The basic ideas underlying the GK system were formulated in the early 1950's when Geary worked as a consultant to FAO. A formal description of the method appeared in a brief paper (Geary, 1958). The method gained prominence through work by Khamis (1970, 1972, 1984), where the method was given a systematic exposition, and therefore the method has come to be known as the Geary-Khamis method. It is the most widely used aggregation method in international comparisons, and has been the main aggregation procedure in all the phases of the International Comparisons Project of the Statistical Offices of the United Nations, E.E.C. and the O.E.C.D. The Geary-Khamis method was also selected for use in the FAO (1986) work on inter-country comparisons of agricultural output. The following is a non-technical description of the Geray-Khamis system.

The GK method employs the repricing and PPP approaches concurrently, and is therefore based on the reference prices of commodities as well as the purchasing power parities. Suppose PPP_j represents the parity of j-th currency with a numeraire currency, called international dollars[4]. Then the international price P_i is defined as an international average of prices of i-th commodity in different countries. As the prices in different countries are expressed in respective national currencies, the GK-method uses national prices after conversion into a common currency using the PPPs. Thus the international price, P_i, of i-th commodity is defined as:

$$P_i = \frac{(p_{i1}q_{i1}/PPP_1) + (p_{i2}q_{i2}/PPP_2) + \ldots + (p_{iM}q_{iM}/PPP_M)}{q_{i1} + q_{i2} + \ldots + q_{iM}} \qquad (4.12)$$

This equation implies that the international price of i-th commodity is calculated by dividing the total value of output of i-th commodity across all countries, converted in international dollars, using PPPs, by the total quantity produced of i-th commodity. Equation 4.12 may be rewritten as:

$$P_i = \sum_{j=1}^{M} (p_{ij}/PPP_j) \frac{q_{ij}}{\sum_{j=1}^{M} q_{ij}} \qquad (4.13)$$

This equation suggests that P_i is a weighted average of national prices p_{ij} after conversion into international dollars using PPP_j.

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4 The term "international dollars" has nothing to do with US dollars, but refers to the numeraire currency selected.

These prices cannot be computed unless the system specifies PPP_j. The GK system defines the parity of j-th currency through the equation:

$$PPP_j^* = \frac{\sum_{i=1}^{N} p_{ij} q_{ij}}{\sum_{i=1}^{N} P_i q_{ij}} \qquad (4.14)$$

The numerator of the equation represents the total value of output in j-th country expressed in national currency, and the denominator is the value of j-th country output evaluated by repricing at international prices P_i, in international dollars. Then PPP_j gives the number of national currency units per international dollar.

The Geary-Khamis system consists of equations 4.12 and 4.14. and requires the computation of the international prices P_i, and parities PPP_j, which are defined using a system of simultaneous equations.

Computational Scheme

Computation of the unknown parities and the international prices requires the solution of a system of (M+N) linear equations in (M+N) unknowns. Khamis (1970) and Prasada Rao (1971) have shown that the system has a unique positive solution[5], if one of the unknowns is fixed at an arbitrary level. For example, if PPP_1 is set to unity, i.e. the currency of country 1 is taken as a numeraire currency, then the remaining parities and international prices are uniquely determined. Obtaining the solution involves the computation of the inverse of a large matrix if the number of countries, M, and the number of commodities, N, are large. The present study covers 103 countries and 185 output commodities, which implies that the computation of GK parities requires the inversion of a matrix of the order 102[6].

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5 The existence of a unique solution requires that the production structures across countries are overlapping. In the present exercise this is easily satisfied.

6 The order is M-1 if one of the countries is selected as the numeraire currency.

A simpler computational scheme can be devised on the basis of the circular nature of the GK equations and parities. An iterative scheme is described below which forms the basis for the computer programmes developed to solve the Geary-Khamis system at the FAO.

Iterative Scheme

Specify any positive initial value for the parities, with $PPP_1 = 1$. In the absence of any additional information an obvious choice is to set all the parities equal to unity[7], i.e. $PPP_1 = 1$, $PPP_2 = 1$, ..., $PPP_M = 1$.

These values are then substituted into the formula for the international prices P_i in equation 4.13. Once these P_i's are computed, they are in turn, used in equation 4.14 to compute the parities PPP_j. After normalizing the computed parities and making $PPP_1 = 1$, these parities are compared with the initial values. If they are identical then the computation is terminated. Otherwise the computation proceeds by repeating the calculation using the new parities.

These steps are repeated until the values converge. Convergence of the iterative procedure is guaranteed by the properties of the GK system discussed in Prasada Rao (1971) and Khamis (1972). Even in cases where M and N are very large, the procedure converges rapidly.

A number of properties of the GK method were discussed in the previous report (FAO, 1986) and the reader is referred to Sections 4.4.1 and 4.4.2 of that paper. More recent work on the GK system can be found in Khamis (1984) and Prasada Rao and Salazar-Carillo (1988, 1990). Alternatives to the GK system based on a similar framework were suggested by Gerardi (1974) and Prasada Rao (1990). The reader is referred to EUROSTAT (1982) for an excellent discussion of the GK system where its superiority was established. The elegance and simplicity of the logic behind the basic

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7 The final solution is independent of the starting values, so there is no harm in setting all parities equal to one. An alternative is to use the official exchange rates as starting values for parities.

Geary equations that determine international prices and purchasing power parities, the usefulness of the results from the system in the computation and presentation of national accounts for different countries in a comparable form make the Geary-Khamis system an obvious choice for most exercises involving inter-country and inter-regional comparisons.

The GK approach is the most suitable method for agricultural output comparisons as shown in Chapter 5, where underlying data consist of prices and quantities of agricultural commodities, produced in different countries. However, the method cannot be applied when either quantity data are not available or quantities are not meaningfully defined, as is the case with aggregate data involving composite commodities, where only values and value shares are meaningful. In such cases an alternative method may be required and the following section describes the EKS method based on Theil-Tornqvist indices, which is found to be suitable for purposes of input price comparisons required for the computation of internationally comparable GDP figures.