THE CONCEPT OF MASS

In the modern language of relativity theory there is only one mass, the Newtonian mass m, which does not vary with velocity; hence the famous formula $E = mc^2$ has to be taken with a large grain of salt.

Lev B. Okun

Mass is one of the most fundamental concepts of physics. Understanding and calculating the masses of the elementary particles is the central problem of modern physics, and is intimately connected with other fundamental problems such as the origin of *CP* violation, the mystery of the energy scales that determine the properties of the weak and gravitational interactions, the compositeness of particles, supersymmetry theory and the properties of the not-yet-discovered Higgs bosons.

But instead of discussing all these subtle and deep connections, I feel obliged to raise and discuss an elementary question—that of the connection between mass and energy. I agree with those readers who think this topic would be more appropriate for high school pupils than for physicists, but just to find out how far off I am, I would like to propose a simple test and tell you about an opinion poll related to it.

The famous Einstein relation between mass and energy is a symbol of our century. Here you have four equations:

$$E_0 = mc^2 \tag{1}$$

$$E = mc^2 \tag{2}$$

$$E_0 = m_0c^2 \tag{3}$$

$$E = m_0c^2 \tag{4}$$

In these equations c is the velocity of light, E the total energy of a free body, E_0 its rest energy, m_0 its rest mass and m its mass.

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Now I ask two simple questions:

▷ Which of these equations most rationally follows from special relativity and expresses one of its main consequences and predictions?

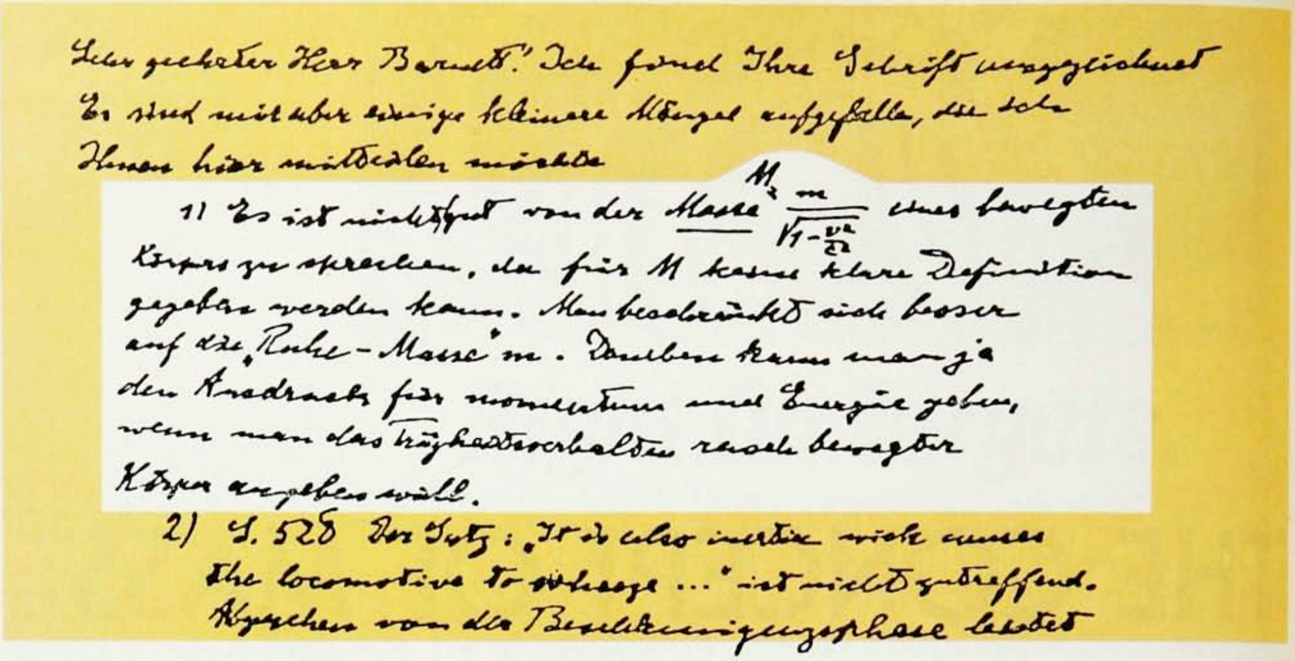
▷ Which of these equations was first written by Einstein and was considered by him a consequence of special relativity?

The correct answer to these two questions is equation 1, while opinion polls that I have carried out among professional physicists have shown that the majority prefers equation 2 or 3 as the answer to both questions. This choice is caused by the confusing terminology widely used in the popular science literature and in many textbooks. According to this terminology the body at rest has a "proper mass" or "rest mass" m_0, whereas a body moving with velocity v has "relativistic mass" or "mass" m, given by

$$m = \frac{E}{c^2} = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

As I will show, this terminology had some historical justification at the beginning of our century, but it has no rational justification today. When doing relativistic physics (and often when teaching relativistic physics), particle physicists use only the term "mass." According to this rational terminology the terms "rest mass" and "relativistic mass" are redundant and misleading. There is only one mass in physics, m, which does not depend on the reference frame. As soon as you reject the "relativistic mass" there is no need to call the other mass the "rest mass" and to mark it with the index 0.

The purpose of this article is to promote the rational terminology. You may wonder whether this subject is really so important. I'm deeply convinced, and I will try to



Sehr geehrter Herr Barnett! Ich fand Ihre Schrift ausgezeichnet
Es sind mir aber einige kleinere Mängel aufgefallen, die ich
Ihnen hier mitteilen möchte

1) Es ist nicht gut von der Masse $M = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}$ eines bewegten
Körpers zu sprechen, da für M keine klare Definition
gegeben werden kann. Man beschränkt sich besser
auf die „Ruhe-Masse" m. Daneben kann man ja
den Ausdruck für momentum und Energie geben,
wenn man das Trägheitsverhalten rasch bewegter
Körper angeben will.

2) S. 528 Der Satz: „It is also inertia which causes
the locomotive to whizz ..." ist nicht zutreffend.
Abgesehen von der Beschleunigungsphase leistet

Letter from Albert Einstein to Lincoln Barnett, 19 June 1948. Einstein wrote in German; the letter was typed and sent in English. The highlighted passage in this excerpt says: "It is not good to introduce the concept of the *mass* $M = m/(1 - v^2/c^2)^{1/2}$ of a moving body for which no clear definition can be given. It is better to introduce no other mass concept than the 'rest mass' m. Instead of introducing M it is better to mention the expression for the momentum and energy of a body in motion." (Reprinted by permission of the Hebrew University of Jerusalem, Israel.)

persuade you, that the use of the proper terminology is extremely important in explaining our science to other scientists, to the taxpayers and especially to students in high schools and colleges. Nonrational, confusing language prevents many students from grasping the essence of special relativity and from enjoying its beauty.

Two fundamental equations

Let us return to equation 1. Its validity is apparent when one recalls two fundamental equations of special relativity for a free body:

$$E^2 - \mathbf{p}^2c^2 = m^2c^4 \tag{5}$$

$$\mathbf{p} = \mathbf{v}\frac{E}{c^2} \tag{6}$$

Here E is the total energy, $\mathbf{p}$ the momentum, $\mathbf{v}$ the velocity and m the ordinary mass, the same as in Newtonian mechanics.

When $\mathbf{v} = 0$, we get $\mathbf{p} = 0$ and $E = E_0$, the energy of the body at rest. Then, from equation 5,

$$E_0 = mc^2$$

This is equation 1. Rest energy was one of Einstein's great discoveries.

Now why have I written m but not m_0 in equation 5? To see the answer, let's consider the case $v \ll c$. In this case

$$\mathbf{p} \simeq \mathbf{v}\frac{E_0}{c^2} = \mathbf{v}m \tag{7}$$

$$E = E_0 + E_{\text{kin}} = \sqrt{\mathbf{p}^2c^2 + m^2c^4} = mc^2 + \frac{\mathbf{p}^2}{2m} + \ \ldots$$

and

$$E_{\text{kin}} = \frac{\mathbf{p}^2}{2m}$$

Thus we obtain in the nonrelativistic limit the well-known Newtonian equations for momentum and kinetic energy. This means that m in equation 5 is the ordinary Newtonian mass. Hence, if I were to use m_0 instead of m, the relativistic and nonrelativistic notations would not match.

If the notation m_0 and the term "rest mass" are bad, why then are the notation E_0 and the term "rest energy" good? The answer is, because mass is a relativistic invariant and is the same in different reference systems, while energy is the fourth component of a four-vector $(E,\mathbf{p})$ and is different in different reference systems. The index 0 in E_0 indicates the rest system of the body.

Let us look again at equations 5 and 6, and consider them in the case when $m = 0$—the extreme "anti-Newtonian" case. We see that in this case the velocity of the body is equal to that of light: $v = c$ in any reference system. There is no rest frame for such bodies. They have no rest energy; their total energy is purely kinetic.

Thus, equations 5 and 6 describe the kinematics of a free body for all velocities from 0 to c, and equation 1 follows from them directly. Every physicist who knows special relativity will agree on this.

On the other hand, every physicist and many nonphysicists are familiar with "the famous Einstein formula $E = mc^2$." But it is evident that equations 1 and 2, $E_0 = mc^2$ and $E = mc^2$, are absolutely different. According to equation 1, m is constant and the photon is massless. According to equation 2, m depends on energy (on velocity) and the photon has mass $m = E/c^2$.

$E = mc^2$ as historical artifact

We have seen the origin of equation 1. Now let us look at the origin of equation 2. It was first written by Henri Poincaré[1] in 1900, five years before Einstein formulated special relativity.[2] Poincaré considered a pulse of light, or a wave train, with energy E and momentum $\mathbf{p}$. (I am using modern terminology.) Recalling that, according to the Poynting theorem, $p = E/c$, and applying to the pulse of light the nonrelativistic Newtonian relation of equation 7, $\mathbf{p} = m\mathbf{v}$, Poincaré concluded that a pulse of light with energy E has mass $m = E/c^2$.

The idea that mass increases with velocity is usually ascribed, following Hendrik Lorentz,[3] to J. J. Thomson. But Thomson, who considered in 1881 the kinetic energy of a freely moving charged body, calculated only the correction proportional to v^2 and therefore derived only the velocity-independent contribution to the mass.[4] In subsequent papers by Oliver Heaviside, George Searle and others, the energy was calculated for various kinds of charged ellipsoids in the whole interval $0 \leqslant v \leqslant c$, but I have not found in the papers I have read any suggestion that mass depends on velocity.[5]

The notion of the dependence of mass on velocity was introduced by Lorentz in 1899 and then developed by him[6] and others in the years preceding Einstein's formulation of special relativity in 1905, as well as in later years. The basis of this notion is again the application of the nonrelativistic formula $\mathbf{p} = m\mathbf{v}$ in the relativistic region, where (as we know now) this formula is not valid.

Consider a body accelerated by some force $\mathbf{F}$. One can show that in the framework of special relativity the formula

$$\frac{d\mathbf{p}}{dt} = \mathbf{F} \tag{8}$$

is valid. If we start from equations 5 and 6, for the case in which the body is massive (as opposed to massless) we can easily obtain

$$\mathbf{p} = m\mathbf{v}\gamma \tag{9}$$

$$E = mc^2\gamma \tag{10}$$

where

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \tag{11}$$

$$\boldsymbol{\beta} = \frac{\mathbf{v}}{c} \tag{12}$$

Substituting equation 9 into equation 8, it is again easy to get the following relation between acceleration $\mathbf{a}$, given by $\mathbf{a} = d\mathbf{v}/dt$, and force $\mathbf{F}$:

$$\mathbf{a} = \frac{\mathbf{F} - (\mathbf{F}\cdot\boldsymbol{\beta})\boldsymbol{\beta}}{m\gamma} \tag{13}$$

We see that in the general case the acceleration is not parallel to the force, unlike the Newtonian situation to which we are accustomed. Hence one cannot cling to the Newtonian relation of proportionality between $\mathbf{a}$ and $\mathbf{F}$,

$$\mathbf{a} = \frac{\mathbf{F}}{m}$$

with mass defined as a scalar, because $\mathbf{a}$ has a nonvanishing component along $\mathbf{v}$. However, when $\mathbf{F}$ is perpendicular to $\mathbf{v}$, one can consider a "transverse mass"

$$m_t = m\gamma$$

and when $\mathbf{F}$ is parallel to $\mathbf{v}$, one can consider a "longitudinal mass"

$$m_l = m\gamma^3$$

These are the very expressions with which Lorentz introduced the two masses. Together with the "relativistic mass" in the relation $\mathbf{p} = m_r\mathbf{v}$, where $m_r = E/c^2$ (which is equal to m_t when $m \neq 0$, but which had a more general meaning applicable also in the case of photons), these masses formed the basis of the language physicists used at the beginning of the century.

Making the trouble even more lasting, however, it was decided to call the "relativistic mass" m_r simply "mass" and to denote it by m, while the normal mass m was nicknamed "rest mass" and denoted m_0.

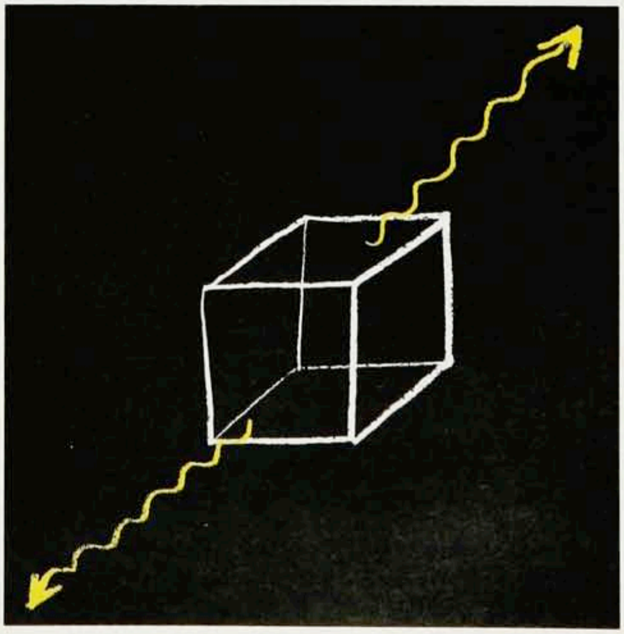
Einstein's papers of 1905 and 1906

In his first paper on relativity Einstein didn't use the term "rest mass," but he did mention the transverse and longitudinal masses.[2] He formulated the famous mass–energy relation in the second of his 1905 papers on relativity[7] in the form

$$\Delta E_0 = \Delta mc^2 \tag{14}$$

Einstein had considered a free body at rest with rest energy E_0 that emits two light waves in opposite directions, as indicated in the figure below. By looking at the same process from a slowly moving frame and applying energy conservation he arrived at equation 14, and, in fact, conjectured equation 1 as universal by writing that "the mass of a body is a measure of its energy content."

From our present point of view we can say that the proof was facilitated by the fact that the two-photon system is at rest with respect to the body and therefore it was easy to see that its mass, which is equal to the sum of the energies of the two photons, is Δm.



***Gedanken* experiment** that Einstein described[7] in 1905. A body at rest with rest energy E_0 emits two equal pulses of light in opposite directions. Applying conservation of energy to the process in stationary and slowly moving reference frames leads to the equation $\Delta E_0 = \Delta mc^2$.

You can also find equation 1 as equation 44 in the famous book *The Meaning of Relativity*,[8] which is based on four lectures Einstein gave at Princeton in 1921. (The figure at the right reproduces the relevant page.)

But in between, Einstein was not absolutely consistent in preferring equation 1 to equation 2. In 1906, for example, he rederived[9] Poincaré's formula (equation 2) by considering a photon (to use modern language) that is emitted at one end of a hollow cylinder and absorbed at the other end of the cavity, as indicated in the figure at the top of page 35. Requiring that the center of mass not move, he essentially equated the product of the large mass M of the cylinder and its small displacement l with the product of the small mass m of the photon and its large displacement L, the length of the cylinder:

$$lM = Lm \tag{15}$$

The small displacement l, however, is a product of the photon's time of flight L/c and the cylinder velocity $v = E/(cM)$, where E is the photon's energy and E/c is both its momentum and the momentum of the cylinder. From equation 15 one immediately obtains equation 2. The conclusion of the paper was that light with energy E transfers mass $m = E/c^2$ (which is the correct expression in this thought experiment), and that to any energy E there corresponds a mass equal to E/c^2 (which we now know is not so correct because the photon is massless).

As we understand it today, the subtle point, which Einstein did not discuss in the 1906 paper, was that in special relativity the absorption of a massless particle changes the mass of the absorbing body. Thus a massless photon may "transfer" nonvanishing mass. In absorbing a massless photon, the end of the cylinder becomes heavier, but its mass increase will be E/c^2 only if it is heavy enough that its recoil kinetic energy is negligible. (For the sake of "physical purity," it is better to consider the cylinder as being cut into two "cups.")

The above inconsistent conclusion was extremely fruitful for Einstein's further thinking, which led him finally to general relativity. It implied that a photon possessing inertial mass $m = E/c^2$ has to possess the same gravitational mass and hence has to be attracted by a gravitational force. This idea served as a sort of a springboard, as Einstein explained in his "Autobiographical Notes."[10] However, when general relativity was ready, Einstein no longer needed this inconsistent conclusion. This is evidenced by equation 44 in *The Meaning of Relativity*, written 15 years after the 1906 paper.

A few years ago I came across a cartoon that showed Einstein contemplating two equations he had written on a blackboard and then crossed out: $E = ma^2$ and $E = mb^2$. This humorous image of how science is done (reproduced at the bottom of page 35) may be closer to reality than is the usual description in books on the history of relativity,[11] which neglects the striking difference between Einstein's papers of 1905 (with $E_0 = mc^2$) and of 1906 (with $E = mc^2$) and presents a "*coup d'etat*" as a quiet evolution.

'Gravitational mass'

Many physicists still believe that gravitational mass is equal to E/c^2 and quite often use this as an argument in favor of equation 2. Contrary to this belief, the gravitational attraction between two relativistic bodies is determined by their energy–momentum tensors, not just by their energies. In the simplest case of a light relativistic body such as a photon or an electron of mass m traveling with energy E and velocity $v = \beta c$ in the gravitational field of a very heavy body of mass M such as the Earth or the Sun, the force acting on the light body has the form

$$\mathbf{F}_g = \frac{-G_N M(E/c^2)[\mathbf{r}(1+\beta^2) - \boldsymbol{\beta}(\boldsymbol{\beta}\cdot\mathbf{r})]}{r^3} \tag{16}$$

Here G_N is Newton's constant, $6.7\times10^{-11}\ m^3\,\mathrm{kg}^{-1}\,\mathrm{sec}^{-2}$. When $\beta \ll 1$, equation 16 coincides with the classical expression

$$\mathbf{F}_g = \frac{-G_N M m \mathbf{r}}{r^3}$$

When $\beta \simeq 1$, however, the force is not directed along the radius $\mathbf{r}$: It has a component along $\boldsymbol{\beta}$ as well. So there is no such notion as "relativistic gravitational mass" entering the coefficient of proportionality between $\mathbf{F}_g$ and $\mathbf{r}$. The so-called gravitational mass of a photon falling vertically toward Earth is, incidentally, given by E/c^2. As you can see from equation 16, however, a horizontally moving photon ($\boldsymbol{\beta} \perp \mathbf{r}$) is twice as heavy. (See the figure on page 36). It is this extra factor of 2 that gives the correct angle

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$$\left.\begin{aligned} I_x &= \frac{m\mathrm{q}_x}{\sqrt{1-q^2}} \\ &\cdot \quad \cdot \quad \cdot \\ &\cdot \quad \cdot \quad \cdot \\ E &= \frac{m}{\sqrt{1-q^2}} \end{aligned}\right\} \quad . \quad . \quad . \quad (43)$$

We recognize, in fact, that these components of momentum agree with those of classical mechanics for velocities which are small compared to that of light. For large velocities the momentum increases more rapidly than linearly with the velocity, so as to become infinite on approaching the velocity of light.

If we apply the last of equations (43) to a material particle at rest ($q = 0$), we see that the energy, E_0, of a body at rest is equal to its mass. Had we chosen the second as our unit of time, we would have obtained

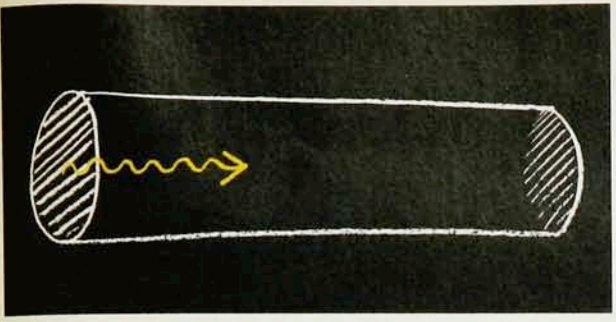
$$E_0 = mc^2 \quad . \quad . \quad . \quad (44)$$

Mass and energy are therefore essentially alike; they are only different expressions for the same thing. The mass of a body is not a constant; it varies with changes in its energy.* We see from the last of equations (43) that E becomes infinite when q approaches 1, the velocity of light. If we develop E in powers of q^2, we obtain,

$$E = m + \frac{m}{2}q^2 + \frac{3}{8}mq^4 + \ldots \quad . \quad (45)$$

* The emission of energy in radioactive processes is evidently connected with the fact that the atomic weights are not integers. Attempts have been made to draw conclusions from this concerning the structure and stability of the atomic nuclei.

Page from Einstein's book *The Meaning of Relativity*,[8] which is based on lectures he gave at Princeton University in May 1921. (Copyright by the estate of Albert Einstein; published by Princeton University Press.)



A light pulse is emitted at one end of a hollow cylinder and absorbed at the other end in a thought experiment described[9] by Einstein in 1906. Taking E/c as the momentum of the photon and requiring that the center of mass of the system not move leads to the conclusion that light with energy E transfers mass $m = E/c^2$.

of deflection of starlight by the Sun: $\theta = 4G_N M_\odot / R_\odot c^2$. With $M_\odot = 2\times10^{30}$ kg and $R_\odot = 7\times10^8$ m, we get $\theta \simeq 10^{-5}$, in agreement with observations.

I have sketched the changes in Einstein's views during the first two decades of our century. But there were many other important protagonists on the stage.[12] Since the beginning of the century experimenters had tried hard to test equations 8–13 for electrons (beta rays and cathode rays) in various combinations of electric and magnetic fields. According to the standard cliché these experiments were done "to test the velocity dependence of longitudinal and transverse masses," but actually they tested the velocity dependence of momentum. The first results "disproved" relativity theory. Gradually technique improved and agreement started to appear. The confirming results were not terribly convincing, however, as you can see from a letter of 10 November 1922 sent to Einstein by the secretary of the Swedish Academy of Sciences[13]:

> . . . the Royal Academy of Sciences decided to award you last year's Nobel Prize for physics, in consideration of your work on theoretical physics and in particular for your discovery of the law of the photoelectric effect, but without taking into account the value which will be accorded your relativity and gravitation theories after these are confirmed in the future.

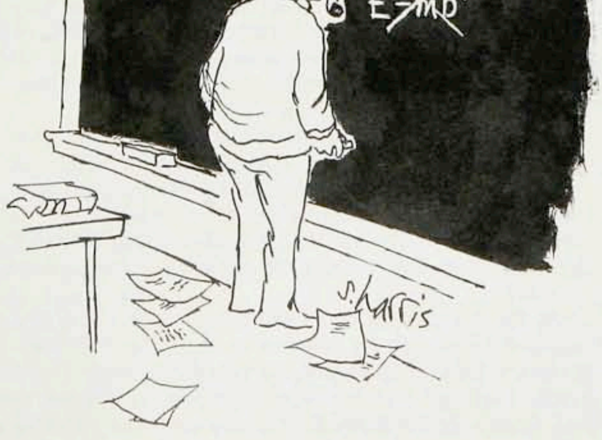
Nor were theorists unanimous in accepting relativity theory or in interpreting its equations. (This article is itself a remote echo of their disputes.) It is well known that the views of Poincaré and Lorentz were different from Einstein's. Important contributions revealing the four-dimensional symmetry of the theory came from Max Planck and especially Hermann Minkowski.[14] But in forming public opinion Gilbert Lewis and Richard Tolman played a particularly important role.[15] It was Tolman who in 1912, starting as before from $\mathbf{p} = m\mathbf{v}$, insisted that m, given by $m_0\gamma$, is *the* mass.[16]

When the 21-year-old student Wolfgang Pauli published in 1921 his encyclopedic article "Relativitätstheorie," which all of us know as the book *The Theory of Relativity*,[17] he discarded the longitudinal and transverse masses as obsolete, but retained the "rest mass" m_0 and the "mass" m, given by $m_0\gamma$, along with the Newtonian relation $\mathbf{p} = m\mathbf{v}$. Pauli's book served as the introduction to relativity for many generations of physicists. It is a great book, but with all its virtues it gave an undesirably long life to the notorious notion that mass depends on velocity, to the term "rest mass" and to the so-called Einstein formula $E = mc^2$.

$E = mc^2$ as an element of mass culture

Not only has this terminology flooded the popular science literature and textbooks, but for a long time it dominated most serious monographs on relativistic physics. To my knowledge, the first authors to ignore this archaic terminology consistently were Lev Landau and Evgenii Lifshitz. In their classic 1940 book *The Classical Theory of Fields*, they called the invariant mass by its correct name, mass.[18] They didn't use the term "relativistic mass" or "rest mass." Their language was consistently relativistic.

In 1949 the introduction of Feynman diagrams generalized this relativistic terminology to include antiparticles.[19] Since then all monographs and scientific papers on elementary particles have used consistently relativistic language. Nevertheless, the popular science literature and high school and college textbooks are still full of archaic notions, terms and notation. (One of the rare exceptions is the 1963 book *Spacetime Physics* by Edwin F. Taylor and John A. Wheeler.) As a result we have a kind of pyramid: At the top are books and articles that use consistently relativistic language and are published in thousands of copies; at the bottom are books and articles that use inconsistently relativistic language and are published in the millions. At the top we have $E_0 = mc^2$; at the bottom $E = mc^2$. In between, all four of the equations listed at the beginning of this article peacefully coexist. I have seen many books in which all the notions, consistent and inconsistent, are so mixed up that one is reminded of nightmare cities in which right- and left-side traffic rules apply simultaneously. The situation is aggravated by the fact that even great scientists such as Landau and Feynman, when addressing nonscientists, have sometimes—though not always—used the equation $E = mc^2$. (Compare, for instance, *The Feynman Lectures on Physics*[20] and Feynman's last published



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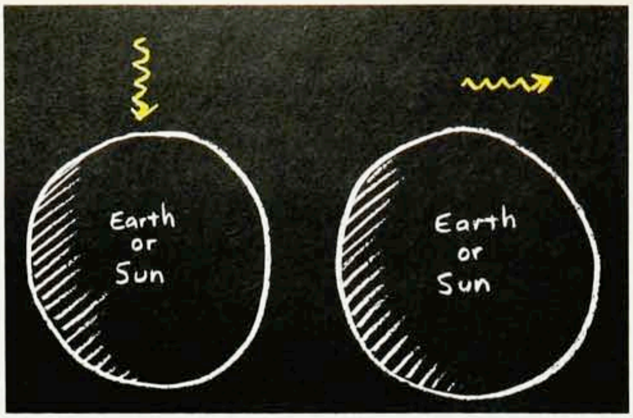

Gravitational force attracting a horizontally moving photon to the Earth or Sun is twice as large as that attracting a vertically moving photon.

lecture, "The Reason for Antiparticles."[21])

The latest example comes from Stephen Hawking's 1988 book *A Brief History of Time.*[23] On the very first page Hawking says: "Someone told me that each equation I included in the book would halve the sales. I therefore resolved not to have any equations at all. In the end, however, I did put in one equation, Einstein's famous equation $E = mc^2$. I hope that this will not scare off half of my potential readers."

I think that in such cases the equation $E = mc^2$, because it is an element of mass culture, is successfully exploited as a kind of "attractor." But the global result of its use is confusion. Readers begin to believe that E/c^2 is a genuine relativistic generalization of inertial and gravitational mass; that whenever you have energy, you have mass (a photon is a counterexample); and that $E = mc^2$ is an inevitable consequence of special relativity (actually it follows from the special and non-natural assumption that $\mathbf{p} = m\mathbf{v}$). The scaffolding used many years ago in the construction of the beautiful building of special relativity has been and is now presented as the central part of the building. The important difference between a Lorentzian scalar and a Lorentzian vector is lost, and with it the four-dimensional symmetry of the theory. The confusion in terminology cannot but lead to confusion in many minds.

"Does Mass Depend on Velocity, Dad?" This is the title of a 1987 *American Journal of Physics* article by Carl Adler.[23] The answers Adler gave to his son were "No!" "Well, yes..." and "Actually, no, but don't tell your teacher." The next day the boy dropped physics. Adler gives several examples of how relativistic mass slowly disappears from university textbooks. There is an interesting quotation in the article from a letter Einstein wrote to Lincoln Barnett in 1948 (the original letter, written in German, is reproduced on page 32):

> It is not good to introduce the concept of the *mass* $M = m/(1 - v^2/c^2)^{1/2}$ of a moving body for which no clear definition can be given. It is better to introduce no other mass concept than the "rest mass" m. Instead of introducing M it is better to mention the expression for the momentum and energy of a body in motion.

In the autumn of 1987 I was asked to be a member of a committee set up by what was then the Ministry of Secondary Education to judge a competition for the best physics textbook for secondary schools. I looked over more than a dozen competing books and was shocked to learn that all were promoting the idea that mass increases with velocity and that $E = mc^2$. I was shocked even more when I discovered that my colleagues on the committee—teachers and specialists in teaching physics—had never heard about the equation $E_0 = mc^2$, where E_0 is rest energy and m is mass. I explained this equation to them, and one of them suggested that I write about the topic in *Physics in the School*, a journal for physics teachers. The next day I asked the assistant editor whether the journal would like to publish such an article, and after three months I got a phone call: The editorial board decided it did not want an article that explained special relativity without using $E = mc^2$.

Every year millions of boys and girls throughout the world are taught special relativity in such a way that they miss the essence of the subject. Archaic and confusing notions are hammered into their heads. It is our duty—the duty of professional physicists—to stop this process.

* * *

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