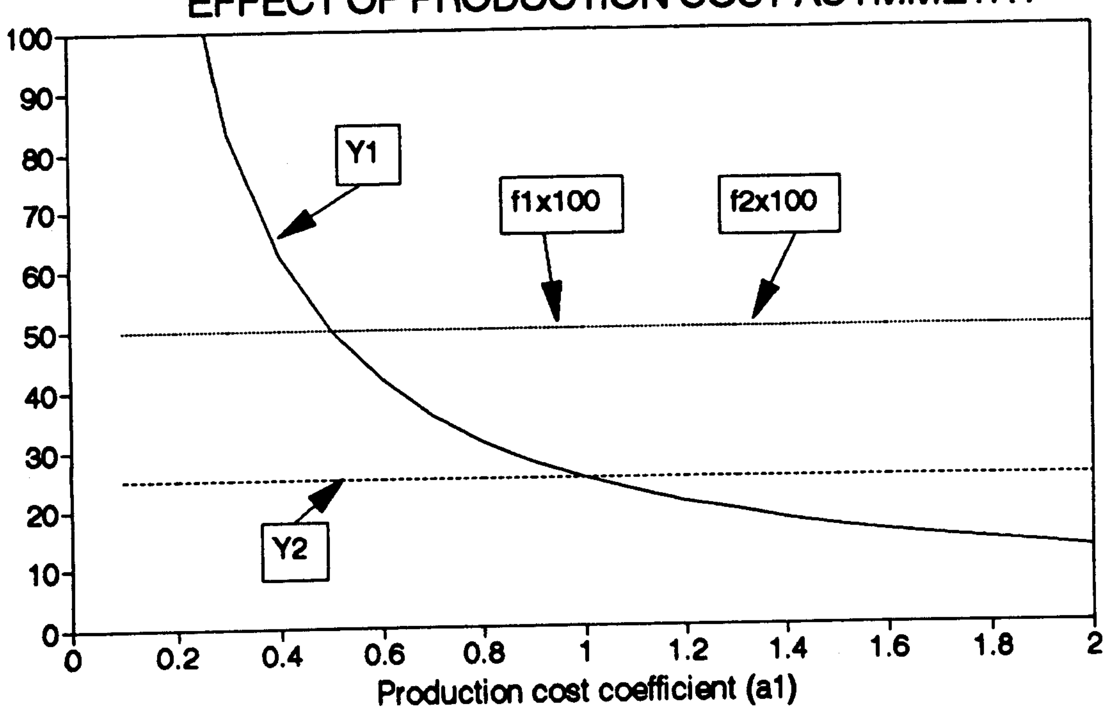

Figure 7
EFFECT OF LOGISTICS COST ASYMMETRY

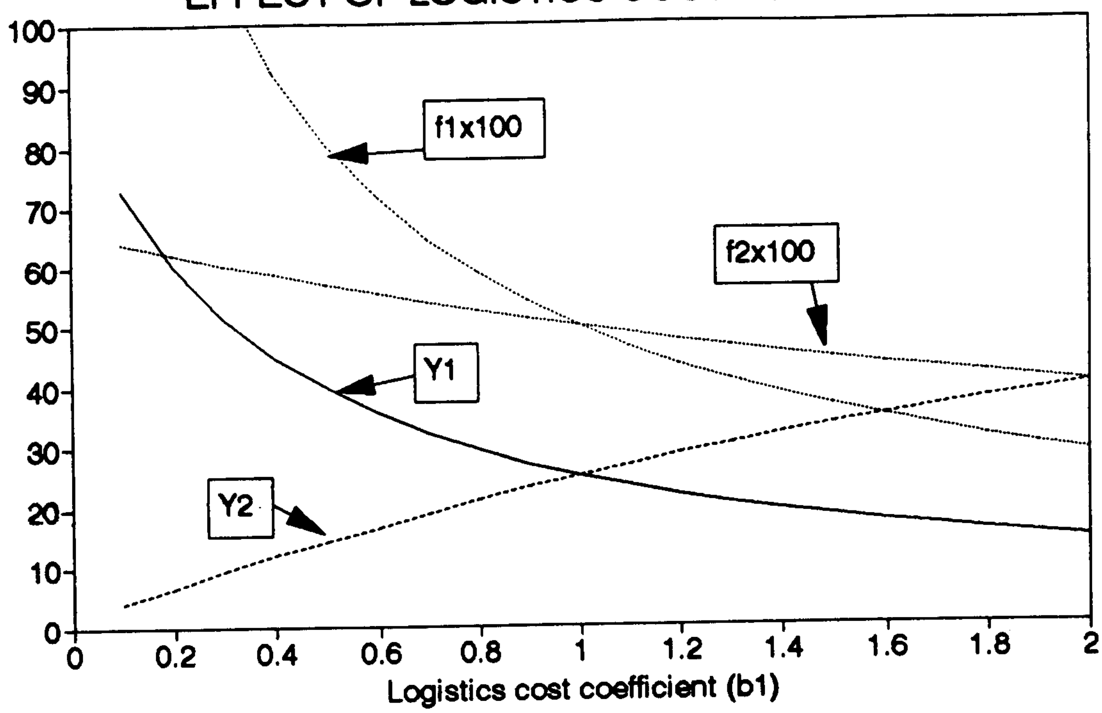

Figure 8
STACKELBERG OPTIMUM