Japan Academy Prize to:

Takuro Mochizuki
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for "Study of Pure Twister *D*-modules"

Outline of the work:

The work of Dr. Takuro Mochizuki concerns all three major branches of mathematics: algebra, geometry and analysis. He has obtained astonishing results on semisimplicity by constructing a very original theory of pure twister *D*-modules. Even if a mathematical object is decomposed into simpler objects, the constituting objects often interact with each other, causing the total structure to remain complicated. However, there are instances where there is no interaction among the constituting objects, in which case the object is said to be "semisimple." The notion of semisimplicity is important in mathematics, because once settled it implies many strong consequences.

Dr. Mochizuki's work has yielded an unexpected result: that semisimple constructible sheaves remain semisimple after various operations. This work is recognized as being highly original, and is considered to be one of the fundamental achievements in mathematics of this century.

Back in the 1970s, Deligne provided an affirmative solution to the Weil conjecture, which had been very influential in number theory. As its application, Beilinson, Bernstein, Deligne, and Gabber showed that constructible perverse sheaves with Galois group action in positive characteristics enjoy the hard Lefschetz theorem, decomposition theorem, and semisimplicity theorem. The corresponding theory in characteristic zero was established by Morihiko Saito as the theory of Hodge *D*-modules. These results are among the top achievements in the twentieth century, and many deep theorems are derived from them.

However, the existence of Galois group action is a very restrictive condition. By masterly manipulating both algebraic and analytic methods, Dr. Mochizuki succeeded in removing such restriction and proving that the hard Lefschetz theorem, decomposition theorem, etc. hold for arbitrary semisimple perverse sheaves. Within Dr. Mochizuki's proof, harmonic metric plays an important role. The existence theorem for harmonic metric on any flat bundle on a quasi-projective manifold had already been shown by Jost-Zuo. However, to apply the theory of harmonic analysis developed by Hodge-Kodaira to an open manifold, one needs a harmonic metric with good behavior at infinity. Dr. Mochizuki succeeded in proving the existence of such a harmonic metric.

As an algebraic tool, he used a notion of twister *D*-module introduced by C. Sabbah, and further constructed the theory of pure twister *D*-modules. He established the hard Lefschetz theorem, decomposition theorem, and semisimplicity theorem for pure twister *D*-modules. Then, he succeeded in showing that all semisimple perverse sheaves carry a pure twister *D*-module structure. He, thus, established that arbitrary semisimple perverse sheaves remain perverse after the operation of taking proper direct images, and that the hard Lefschetz and decomposition theorems hold for such sheaves.

Recently, Dr. Mochizuki has generalized this theory to a much wider class: the case of irregular singularities, which includes exponential functions. This magnificent result gives much wider and deeper applications to his theory.

References

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