Lecture 6

Real fluids

– viscosity and turbulence

Poiseuille's equation and Reynolds numbers are not discussed in the text.

You should read the relevant sections of the notes on the web, at

http://www.physics.usyd.edu.au/teach_res/jp/fluids/wfluids.htm

So far we have considered ideal fluids: fluids which have no internal friction (*nonviscous*) and which flow in steady, *laminar* flow.

In real situations, these assumptions often cannot be made.

Viscosity

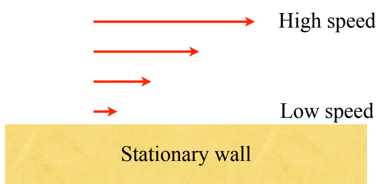
Real fluids have different *viscosity*.

In liquids, viscosity is due to adhesion forces between the liquid molecules.

Viscosity is a *dissipative* effect.

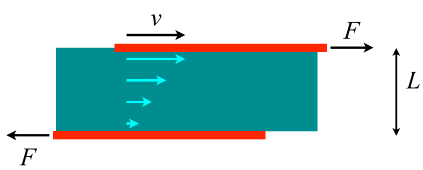
When a viscous (sticky) fluid flows past a stationary wall, the fluid next to the wall does not move*, but away from the wall the speed is non-zero

⇒ *velocity gradient*.



* In general: when a viscous fluid is near a solid wall, its velocity will match that of the wall

Consider a viscous fluid between two parallel plates of area A, where one plate moves with velocity v.



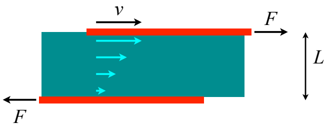
The fluid in contact with each surface has the same velocity as the surface. The flow in between increases linearly with distance, so the velocity gradient is v/L.

The force needed to keep the top plate moving is proportional to the area:

$$F \propto A$$

Experimentally it is found that the velocity gradient is proportional to the stress F/A:

$$\frac{F}{A} = \eta \frac{v}{L}$$



The coefficient η is called the *coefficient of viscosity*, and is different for different liquids.

Fluids which flow easily (water, petrol) have smaller viscosities than "thick" liquids (honey, glycerine).

Viscosity is highly dependent on temperature. The viscosity of a liquid *decreases* as T increases, while for a gas η *increases* as T increases.

Viscosity has units of Pa.s (=N m^{-2} s).

Liquid	**η (mPa.s)**
water (0° C)	1.8
water (20° C)	1.0
water (100° C)	0.3
blood plasma (37° C)	~ 1.5
engine oil (AE10)	~ 200
air	0.018
honey	2,000 – 10,000

For a *Newtonian fluid*, the viscosity η is independent of speed v, and the force is proportional to the speed.

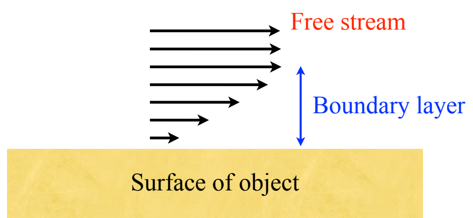
Not all liquids are Newtonian, particularly "thick" liquids like colloidal suspensions.

Non-Newtonian fluids have viscosity which changes with the applied shear force.

e.g.

- hair gel or toothpaste, where the viscosity decreases when force is applied
- corn flour + water mixture (*oobleck*), where the viscosity increases when force is applied

Viscosity means that when a fluid moves over a surface, there is a thin layer near the surface which is nearly at rest: a *boundary layer*.



Flow through a pipe

The rate of flow through a pipe for a viscous liquid is described by *Poiseuille's law*. We are not going to derive it here; instead, here is a motivation for the form of the law.

Since viscosity restricts the velocity gradient, a liquid must flow faster through a *wide* pipe than a *narrow* one.

⇒ flow rate ∝ R

Similarly, we can guess:

- the bigger the pressure difference, the higher the flow

  ⇒ flow rate ∝ Δp

- the longer the pipe, the greater the friction

  ⇒ flow rate ∝ $1/L$

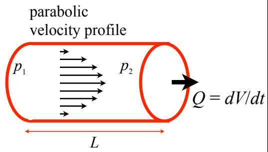
- the more viscous the liquid, the lower the flow

  ⇒ flow rate ∝ $1/\eta$

The *volume flow rate* $Q = dV/dt$ is

$$\frac{dV}{dt} = \frac{\pi}{8}\frac{R^4}{\eta}\frac{\Delta p}{L}$$

– *Poiseuille's law*.



Poiseuille's law is only applicable to laminar flow in Newtonian fluids.

The *volume flow rate* $Q = dV/dt$ is

$$\frac{dV}{dt} = \frac{\pi}{8}\frac{R^4}{\eta}\frac{\Delta p}{L}$$

Consequences:

- high viscosity ⇒ low flow rate
- $\Delta p/L$ is the pressure *gradient*: the bigger the pressure difference, the faster the flow
- the radius of the pipe makes a *large* difference to the flow rate

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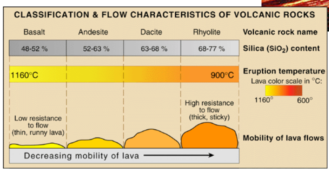
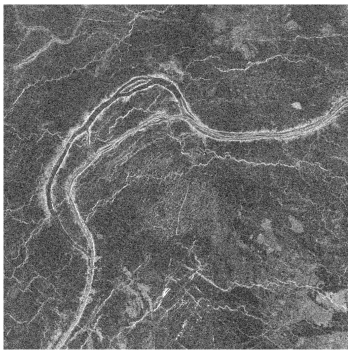
Applications of Poiseuille's law:

- *Irrigation pipes*: Since $Q \propto \Delta p/L$, it is uneconomical to spray irrigation too far from the river
- *Blood flow*: Any constriction of the blood vessels – like cholesterol build-up on the walls of arteries – increases the resistance ⇒ heart has to work harder to produce same flow rate.

The viscosity of lava affects how volcanoes behave.





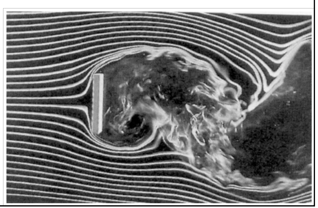
A lava channel on Venus, Baltis Vallis, up to 2 km wide and 6800 km in length, must have been formed by lava with very low viscosity which flowed for a very long time.

Turbulence

So far we have only talked about laminar flow. When the motion becomes too violent, eddies and vortices occur: the motion becomes *turbulent*.

The flow pattern is no longer stable, but becomes irregular and chaotic.

Turbulence dissipates energy.



When does a fluid become turbulent?

We can guess some of the factors:

- *Speed of flow*: fast flow gets turbulent more easily
- *Stickiness of fluid*: thick liquids like honey don't get turbulent as easily as thin ones.

The nature of the flow depends on a dimensionless quantity called the *Reynolds number*:

$$R_e = \frac{\rho v L}{\eta}$$

As predicted, it depends on the velocity v and the viscosity (actually the *kinematic viscosity*, η/ρ).

Unexpectedly, it also depends on the *size* of the system L.

The Reynolds number is not a precise quantity. L and v are "typical" values of size and speed. Often it's not clear which length you should use.

For fluids flowing through a pipe, L turns out to be the pipe diameter.

As a rule of thumb,

- $R_e < \sim 2000 \Rightarrow$ laminar flow
- $R_e > \sim 2000 \Rightarrow$ turbulent flow

$R_e = \frac{\rho v L}{\eta}$

- *Sydney Harbour ferry*:
  $v \sim 5$ ms^{-1}, $L \sim 20$ m
  so $R_e \sim 5 \times 20 \times 10^3 / 10^{-3} = 10^8$
  ⇒ turbulent flow

$R_e = \frac{\rho v L}{\eta}$

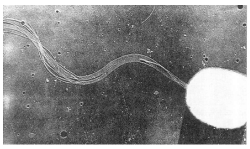
- *Blood circulation*:
  $v \sim 0.2$ ms^{-1}, $L \sim 10$ mm for the aorta;
  assume $\eta_{blood} \sim$ water = 10^{-3} Pa s

  so $R_e \sim 0.2 \times 0.1 \times 10^3 / 10^{-3} = 2000$
  ⇒ right on boundary of turbulent flow

$R_e = \frac{\rho v L}{\eta}$

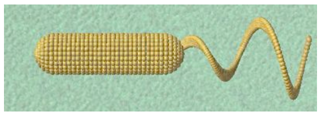
- *Bacterium*:
  $v \sim 30 \times 10^{-6}$ ms^{-1}, $L \sim 1$ μm; $\eta_{water} = 10^{-3}$ Pa s

  so $R_e \sim 30 \times 10^{-6} \times 1 \times 10^{-6} \times 10^3/10^{-3} = 3 \times 10^{-5}$
  ⇒ very low Reynolds number



Flow patterns are very different in systems with low and with high Reynolds numbers.

In particular, the flow in very low Reynolds number situations is perfectly *reversible*.



see "Micro-robot olympics reveal champion swimmer", New Scientist 12 December 2007
http://technology.newscientist.com/article/dn13041-microrobot-olympics-reveal-ch

In modelling a flow system, the flow patterns will be similar if the Reynolds numbers for both are equal; thus

$$R_e = \frac{\rho v L}{\eta} = R_{e_m} = \frac{\rho_m v_m L_m}{\eta_m}$$

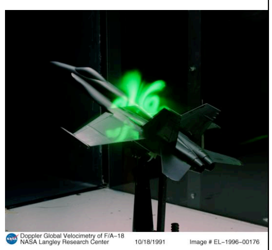
If the same fluid is used for model and prototype, then flow similarity is achieved if

$$vL = v_m L_m$$

so since $L_m < L$, then $v_m > v$, i.e. a scaling *down* of size requires a scaling *up* of velocity.

In wind tunnels, scale models of aircraft are often tested at higher air pressure to reproduce the same fluid flow.

e.g. a 1/4 scale aircraft would be tested at 4 atmospheres pressure.



The fact that the Reynolds number depends on *size* means that it's very hard to make scale models of anything to do with water.

The human brain is surprisingly good at estimating the Reynolds number of a situation.

Summary

Energy dissipation:

Both viscosity and turbulence dissipate energy.

Viscous effects are important in low Reynolds number situations: in thick liquids (η large), or small, slow flow systems.

Turbulence can be responsible for energy loss in high Reynolds number situations.

Static fluids
- variation of p in static fluid
- buoyancy
- surface tension

Ideal fluids
- mass conservation: continuity: flow rate
- energy conservation: Bernoulli's equation

Real fluids
- viscosity: internal friction (qualitative)
- turbulence: chaotic eddies (qualitative)
- use Reynold's number