# Chapter 2

# The global energy balance

We consider now the general problem of the radiative equilibrium temperature of the Earth. The Earth is bathed in solar radiation and absorbs much of that incident upon it. To maintain equilibrium it must warm up and radiate energy away at the same rate as it is received, as depicted in Fig.2.1. We will see that the emission temperature of the Earth is 255 K and that a body at this temperature radiates energy primarily in the infrared (IR). But the atmosphere is strongly absorbing at these wavelengths due to the presence of trace gases — principally the triatomic molecules $H_2O$ and $CO_2$ — which absorb and emit in the infrared, this raising the surface temperature above that of the emission temperature, a mechanism that has become known as the 'greenhouse effect'.

## 2.1 Planetary emission temperature

The Earth receives almost all of its energy from the Sun. At the present time in its evolution the Sun emits energy at a rate of $Q = 3.87 \times 10^{26}$W. The flux of solar energy at the Earth — called the 'solar constant' — depends on the distance of the Earth from the Sun, $r$, and is given by the inverse square law: $S_0 = \frac{Q}{4\pi r^2}$ . Of course, because of variations in the Earth's orbit (see Sections 5.1.1 and 12.3.5) the solar constant is not really constant; the terrestrial value $S_0 = 1367\ \mathrm{W\,m^{-2}}$ set out in Table 2.1, along with that for other planets, is an average corresponding to the average distance of Earth from the Sun, $r = 150 \times 10^9$ m.

The way in which radiation interacts with an atmosphere depends on

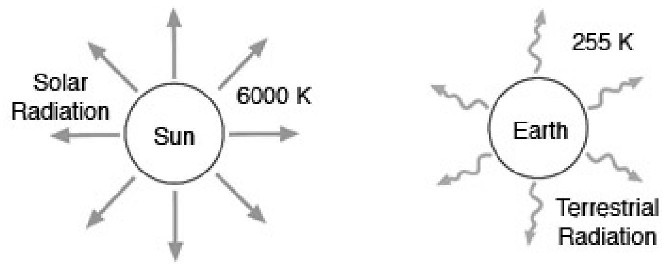


Figure 2.1: The Earth radiates energy away at the same rate as it is received from the Sun. The Earth's emission temperature is 255 K; that of the Sun, 6000 K. The outgoing terrestrial radiation peaks in the infrared; the incoming solar radiation peaks at shorter wavelengths, in the visible.

| | $r$ | $S_0$ | $\alpha_p$ | $T_e$ | $T_m$ | $T_s$ | $\tau$ |
|---|---|---|---|---|---|---|---|
| | $10^9$ m | $W\,\mathrm{m}^{-2}$ | | K | K | K | Earth days |
| Venus | 108 | 2632 | 0.77 | 227 | 230 | 760 | 243 |
| Earth | 150 | 1367 | 0.30 | 255 | 250 | 288 | 1.00 |
| Mars | 228 | 589 | 0.24 | 211 | 220 | 230 | 1.03 |
| Jupiter | 780 | 51 | 0.51 | 103 | 130 | 134 | 0.41 |

Table 2.1: Properties of some of the planets. $S_0$ is the solar constant at a distance $r$ from the Sun, $\alpha_p$ is the planetary albedo, $T_e$ is the emission temperature computed from Eq.(2.4), $T_m$ is the measured emission temperature and $T_s$ is the global mean surface temperature. The rotation period, $\tau$, is given in Earth days.

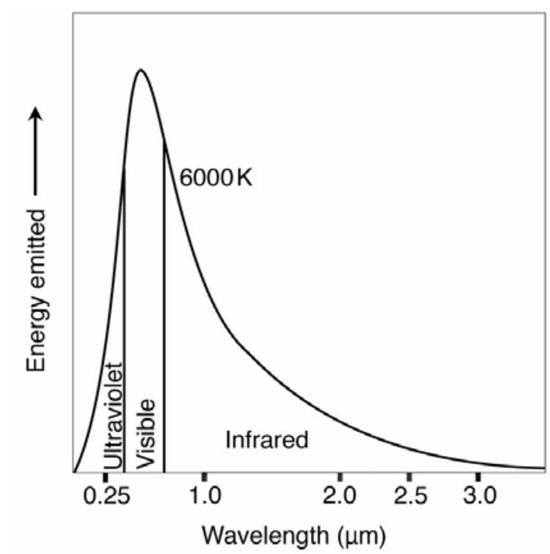


Figure 2.2: The energy emitted from the sun plotted against wavelength based on a black body curve with $T = T_{Sun}$. Most of the energy is in the visible and 95% of the total energy lies between 0.25 and 2.5 $\mu$m ($10^{-6}$ m).

the wavelength as well as the intensity of the radiative flux. The relation between the energy flux and wavelength — the spectrum — is plotted in Fig.2.2. The Sun emits radiation that is primarily in the visible part of the spectrum, corresponding to the colors of the rainbow — red, orange, yellow, green, blue, indigo and violet — with the energy flux decreasing toward longer (infrared, IR) and shorter (ultraviolet, UV) wavelengths.

Why does the spectrum have this pattern? Such behavior is characteristic of the radiation emitted by incandescent material, as can be observed, for example, in a coal fire. The hottest parts of the fire are almost white and emit the most intense radiation, with a wavelength that is shorter than that coming from the warm parts of the fire, which glow red. The coldest parts of the fire do not seem to be radiating at all, but are, in fact, radiating in the infrared. Experiment and theory show that the wavelength at which the intensity of radiation is a maximum, and the flux of emitted radiation, depend only on the temperature of the source. The theoretical spectrum,

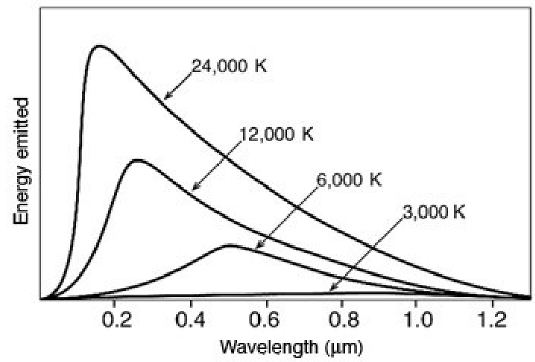


Figure 2.3: The energy emitted at different wavelengths for black bodies at several temperatures. The function $B_\lambda(T)$, Eq.(13.1) is plotted.

one of the jewels of physics, was worked out by Planck[1], and is known as the 'Planck' or 'blackbody' spectrum. (A brief theoretical background to the Planck spectrum is given in Appendix 13.1.1). It is plotted as a function of temperature in Fig.2.3. Note that the hotter the radiating body, the more energy it emits at shorter wavelengths. If the observed radiation spectrum of the Sun is fitted to the black body curve by using $T$ as a free parameter, we deduce that the blackbody temperature of the sun is about 6000 K.

Let us consider the energy balance of the Earth as in Fig.2.5, which shows the Earth intercepting the solar energy flux and radiating terrestrial energy away. If at the location of the (mean) Earth orbit, the incoming solar energy

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[1] In 1900 Max Planck (1858-1947) combined the formulae of Wien and Rayleigh describing the distribution of energy as a function of wavelength of the radiation in a cavity at temperature $T$, to arrive at what is now known as Planck's radiation curve. He went on to a complete theoretical deduction, introduced quanta of energy and set the scene for the development of Quantum Mechanics.

| Type of surface | Albedo (%) |
|---|---|
| Ocean | $2-10$ |
| Forest | $6-18$ |
| Cities | $14-18$ |
| Grass | $7-25$ |
| Soil | $10-20$ |
| Grassland | $16-20$ |
| Desert (sand) | $35-45$ |
| Ice | $20-70$ |
| Cloud (thin, thick stratus) | $30,\ 60-70$ |
| Snow (old) | $40-60$ |
| Snow (fresh) | $75-95$ |

Table 2.2: Albedos for different surfaces. Note that the albedo of clouds is highly variable and depend on the type and form. See also the horizontal map of albedo shown in Fig.2.4.

flux is $S_0 = 1367\ \mathrm{W\,m^{-2}}$, then, given that the cross-sectional area of the Earth intercepting the solar energy flux is $\pi a^2$ where $a$ is the radius of the Earth (Fig. 2.5),

$$\text{Solar power incident on the Earth} = S_0 \pi a^2 = 1.74 \times 10^{17}\ \mathrm{W}$$

using the data in Table 1.1. Not all of this radiation is absorbed by the Earth; a significant fraction is reflected. The ratio of reflected to incident solar energy is called the *albedo*, $\alpha$. As set out in Table 2.2 and the map of surface albedo shown in Fig.2.4, $\alpha$ depends on the nature of the reflecting surface and is large for clouds, light surfaces such as deserts and (especially) snow and ice. Under the present terrestrial conditions of cloudiness and snow and ice cover, on average a fraction $\alpha_p \simeq 0.30$ of the incoming solar radiation at the Earth is reflected back to space; $\alpha_p$ is known as the *planetary albedo* (see table 2.1). Thus

$$\text{Solar radiation absorbed by the Earth} = (1 - \alpha_p) S_0 \pi a^2 = 1.22 \times 10^{17}\ \mathrm{W}\ . \tag{2.1}$$

In equilibrium, the total terrestrial flux radiated to space must balance the solar radiation absorbed by the Earth. If, in total, the spinning Earth radiates

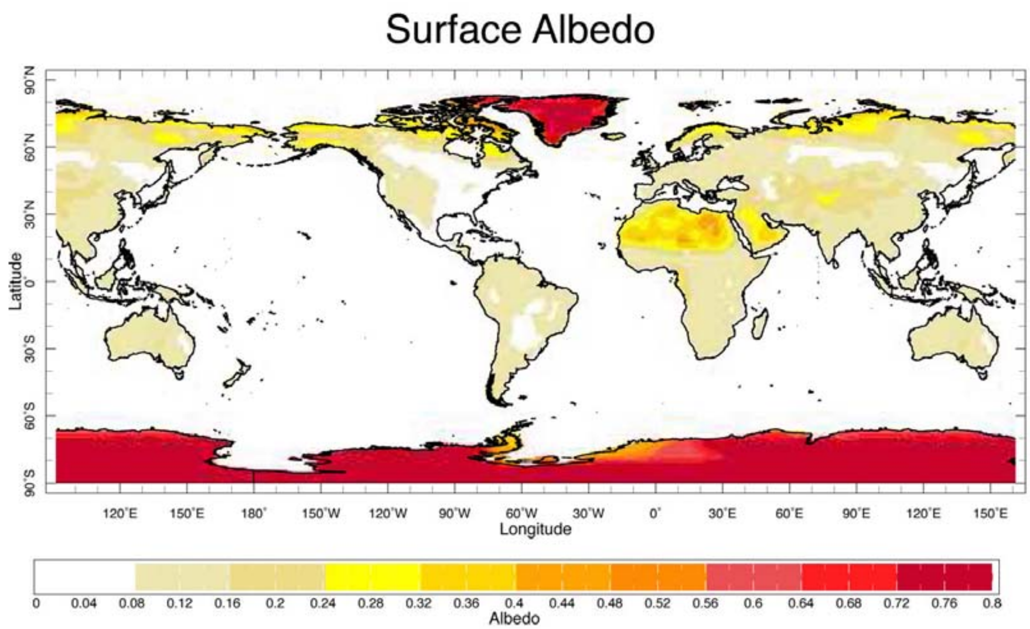


Figure 2.4: The albedo of the earth's surface. Over the ocean the albedo is small (2-10%). It is larger over the land (typically 35-45% over desert regions) and is particularly high over snow and ice (80% or so): see Table 2.2.

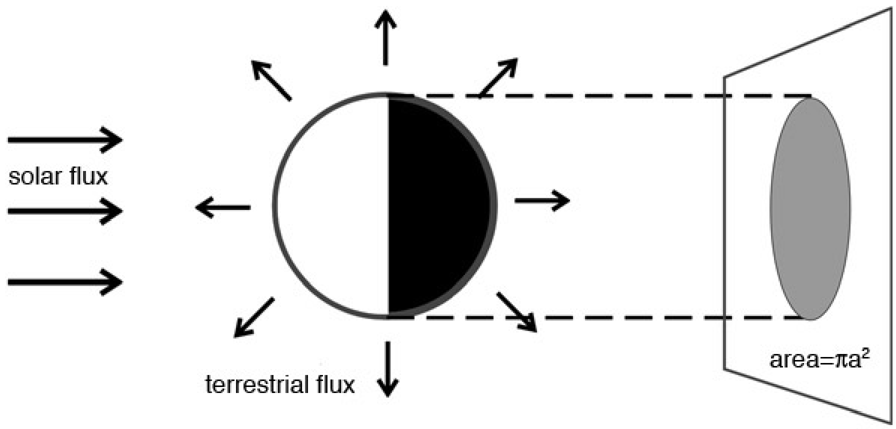


Figure 2.5: The spinning Earth is imagined to intercept solar energy over a disk of radius 'a' and radiate terrestrial energy away isotropically from the sphere. Modified from Hartmann, 1994.

in all directions like a blackbody of uniform temperature $T_e$ (known as the '*effective planetary temperature*', or '*emission temperature*' of the Earth) the Stefan-Boltzmann law gives:

$$\text{Emitted radiation per unit area} = \sigma T_e^4 \tag{2.2}$$

where $\sigma = 5.67 \times 10^{-8} \mathrm{W\,m^{-2}\,K^{-4}}$ is the Stefan-Boltzmann constant. So

$$\text{Emitted terrestrial radiation} = 4\pi a^2 \sigma T_e^4 \ . \tag{2.3}$$

Note that Eq.(2.3) is a **definition** of *emission temperature* $T_e$ — it is the temperature one would infer by looking back at Earth if a black body curve were fitted to the measured spectrum of outgoing radiation.

Equating Eq.(2.1) with Eq.(2.3) gives

$$T_e = \left[\frac{S_0(1-\alpha_p)}{4\sigma}\right]^{\frac{1}{4}} \ . \tag{2.4}$$

Note that the radius of the Earth has cancelled out: $T_e$ depends only on the planetary albedo and the distance of the Earth from the Sun. Putting in numbers we find that the Earth has an emission temperature of 255 K.

Table 2.1 lists the various parameters for some of the planets and compares approximate measured values, $T_m$, with $T_e$ computed from Eq.(2.4). The agreement is very good, except for Jupiter where it is thought that about one-half of the energy input comes from the gravitational collapse of the planet.

However, as can be seen from Table 2.1, the emission temperature of Earth is nearly 40 K cooler than the globally averaged observed surface temperature which is $T_s = 288\,\mathrm{K}$. As we shall discuss in Section 2.3, $T_s \neq T_e$ because 1) radiation is absorbed within the atmosphere, principally by its water vapor blanket and 2) fluid motions — air currents — carry heat both vertically and horizontally.

## 2.2 The atmospheric absorption spectrum

A property of the black body radiation curve is that the wavelength of maximum energy emission, $\lambda_m$, satisfies

$$\lambda_m T = \text{constant} \ . \tag{2.5}$$

This is known as *Wien's displacement law*. Since the solar emission temperature is about 6000 K and the maximum of the solar spectrum is (see Fig.2.2) at about $0.6\,\mu\mathrm{m}$ — i.e., in the visible — and we have determined $T_e = 255\,\mathrm{K}$ for the Earth, it follows that the peak of the terrestrial spectrum is at

$$\lambda_m^{earth} = 0.6\,\mu\mathrm{m} \times \frac{6000}{255} \simeq 14\,\mu\mathrm{m}.$$

Thus the Earth radiates to space primarily in the infrared. Normalized (see Eq.13.1.1 of the Appendix) blackbody spectra for the Sun and Earth are shown in Fig.2.6. The two spectra hardly overlap at all, which greatly simplifies thinking about radiative transfer.

Also shown in Fig.2.6 is the atmospheric absorption spectrum; this is the fraction of radiation at each wavelength that is absorbed on a single vertical path through the atmosphere. From it we see that:

- the atmosphere is almost completely transparent in the visible, at the peak of the solar spectrum.

- the atmosphere is very opaque in the UV.

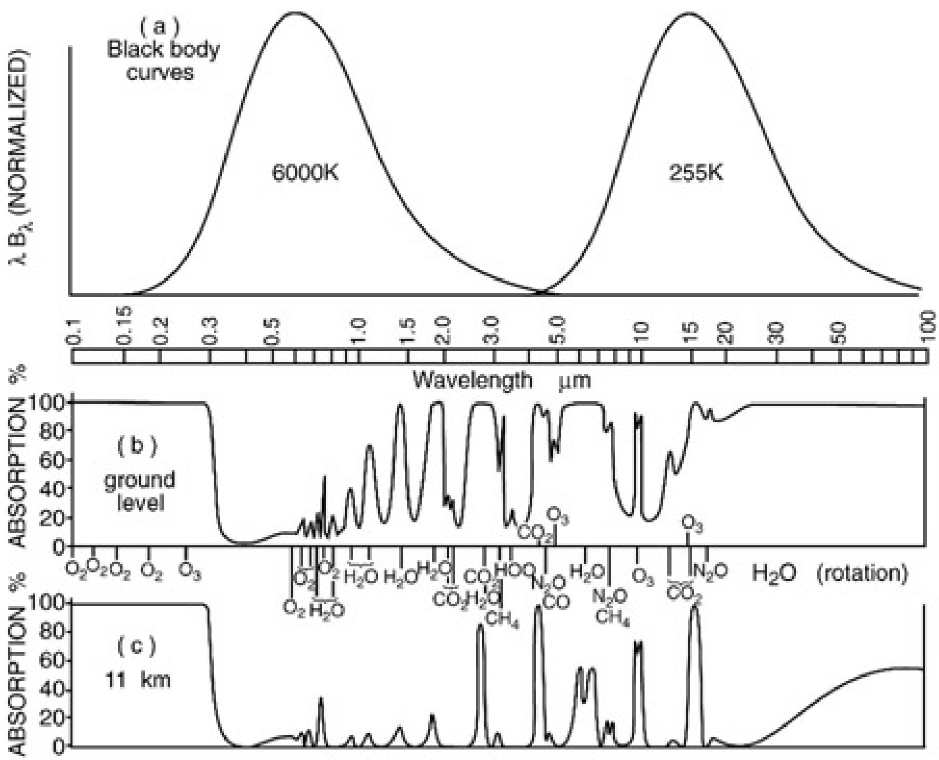


Figure 2.6: (a) The normalized blackbody emission spectra, $T^{-4}\lambda B_\lambda$, for the Sun ($T = 6000\,\mathrm{K}$) and Earth ($T = 255\,\mathrm{K}$) as a function of ln$\lambda$ (top) where $B_\lambda$ is the black body function (see Eq.(13.2)) and $\lambda$ is the wavelength (see the Appendix for further discussion.) (b) The fraction of radiation absorbed while passing from the ground to the top of the atmosphere as a function of wavelength. (c) The fraction of radiation absorbed from the tropopause (typically at a height of 11 km) to the top of the atmosphere as a function of wavelength. The atmospheric molecules contributing the important absorption features at each frequency are also indicated. After Goody and Yung (1989).

- the atmosphere has variable opacity across the IR spectrum — it is almost completely opaque at some wavelengths, transparent at others.

- $N_2$ does not figure at all in absorption, and $O_2$ absorbs only in the far UV (where there is little solar energy flux) and, a little, in the near IR: the dominant constituents of the atmosphere are incredibly transparent across almost the whole spectral range of importance.

- the absorption of terrestrial radiation is dominated by triatomic molecules — $O_3$ in the UV, $H_2O$, $CO_2$ and others in the IR because it so happens that triatomic molecules have rotational and vibrational modes that can easily be excited by radiation with wavelengths in the IR. These molecules are present in tiny concentrations (see Table 1.2) but play a key role in the absorption of terrestrial radiation (see Fig.2.6). They are known as Greenhouse gases. This is the fundamental reason why atmospheric radiation may be so vulnerable to the human-induced changes in composition shown in Fig.1.3.

## 2.3 The greenhouse effect

The global average mean surface temperature of the earth is 288 K (Table 2.1). Above we deduced that the emission temperature of the Earth is 255 K, considerably lower. Why? We saw from Fig.2.6 that the atmosphere is rather opaque to IR, so we cannot think of terrestrial radiation as being radiated into space directly from the surface. Much of the radiation emanating from the surface will be absorbed, primarily by $H_2O$, before passing through the atmosphere. On average, the emission to space will emanate from some level in the atmosphere (typically about 5 km, in fact) such that the region above that level is mostly transparent to IR. It is this region of the atmosphere, rather than the surface, that must be at the emission temperature. Thus radiation from the atmosphere will be directed downward, as well as upward, and hence the surface will receive not only the net solar radiation, but IR from the atmosphere as well. Because the surface feels more incoming radiation than if the atmosphere were not present (or were completely transparent to IR) it becomes warmer than $T_e$. This has become known as the 'greenhouse effect'[2].

[2] It is interesting to note that the domestic greenhouse does not work in this manner! A greenhouse made of plastic window panes, rather than conventional glass, is effective

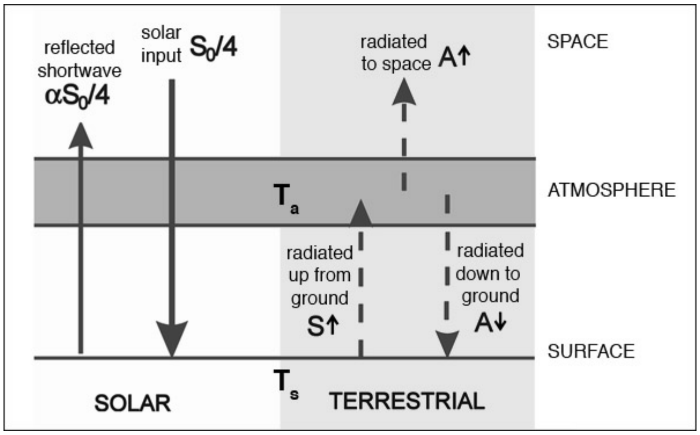


Figure 2.7: The simplest greenhouse model, comprising a surface at temperature $T_s$, and an atmospheric layer at temperature $T_a$, subject to incoming solar radiation $\frac{S_o}{4}$. The terrestrial radiation upwelling from the ground is assumed to be completely absorbed by the atmospheric layer.

### 2.3.1 A simple greenhouse model

Consider Fig.2.7. Since the atmosphere is thin, let us simplify things by considering a planar geometry, in which the incoming radiation per unit area is equal to the average flux per unit area striking the Earth. This average incoming solar energy *per unit area of the Earth's surface* is

$$\text{average solar energy flux} = \frac{\text{intercepted incoming radiation}}{\text{Earth's surface area}} = \frac{S_0 \pi a^2}{4\pi a^2} = \frac{S_0}{4}. \tag{2.6}$$

We will represent the atmosphere by a single layer of temperature $T_a$, and, in this first calculation, assume 1) that it is completely transparent to shortwave solar radiation, and 2) that it is completely opaque to IR (*i.e.*, it absorbs all the IR radiating up from the ground) so that the layer that is emitting to space is also "seen" by the ground. Now, since the whole Earth-atmosphere system must be in equilibrium (on average), the net flux into the system must vanish. The average net solar flux per unit area is, from Eq.(2.6),

even though plastic (unlike glass) does not have significant absorption bands in the IR. The greenhouse works because its windows allow energy in and its walls prevent the warm air from rising or blowing away.

and allowing for reflection, $\frac{1}{4}(1-\alpha_p) S_0$, while the terrestrial radiation being emitted to space per unit area is, using Eq.(2.2):

$$A \uparrow = \sigma T_a^4 .$$

Equating them, we find:

$$\sigma T_a^4 = \frac{1}{4}(1-\alpha_p) S_0 = \sigma T_e^4 , \tag{2.7}$$

using the definition of $T_e$, Eq.(2.4). We see that the atmosphere is at the emission temperature (naturally, because it is this region that is emitting to space).

At the surface, the average incoming shortwave flux is also $\frac{1}{4}(1-\alpha_p) S_0$, but there is also a downwelling flux emitted by the atmosphere,

$$A \downarrow = \sigma T_a^4 = \sigma T_e^4 .$$

The flux radiating upward from the ground is

$$S \uparrow = \sigma T_s^4 ,$$

where $T_s$ is the surface temperature. Since, in equilibrium, the net flux at the ground must be zero,

$$S \uparrow = \frac{1}{4}(1-\alpha_p) S_0 + A \downarrow ,$$

whence

$$\sigma T_s^4 = \frac{1}{4}(1-\alpha_p) S_0 + \sigma T_e^4 = 2\sigma T_e^4 , \tag{2.8}$$

where we have used Eq.(2.7). Therefore

$$T_s = 2^{\frac{1}{4}} T_e . \tag{2.9}$$

So the presence of an absorbing atmosphere, as depicted here, increases the surface temperature by a factor $2^{\frac{1}{4}} = 1.19$. This arises as a direct consequence of absorption of terrestrial radiation by the atmosphere, which, in turn, reradiates IR back down to the surface, thus increasing the net downward radiative flux at the surface. Note that $A \downarrow$ is of the same order — in fact, in this simple model, equal to — the solar radiation that strikes the ground. This is true of more complex models and indeed observations show that the

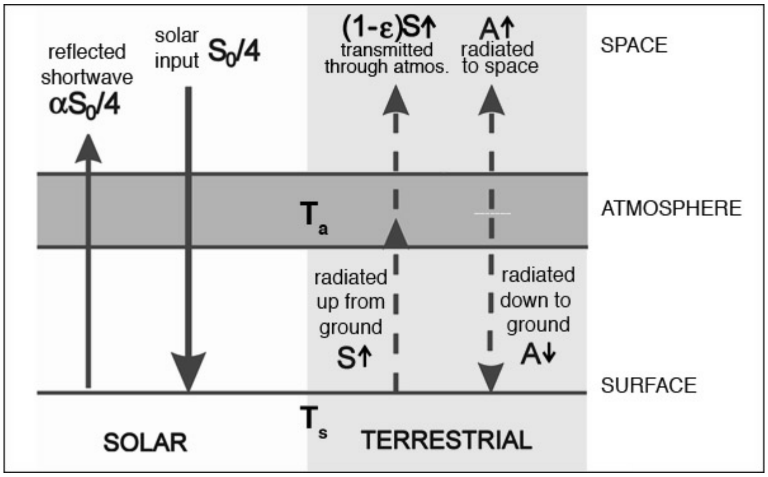


Figure 2.8: A leaky greenhouse. In contrast to Fig.2.7, the atmosphere now absorbs only a fraction, $\varepsilon$, of the terrestrial radiation upwelling from the ground.

downwelled radiation from the atmosphere can exceed that due to the direct solar flux.

Applying this factor to our calculated value $T_e = 255\,\mathrm{K}$, we predict $T_s = 2^{\frac{1}{4}} \times 255 = 303\,\mathrm{K}$. This is closer to the actual mean surface temperature of 288 K but is now an overestimate! The model we have discussed is clearly an oversimplification:

- For one thing, not all the solar flux incident on the top of the atmosphere reaches the surface—typically, some 20-25% is absorbed within the atmosphere (including by clouds).

- For another, we saw in Section (2.2) that IR absorption by the atmosphere is incomplete. The greenhouse effect is actually less strong than in the model assumed above and so $T_s$ will be less than the value implied by Eq.(2.9). We shall analyze this by modifying Fig.2.7 to permit partial transmission of IR through the atmosphere — a leaky greenhouse model.

### 2.3.2 A leaky greenhouse

Consider Fig.2.8. We suppose the atmosphere has absorptivity $\epsilon$, *i.e.*, a fraction $\epsilon$ of the IR upwelling from the surface is absorbed within the atmosphere (so the case of Fig.2.7 corresponds to $\epsilon = 1$). Now, again insisting that, in equilibrium, the net flux at the top of the atmosphere vanishes gives

$$\frac{1}{4}(1-\alpha_p)S_0 = A\uparrow + (1-\epsilon)\, S\uparrow \ . \tag{2.10}$$

Zero net flux at the surface gives

$$\frac{1}{4}\,(1-\alpha_p)\, S_0 + A\downarrow = S\uparrow \ . \tag{2.11}$$

Since at equilibrium, $A\uparrow = A\downarrow$, we have

$$S\uparrow = \sigma T_s^4 = \frac{1}{2\,(2-\epsilon)}\,(1-\alpha_p)\, S_0 = \frac{2}{(2-\epsilon)}\sigma T_e^4. \tag{2.12}$$

Therefore,

$$T_s = \left(\frac{2}{2-\epsilon}\right)^{\frac{1}{4}} T_e \ . \tag{2.13}$$

So in the limit $\epsilon \to 0$ (transparent atmosphere), $T_s = T_e$, and for $\epsilon \to 1$ (opaque atmosphere), $T_s = 2^{\frac{1}{4}} T_e$, as found in section 2.3.1. In general, when $0 < \epsilon < 1$, $T_e < T_s < 2^{\frac{1}{4}} T_e$. So, of course, partial transparency of the atmosphere to IR radiation—a "leaky" greenhouse—reduces the warming effect we found in Eq.(2.9).

To find the atmospheric temperature, we need to invoke *Kirchhoff's law*[3], *viz.*, that the emissivity of the atmosphere is equal to its absorptivity. Thus,

$$A\uparrow = A\downarrow = \epsilon\sigma T_a^4. \tag{2.14}$$

We can now use Eqs. (2.14), (2.10), (2.11) and (2.12) to find

$$T_a = \left(\frac{1}{2-\epsilon}\right)^{\frac{1}{4}} T_e = \left(\frac{1}{2}\right)^{\frac{1}{4}} T_s \ .$$

So the atmosphere is, for $\epsilon < 1$, cooler than $T_e$ (since the emission is then only partly from the atmosphere). Note, however, that $T_a < T_s$: the atmosphere is *always* cooler than the ground.

[3] Kirchhoff's law states that the emittance of a body — the ratio of the actual emitted flux to the flux that would be emitted by a black body at the same temperature — equals its absorptance.

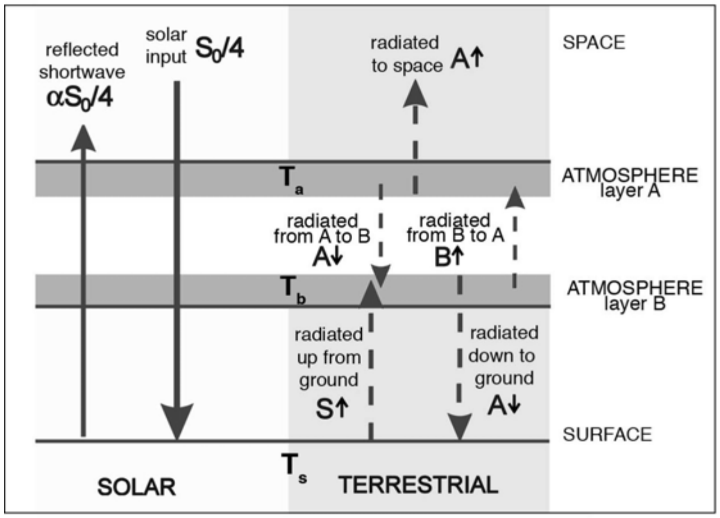


Figure 2.9: An 'opaque' greenhouse made up of two layers of atmosphere. Each layer completely absorbs the IR radiation impinging on it.

### 2.3.3 A more opaque greenhouse

Above we considered a leaky greenhouse. To take the other extreme, suppose that the atmosphere is so opaque that even a shallow layer will absorb *all* the IR passing through it. Now the assumption implicit in Fig.2.7—that space and the surface both "see" the same atmospheric layer—is wrong. We can elaborate our model to include a second, totally absorbing, layer in the atmosphere, as illustrated in Fig.2.9. Of course, to do the calculation correctly (rather than just to illustrate the principles) we would divide the atmosphere into an infinite number of infinitesimally thin layers, allow for the presence of cloud, treat each wavelength in Fig.2.6 separately, allow for atmospheric absorption layer-by-layer — which depends on the vertical distribution of absorbers, particularly $H_2O$, $CO_2$ and $O_3$ (see section 3.1.2) — and do the required budgets for each layer and at the surface (we are not going to do this). An incomplete schematic of how this might look for a rather opaque atmosphere is shown in Fig.2.10.

The resulting profile—which would be the actual mean atmospheric temperature profile *if heat transport in the atmosphere occurred only through radiative transfer*—is known as the **radiative equilibrium temperature**

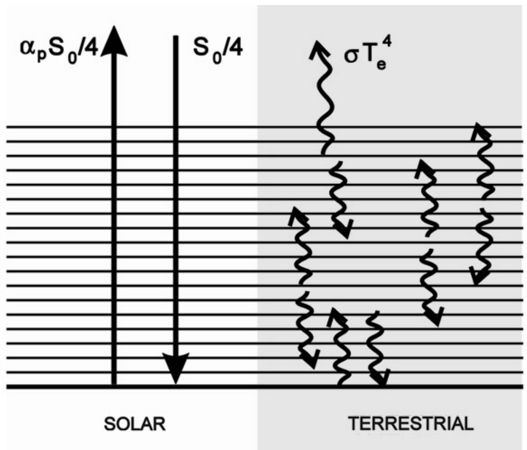


Figure 2.10: Schematic of radiative transfer model with many layers.

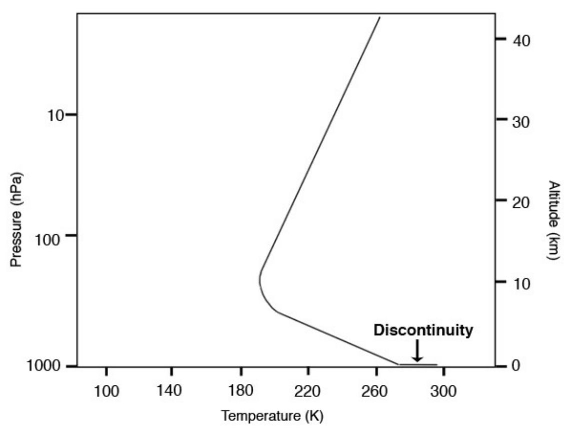


Figure 2.11: The radiative equilibrium profile of the atmosphere obtained by carrying out the calculation schematized in Fig.2.10. The absorbers are $H_2O$, $O_3$ and $CO_2$. The effects of both terrestrial radiation and solar radiation are included. Note the discontinuity at the surface. Modified from Wells (1997).

**profile.** It is shown in Fig.2.11. In particular, note the presence of a large temperature discontinuity at the surface in the radiative equilibrium profile which is not observed in practice. (Recall from our analysis of Fig.2.8 that we found that the atmosphere in our slab model is always colder than the surface.) The reason this discontinuity is produced in radiative equilibrium is that, while there is some absorption within the troposphere, both of solar and terrestrial radiation, most solar radiation is absorbed at the surface. The reason such a discontinuity is not observed in nature is that it would (and does) leads to *convection* in the atmosphere, which introduces an additional mode of *dynamical* heat transport. Because of the presence of convection in the lower atmosphere, the observed profile differs substantially from that obtained by the radiative calculation described above. This is discussed at some length in Chapter 4.

Before going on in Chapter 3 to a discussion of the observed vertical profile of temperature in the atmosphere, we briefly discuss what our simple greenhouse models tell us about climate feedbacks and sensitivity to changes in radiative forcing.

### 2.3.4 Climate feedbacks

The greenhouse models described above illustrate several important radiative feedbacks that play a central role in regulating the climate of the planet. Following Hartmann (1994) we suppose that a perturbation to the climate system can be represented as an additional energy input $dQ$ (units $\mathrm{W\,m^{-2}}$) and study the resultant change in global-mean surface temperature, $dT_s$. Thus we define $\frac{dT_s}{dQ}$ to be a measure of climate sensitivity.

The most important negative feedback regulating the temperature of the planet is the dependence of the outgoing longwave radiation on temperature. If the planet warms up then it radiates more heat back out to space. Thus, using Eq.(2.2) and setting $\delta Q = \delta\left(\sigma T_e^4\right) = 4T_e^3 \delta T_s$, where it has been assumed that $T_e$ and $T_s$ differ by a constant, implies a climate sensitivity associated with black body radiation of:

$$\frac{\partial T_s}{\partial Q}_{\mathrm{BB}} = \left(4\sigma T_e^3\right)^{-1} = 0.26 \frac{\mathrm{K}}{\mathrm{W\,m^{-2}}}. \tag{2.15}$$

where we have inserted numbers setting $T_e = 255\,\mathrm{K}$. Thus for every $1\mathrm{W\,m^{-2}}$ increase in the forcing of energy balance at the surface, $T_s$ will increase by about a quarter of a degree. This is rather small when one notes that a $1\,\mathrm{W\,m^{-2}}$ change in surface forcing demands a change in solar forcing of about $6\,\mathrm{W\,m^{-2}}$ on taking in to account geometrical and albedo effects — see Q7 at the end of the Chapter.

A powerful positive climate feedback results from the temperature dependence of saturated water vapor pressure, $e_s$, on $T$; see Eq.(1.4). If the temperature increases, the amount of water that can be held at saturation increases. Since $H_2O$ is the main greenhouse gas, this further raises surface temperature. From Eq.(1.4) we find that:

$$\frac{de_s}{e_e} = \beta dT$$

and so, given that $\beta = 0.067\ {}^{\circ}\mathrm{C}^{-1}$, a $1\,^{\circ}\mathrm{C}$ change is temperature leads to a full 7% change in saturated specific humidity. The observed relative humidity of the atmosphere (that is the ratio of actual to the saturated specific humidity — see Section 5.3) does not vary significantly, even during the seasonal cycle when air temperatures vary markedly. One consequence of the presence of this blanket of $H_2O$ is that the emission of terrestrial radiation from the surface depends much more weakly on $T_s$ than suggested by the Stefan-Boltzmann law. When Stefan-Boltzmann and water vapor feedbacks are combined, calculations show that the climate sensitivity is:

$$\frac{\partial T_s}{\partial Q}_{\mathrm{BB\ and\ H_2O}} = 0.5\frac{\mathrm{K}}{\mathrm{W\,m^{-2}}},$$

twice that of Eq.(2.15).

The albedo of ice and clouds also play a very important role in climate sensitivity. The primary effect of ice cover is its high albedo relative to typical land surfaces or the ocean — see Table 2.2 and Fig.2.4. If the surface area of sea-ice, for example, were to expand in to low albedo regions, the amount of solar energy absorbed at the surface would be reduced, so causing further cooling and enhancing the expansion of ice. Clouds, due to their high reflectivity, typically double the albedo of the earth from 15 to 30% and so have a major impact on the radiative balance of the planet. However it is not known to what extent the amount or type of cloud (both of which are important for climate, as we will see in Chapter 4) is sensitive to the state of the climate or how they might change as the climate evolves over time.

Unfortunately our understanding of cloud/radiative feedbacks is one of the greatest uncertainties in climate science.

## 2.4 Further reading

More advanced treatments of radiative transfer theory can be found in the texts of Houghton (1986) and Andrews (2000). Hartmann (1994) has a thorough discussion of greenhouse models, radiative/convective processes and their role in climate and climate feedbacks.

## 2.5 Problems

1. At present the emission temperature of the Earth is 255 K, and its albedo is 30%. How would the emission temperature change if:

   (a) the albedo were reduced to 10% (and all else were held fixed);

   (b) the infrared absorptivity of the atmosphere — $\epsilon$ in Fig.2.8 — were doubled, but albedo remains fixed at 30%.

2. Suppose that the Earth is, after all, flat. Specifically, consider it to be a thin circular disk (of radius 6370 km), orbiting the Sun at the same distance as the Earth; the planetary albedo is 30%. The vector normal to one face of this disk always points directly towards the Sun, and the disk is made of perfectly conducting material, so both faces of the disk are at the same temperature. Calculate the emission temperature of this disk, and compare with Eq.(2.4) for a spherical Earth.

3. Consider the thermal balance of Jupiter.

   (a) Assuming a balance between incoming and outgoing radiation, and given the data in Table 2.1, calculate the emission temperature for Jupiter.

   (b) In fact, Jupiter has an internal heat source resulting from its gravitational collapse. The measured emission temperature $T_e$ defined by

$$\sigma T_e^4 = \begin{array}{l}\text{(outgoing flux of planetary}\\ \text{radiation per unit surface area)}\end{array}$$

is 130 K. Comment in view of your theoretical prediction in part (a). Modify your expression for emission temperature for the case where a planet has an internal heat source giving a surface heat flux $Q$ per unit area. Calculate the magnitude of Jupiter's internal heat source.

(c) It is believed that the source of $Q$ on Jupiter is the release of gravitational potential energy by a slow contraction of the planet. On the simplest assumption that Jupiter is of uniform density and remains so as it contracts, calculate the annual change in its radius $a_{jup}$ required to produce your value of $Q$. (Only one half of the released gravitational energy is convertible to heat, the remainder appearing as the additional kinetic energy required to preserve the angular momentum of the planet.)

[A uniform sphere of mass $M$ and radius $a$ has a gravitational potential energy of $-\frac{3}{5}G\frac{M^2}{a}$ where $G$ is the gravitational constant $= 6.7 \times 10^{-11}\,\mathrm{kg}^{-1}\,\mathrm{m}^3\,\mathrm{s}^{-2}$. The mass of Jupiter is $2 \times 10^{27}$ kg and its radius is $a_{jup} = 7.1 \times 10^7$ m.]

4. Consider the "two-slab" greenhouse model illustrated in Fig.2.9 in which the atmosphere is represented by two perfectly absorbing layers of temperature $T_a$ and $T_b$.

   Determine $T_a$, $T_b$, and the surface temperature $T_s$ in terms of the emission temperature $T_e$.

5. Consider an atmosphere that is completely transparent to shortwave (solar) radiation, but very opaque to infrared (IR) terrestrial radiation. Specifically, assume that it can be represented by $N$ slabs of atmosphere, each of which is completely absorbing of IR, as depicted in Fig.2.12 (not all layers are shown).

   (a) By considering the radiative equilibrium of the surface, show that the surface must be warmer than the lowest atmospheric layer.

   (b) By considering the radiative equilibrium of the $n^{th}$ layer, show that, in equilibrium,

$$2T_n^4 = T_{n+1}^4 + T_{n-1}^4 \, , \tag{2.16}$$

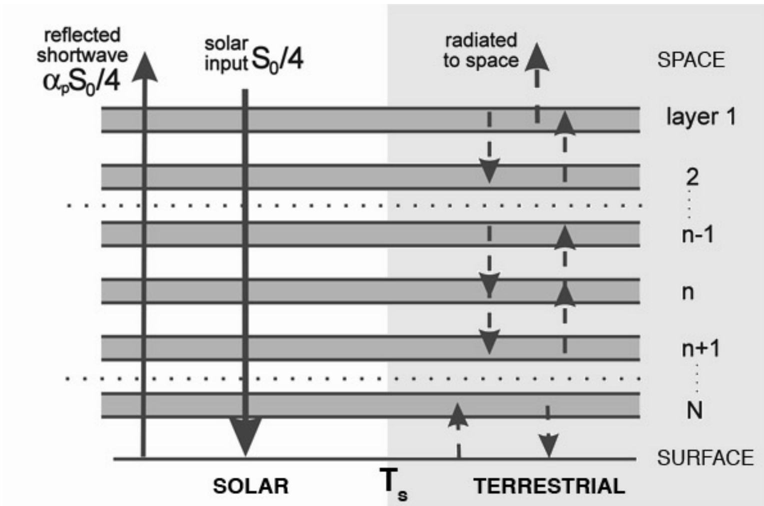


Figure 2.12: An atmosphere made up of $N$ slabs each which is completely absorbing in the IR.

where $T_n$ is the temperature of the $n^{th}$ layer, for $n > 1$. Hence argue that the equilibrium surface temperature is

$$T_s = (N+1)^{\frac{1}{4}} T_e ,$$

where $T_e$ is the planetary emission temperature. [Hint: Use your answer to part (a); determine $T_1$ and use Eq.(2.16) to get a relationship for temperature differences between adjacent layers.]

6. Determine the emission temperature of the planet Venus. You may assume the following: the mean radius of Venus' orbit is 0.72 times that of the Earth's orbit; the solar flux $S_o$ decreases like the square of the distance from the sun and has a value of $1367 \mathrm{W\,m^{-2}}$ at the mean Earth orbit; Venus planetary albedo = 0.77.

   The observed mean surface temperature of the planet Venus is about 750 K — see Table 2.1. How many layers of the $N-$layer model considered in Question 5 would be required to achieve this degree of warming? Comment.

7. Climate feedback due to Stefan-Boltzmann.

(a) Show that the globally-averaged incident solar flux at the ground is $\frac{1}{4}(1-\alpha_p)S_0$.

(b) If the outgoing longwave radiation from the earth's surface were governed by the Stefan-Boltzmann law, then we showed in Eq.(2.15) that for every $1\mathrm{W\,m^{-2}}$ increase in the forcing of the surface energy balance, the surface temperature will increase by about a quarter of a degree. Use your answer to (a) to estimate by how much one would have to increase the solar constant to achieve a $1\,^{\circ}\mathrm{C}$ increase in surface temperature? You may assume that the albedo of earth is 0.3 and does not change.