SUBSURFACE LAND DRAINAGE BY TUBE WELLS

WELL SPACING EQUATIONS FOR FULLY AND PARTIALLY PENETRATING WELLS IN UNIFORM OR LAYERED AQUIFERS WITH OR WITHOUT ANISOTROPY AND ENTRANCE RESISTANCE

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On website https://www.waterlog.info , explanation of WellDrain model at https://www.waterlog.info/weldrain.htm
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1. INTRODUCTION

Subsurface drainage of agricultural land can be done by horizontal pipe drainage systems, but when the aquifer is deep drainage by wells (vertical drainage) may be a feasible alternative because the well spacing can be quite wide achieving the same effect on the lowering of the water table.

The law of Darcy (Figure 1) states:

$$\frac{\delta J}{\delta X} = -\frac{V_x}{K_x} \qquad (1)$$

where:

- J is the level of the water table at distance X, taken with respect to the level of the impermeable base of the aquifer (m)
- X is a distance in horizontal direction (m)
- Vx is the apparent flow velocity at X in horizontal X-direction (m/day)
- Kx is the horizontal hydraulic conductivity (m/day)
- δX is a small increment of distance X (m)
- δJ is the increment of level J over increment δX (m)
- δJ/δX is the gradient of the water table at X (m/m)

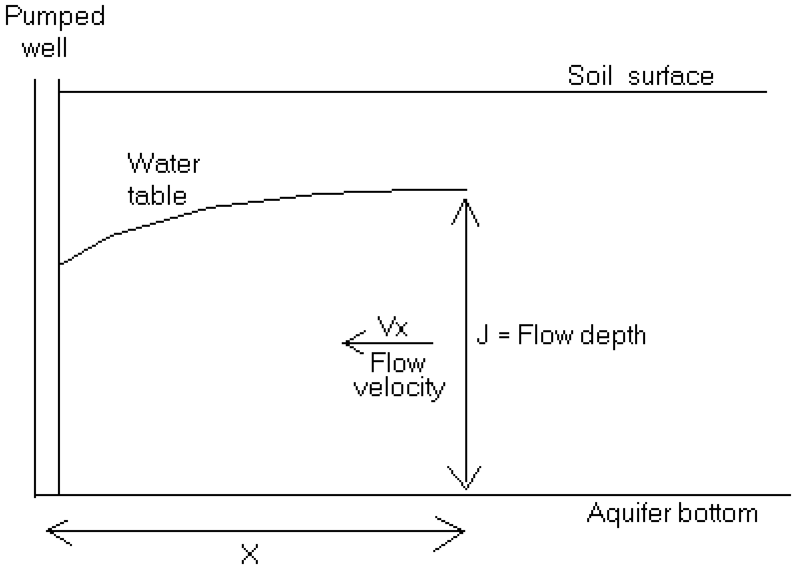

Figure 1. Cross-section over a well

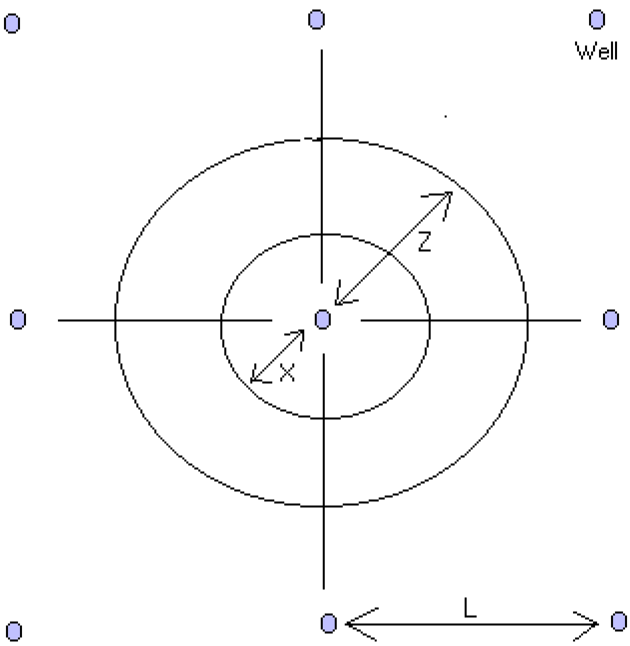

Figure 2. Square grid of wells with well spacing L and radius of influence Z

The velocity of flow to a fully penetrating well, i.e. a well reaching the bottom of the aquifer, in a homogeneous aquifer can be found from the water balance (Figure 2) as:

$$V_x = \frac{\pi(Z^2-X^2)R}{2\pi X.J} \qquad (2)$$

where:

- R is the long term average (quasi steady) recharge by downward percolating water stemming from rain or irrigation water equalling the long term pumping rate from the well (m/day)
- Z the radius of a circle around the well to where the influence of the well extends, the zone of influence (m)

The factor $\pi(Z^2-X^2)R$ represents the amount of water (m^3/day) stemming from the surface of a segment between the circles with radius Z and X, and the factor $2\pi X.J$ is the cylindrical cross-section (m^2/day) of flow at distance X.

When the wells are arranged in a square pattern we may equate the surface area under the radius of influence of the well with the surface area geometrically associated with a well:

$$\pi Z^2 = L^2 \text{ so that } Z = 0.56\ L \text{ and } L = 1.77\ Z \qquad (3)$$

For a regular triangular pattern we find L = 1.73 Z and L = 0.58 Z. The difference is relatively small.

When the well pattern is somewhat irregular we may use: $L=0.57\sqrt{(A/N)}$, where A is the area served by N wells, and $Z=\{\sqrt{(A/N)}\}/1.75$ using factors intermediate between those found before.

2. SOLUTIONS

The previous equations can be solved analytically (Boehmer and Boonstra 1994, Oosterbaan 1986) for fully penetrating wells in a uniform aquifer as in Figure 1:

$$Q = \frac{\Pi}{\ln(Z/W_r)} \{2K_b(D_2-D_w)(J_z-J_r) + K_a(J_z-J_r)^2\} \qquad (4)$$

where: Q is the well discharge (m3/day), J_z and J_r are the heights of the water table above the impermeable layer at distance Z (the sphere of influence of the well) and at W_r (i.e. at the circumference of the well, W_r being the well radius), K_b and K_a are the hydraulic conductivity below and above the water level in the well respectively. The distance Z may be taken as $L/\sqrt{\pi}$, where L is the well spacing in an equidistant square network (see Fig. 3). The symbols D_2 and D_w are also shown in Fig.3.

However, for partially penetrating wells and heterogeneous aquifers an analytical solution is difficult to find, the more so if one wishes to know the shape of the water table from the well to the end of the zone of influence. The model WellDrain on website www.waterlog.info/weldrain.htm therefore uses a numerical solution using the following input data:

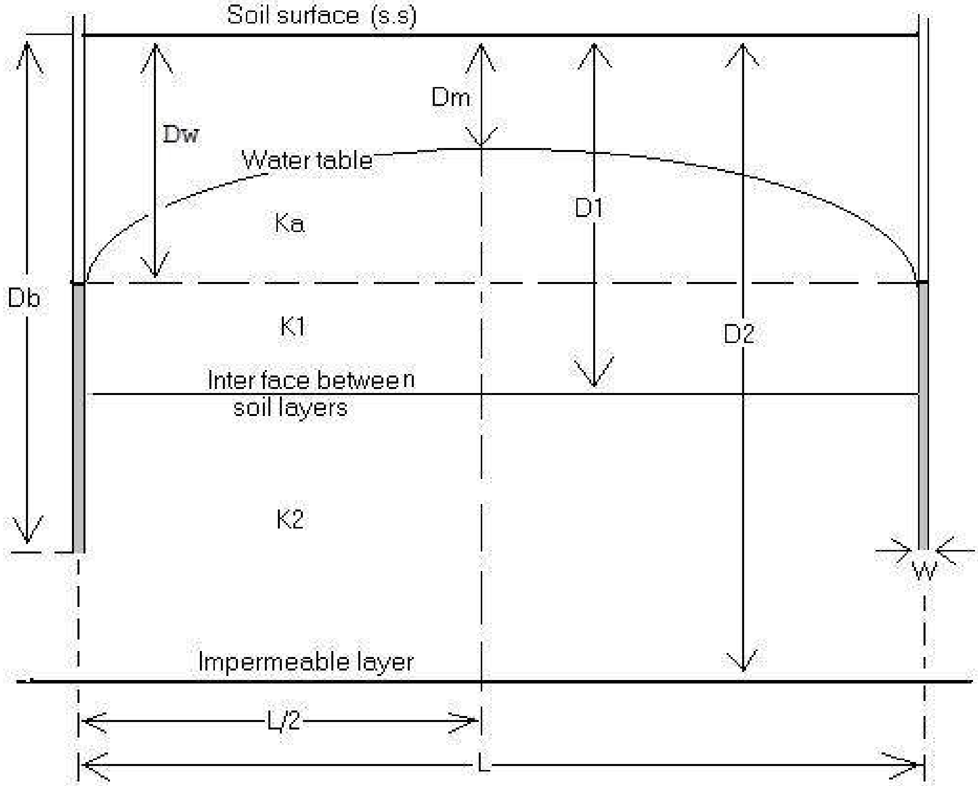

Figure 3. Illustration of input data for the WellDrain program

Long term average recharge or discharge	R	(m/day)
Bottom depth of 1st layer below the soil surface (s.s.)	D1	(m)
Bottom depth of 2nd layer below s.s.	D2	(m)
Depth water level in well below s.s. (long term average)	Dw	(m)
Depth of well bottom below s.s	Db	(m)
Entrance resistance at the well	E	(day/m)
Diameter of well screen	W	(m)
Permeability above water level in well	Ka	(m/day)
Horizontal permeability, 1st soil layer	Kb1	(m/day)
Vertical permeability, 1st soil layer	Kv1	(m/day)
Horizontal permeability, 2nd soil layer	Kb2	(m/day)
Vertical permeability, 2nd soil layer	Kv2	(m/day)
Depth water table midway between wells (long term average)	Dm	(m)
Spacing between wells	L	(m)

See figure 3.

The depth D1 can be greater or smaller than Dw. It can even be zero, meaning that one soil layer is not present. Also it can be equal to D2 meaning that another soil layer is not present.

3. PARTIALLY PENETRATING WELLS

When the wells penetrate only partially into the aquifer (Figure 4) the flow in the neighbourhood of the well moves radially up from the underground. As the flow is also radial in a horizontal plane, it becomes spherical flow (Oosterbaan 1986).

To account for the radially upward moving water, the horizontal impermeable is partly replaced by an imaginary impermeable layer sloping downward from the well bottom to the real impermeable layer with a slope at an angle whose tangent equal to $\frac{1}{2}\pi$, much in the same way as described by Oosterbaan (2002)using the Hooghoudt principle.

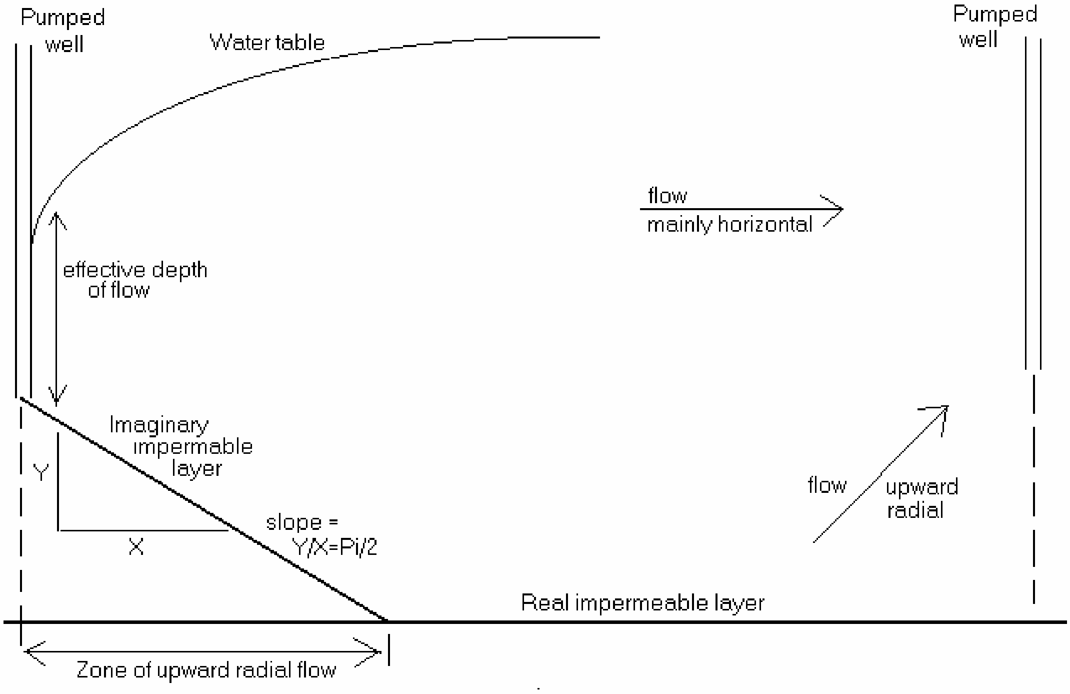

Figure 4. Illustrating the imaginary impermeable layer in the presence of partially penetrating wells.

4. TRANSMISSIVITY

Transmissivity is defined as the product of horizontal hydraulic conductivity and thickness of the saturated soil layer. Since the water table gets higher away from the drain, the transmissivity increases away from the drain. When partially penetrating wells are present there is a further increase in transmissivity because the imaginary impermeable layer descends away from the drain. Figure 5 gives an example.

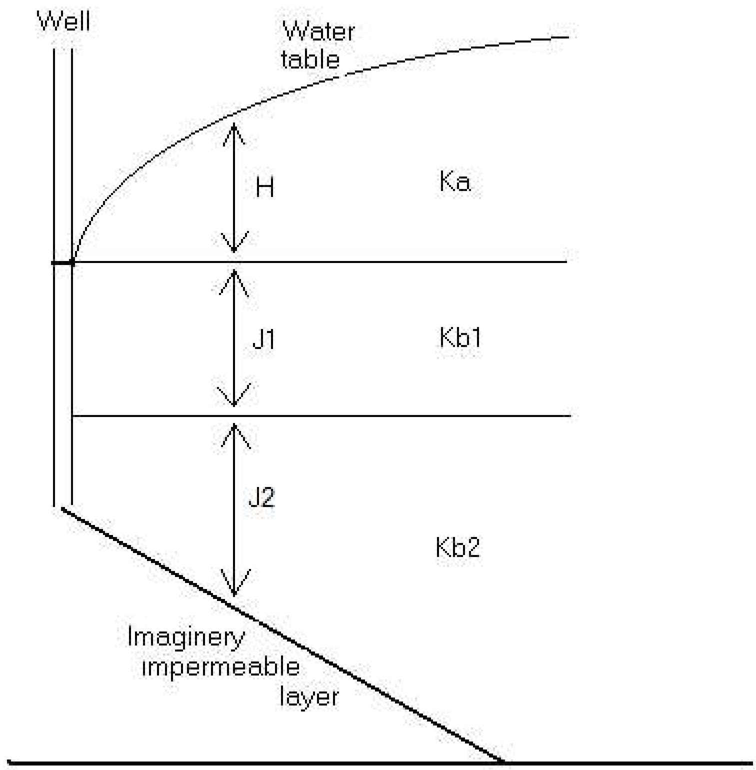

Figure 5. The transmissity in the cross-section shown equals
$T = H.K_a + J_1.K_{b1} + J_2.K_{b2}$

5. EQUIVALENT HYDRAULIC CONDUCTIVITY

With a numerical approach, the discharge weighted average transmissivity (T_{av}) can be calculated. It is found dividing the the distance from the well to the point midway between the wells into small steps and determining in each corresponding vertical section the transmissivity below the waterlevel in the well (in m^2/day) and the discharge (in m^2/day). Next these two quantities are multiplied and the products are totalized. Finally T_{av} is obtained by dividing this sum of products by the sum of the discharges.

Using $K_e = T_{av}/(D_2-D_w)$, where K_e is the equivalent hydraulic conductivity (m/day), one will be able to use the well flow equations for fully penetraing wells (Eq. 4) to a situation with partially penetrating wells replacing K_b by K_e.

The meaning of the symbols D_2 and D_m can be seen in Figure 3. Note that J_z-J_r in Eq. 4 equals H in Fig. 5.

6. ENTRANCE RESISTANCE

When entrance resistance is present, the water level just outside the well is higher than inside by a difference Fe (m), the entrance head (Figure 6).

Entrance resistance is defined as E = Fe/Q (day/m), where Q is the flow entering the well in m3/day per m length of submerged part of the well. Hence $Q = \pi R.Z^2/(Db-Dw)$.

Therefore $Fe = \pi E.R.Z^2/(Db-Dw)$.

The effect of entrance resistance is somewhat diminished by an increased transmissivity.

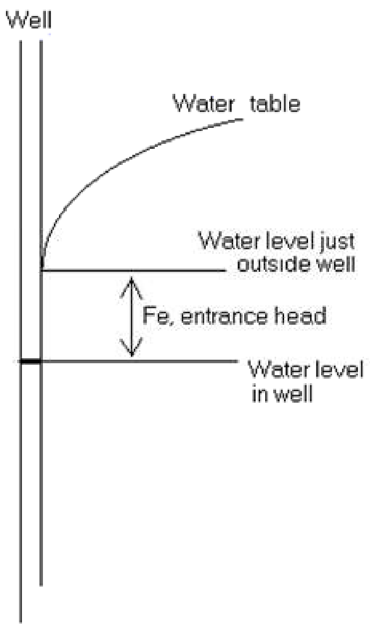

Figure 6. Entrance head due to entrance resistance.

7. ANISOTROPY

The hydraulic conductivity of the soil may change with depth and be different in horizontal and vertical direction. We will distinguish a horizontal conductivity K_a of the soil above drainage level, and a horizontal (K_b) and vertical (K_v) conductivity below drainage level. The following principles are only valid when $K_v>R$, otherwise the recharge R percolates downwards only partially and the assumed water balance is not applicable.

The effect of the conductivity K_v is taken into account by introducing the anisotropy ratio $A=\sqrt{(K_b/K_v)}$, as described for example by Boumans (1979). The conductivity K_b is divided by this ratio, yielding a transformed conductivity: $K_t = K_b/A = \sqrt{(K_b.K_v)}$. As normally $K_v<K_b$, we find $A>1$ and $K_t<K_b$. On the other hand, the thickness J of the aquifer below the bottom level of the well is multiplied with the ratio. Hence the transformed depth is: $J_t=A.J$

When A>1, the zone of upward radial flow increases. The effect of the transformation is that the extended area of upward radial flow and the reduced conductivity K_t increase the resistance to the flow and enlarges the height of the water table. In extreme cases the extended area reaches beyond the zone of influence, which means that the lower part of the aquifer does not contribute to the flow of water.

8. LAYERED (AN)ISOTROPIC SOILS

The soil may consist of distinct (an)isotropic layers. In the following model, three layers are discerned.

The first layer reaches to a depth Wd below the soil surface, corresponding to the water level in the well, and it has an isotropic hydraulic conductivity Ka. The layer represents the soil conditions above drainage level.

The second layer has a reaches to depth D1 below the soil surface ($D_1>D_w$). It has horizontal and vertical hydraulic conductivities K_{b1} and K_{v1} respectively with an anisotropy ratio $A_1=\sqrt{(K_{b1}/K_{v1})}$. The transformed conductivity is $K_{t1} = K_{b1}/A_1$. The thickness $J_1 = D_1 - D_w$ is transformed to $J_{t1} = A_1.J_1$.

The third layer rests on the impermeable base at a depth D_2 ($D_2>D_1$). It has a thickness $J_2 = D_2 -D_1$ and horizontal and vertical hydraulic conductivities K_{b2} and K_{v2} respectively with an anisotropy ratio $A_2 = \sqrt{(K_{b2}/K_{v2})}$,. The transformed conductivity is $K_{t2} = K_{b2}/A_2$, and the transformed thickness is $J_{t2} = A_2.F_2$

The transmissivity of the aquifer now is:

$$T = H.K_a + J_{t1}.K_{t1} + J_{t2}.K_{t2}$$

9. RECTANGULAR WELL FIELD

When the wells are placed in a rectangular well field with length A and width B (A>L>B), we have:

$$L = \sqrt{A.B}$$

The height H_A of the water table midway between the distance A is somewhat larger than the height H_L midway between the distance L. On the other hand, the height H_B of the water table midway between the distance B is somewhat smaller than the height H_L midway between the distance.

The difference $\Delta H_A=H_A-H_L$ is:

$$\Delta H_A = B(A-L)R/T_A$$

where

$$T_A = (H_L+0.5\Delta H_A)K_a + J_{t1}.K_{t1} + J_{t2}.K_{t2}$$

The difference $\Delta H_{\mathbf{B}}=H_L-H_B$ is:

$$\Delta H_{\mathbf{B}} = A(L-B)R/T_B$$

where

$$T_B = (H_L-0.5\Delta H_B)K_a + J_{t1}.K_{t1} + J_{t2}.K_{t2}$$

Hence, the depth of the water table D_A midway between the larger distance A is somewhat smaller than the depth D_m midway between the distance L, and the depth of the water table D_B midway between the shorter distance B is somewhat smaller than the depth D_m midway between the distance L.

The calculation of $\Delta H_{\mathbf{A\ and}}\ \Delta H_{\mathbf{B}}$ requires a procedure of trial and error because these values are needed in advance for the calculation of T_A and T_B. When the aquifer is thick, the transmissivities T_A and T_B can be approximated simply by

$$T_A = T_B = H_L.K_a + J_{t1}.K_{t1} + J_{t2}.K_{t2}$$

without committing a large error.

REFERENCES

Boehmer, W.K. and J.Boonstra (1994). Tubewell Drainage systems. In: H.P.Ritzema (Ed), Drainage Principles and Application, p. 931-964, ILRI Publ 16., Wageningen, The Netherlands.

Boumans, J.H. (1979). Drainage calculations in stratified soils using the anisotropic soil model to simulate hydraulic conductivity conditions. In: J.Wesseling (Ed.), Proceedings of the International Drainage Workshop, p. 108-123. Publ. 25, ILRI, Wageningen, The Netherlands.

Oosterbaan, R.J. (1986). Tube-well Spacing Formulas for Subsurface Drainage. In: K.V.H.Smith and D.W.Rycroft (Eds.), Proceedings of the 2nd International Conference, Southampton University, UK.

Oosterbaan, R.J. (2002). The Energy Balance of Groundwater Flow Applied to Subsurface Drainage in Anisotropic Soils by Pipes or Ditches with Entrance Resistance. On website www.waterlog.info/articles.htm.