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INCENTIVES IN TEAMS

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This paper analyzes the problem of inducing the members of an organization to behave as if they formed a team. Considered is a conglomerate-type organization consisting of a set of semi-autonomous subunits that are coordinated by the organization's head. The head's incentive problem is to choose a set of employee compensation rules that will induce his subunit managers to communicate accurate information and take optimal decisions. The main result exhibits a particular set of compensation rules, an optimal incentive structure, that leads to team behavior. Particular attention is directed to the informational aspects of the problem. An extended example of a resource allocation model is discussed and the optimal incentive structure is interpreted in terms of prices charged by the head for resources allocated to the subunits.

1. INTRODUCTION

THE PROBLEM OF INCENTIVES appeared in economic theory with the debate known as the Socialist Controversy.[2] Although this debate focused on the informational and computational feasibility of a centrally directed economic system making the vast number of computations required for an efficient allocation, the incentive question interlaced much of the discussion. In competitive free enterprise economies, informational efficiency is achieved through individual decision making in markets where the agents are motivated by self-interest. For centrally planned economies, a dilemma appears: Either the Central Planning Bureau (CPB) provides detailed instructions to each agent, in which case the incentive problem involves guaranteeing the quality (accuracy and completeness) of a tremendous quantity of information, or alternatively, decision making is decentralized in the manner of Market Socialism (thus easing the informational burden on the CPB) and the incentive problem involves motivating the agents to behave in the prescribed way, as, say, Lange-Lerner civil servants.

The incentive problem is not limited to centrally planned economies, but is encountered in any large organization of which the centrally planned economy may be considered an extreme example. As Charles Schultze points out, "the problem of incentives is... an aspect of social behavior which should be taken into account at every stage of public policy formation" [**9**, p. 203]. The elements of an incentive problem are an organization consisting of many members with different information and decision possibilities, and some clear organizational objective that may not be coincident with the members' individual objectives. Employees of a large organization who are far down the hierarchy, for example, might be

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[2] For a description of the debate, see Ward [**10**, Ch. II).

expected to pursue objectives quite different from the organization's goals, particularly when the organization's leaders are limited informationally and hence unable to monitor their employees' actions.

The *theory of teams*, developed by Marschak and Radner [**3**, **4**, 7, **8**], analyzes organizational decision making where different members' decisions may depend on different variables, but are guided by a common organization goal. Thus, in standard team problem formulations, there is no incentive problem since there is no conflict of interest. However in this paper the methods of team theory are used in viewing the incentive problem as a problem of team formation or of the formulation of mechanisms to induce employees to make decisions conforming to the organizational interest, even when payoffs are uncertain or unknown and decisions depend on incomplete information.

In Section 2, the incentive problem in a team model of a general organization is formulated and the issue of information necessary to induce team behavior is raised. In Section 3, a special type of organization, called a conglomerate, is discussed in terms of the incentive problem. The main result of the paper is given in a theorem that exhibits a system of compensation rules for employees of a conglomerate organization that will induce them to behave as a team, i.e., to send optimal information and make optimal decisions from the point of view of the organization objective. A crucial point of this result is that these rules do not require the organization leader to possess any additional information in order to compensate his employees or even to have knowledge of the true accuracy or completeness of his information. The optimal compensation rules will elicit accurate and complete information messages from the employees. Finally, in Section 4 an extensive example is given of this result in a resource allocation model where the optimal compensation rules are interpreted in terms of prices charged by a resource manager for resources allocated to a collection of enterprises.

2. THE INCENTIVE PROBLEM IN A GENERAL ORGANIZATION

A. *The Team Model*

A team decision problem is, roughly speaking, a multi-person joint decision problem in which the decision makers base their decision choices on different information, yet are motivated by a common goal. In a team model the information available to the decision makers is acquired through observations of the environment and communication with each other. Thus, given a payoff function reflecting the team's objectives or goals, the task of its members is to select rules for acquiring information through observation and communication and for making decisions based on this information that maximize the payoff.

The assumption of a common objective and different information available to the decision makers lends the team model a particular appropriateness for analyzing large-scale organizations including centrally planned economies with Lange-Lerner bureaucrats.

We may consider a team decision problem arising in an organization in which the payoff function reflects the goals or preferences of the organization's leader or head and the remainder of the decision makers are employees of the organization. If the employees also share the head's preferences or behave as if they did, the model of such an organization is a team model.

However, in general, employees may not share the head's preferences (or behave as if they did). Their behavior might more accurately be analyzed in terms of the compensation they receive from the organization for their participation in it. If their compensation is determined by the organization head in accordance with some specified set of rules, and if it is assumed that the employees behave in such a way as to maximize their compensation, the decision problem of the organization may formally be represented as an n-person game.

Typically, the head of an organization has some latitude in selecting the rules for compensating his employees and it would be desirable for him to select rules, if they exist, that will induce his employees to behave as if they were members of a team. Any set of compensation rules is called an *incentive structure*, and thus, we may view the organization head's incentive problem as finding an optimal incentive structure, i.e., one inducing his employees to behave as if they formed a team.

Formally,[3] we summarize the discussion as follows: (i) Let $I = \{0, 1, \ldots, n\}$ denote a set of decision makers, where $i = 0$ denotes the organization head, and $i = 1, \ldots, n$ denote his employees; (ii) let $(S, \mathcal{S}, P)$ denote a probability space of alternative states of the environment relevant to the organization's decision problem; (iii) let $\{B_i, i \in I\}$ denote $n + 1$ sets of the decision makers alternative strategies; and (iv) let $\omega_0 : B \times S \to R$ denote a real-valued payoff function[4] defined on the set of joint strategies $B \equiv \times_{i=0}^{n} B_i$ and the state space S. The strategy sets $B_i, i \in I$, are composed of strategies β_i that specify, for example, rules for observation of the state s that has obtained in a particular instance, rules for communicating with the other decision makers, and rules for making decisions based on the information acquired through observation and communication. The payoff function ω_0 is interpreted as reflecting the organization head's preferences. Thus, the general organization team model may be denoted (in normal form) as

$$T = [I, (S, \mathcal{S}, P), \{B_i, i \in I\}, \omega_0]. \tag{2.1}$$

The team's objective is assumed to be to choose a joint strategy $\beta^* \in B$, if one exists, that maximizes the expected value of the team payoff function ω_0:

$$\bar{\omega}_0(\beta^*) \equiv \int_S \omega_0(\beta^*, s)\, dP(s) = \max_{\beta \in B} \int_S \omega_0(\beta, s)\, dP(s) \equiv \max_{\beta \in B} \bar{\omega}_0(\beta). \tag{2.2}$$

[3] This formulation is based on that of Radner in [**6**].

[4] The payoff function is assumed to be P-integrable for every β in B, i.e., $\int_S \omega_0(\beta, s)\, dP(s)$ exists and is finite for all $\beta \in B$.

For the remainder of the paper, in order to have a well-defined problem, we assume that there exists at least one joint strategy β^* that maximizes $\bar{\omega}_0$ and furthermore, that each employee's strategy β_i^*, $i = 1, \ldots, n$ maximizes $\bar{\omega}_0$ uniquely[5] over all β_i in B_i given that all the other decision makers have chosen β_j^*, $j \neq i$:

ASSUMPTION A: *There exists a* $\beta^* \in B$ *such that*

$$\bar{\omega}_0(\beta^*) \geqslant \bar{\omega}_0(\beta) \qquad \textit{for all } \beta \in B, \qquad \textit{and} \tag{2.3a}$$

$$\bar{\omega}_0(\beta^*) > \bar{\omega}_0(\beta^*/\beta_i) \qquad \textit{for all } \beta_i \in B_i,\ \beta_i \neq \beta_i^* \qquad (i = 1, \ldots, n), \tag{2.3b}$$

where $\beta^*/\beta_i \equiv (\beta_0^*, \ldots, \beta_i, \ldots, \beta_n^*)$.

If $\omega_i : B \times S \to R$, $i = 1, \ldots, n$ denote n payoff functions interpreted as a set of employee compensation rules, then

$$G = [I, (S, \mathscr{S}, P), \{B_i, i \in I\}, \{\omega_i, i \in I\}] \tag{2.4}$$

is an $(n + 1)$-person game in normal form. We assume that this game is played non-cooperatively, i.e., that each player i chooses his own strategy assuming that all the other players are doing likewise. An equilibrium joint-strategy is a Nash equilibrium, i.e., $\hat{\beta}$ is a Nash equilibrium if

$$\bar{\omega}_i(\hat{\beta}) = \max_{\beta_i \in B_i} \bar{\omega}_i(\hat{\beta}/\beta_i) \qquad \text{for all } i \in I. \tag{2.5}$$

A set $W = \{\omega_i, i = 1, \ldots, n\}$ of employee payoff functions is called an *incentive structure*, and an incentive structure $W^* = \{\omega_i^*, i = 1, \ldots, n\}$ is called optimal if, for the joint strategy β^* satisfying Assumption A,

$$\bar{\omega}_i^*(\beta^*) = \max_{\beta_i \in B_i} \bar{\omega}_i^*(\beta^*/\beta_i) \qquad \text{uniquely}^6 \text{ for all } i = 1, \ldots, n. \tag{2.6}$$

The *incentive problem* of the organization head is then to find an optimal incentive structure W^*, or equivalently, since every incentive structure W defines an $(n + 1)$-person game, to choose the optimal game for his organization to play.[7]

B. *Information and the Incentive Structure: Two Common Systems*

As posed above, the incentive problem is trivial. All the organization head need do is specify that his employees receive more compensation when they make the "correct" decisions than when they do not. Formally, define the incentive structure

[5] Up to equivalence classes. We say that two strategies β_i' and β_i'' are equivalent and write $\beta_i' = \beta_i''$ if and only if $\bar{\omega}_0(\beta/\beta_i') = \bar{\omega}_0(\beta/\beta_i'')$ for all β in B where $(\beta/\beta_i') = (\beta_0, \ldots, \beta_i', \ldots, \beta_n)$.

[6] Thus, β^* is not only a Nash equilibrium, but also satisfies $\bar{\omega}_i^*(\beta^*) > \bar{\omega}_i^*(\beta^*/\beta_i)$ for all $\beta_i \in B_i$ such that $\beta_i \neq \beta_i^*$, $i = 1, \ldots, n$.

[7] It should be noted that we exclude any possibility of bargaining between the employees and the employer in establishing the payoff *functions* ω_i, $i = 1, \ldots, n$. I am indebted to J. Marschak for this point.

$W^0 = \{\omega_i^0, i = 1, \ldots, n\}$ by

$$\omega_i^0(\beta, s) = \begin{cases} 1 \text{ if } \beta_i = \beta_i^*, \\ 0 \text{ otherwise} \end{cases} \qquad (i = 1, \ldots, n). \tag{2.7}$$

This incentive structure is, of course, the most familiar scheme of employee compensation, namely, that of a paid worker who receives his wage (1) if he does "the job he was hired to perform," but is fired (receives 0) if he does not.[8] We call W^0 the paid worker incentive structure.

Three properties of W^0 are noteworthy. First, since the compensation of employee i depends only on his strategy decision, if another employee chooses a non-optimal strategy, it has no effect on i's compensation. Second, the incentive structure W^0 distinguishes, on the basis of the compensation paid, between optimally and non-optimally performing employees. These two properties seem desirable. However, the third property of W^0 is that the head must have complete information in order to make the specified compensation. He must not only know what all the decisions of his employees actually were, but also, what the optimal decisions should have been. In a complex organization, this information requirement is quite unreasonable, especially if the head's information depends on his employee's messages. This property suggests that the class of allowable incentive structures should be restricted in some way to account for informational limitations and dependencies.

If the organization head knows or learns the payoff resulting from the joint strategy chosen by the organization, the compensation rules might be restricted to be functions of the organization payoff. In this case, one can also find a common example of an optimal incentive structure. Suppose each employee receives a fixed constant amount (his basic salary) plus some non-zero proportion of the organization payoff (his profit share). Formally, define the incentive structure $W^I = \{\omega_i^I, i = 1, \ldots, n\}$ by

$$\omega_i^I(\beta, s) = \alpha_i \omega_0(\beta, s) + A_i \qquad (i = 1, \ldots, n), \tag{2.8}$$

where α_i is a positive constant and A_i is any constant.[9]

This type of compensation scheme is commonly encountered as a "profit-sharing" plan; we call W^I the profit-sharing incentive structure. In comparison with the paid worker scheme, this incentive structure does not require extensive

[8] Any positive linear transformation of ω_i^0 will lead to the same behavior, and thus there is no necessary abuse of terminology in referring to 1 as the wage. One could, in fact, allow bargaining between the employer and employee over the units of the number 1. See Footnote 7. It is probably simpler, however, to view the compensation as a "success indicator" rather than any monetary remuneration.

[9] See Footnote 8. Since the compensation may be viewed as "success indicators" there is no reason to require the sum of all compensation to equal any prescribed fraction of the total organization payoff $\omega_0(\beta, s)$. Also, although any positive α_i would work in defining ω_i^I, it might reasonably be expected that the strength of the incentive system in motivating team behavior would be positively related to the size of α_i. However, such considerations are outside the scope of the methodology employed here as indeed in most economic theory, where it is assumed, for example, that as long as an extra penny can be gained, a profit-maximizing producer will leap to garner it.

information of the organization head. However, it does not discriminate between optimally and non-optimally performing employees. If any employee chooses a non-optimal strategy, the foregone organizational payoff is shared by all in proportion to their fixed shares α_i, $i = 1, \ldots, n$.

This brief discussion of the two common incentive structures—paid worker and profit-sharing—has focussed on the twin issues of the information required to make the compensation and the ability of the incentive structure to discriminate optimal from non-optimal performance. In the next section the incentive problem is considered for a special class of organizations called conglomerates and these two aspects are discussed in detail.

3. THE INCENTIVE PROBLEM IN A CONGLOMERATE

A. *The Model of a Conglomerate*

A conglomerate is an organization consisting of many partially autonomous units linked only through a central administration. An example of such an organization might be a large firm with many plants independently producing and marketing a wide variety of products, or a national economy with many sectors producing commodities in accordance with a centrally formulated national plan.

Let us suppose that the organization consists of $n + 1$ components; n of these (called subunits) are managed by employees (subunit managers) and the remaining component, the central administration or center, is run by the organization's leader or head. In contrast with a fully integrated organization, the subunits are independent of each other. Their performance depends only on their own manager's decision and that of the head. In other words, the subunits are linked only through the coordinating decision of the head.

Associated with each component of the organization is a payoff function (e.g., a profit function) that specifies the payoff to the subunit or center resulting from the subunit manager's decision, the head's decision, and an environmental state variable. The payoff to the organization is the sum of the component's payoffs, and it is this payoff that the head desires to maximize.

Uncertainty is conceptualized in terms of the environmental state variables. Each subunit manager and the head observes his own state variable, but it is assumed that his observation provides no information regarding the other components' state variables. Although the head is initially ignorant of the subunits' state variables, he may gain information regarding these variables through direct communication with the subunit managers. The subunit managers, however, gain additional information only through the head's messages to them; they do not communicate directly with each other.

The decision process is as follows: At the beginning of the decision period all managers make their observations of their own state variables. Then each subunit manager and the head communicate with each other. Finally, on the basis of the information acquired through observation and communication, the managers and the head make their decisions.

Formally, the conglomerate organization is specified by the following conditions on the general organization team model,

$$T = [I, (S, \mathscr{S}, P), \{B_i, i \in I\}, \omega_0].$$

CONDITION S.1: $I = \{0, 1, \ldots, n\}$, $i = 1, \ldots, n$ denote the subunit managers; $i = 0$ denotes the head.

CONDITION S.2: $(S, \mathscr{S}, P) = (\bigtimes_{i=0}^{n} S_i, \sigma[\bigtimes_{i=0}^{n} \mathscr{S}_i], \prod_{i=0}^{n} P_i)$ where $(S_i, \mathscr{S}_i, P_i)$ is the probability space of the ith component's environmental state variable and $\sigma[\bigtimes_{i=0}^{n} \mathscr{S}_i]$ is the σ-algebra of subsets of S generated by the σ-algebrae $\mathscr{S}_i$, $i = 0, \ldots, n$.

Note that Condition S.2 formalizes in part the concept of the subunits' independence by specifying that their environments are mutually independent. This condition is necessary to rule out informational externalities of the following type. If s_i and s_j are correlated, then it is possible for manager i to learn more about s_j through a message from manager j to manager i relayed through the head than the head has learned from the same message.

CONDITION S.3: Each strategy set B_i contains strategies β_i consisting of three parts—an observation strategy ζ_i, a message strategy γ_i, and a decision strategy δ_i: $\beta_i = (\zeta_i, \gamma_i, \delta_i)$. The observation strategy ζ_i is defined on S_i and the message and decision strategies, γ_i and δ_i, are defined on an information set Y_i, $i = 0, \ldots, n$. The subunits communicate only with the head and thus, for any specified set of observation and message strategies (ζ_i, γ_i), $i = 0, \ldots, n$, and state s, the information of the managers and the head can be expressed as

$$\text{(3.1)} \qquad \begin{aligned} y_i(s) &= [\zeta_i(s_i), \gamma_0^i(y_0(s))] \in Y_i \qquad (i = 1, \ldots, n), \\ y_0(s) &= [\zeta_0(s_0), \{\gamma_i(y_i(s))\}_{i=1}^{n}] \in Y_0, \end{aligned}$$

where $\gamma_0 = (\gamma_0^1, \ldots, \gamma_0^n)$ and γ_0^i is the function specifying the message from the head to the ith subunit.

CONDITION S.4:

$$\omega_0(\beta, s) = \sum_{i=1}^{n} v_i[\delta_i(y_i(s)), \delta_0(y_0(s)); s_i] + v_0[\delta_0(y_0(s)), s_0]$$

where the $y_i(s)$, $i = 0, \ldots, n$ are defined in (3.1).

Additionally, we assume the following:

CONDITION S.5: The portion $v_i[\delta_i(y_i(s)), \delta_0(y_0(s)); s_i]$ of the organization payoff accrues directly to the ith subunit.

B. *Incentives in a Conglomerate*

Although under Condition S.5 each subunit's payoff accrues directly to the subunit, if each subunit manager attempts to maximize his subunit's expected payoff, in general it is not necessary that this will maximize the organization expected payoff. The difficulty arises for the following reason. Let β^* be an optimal joint strategy of the organization. The decision of the head $\delta_0^*(y_0(s))$ affects the ith subunit's payoff and is a function, in part, of the ith subunit's message $\gamma_i(y_i)$. Now, although the optimal message strategy γ_i^* maximizes the organization payoff, another message strategy, say, $\hat{\gamma}_i$ may lead to a higher expected payoff to the ith subunit. In other words, it may pay for the ith subunit manager to lie and send false information to the head.

The incentive problem facing the head is to devise a set of compensation rules that will induce the subunit managers not only to take the optimal decision but to send him the optimal (truthful) information as well. Furthermore, the compensation rules must not require any additional information of the head. Given a set of compensation rules, an incentive structure is defined by the addition of each compensation function to the corresponding subunit's payoff function, so that the total amount received by the ith subunit is his own subunit's payoff plus the compensation received from the head.

The class of all incentive structures requiring no additional information of the head is given by

$$\mathscr{I} = [W = \{\omega_i, i = 1, \ldots, n\}] \tag{3.2}$$

where

$$\omega_i(\beta, s) = v_i[\delta_i(y_i(s)), \delta_0(y_0(s)); s_i] + C_i(y_0(s)) \qquad (i = 1, \ldots, n),$$

and C_i is a function from Y_0 to R. For any s, the compensation $C_i(y_0(s))$ may be positive or negative. If positive it is interpreted as a bonus paid and if negative a charge assessed the ith subunit by the organization head.

Given a conglomerate model, the main result of this paper is that it is possible for the head to solve the incentive problem; that is, there exist compensation functions that define an optimal incentive structure.

Consider the functions C_i^{II} defined by

$$C_i^{\text{II}}(y_0) = \sum_{j \neq i} E[v_j[\delta_j^*(y_j^*(s)), \delta_0^*(y_0^*(s)); s_j] \mid y_0^*(s) = y_0] - A_i \tag{3.3}$$

where

$$y_j^*(s) = [\zeta_j^*(s_j), \gamma_0^{j*}(y_0^*(s))] \qquad (j = 1, \ldots, n), \tag{3.4}$$

$$y_0^*(s) = [\zeta_0^*(s_0), \{\gamma_j^*(y_j^*(s))\}_{j=1}^n], \quad \text{and}$$

$$A_i \text{ is any constant} \qquad (i = 1, \ldots, n).$$

The first term on the right hand side of (3.3) is the conditional expected value of the sum of all subunits' payoff excepting the ith under the assumption that the optimal joint-strategy $\beta^* = (\zeta^*, \gamma^*, \delta^*)$ is being played, given the actual information of the head y_0. In other words, after all the messages have been exchanged, the head asks himself the question: "Given my information y_0, and assuming that my subordinates have sent me the optimal information, i.e., $y_0 = y_0^*(s)$, what do I expect the subunits' payoffs to be if we together choose the optimal joint decision strategy $\delta^* = (\delta_0^*, \ldots, \delta_n^*)$?"

Since A_i is a constant, it may be interpreted as a fixed charge (say, a "franchise" price) levied in each period against the ith subunit. Offsetting this charge is an amount that depends on the head's expectation of the overall success of the organization.

If the functions C_i^{II} defined in (3.3) are chosen as the compensation functions, the incentive structure W^{II} defined by:

$$(3.5) \qquad W^{\text{II}} = \{\omega_i^{\text{II}}, i = 1, \ldots, n\} \quad \text{where}$$

$$\omega_i^{\text{II}}(\beta, s) = v_i[\delta_i(y_i(s)), \delta_0(y_0(s)); s_i] + C_i^{\text{II}}(y_0(s))$$

can be shown to be an optimal incentive structure.

THEOREM 1: *Given the organization model $T = [I, (S, \mathscr{S}, P), \{B_i, i = 0, \ldots, n\}, \omega_0]$ with the conglomerate specifications S.1–S.5, if T satisfies Assumption A, then W^{II} (defined by (3.3)–(3.5)) is an optimal incentive structure in the class $\mathscr{I}$.*

PROOF: See Appendix.

Since each subunit's total payoff under the incentive structure W^{II} consists of its own contribution to the total payoff less an incremental amount, W^{II} is called the own profit incentive structure.

Although Condition S.5 specified that the subunit's payoff v_i accrued directly to the subunit, we might consider a variation of the conglomerate in which the payoffs of all the subunits accrue directly to the head.

CONDITION S.5′: The portions $v_i[\delta_i(y_i(s)), \delta_0(y_0(s)); s_i], i = 0, 1, \ldots, n$, of the of the organization payoff accrue directly to the organization head.

In such a case a richer information is available to the head on which he could base the compensation to his subunit managers. His compensation rules may depend on the information y_0 he has when he makes his decision and also on the set of payoffs to the subunits, v_i, $i = 0, 1, \ldots, n$. The class of all such compensation rules or incentive structures is defined by

$$(3.6) \qquad \mathscr{I}' = [W = \{\omega_i, i = 1, \ldots, n\} \mid \omega_i(\beta, s) = g_i(y_0(s), v_0(\beta, s), \ldots, v_n(\beta, s))]$$

where g_i is any real-valued function defined on $Y_0 \times R^n$. It is evident that both the profit-sharing (W^{I}) and own profit (W^{II}) incentive structures belong to $\mathscr{I}'$ and thus are both optimal for the variant of the conglomerate organization satisfying S.5′ instead of S.5. A comparison of these two incentive structures illustrates the trade-offs between the information required of the organization head and other properties of an incentive structure.

First of all, W^{I} (profit-sharing) requires less information of the head than W^{II} (own profit). But, the compensation to each subunit under W^{II} is independent of the decision strategy choices of the other subunit managers, unlike W^{I}, although under both schemes the subunit compensations depend on the message strategy choices of all the managers. This feature appears unavoidable; as long as the head's optimal decisions and compensation depend on information from the subunit managers it seems impossible to uncouple the payoff to a subunit from the other subunit managers' messages. However, under the conditions of Theorem 1, at least the subunit managers can be provided with an incentive to make optimal decisions and send accurate and complete information. In fact, under W^{I} and W^{II} especially, although the compensations to the subunits depend on all information sent to the head, he need not know the quality of this information.

4. AN EXAMPLE: RESOURCE ALLOCATION IN A TEAM[10]

A. *The Resource Allocation Model*

Consider a conglomerate organization consisting of n enterprises and a central resource allocation board managed by the organization head or the resource manager. Each enterprise manager produces an output θ_i according to a quadratic[11] production function f_i:

$$(4.1)\qquad \theta_i = f_i(L_i, K_i; s_i) = 2(\mu_{iK}, \mu_{iL})\begin{pmatrix} K_i \\ L_i \end{pmatrix} - (K_i, L_i)Q_i\begin{pmatrix} K_i \\ L_i \end{pmatrix} \qquad (i = 1, \ldots, n)$$

where K_i is the input of the resource allocated to the enterprise by the resource manager, L_i is the level of a decision variable chosen by the enterprise manager, $s_i \equiv (\mu_{iK}, \mu_{iL}) \equiv \mu_i$ is a pair of random variables, and Q_i is a fixed 2×2 positive definite matrix. The total supply of the resource $\kappa \equiv s_0$ is also assumed to be a random variable. The state of the environment for this model may then be represented by the $(n + 1)$-tuple of random vectors:

$$(4.2)\qquad s = [s_0, s_1, \ldots, s_n] \equiv [\kappa, \mu_1, \ldots, \mu_n].$$

We assume that the n random vectors μ_i, $i = 1, \ldots, n$ and the random variable κ are independently distributed with finite means and variance-covariance matrices. The organization payoff is a weighted sum of the n enterprise outputs:

$$(4.3)\qquad \omega_0(\beta, s) = \sum_{i=1}^{n} w_i\theta_i = \sum_{i=1}^{n} w_i f_i(L_i, K_i; s_i)$$

[10] This model is fully developed in Groves and Radner [2].

[11] The assumption of a quadratic production function may be viewed as a second order approximation of a general production function. For an analysis of the model from this point of view, see Groves [1, III].

where w_i is the fixed weight assigned to the output θ_i of the ith enterprise. We assume the portion $w_i f_i(L_i, K_i, s_i)$ of the total payoff accrues directly to the ith enterprise manager; it may be interpreted as the ith enterprise's "gross profit."

The strategies available to the managers are defined by the following restrictions: Each enterprise manager observes his own technological coefficients $\mu_i \equiv s_i$ and the resource manager observes the total resource supply $\mu_i \equiv s_0$. Based on the supply observation, the resource manager sends messages to the enterprise managers who each then return some message to the resource manager. At the conclusion of this single exchange of messages, the managers make their decisions. Each enterprise manager chooses the level of his decision variable L_i and the resource manager makes an allocation $(K_1, \ldots, K_n)$ of the supply κ of the scarce resource. Since the quantity allocated cannot exceed the supply, the resource manager's decision (allocation) is constrained by the relation[12]

$$\sum_{i=1}^{n} K_i \leqslant \kappa. \tag{4.4}$$

It may be easily verified that all the specifications S.1–S.5 of the conglomerate organization model given in Section 3 are satisfied by the resource allocation model. Thus, if Assumption A (2.3) is satisfied, Theorem 1 is applicable to this example and would guarantee the existence of an optimal incentive system. Theorem 2 establishes Assumption A for this example.

THEOREM 2: *Under the specifications of the resource allocation model given above, if* $\hat{\gamma} = [\{\hat{\gamma}_i(\cdot)\}_{i=1}^n, \{\hat{\gamma}_0^i(\cdot)\}_{i=1}^n]$ *is the joint message strategy where*

$$\begin{aligned} &\hat{\gamma}_i(y_i) = \mu_i && (i = 1, \ldots, n), \\ &\hat{\gamma}_0^i(y_0) = \kappa, \end{aligned} \tag{4.5}$$

and $y_i = (\mu_i, \kappa)$, $y_0 = \kappa$, *then there exists a joint decision strategy* $\hat{\delta}$ *such that the joint strategy* $\hat{\beta} = (\hat{\gamma}, \hat{\delta})$ *is optimal, i.e., (i)* $\bar{\omega}_0(\hat{\beta}) \geqslant \bar{\omega}_0(\beta)$ *for all* $\beta \in B$. *Furthermore, each individual strategy* $\hat{\beta}_j = (\hat{\gamma}_j, \hat{\delta}_j)$ *maximizes* $\bar{\omega}_0(\hat{\beta}/\beta_j)$ *uniquely*[13] *in* B_j, $j = 1, \ldots, n$; *i.e., (ii)* $\bar{\omega}_0(\hat{\beta}) > \bar{\omega}_0(\hat{\beta}/\beta_j)$ *for all* $\beta_j \in B_j$, $\beta_j \neq \hat{\beta}_j$, $j = 1, \ldots, n$.

PROOF: See Appendix.

It should be noted that the messages specified by the joint message strategy $\hat{\gamma}$ are the observations of the individual managers. The message $\hat{\gamma}_0^i(y_0) = \kappa$ is the resource manager's observation of the total supply, and the message $\hat{\gamma}_i(y_i) = \mu_i$ is the ith enterprise manager's observation of his technical coefficients.

[12] Although it would be reasonable to impose non-negativity constraints on the allocations as well, this has been neglected. This approach is justified if, for the optimal decision rules, the probability of negative allocations is negligible.

[13] Up to equivalence classes; see Footnote 5.

B. *The One-Stage Lange-Lerner Message Strategy*

Although the joint strategy $\hat{\beta}$ of Theorem 2 is optimal, there exist other optimal joint strategies. Of particular interest is one resulting from the simple exchange of price and demand messages. This joint message strategy is defined by the following process: After all managers have made their observations, the resource manager sends a price to the enterprises based on his observation of the total supply κ and calculated using any one-to-one function Ψ from R to R; i.e., let

$$\gamma_0^{i*}(\kappa) = \Psi(\kappa) = \psi \qquad (i = 1, \ldots, n). \tag{4.6}$$

Next, each enterprise manager, upon receipt of this message, forms the "profit" function,

$$\Pi_i(L_i, K_i) = w_i f_i(L_i, K_i; s_i) - \psi K_i. \tag{4.7}$$

He then maximizes Π_i with respect to L_i and K_i; this yields two functions, $\hat{L}_i$ and $\hat{K}_i$ of $\mu_i \equiv s_i$ and ψ. The value $K_i(\mu_i, \psi)$ may be interpreted as the ith enterprise manager's "profit-maximizing" demand for the resource; it is this message that he returns to the resource manager,

$$\gamma_i^*(y_i) = \hat{K}_i(y_i) \quad \text{where} \quad y_i = (\mu_i, \psi). \tag{4.8}$$

With this simple exchange of messages, communication ceases and the managers make their decisions. It can be shown that given the joint message strategy $\gamma^* = [\{\gamma_i^*, i = 1, \ldots, n\}, \{\gamma_0^{i*}, i = 1, \ldots, n\}]$, there exists a unique optimal joint strategy δ^* and furthermore, that the combined joint strategy $\beta^* = (\gamma^*, \delta^*)$ is optimal.[14]

The message process described above is a one-round tâtonnement process and since such information procedures were suggested by Lange and Lerner in their discussions of market socialism, we call γ^* the one-stage Lange-Lerner message strategy.

C. *An Optimal Incentive Structure*

Since Theorem 2 verifies Assumption A, Theorem 1 may be applied to establish that the own profit incentive structure is optimal for the resource allocation model. Thus, by paying (or assessing) the enterprise managers the amount $C_i^{II}(y_0)$ specified in equation (3.3), the resource manager can insure that he will receive the correct demand messages and that the enterprise managers will make the correct decisions as well.

Although the formula for calculating the payment $C_i^{II}(y_0)$ is awkward, it can be given a simple, economically meaningful interpretation. Since the decision of the resource manager is an allocation of the scarce resource κ, the negative of the quantity $C_i^{II}(y_0)$ may be interpreted as the "cost" of the resource allocated to

[14] See Groves and Radner [2]. The proof involves showing that the optimal decision rules δ^* for the Lange-Lerner information structure are identical to $\hat{\delta}$ which are shown in Theorem 2 to be optimal.

enterprise i.[15] In fact, we may define "prices" p_i^*, $i = 1, \ldots, n$, by

$$(4.9) \qquad p_i^*(y_0) = \frac{-C_i^{\mathrm{II}}(y_0)}{\kappa_i^*(y_0)} \quad \text{for all} \quad y_0 \in Y_0 \qquad (i = 1, \ldots, n);$$

write $C_i^{\mathrm{II}}(y_0)$ as

$$(4.10) \qquad C_i^{\mathrm{II}}(y_0) = -p_i^*(y_0)\kappa_i^*(y_0)$$

and observe that the resource manager, by setting prices according to equation (4.9) and charging these prices for the resources allocated has, in effect, instituted an optimal incentive structure.

While it is perhaps suggestive to represent the own profit incentive structure in terms of prices charged for resources allocated, it should be observed that these prices are, in general, not only different for the different enterprises, but also that the rules (functions) by which they are calculated are different for each enterprise. This non-anonymity property is unconventional, but has some parallel to the situation of a discriminating monopolist. However, two special cases of the model for which the optimal price rules will be the same for all enterprises (even though the particular prices charged in any situation may be different) are the case of a continuum of infinitesimal enterprises and the case when all enterprises are identical. The first case will not be pursued further as it involves extensive elaboration and the point is obvious anyway. The case of identical enterprises is interesting since the optimal price rule (which is the same for all enterprises) is a quantity discount rule.

That is, suppose that the production functions of all n firms are identical, that the random production parameters μ_i, $i = 1, \ldots, n$, have identical distributions, and that the weights w_i, $i = 1, \ldots, n$, of the enterprise outputs in the payoff function ω_0 are the same for all enterprises. Then, it can be shown that the optimal price rules (equation 4.9) take the form[16]

$$(4.11) \qquad p_i^*(y_0) = a(y_0) - b\kappa_i^*(y_0),$$

where b is positive.[17]

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APPENDIX

PROOF OF THEOREM 1: By the definition of the functions C_i^{II} [equation (3.3)], W^{II} is in the class $\mathscr{I}$. To show that W^{II} is optimal, we must show that β_i^* maximizes $\bar{\omega}_i^{\mathrm{II}}(\beta^*/\beta_i)$ uniquely for every $i = 1, \ldots, n$. Since β_i^* maximizes $\bar{\omega}_0(\beta^*/\beta_i)$ uniquely (by Assumption A), it is clearly sufficient to show that

$$(\mathrm{A.1}) \qquad \bar{\omega}_i(\beta^*/\beta_i) + A_i = \bar{\omega}_0(\beta^*/\beta_i) \quad \text{for all} \quad \beta_i \in B_i \qquad (i = 1, \ldots, n),$$

[15] Since the expected value of the quantity $C_j^{\mathrm{II}}(y_0)$ can be arbitrarily set by suitably choosing the constant A_j, there is no necessary abuse of terminology in referring to $C_j^{\mathrm{II}}(y_0)$ as a cost.

[16] We must also take $A_i = A_j$ for all $i, j = 1, \ldots, n$. See equation (3.3).

[17] See Groves [**1**, IV].

where A_i is the constant term of the function C_i^{II}. Since

$$\varpi_i(\beta^*/\beta_i) + A_i = E\{v_i[\delta_i(\hat{y}_i), \delta_0^*(\hat{y}_0); s_i]\} + \quad E[E\{v_j[\delta_j^*(y_j^*), \delta_0^*(y_0^*); s_j] \mid y_0^*(s) = \hat{y}_0\}]$$

and

$$\varpi_0(\beta^*/\beta_i) = E\{v_i[\delta_i(\hat{y}_i), \delta_0^*(\hat{y}_0); s_i]\} + \sum_{j \neq i} E[v_j[\delta_j^*(\hat{y}_j), \delta_0^*(\hat{y}_0); s_j]],$$

to show (A.1) we must show

$$\text{(A.2)} \qquad E[v_j[\delta_j^*(\hat{y}_j), \delta_0^*(\hat{y}_0); s_j]] = E[E\{v_j[\delta_j^*(y_j^*), \delta_0^*(y_0^*); s_j] \mid y_0^*(s) = \hat{y}_0\}]$$

for all $j \neq i$, for all $\beta_i = (\zeta_i, \gamma_i, \delta_i) \in B_i$, where

$$y_j^*(s) = [\zeta_j^*(s_j), \gamma_0^{j*}(y_0^*(s))] \qquad (j = 1, \ldots, n),$$

$$y_0^*(s) = [\zeta_0^*(s_0), \{\gamma_j^*(y_j^*(s))\}_{j=1}^n],$$

$$\hat{y}_j(s) = [\zeta_j^*(s_j), \gamma_0^{j*}(\hat{y}_0(s))] \qquad (j = 1, \ldots, n; j \neq i),$$

$$\hat{y}_i(s) = [\zeta_i(s_i), \gamma_0^{i*}(\hat{y}_0(s))],$$

$$\hat{y}_0(s) = [\zeta_0^*(s_0), \gamma_1^*(\hat{y}_1(s)), \ldots, \gamma_i(\hat{y}_1(s)), \ldots, \gamma_n^*(\hat{y}_n(s))].$$

For every $s \in S$, define

$$\text{(A.3)} \qquad \begin{aligned} A(s) &= \{s' \in S \mid \hat{y}_0(s') = \hat{y}_0(s)\}, \\ B(s) &= \{s' \in S \mid y_0^*(s') = \hat{y}_0(s)\}, \end{aligned}$$

and let $A_j(s)$ and $B_j(s)$ be the projections of $A(s)$ and $B(s)$ onto S_j respectively, where, recall, $S = \times_{j=0}^n S_j$.

LEMMA: $A_j(s) = B_j(s)$ for all $s \in S, j = 0, \ldots, n, j \neq i$.

PROOF: (i) Let $j = 0$. Now $s_0' \in A_0(s)$ iff $\zeta_0^*(s_0') = \zeta_0^*(s_0)$ and $s_0' \in B_0(s)$ iff $\zeta_0^*(s_0') = \zeta_0^*(s_0)$. Thus, $A_0(s) = B_0(s)$. (ii) Let $j = 1, \ldots, n, j \neq i$. Now $s_j' \in A_j(s)$ iff

$$\text{(A.4)} \qquad \gamma_j^*[\zeta_j^*(s_j'), \gamma_0^{j*}(\hat{y}_0(s))] = \gamma_j^*[\zeta_j^*(s_j), \gamma_0^{j*}(\hat{y}_0(s))].$$

Also, $s_j' \in B_j(s)$ iff (A.4) holds. Thus, $A_j(s) = B_j(s)$. *QED* for Lemma.

Returning to the proof of Theorem 1, consider the r.h.s. of (A.2), for $j \neq i, j \neq 0$ [the proof for $j = 0$ is strictly analogous].

$$\begin{aligned} \text{r.h.s. (A.2)} &= E[E\{v_j[\delta_j^*(y_j^*(s')), \delta_0^*(y_0^*(s')); s_j'] \mid y_0^*(s') = \hat{y}_0(s)\}] \\ &= E[E\{v_j[\delta_j^*\{\zeta_j^*(s_j'), \gamma_0^{j*}(\hat{y}_0(s))\}, \delta_0^*(\hat{y}_0(s)); s_j'] \mid s' \in B(s)\}] \\ &= E[E\{v_j[\delta_j^*\{\zeta_j^*(s_j'), \gamma_0^{j*}(\hat{y}_0(s))\}, \delta_0^*(\hat{y}_0(s)); s_j'] \mid s_j' \in B_j(s)\}] \end{aligned}$$

where the last equality follows by the independence of the distributions of s_k and s_j, $k \neq j$.

Consider the l.h.s. of (A.2). By the theorem of iterative expectations (cf. [**5**, p. 121]):

$$\begin{aligned} \text{l.h.s. (A.2)} &= E[E\{v_j[\delta_j^*(\hat{y}_j(s')), \delta_0^*(\hat{y}_0(s')); s_j'] \mid s' \in A(s)\}] \\ &= E[E\{v_j[\delta_j^*\{\zeta_j^*(s_j'), \gamma_0^{j*}(\hat{y}_0(s))\}, \delta_0^*(\hat{y}_0(s)); s_j'] \mid s' \in A(s)\}] \\ &= E[E\{v_j[\delta_j^*\{\zeta_j^*(s_j'), \gamma_0^{j*}(\hat{y}_0(s))\}, \delta_0^*(\hat{y}_0(s)); s_j'] \mid s_j' \in A_j(s)\}] \end{aligned}$$

where the last equality follows by the independence of s_k and s_j, $k \neq j$. Thus, for $j \neq i$, l.h.s. of (A.2) = r.h.s. of (A.2) since $A_j(s) = B_j(s)$ by the Lemma. *QED* for Theorem 1.

PROOF OF THEOREM 2: A detailed proof of the first part of the Theorem may be found in Groves and Radner [**2**]. It is sufficient for our purposes to note (i) that once the joint message strategy $\hat{\gamma}$ is specified, by Radner's Theorem [**8**, Theorem 4.3] there exists a unique optimal joint decision strategy $\hat{\delta}$ given $\hat{\gamma}$, and (ii) under the restrictions on communication, no joint message strategy other than $\hat{\gamma}$ can give more information to any manager, since $\hat{\gamma}$ gives the resource manager a complete knowledge of the state s that has obtained and since, under $\hat{\gamma}$, the resource manager sends his complete information, κ, at the time he communicates with the enterprise managers.

To prove the second part of the theorem, consider any joint strategy $\hat{\beta}/\beta_j$ where β_j is not equivalent to $\hat{\beta}_j$. Define the joint strategy $\tilde{\beta} = (\tilde{\gamma}, \tilde{\delta})$ by $\tilde{\gamma} = (\hat{\gamma}/\gamma_j)$ and $\tilde{\delta}$ = the unique optimal joint decision strategy given $\tilde{\gamma}$. The existence of $\tilde{\delta}$ is assured by Radner's Theorem [**8**, Theorem 4.3]. By part (i) of Theorem 2, $\bar{\omega}_0(\hat{\beta}) \geqslant \bar{\omega}_0(\tilde{\beta})$ and by Radner's Theorem, $\bar{\omega}_0(\tilde{\beta}) > \bar{\omega}_0(\tilde{\gamma}, \delta)$ for all δ not equivalent to $\tilde{\delta}$. Let $\delta' = (\hat{\delta}/\delta_j)$. Then, since $\tilde{\gamma} = (\hat{\gamma}/\gamma_j)$,

$$(\tilde{\gamma}, \delta') = (\hat{\gamma}/\gamma_j, \hat{\delta}/\delta_j) = (\hat{\beta}/\beta_j).$$

Furthermore, δ' is not equivalent to $\hat{\delta}$ since β_j is not equivalent to $\hat{\beta}_j$. Thus

$$\bar{\omega}_0(\hat{\beta}) \geqslant \bar{\omega}_0(\tilde{\beta}) > \bar{\omega}_0(\tilde{\gamma}, \delta') = \bar{\omega}_0(\hat{\beta}/\beta_j) \qquad \text{for all} \qquad \beta_j \in B_j$$

where β_j is not equivalent to $\hat{\beta}_j$. *QED* for Theorem 2.

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