A PROPOSED CONTEXTUAL EVALUATION OF REFERENDUM QUORUM USING FUZZY LOGICS[1]

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Abstract:

This conceptual article proposes a new approach for referendum quorum size calculation, based on criteria that do not take into account voters' preferences or levels of information. Although some political commissions and plenty of past researches support the absence of a quorum in a referendum, the herein model relies on voting exogenous variables, referring some objective criteria of quorum calculation. A Mamdani fuzzy inference system is used to build a controller that yields the value of the output based on three inputs: Type, Discrepancy and Age. The results obtained through elementary simulations are of a wide range, from facilitating to obstructing the usage of a referendum depending on the context.

Key words: quorum; referendum; fuzzy logic; fuzzy sets; Mamdani fuzzy inference system

1. Introduction

The etymology of word *referendum* leads to the gerund of Latin verb *referre*, thus implying an action of restoring, of turning back to the people's opinion. An exact definition is that of direct voting that establishes the community, region or country electorate's will in a rather contextual matter. Examples are the constitution modification or the sovereignty debate (when a referendum becomes a plebiscite) that are forms of participative or direct democracy (DD).

DD becomes more prominent nowadays, especially from the perspective of using the institution of referendum. Although skeptics are afraid of voter's lack of information, high costs or the cleavage between mass and elites, the economic perspectives are encouraging for a functional DD (Matsusaka 2005). Low turnout in case of a referendum is sometimes an important issue, especially after excessive use of this instrument, but not as important as the problem of informed citizens in Switzerland of 1981-1999, a state with great appetite for referendums, where some tedious manifestations appeared because of the lack of information (Lutz 2007). In the same state and similar period, there were identified some aspects of DD that have an economic positive impact: preliminary debates lead to better information, the citizens take greater responsibility and accept a relaxation of their principles, and, as a direct impact, tax evasion is lower while some public services are significantly improved (Feld and Kirchgassner 2000). In an inter-country DD analysis, the

impact over fiscal policies and government efficiency is reconfirmed, but not the over productivity and individual happiness self-perception; in exchange it is emphasized the need for guiding referendums towards contextual problems (Blume, Müller and Voigt 2009).

The issue of the referendum quorum (RQ), meaning establishing a relative (percentage) or absolute (number) level for validating a referendum by representative participation, is an important topic of the recent or old DD debates. Subject of specific political decisions paradoxes, belonging to the class of composition and decomposition (Nurmi 1998), RQ is not recommended for two reasons (Venice Commission 2007): i) "a turn-out quorum (threshold, minimum percentage), because it assimilates voters who abstain to those who vote no" and ii) "an approval quorum (approval by a minimum percentage of registered voters), since it risks involving a difficult political situation if the draft is adopted by a simple majority lower than the necessary threshold". To prevent the bias towards acceptance, the result of the non-valid referendum should take rejection into account, while the settling of RQ should encompass the collective referendums' memory and relate to the paradox that the turn-out may exceed a non-imposed quorum. Moreover, a non-optimal vote relying on tendencies and preferences of citizens is preferred when targeting the result instead of the turn-out (Zwart 2010). Reclaiming the idea of quorum paradox, Herrera and Matozzi (2010) propose settling it to a level that is half of the desired turn-out target. If there is an established RQ to assuring legitimacy for the referendum's political decision, then it has negative effects on turn-out and representation rather than promoting absenteeism; it opposes status-quo, hardly disadvantaging minorities, and it facilitates pressure over voters in an undemocratic way (Aguiar-Conraria and Magalhães 2010).

With all these many scientific and socio-politic (e.g. Venice Commission) advocacies for not imposing an RQ, the commitment to support the contrary ought to be questioned. There are three reasons to defend an RQ: i) legitimacy and preventing the tyranny of minority, the need for representation; ii) context and iii) objective criterion, detached from voting preference, intention or turn-out. Proposals for abandoning RQ are based on citizen's inclination to participate or not and on voter's preference for yes or no. On the contrary, RQ should be established on objective and exogenous criteria, aiming to prevent referendum's trifling or abuse in non-consolidated states subjects to the rule of law, see pre-Nazis Germany (Zurcher 1935) or, in a more recent time, see post-communist contemporaneous Romania (Scheppele 2012).

RQ is more important in the context of E-democracy, where online petitions are solved using E-referendum. Discussions on this issue and a conceptual model have been presented in a previous paper (Turcoane 2014b).

This paper is structured as follows: section 2 briefly presents the Mamdani fuzzy inference system (MFIS) relying on a human-like decision making controller; the herein introduced model for evaluation of RQ is discussed in section 3; results and examples are in section 4 and the last section presents the conclusions of this conceptual exploration.

2. Fuzzy logic and MFIS

Fuzzy logic (FL) is an extension of the binary logic, which, from a mathematical point of view, works with two values: 0 and 1. FL is a multi-value logic that uses the whole range between 0 (nothing, absence) and 1 (absolute, certain thing) to negotiate with problems that naturally deal, under the constraint of imprecision and well-defined criteria,

with classes of values and possibility rather than with random variables (Zadeh 1965). FL have been already used in social (Montero 2008) or political (Nurmi and Kacprzyk 2007) sciences. The role of uncertainty is discussed using propositional logic from political perspective (Sen 2009) or even under fuzzy aspects in social systems (Treadwell 1995). In order to build a decisional model (MFQ) for settling RQ based on FL, it will be used an MFIS that usually guides technological processes formalized in an imprecise linguistic-human manner. MFIS gives remarkable results in the case of vaguely defined problems, as the controversial issue of RQ. There are also examples of using an MFIS in supply chain management (Ayadi, Cheikhrouhou and Masmoudi 2013) or in web shopping analysis (Liu, Geng and Zhang 2005).

Figure 1 illustrates the steps on an MFIS (Mamdani and Assilian 1975, MathWorks 2015), which is based on the conversion of input ordinary crisp values to fuzzy values that are part of fuzzy sets and which, filtered through a knowledge base given by fuzzy rules (FR), are transformed and aggregated in a final fuzzy set that is in the end subject to re-conversion to a crisp result.

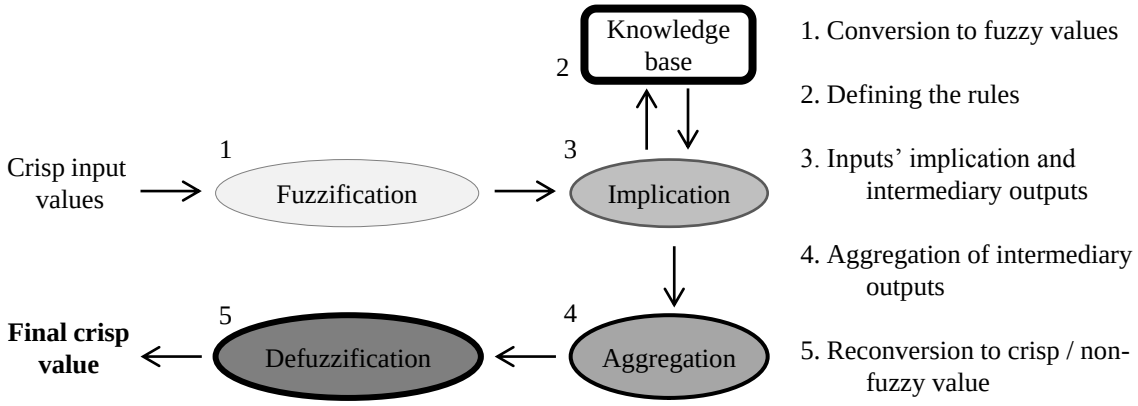

Figure 1. MFIS Without insisting on further details of MFIS formalization, see (MathWorks 2015), section 3 will build and explain MFQ in a practical manner, thus exemplifying the inferential system briefly and visually introduced in Figure 1.

3. MFQ

Prior to build an MFIS, there must be identified the inputs and output, and there must be created their fuzzy sets (FS). FS are an extension of the classic logic sets and they are described through membership functions (usually denoted by μ) that take values in the range of [0;1] or [0%;100%]. Any of the MFQ inputs or output will be built as a set made out of other fuzzy (sub)sets that define the variables of the model using a linguistic and qualitative-quantitative approach. While the output is easily identifiable as the (dimension of) Quorum (i.e. RQ), the inputs are far more difficult to be established. Apart from previous researches, see section 1, not the pattern of the voter is the key to determining the inputs, but some electorate exogenous variables that are subject to a flexible approach depending on the context.

This paper proposes three inputs for MFQ: Type, Discrepancy and Age. While the inputs may not be found reasonable and viable to dictate the value of RQ, although they may be found in different forms in Venice Commission's code for referendums (2007), the author believes that at least the model may give incentives to others for further explorations

in this area. Let us explain each of the inputs, using examples to better understand MFQ.

Type represents the class, kind or category of law, decision, decree (generic denoted as draft) that is subject of the referendum and it has two subtypes: proposal and abrogation. The former materializes a pure original political statement that is supposed to be enacted; the latter embodies a radical contestation of a functioning / active draft. It is true that labeling one of the two subtypes in a crisp (i.e. binary) way may not be difficult using classification algorithms or plain human approach (e.g. by an appointed organism). However, the classification of the draft should be based on some computational approaches when seeking for accuracy and reliability, especially for the fuzzy way (i.e. the draft is neither proposal nor abrogation, but somewhere in between). There already exists enough literature in this area, from binary classification based on prior knowledge and support vector machines (Lauer and Bloch 2008) or based on knowledge sets (Orchel 2015) to fuzzy concepts in text classification (Li and Tsai 2013). The Venice Commission (2007) also discusses the problem of presenting texts in various forms, but valid from unity point of view; these make easier the draft's classification.

From this perspective, Figure 2 illustrates the FS of Type, which allows a binary and as well a fuzzy representation of the draft. If proposal is 0 then abrogation is 1 and vice versa; both are half present at the set interval's middle, i.e. μproposal(0.5) = μabrogation(0.5) = 0.5. The scale suggested for Type is 0–1, but it could be 0–100 or 0%–100%. Actually, the scale for the inputs or output should be intuitive and simple and this affects the model only relatively.

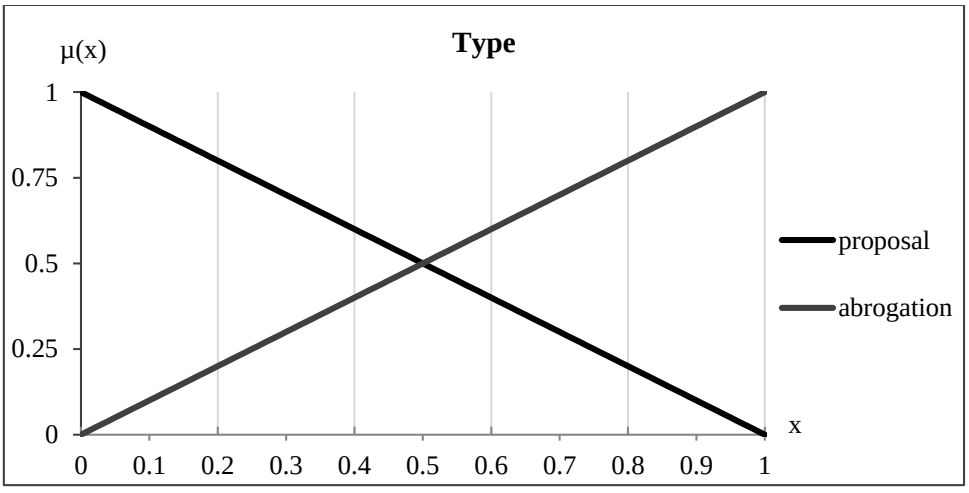

Figure 2. FS of *Type*

An example would be the intention of the European Commission or some other supranational body to legislate a *draft*, the newly digital decentralized currencies (e.g. Bitcoin). Although there is some guidance provided in US by the treasury or other exchange commissions, legislation in this area would practically be classified as a pure proposal. However, there are some laws and rules regarding digital economy and they may be somehow or partially abrogated by the new *draft*. The author doubts that a pure proposal exists, given the fundamentals of any society, global or not. An abrogation example is the impeachment and dismiss of a representative such as a mayor or a president.

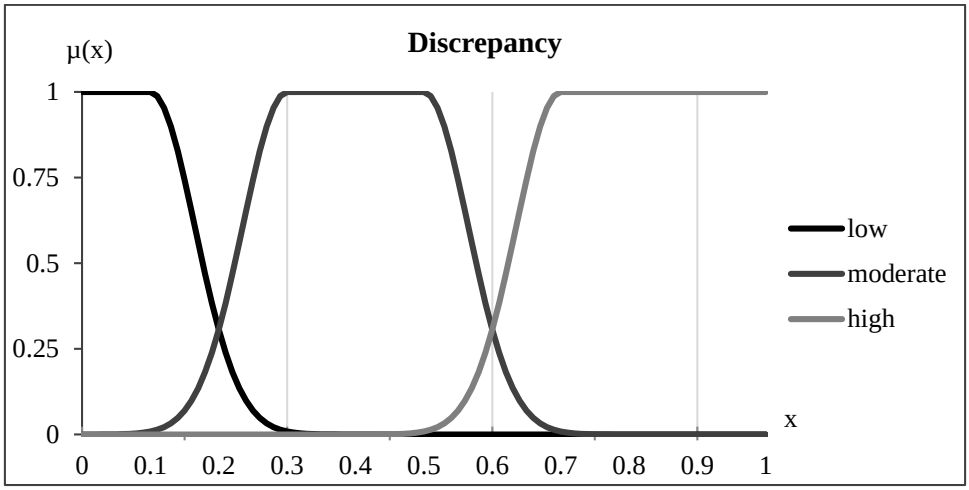

Figure 3. FS of *Discrepancy*

The second input is Discrepancy and it refers the inconsistency of the draft with other laws and rules. The Venice Commission also points in this direction, specifying that both referendum and "texts put to a referendum must not be contrary to international law or to the Council of Europe's statutory principles (democracy, human rights and the rule of law)". Inconsistency would never be eradicated in a system that is based on self-optimization and perpetual search for better solutions under the constraint of epistemic uncertainty, as it might be E-democracy (Turcoane 2014b). But even in nowadays representative democracy there are plenty of judicial inconsistencies (Niblett 2013, Fischman 2014) and examples of law promulgations that contradict the constitution in young states subject to the rule of law (APADOR-CH 2015). The last reference proves that this kind of discrepancy is observable in some cases using a simple human approach, but ways to measure it by mathematical (Doder, et al. 2010) and computational (Olson and Fusco 2012, McAreavey, Liu and Miller 2014) methods have already been proposed.

Figure 3 illustrates Discrepancy in a fuzzy approach with three FS that are not symmetric as in the case of Type and that are also based on author's perception (using the same scale), assuming that low and moderate classes of values are not as important as high level of inconsistency that may drastically affect the system output, see also the knowledge base of MFQ.

Resuming the example with digital currency, inconsistency at high level would occur if a draft enacted the new currency as the only one sanctioned by governmental bodies. Moderate discrepancy is when digital currencies are not allowed on the market as this contradicts the economic freedom (but only in virtual environment); low level appears, in author's opinion, when any digital currency should rely on centralized bodies' decisions.

The third and last input of MFQ is Age and it takes into account the draft's duration from its beginning to any given time, being inspired by the concept of product lifecycle found in engineering, software development, marketing etc. This paper sticks to the classical approach with four stages (Productlifecyclestages 2015): introduction, growth, maturity and decline. The first stage, of introduction, includes research and innovation, which are similar in politics to deliberation and decision; the correspondent for it in MFQ is the fuzzy subset new, which is described by a descendent slope immediately after its inception. Mature is the second subset of Age and it encompasses growth and maturity of a product, while old represents the fourth stage, decline, which is adjusted here based on the idea of a lifespan

extension of the product that still proves its usefulness (Bakker, et al. 2014).

Figure 4 illustrates Age and its fuzzy subsets (i.e. new, mature and old) using a time scale that has its units expressed in years (different approach from the other two inputs). In author's opinion, a draft loses its newness immediately after its promulgation, becoming mature between its age of three to five years and entering the old / obsolete stage after six or seven years. This way of building the third input fits to an accelerated rate of development of new products and of society itself, subject to rapid technological transformations. It is a common thing to acknowledge the fact that predictability must play an important role in legislation (with consequences in economics, research etc.), but Age is developed so as to also respect the idea of innovation.

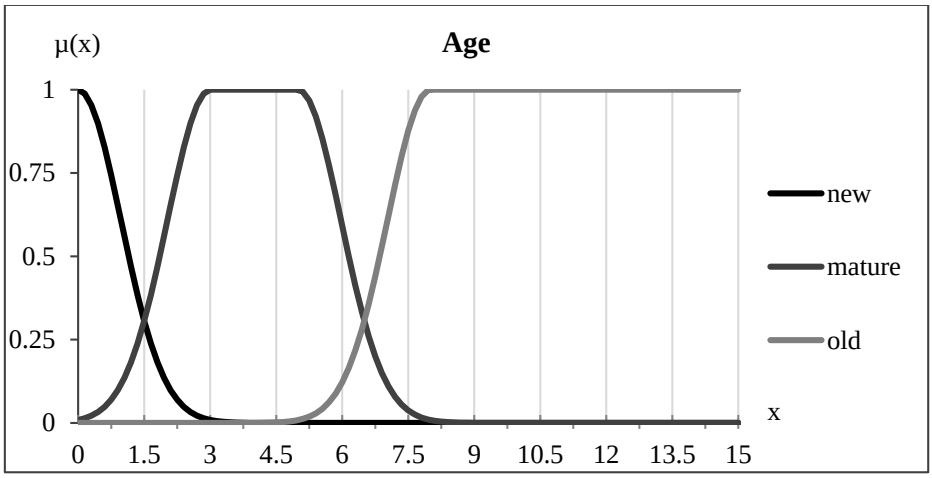

Figure 4. Fs of *Age*

A final example regarding the inputs and digital currency will put the latter in the origin point of the Age's scale, i.e. μnew(0) = 1; μmature(0) = 0 and μold(0) = 0. On the contrary, if the draft is about actual monetary legislation then it will be located somewhere on the scale that defines the FS of old.

The final variable of MFQ to be discussed is Quorum, which represents the value of RQ, the expected result. Subjectively chosen, the three subsets of the output are yet common and intuitive: low, medium, high. Figure 5 illustrates the output and its FS, using the same scale as for the first two inputs, i.e. Type and Discrepancy.

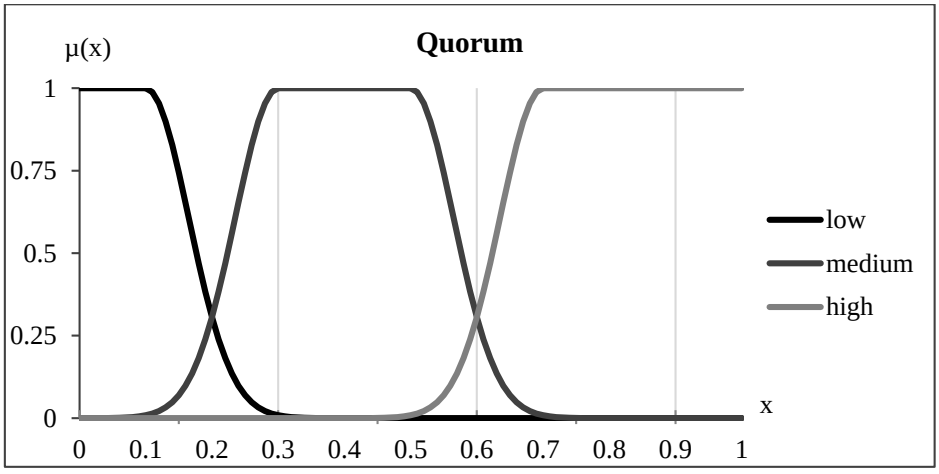

Figure 5. FS of *Quorum*

Table 1 presents the values of parameters of Matlab's functions (MathWorks 2015) used to build the FS of the inputs and the output of MFQ. There are two main types of membership functions in Table 1: triangular-shape and Gaussian-shape. In order to build a Gaussian curve one needs a standard deviation or sigma value and a mean value. The mean value is any bound of the crisp interval of FS of the inputs and the output. For a 0–1 scale, the sigma value is 0.065 (see Figure 2, Figure 3 and Figure 5) and it is based on a proxy determined in some other extended exploration (Turcoane 2014a). Extrapolating this value to a 0–15 scale, the sigma value becomes 0.975 as it is shown in Figure 4.

Table 1. FS of the inputs and the output of MFQ

Input	Fuzzy subsets	Matlab function
Type	proposal	trimf(0 0 1)
	abrogation	trimf(0 1 1)
Discrepancy	low	gauss2mf(0.065 0.0 0.065 0.1)
	moderate	gauss2mf(0.065 0.3 0.065 0.5)
	high	gauss2mf(0.065 0.8 0.065 1.0)
Age	new	gaussmf(0.975 0.0)
	mature	gauss2mf(0.975 3.0 0.975 5.0)
	old	gauss2mf(0.975 8.0 0.975 15.0)
Quorum	low	gauss2mf(0.065 0.0 0.065 0.1)
	medium	gauss2mf(0.065 0.3 0.065 0.5)
	high	gauss2mf(0.065 0.7 0.065 1.0)

The inputs and the output proposed in this paper are built based more on the author's educated guess rather than on self-evidence truth. However, these variables of MFQ should be subject to deliberation and practical experience refinement (e.g. using artificial intelligence to identify real life patterns), not to a proposal from an individual or a group (even one of certified scholars). Using the same principle, the FR of MFQ would derive from true life experiments, while this article only proposes a starting point in defining the knowledge base. There are ten FR that follow the *if–then* statement paradigm and each of them is briefly explained.

FR 1) If (Type is proposal) and (Discrepancy is low) then (Quorum is low). Any proposal that does not contradict any other rule or law requires a low representation and participation to be enacted; it will bring nothing but added value.

FR 2) If (Type is proposal) and (Discrepancy is not low) then (Quorum is high). Any proposal that, although innovative, contradicts the already implemented system demands a high level of approval from citizens; the system should not easily become unstable.

FR 3) If (Type is abrogation) and (Discrepancy is low) and (Age is new) then (Quorum is high). Any *draft* that does not contradict the system cannot be dismissed without high representation if the *draft* is not old / obsolete or if it is not mature; otherwise it will bring instability to the system.

FR 4) If (Type is abrogation) and (Discrepancy is low) and (Age is mature) then (Quorum is medium). A medium RQ is necessary to abrogate a mature *draft*, if the *draft* does not contradict the already implemented system, thus proving its utility.

FR 5) If (Type is abrogation) and (Discrepancy is low) and (Age is old) then (Quorum is low). An old *draft* may be abrogated as any system needs a refresh; having a low inconsistency and being old, the *draft* requires a low RQ to be dismissed.

FR 6) If (Type is abrogation) and (Discrepancy is moderate) and (Age is new) then (Quorum is high). Although inconsistent with the system, if a *draft* is new but there is need to reshape the system then it demands a high representation for abrogation (in order to support stability).

FR 7) If (Type is abrogation) and (Discrepancy is moderate) and (Age is mature) then (Quorum is medium). A *draft* requires medium RQ to be dismissed, if it is mature (granting stability) but it has an evident discrepancy.

FR 8) If (Type is abrogation) and (Discrepancy is moderate) and (Age is old) then (Quorum is low). An inconsistent and old *draft* should be easily abrogated, thus improving the system.

FR 9) If (Type is abrogation) and (Discrepancy is high) and (Age is new) then (Quorum is medium). A brand new *draft*, even with a high discrepancy, needs a medium representation to be abrogated, in order to support the idea of predictability and stability.

FR 10) If (Type is abrogation) and (Discrepancy is high) and (Age is not new) then (Quorum is low). Inconsistent with the system and already mature and old, a *draft* should be easily dismissed.

As can be seen in the ten FR, this research emphasizes the need for a stable system, which is also subject to flexibility and innovation. This section has tried to describe a model that is not indebted to citizen's preference or voter's behavior, but it is objectively bound to the context of the matter deliberated in a referendum.

4. Results of simulations and discussions

Using the Matlab's Fuzzy Toolbox (MathWorks 2015) it is easy to simulate the behavior of MFQ. For minimization and maximization, i.e. minimum and maximum of RQ, a fuzzy optimization algorithm was applied (Turcoane 2014a).

Table 2 presents different combinations of the inputs and the outputs yielded by MFQ.

Table 2. Output values

Simulations	Type	Discrepancy	Age	Quorum
1. Maximization	0	0.3624	0.5	**0.80941042**
	0	0.85	0.5	**0.80941042**
2. Minimization	0	0	0.5	**0.09311753**
	0	0.05	15	**0.09311753**
	1	0.87	11.5	**0.09311753**
3. Other high values	1	0	3.57	0.80935687
	1	0.05	0	0.7975
4. Average values	0.5	0.5	7.5	0.575
	0.5	0.5	1	0.501
	1	0	8	0.4
	0.9	0.5	7.5	0.227
	1	0.25	3	0.227
5. Other low values	1	0.85	4.02	0.09323834
	1	0.05	4	0.09324849

First of all, one notices very easily that values of minimum and maximum RQ are both yielded by different input combinations. Thus, the maximum value of 0.80941042 is determined using significantly different values of Discrepancy. The same thing also applies

for minimization, but more keenly, where the value of 0.09311753 is yielded by significantly disparate value of all the three inputs.

The optimum values of RQ empirically prove the nonlinearity of an MFIS and of the herein proposed MFQ. An MFIS is subject to a human approach that deals with gradual truth and partial belief as components of the uncertainty of the model. The if-then rules give an easy way for non-academics to control the model that is rather suitable for a non-classical mathematical approach. A standard mathematical or computational procedure would hardly fit the prototype described by MFQ.

Secondly, insignificant changes of the output minimum and maximum are identified by MFQ in Table 2. This proves that MFQ provides accurate results when needed. Moreover, the values of inputs could be provided to MFQ with a desired precision that affects the output accuracy; the latter may be adjusted to a round number or should provide a precise RQ, depending on practical decisions.

Thirdly, average values are also yielded in a large range with different input combinations, proving that not only extreme values may occur.

From sociopolitical point of view, MFQ is capable to provide a large range of results based on the contextual problems which need to be identified by FR. This means that RQ is a flexible concept, not a fix point given by a pure political deliberation that can be subject to non-democratic diversion. On one hand, RQ provides incentives to using a referendum in contextual matters (see the low outputs). On the other hand, the institution of referendum could not be used as a non-democratic instrument and a bagatelle mastered by some politicians (see the high outputs).

5. Conclusions

This conceptual paper has introduced a flexible quantitative-qualitative model of evaluating the size of a referendum quorum based on the contextual drafted matter, arguing against models relying on voters' preferences or behavior. Moreover, the quorum should not be determined by simple political debate or quarrel that seeks for individual or group favoritism and it definitely should not be a fix percentage or number. The quorum must give incentives for participation when the system requires enhancement or rejuvenating and it must stop abusing the institution of referendum when stability is needed. This exploration has proposed three variables that define the draft to evaluate the context of the referendum: type, inconsistency / discrepancy and age. Using these three variables, an objective evaluation of the quorum size is achievable depending on the context. While there are still many issues to address regarding the proposed model (e.g. deliberation over the knowledge base, setting up the inputs and the output), the author believes that this research will give incentives to further investigations from others.

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