THE KINEMATICS OF THE QUIXOTE AND THE IDENTITY OF THE "PLACE IN LA MANCHA": A SYSTEMIC APPROACH

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ABSTRACT

A multidisciplinary research team using systemic methodologies to analyze the novel *The ingenious hidalgo Don Quixote of La Mancha* has discovered the underlying kinematics in Cervantes' book: an architecture emerging when the riding times between actual geographic places, expressed in days and nights on horseback, were systemically related to the respective real distances expressed in kilometers. Building on this kynematics, two things were studied: first, the ground that – according to the novel – Rosinante and Sancho's donkey could cover in a day's time; and second, the identity of Cervantes' mysterious "place in La Mancha", clearly located in the region known as "Campo de Montiel", and localised within a reasonable degree of certitude to be one of the three following villages of Villanueva de los Infantes, Alhambra and Carrizosa (in this order) in the Spanish province of Ciudad Real.

Keywords: Quixote; Place in La Mancha; Velocity.

INTRODUCTION

According to Parra-Luna et al.(2005), in the four centuries since Cervantes published his novel, no-one has identified the "place in La Mancha" whose name the author cared not to remember, and more recent papers proposing new places likewise lack scientific substantiation. In fact, there is no known attempt to rigorously apply scientific method to determine the exact location of Don Quixote's "place in La Mancha".

And yet, today such an approach is perfectly possible thanks to the methodological paradigms proposed in Systems Theory, among others. Indeed, the systemic approach is generally characterized by providing for the best possible selection of variables that not only represent the object studied as a whole, but are themselves inter-related in complex ways. In the present case, the analysis involves 24 possible villages, 96 decisive distances and some different assumptions about the speed at which Don Quixote and Sancho Panza were able to ride Rosinante and the donkey, all of which must be taken into account in the systemic whole. All of this precludes simplistic approaches and conclusions obtained on the basis of a single village; rather, the solution should be found by optimizing plausible assumptions and taking account of all the potential candidates.

From the vantage of the first objective, the present study entails "revisiting" research conducted at the Complutense University of Madrid (Parra-Luna et al., 2005) and published under the title *EL QUIJOTE COMO UN SISTEMA DE DISTANCIAS/TIEMPOS: HACIA LA LOCALIZACIÓN DEL LUGAR DE LA MANCHA (The novel Don Quixote as a time/distance system: locating the place in La Mancha),* Ed. Complutense, Madrid. The reply implicit in this article reformulates some of the initial assumptions on which that research was based for the sole purpose of verifying its core hypothesis (location of the "place in La Mancha"), while pursuing a higher degree of accuracy and scientific rigor, whatever the new findings show. The existence of a tacit kinematics in the work is instrumental to test the prior research.

Another recent paper (Girón & Ríos) introduces a statistical study of the problem. Nevertheless, their statistical approach parts from the dispersion of the data between the different villages. This don't seem reasonable if we consider that "place in La Mancha" is a village, not the mean of a set of several villages. They defer to a future work the consideration of the uncertainty of the duration of the displacements of Don Quixote and Sancho Panza. In this paper, we part from this uncertainty.

THE DISTANCE / RIDING TIME SYSTEM

But is the existence of a kinematics in Don Quixote, of any sort, let alone able to attain the above two objectives, even imaginable? In fact, it is both imaginable and verifiable, thanks to the facts that Cervantes himself provides:

First, the point of departure: Campo de Montiel, a historic/geographic region with a total of 24 villages at the time the novel was written, all at empirically measurable distances from one another. Second, "theoretical" distances (expressed in riding time): between the enigmatic "place in La Mancha" (hereafter abbreviated "L") and three places (Puerto Lápice, Sierra Morena and El Toboso) explicitly cited by Cervantes, and one (Munera) implicitly mentioned (Parra-Luna et al, 2005); precisely, we take a point to 2 kilometers at SE of Puerto Lapice (end of chapter X) and a point beside Venta de Cardenas in Sierra Morena from Parra-Luna et al. (2005); and all of these are real geographic sites with their respective distances to the region's 24 villages.

Moreover, these theoretical riding times can be quantified to a reasonable degree of accuracy. These data were structured on the basis of these premises and the information gathered during the research effort described above. Parra-Luna et al. 2005, announced some of the quantitative keys implicit in the novel: among others, the so-called topological solution, which is the one reconsidered here under more certain and verifiable assumptions.

THE REAL DISTANCES BETWEEN VILLAGES AND THE FOUR POINTS OF REFERENCE

Measuring the distances between villages was an exercise fraught with difficulties due to the inability to objectively establish what would have been the most logical routes – and respective distances - in the late sixteenth century (differences between maps, new roads, short-cuts, paths and so on) and indeed, the want of a more verifiable system of distances left the researchers with an uncomfortable sensation of uncertainty. Based on the old adage that "La Mancha has no roads, for it is nothing but a road" (plain that it is), this problem was circumvented by taking into account the distances between villages according with the

Directorio Cartográfico de España (http://www.dices.net/mapas) to reach a consensus on the question with the highest possible degree of objectivity. Table 1 shows then these distances.

	Puerto Lapice	Venta Cardenas	El Toboso	Munera
Albaladejo	95.188	65.346	101.18	54.73
Alcubillas	67.685	49.276	85.672	65.825
Alambra	56.999	66.803	68.384	52.594
Almedina	88.094	53.253	99.783	62.564
Cañamares	89.107	75.591	89.989	41.049
Carrizosa	66.389	64.916	75.66	49.978
Castellars	87.081	23.33	111.664	90.125
Cozar	80.184	46.214	96.163	67.315
Fuenllana	75.561	60.944	85.004	52.462
Membrilla	38.775	62.718	67.934	76.72
Montiel	84.304	63.932	91.174	50.843
Ossa	71.483	89.745	64.561	25.566
Puebla P.	93.651	52.554	105.396	65.909
Ruidiera	61.998	81.657	61.677	36.739
S.C. Cañamos	90.336	60.63	98.517	56.505
Solana	45.57	62.2	67.619	67.22
Terriches	93.502	62.624	100.727	56.305
Torre J.A.	86.97	42.487	103.549	72.268
Torres M.	81.633	55.227	93.048	58.728
Torrenueva	75.111	27.071	102.853	89.865
Villahermosa	79.93	66.839	85.672	46.914
Villamanrique	92.691	46.459	107.087	71.003
Villa. Fuente	103.733	67.461	109.251	59.233
Villa. Infantes	73.981	55.26	86.755	58.224

Table 1. Distances in Km from the villages of Campo de Montiel to the four points of reference: Puerto Lapice, Venta de Cardenas , el Toboso and Munera.

Then, we have as premises the conclusions of Parra-Luna et al. (2205), and other in the parent paper, who infere the following times of displacement from L to the four points of reference:
To 2 kms. at SE of Puerto Lápice (2SEPL), 22.5 hours ± 15% (from 19.1 to 25.9 hours).
To El Toboso (ET), 25 hours ± 10% (from 22.5 to 27.5 hours).
To Venta de Cárdenas (VC), 20 hours ± 15% (from 17 to 23 hours).
To Munera (M), 20 hours ± 10% (from 18 to 22 hours).

We also assume:
a) That Cervantes based his estimations on the geographical reality.
b) That the geographical irregularities are distributed in a way enough homogeneous so that we can take displacement in straight-line at a constant velocity on a plane as an approximation.
c) That the "*Lugar de la Mancha*" is an existent locality.

THE BOUNDING OF THE "PLACE IN LA MANCHA"

From these premises we can bound the zone in which **L** would have to be. To this end, being k>1 the quotient between its distances to the point of reference which position is given by the vector $\mathbf{r_2}$ and to the point of reference which position is given by the vector $\mathbf{r_1}$, that is to say, $k=|\mathbf{L}-\mathbf{r_2}|/|\mathbf{L}-\mathbf{r_1}|$, **L** will have to be in the circumference of center in $\mathbf{r_2}+(\mathbf{r_1}-\mathbf{r_2})k^2/(k^2-1)$ and radius $|\mathbf{r_1}-\mathbf{r_2}|k/(k^2-1)$, as you can algebraically verify (Appendix 1 in http://www.uv.es/pla/Quixote/append1.htm). Of course, if k=1 then **L** will have to be in the perpendicular bisector of $\mathbf{r_1}$ y $\mathbf{r_2}$, given by $(\mathbf{L}-(\mathbf{r_1}+\mathbf{r_2})/2)\cdot(\mathbf{r_1}-\mathbf{r_2})=0$.

By assuming constant velocity, the quotient between the distances will be equal to the quotient between the times of displacement. Thus, taking in twos the four points of reference previously indicated, the quotients between their distances will have to be between the quotient of the ends of the intervals which we have taken for the corresponding times of displacement:
Between 2SEPL and ET, between k=1.44 in a way and k=1.15 in the another (remember that we always divide the major by the minor time so that k>1).
Between 2SEPL and VC, between k=1.20 in a way and k=1.52 in the another.
Between 2SEPL and M, between k=1.15 in a way and k=1,44 in the another.
Between ET and VC, between k=1.02 in a way and k=1.62 en el otro.
Between ET and M, between k=1.02 and k=1.53
Between VC and M, between k=1.29 in a way and k=1.28 in the another.

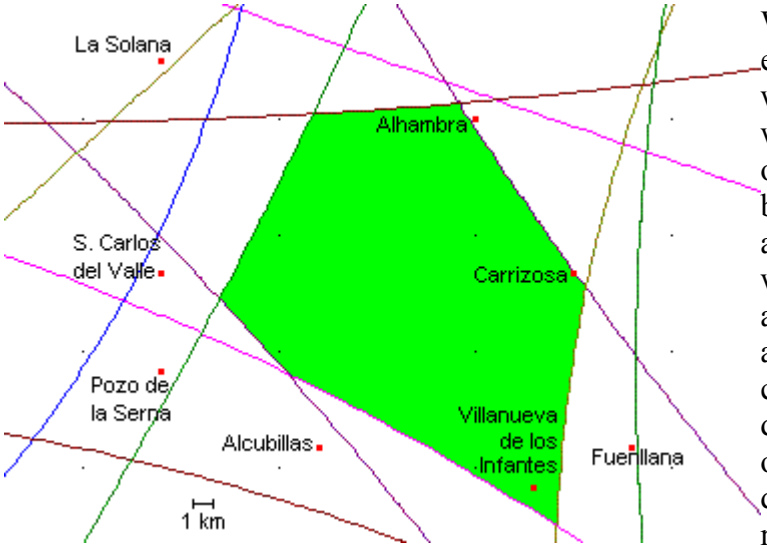

Figure 1: *Localities in the bounded zone.*

We get so two circumferences for each pair of points of reference, which will bound a zone in which L will be able to be. The intersection of the six zones so gotten will bound the position of L in accordance with the premises from which we have started. Note that although we would be geometrically able to point to another zone compatible with the times of displacement, this zone would be out of La Mancha, and to be displaced from it to the points of reference you would require high velocities typical of a jet and not of the mounts of Don Quixote and Sancho Panza.

To make the calculations we have taken the geographical coordinates the with a precision of one kilometer, provided that we can estimate that they have an extension of this order.

Now then, provided that L has to be a locality, we must examine which localities are in the bounded zone. As you can see in the enlarged drawing of the zone in the Figure 1, the only localities which exist in it are **Villanueva de los Infantes**, **Carrizosa** and **Alhambra**. In fact,

Alhambra is lightly out of the zone, but its distance to the boundary is lower than the distance of one kilometer which we had taken as precision for the position of the localities.

DETERMINATION OF THE VELOCITY

For each locality and point of reference we can get an interval of possible velocities by dividing their distance by the ends of the corresponding interval of times of displacement. The intersection of the intervals of possible velocities will give us the set of velocities which were compatible with every interval of times of displacement. If this intersection were the empty set, the locality would not compatible.

We will study the three localities which exist in the bounded zone together with several near localities which we will use as a contrast. From the distances in kilometers to the four points of reference we get the intervals of possible velocities, in kilometers/hour:

DISTANCE	La Solana	Alhambra	Carrizosa	Villanueva I.	Alcubillas	Fuenllana
to 2SEPL	1.80-2.41	2.20-3.98	2.56-3.48	2.87-3.90	2.61-3.54	2.91-3.95
to ET	2.47-3.02	2.48-3.03	2.76-3.38	3.16-3.87	3.12-3.82	3.09-3.78
to VC	2.69-3.64	2.92-3.95	2.80-3.79	2.41-3.26	2.13-2.88	2.64-3.57
to M	3.05-3.73	2.37-2.90	2.29-2.81	2.64-3.22	3.01-3.68	2.40-2.93

Therefore their intersections, approximating with only one decimal numeral, are:

Not exist	2.9	2.8	3.2	Not exist	Not exist

As it was predictable, only the three localities which exist in the bounded zone have velocities which are compatible with the considered intervals of times of displacement.

STATISTICAL SIMULATION

The algorithm used to simulate a great number (100000) of trips of D. Quixote and Sancho from the "Place in la Mancha" (LM) to the 4 referencie points (P. Lápice, El Toboso, Munera, Venta de Cárdenas) takes as data the following variables:

T(4) : Estimated travelling time from the "Place in la Mancha" (LM) to the 4 referencie points (P. Lápice, El Toboso, Munera, Venta de Cárdenas)(hours). Considered as a random variable normally distributed with a mean and a standard deviation estimated by experts (The values stated above in this paper).
V(21) : The 21 possible velocities for travels of D. Quixote and Sancho (Km/h).
L(24,2): Longitude and latitude of the 24 candidate villages to LM (Km).
R(4,2): Longitude and latitude of the 4 reference points.

From such data, the following variables are calculated:
D(24,4): Real distances from the 24 candidate villages to the 4 reference points (Km).
C(4,21): Calculated distances from LM to the 4 reference points for each of the 21 possible velocities (hours). $C(j,k) = T(j) * V(k)$
B(24,4,21): Estimated distance discrepancies, travelling from LM to the 4 points of reference, between real values and calculated values for each of the 21 possible velocities (hours). $B(i,j,k) = D(i,j) - C(j,k)$
N(24,21): Norm-2 of the distance discrepancies, in relative value.

$$N(i,k) = \sum_{j=1}^{4} \left(\frac{B(i,j,k)}{D(i,j)} \right)^2$$

X: Index number of the village selected as LM.

Y: Index number of the velocity corresponding to X.

$$N(X,Y) = \underset{\substack{i=1,24 \\ j=1,21}}{Min}\left(N(i,j)\right)$$

A(24,21): Times that a given selection village-velocity is observed in *r* simulations with different values of T randomly obtained following a Gaussian distribution.

AL(24): Times that a given village is selected.

P(24,21,2): Confidence intervals (95%) for the probability of each combination village-velocity (Using the Central Limit Theorem).

PL(24,2): Confidence intervals (95%) for the probability of each village.

PM(24): Average probability for each village.

PC(24,21,2): Confidence intervals (95%) for the conditioned probabilities of each velocity in each village. *PC(i,j,1)=P(i,j,1)/PL(i,2) ; PC(i,j,2)=P(i,j,2)/PL(i,1)*

VI(24,2): Confidence intervals (95%) for the average velocity from each village.

$$VI(i,k) = \sum_{j=1}^{21} V(j) \cdot PC(i,j,k)$$

VM(2): Confidence interval (95%) for the average velocity of the travelers considering all possible velocities.

$$VM(k) = \sum_{i=1}^{24} VI(i,k) \cdot PM(i)$$

The detailed algorithm can be found in (Appendix 2 in http://www.uv.es/~Caselles/Quixote/append2.htm)

CONCLUSIONS

In accordance with the premises from which we have started and with the considered margin of error:

a) The "Place in La Mancha" has to be Villanueva de los Infantes, Alhambra or Carrizosa.

b) With a reliability of 95%,

- the probability of Villanueva de los Infantes is between 0.326 and 0.330 with a velocity between 3.1 and 3.2 kms/h.

- the probability of Alhambra is between 0.257 and 0.262 with a velocity between 2.7 and 2.9 kms/h.

- the probability of Carrizosa is between 0.223 and 0.227 with a velocity between 2.8 and 3.0 kms/h.

c) With a reliability of 95%, the global mean velocity has to be between 2.9 and 3.1 kms/h.

These results are compatible with those of Parra-Luna et al. (2005).

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