Generalizations of Homophonic Truth-sentences[1]

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I

Homophonic truth-sentences are of three types. The following three sentences illustrate these types.

'Snow is white' is true if and only if snow is white

The proposition that snow is white is true if and only if snow is white

It is true that snow is white if and only if snow is white.

Sentences of the first two types are *predicate* (homophonic truth-) sentences. Sentences of the first type are *sentential* predicate sentences. Sentences of the second type are *propositional* predicate sentences. Sentences of the third type are *operator* (homophonic truth-) sentences. Homophonic truth-sentences have been of interest to philosophers for various reasons. One of the most important of these reasons is that they have seemed to many philosophers to suggest that the adjective 'true' is in some sense redundant, and that it may be possible to eliminate this adjective from our discourse without loss of content – and without introducing some new adjective or adjectival phrase (such as 'in correspondence with reality') to take its place. Homophonic truth-sentences suggest that it may be possible to eliminate 'true' because 'true' occurs in the left-hand constituent of each homophonic truth-sentence and not in the right-hand constituent[2] ; nevertheless, there seems to be an obvious sense in which the right-hand constituent of a homophonic truth-sentence contains no less information than the left-hand constituent.

And why would one wish to be able to eliminate 'true' from our discourse? Many philosophers believe – I do myself – that the perfect definition is the eliminative definition. Suppose, for example, that our vocabulary comprises four items, A, B, C, and D, and that we regard D as somehow philosophically problematical. If we can find a systematic method of translating all our sentences into sentences that convey exactly the same information but contain only the vocabulary items A, B, and C, then the theory that the perfect definition is the eliminative definition tells us that we have thereby achieved a perfect understanding of D. (The theory does not demand that we

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1 This paper is dedicated to the memory of Herbert Heidelberger.

2 More exactly: 'true' occurs one fewer times in the right-hand constituent of a homophonic truth-sentence than in its left-hand constituent.

find some phrase containing only A, B, and C with which to replace D at all its occurrences; it demands only that we find some systematic way to replace each sentence containing D with a sentence that contains only the other vocabulary items.) And almost all philosophers – from the pre-Socratics to Tarski and Heidegger – have regarded truth as a philosophically problematical concept. Analytical philosophers, at least, generally believe that the way to give an account of a philosophically problematical concept is to provide a non-trivial definition of the word that expresses that concept, and 'true' is the word that expresses the concept of truth.[3] To show how systematically to eliminate 'true' from our discourse, therefore, would be – at least so many philosophers would agree – to give an account of the philosophically problematical concept of truth.

Homophonic truth-sentences suggest that it may be possible to eliminate 'true' from our discourse, but they do no more than suggest this. This is because 'true' occurs in sentences that are not of the forms exhibited by the left-hand constituents of homophonic truth sentences. There are for example, such sentences as 'The Axiom of Choice is true'; and there are generalizations like 'Only true propositions follow logically from true propositions' or 'Some of the things Alice said were true and some were not' or 'An analytic sentence is one that is true in virtue of its grammatical structure and the meanings of the words it contains'. Nevertheless, the suggestion is a powerful and attractive one, since it is hard to avoid the impression that each homophonic truth-sentence is a particular instance of a general thesis. For example, '"Snow is white" is true if and only if snow is white' and '"Cows are purple" is true if and only if cows are purple' *seem* to be in some sense two instances of some one general thesis. And it is very tempting to believe that if we could find these general theses (there would be three, corresponding to the three types of homophonic truth-sentences), they would show us how to eliminate every occurrence of 'true' from our discourse. Can such generalizations be found, and (if they can be found) will they indeed show us how to eliminate 'true' from our discourse? Let us see.

Let us first consider the *predicate* 'is true'. To eliminate a predicate systematically from our discourse, it is necessary to find some other predicate with which it may be replaced at any of its occurrences, and which in some sense has the same content as the original. We may, for example, replace the predicate 'is a sphere' with the predicate 'is a surface such that, for some point, it comprises all and only those points equidistant from that point'. It may be hard to spell out the sense in which 'is a surface such that, for some point, it comprises all and only those points equidistant from that point' "has the same content as" 'is a sphere', but it does seem that there is some important sense in which these predicates have the same content. Can something comparable be done for the predicate 'is true'? Let us first consider the case in which this predicate is a predicate of propositions (non-linguistic objects of affirmation, denial, and acceptance or belief). Can 'is (a) true (proposition)' be eliminated from our discourse about propositions – and do propositional predicate sentences provide some sort of clue as to how this might be done? The idea that suggests itself is this: find a biconditional whose left-hand constituent is 'x is true' and from which all propositional predicate homophonic truth-sentences (and no false sentences) follow by universal instantiation. In such a biconditional, it would seem, the right-hand constituent will contain the variable 'x' free and will not contain 'true'. What would such a biconditional be? Certainly not this sentence, which is not even well formed:

x is true if and only if x.

But if not this sentence, what? The problem of finding a biconditional from which all propositional predicate homophonic truth-sentences (and no false sentences) follow by universal instantiation seems insoluble. If we say this, however, we shall no doubt be told that the problem is insoluble only because we have made it so: we have made it insoluble by treating 'x' as a nominal variable; and (we shall be told) there are other kinds of variables than nominal variables. There are also predicate variables, and among these there are 0-place predicate variables, or sentential variables. (Nominal variables have the syntax of names or terms; sentential variables have the syntax of sentences.) If we have sentential variables ('p', 'q' and so on) at our disposal – and "sentential quantifiers" to bind them – , we can find the general sentence we want. It will not, of course, be 'p is true if and only if p', for this sentence, too, is not well formed – and its left-hand constituent is not 'x is true'. In order to get the general sentence we want, we need some construction that connects terms and sentences (or sentential variables). The obvious candidate for this construction is the operator 'the proposition that', which takes a sentence (or sentential variable) and makes a term. We can use 'the proposition that' and sentential variables to construct various sentences that are in some sense generalizations of homophonic truth sentences. For example:

$\forall p$: The proposition that p is true . if and only if p.

But this sentence will not help us to find an equivalence for 'x is true', for it contains no nominal variables. The best solution to our problem, I think, is this

x is true if and only if $\exists p$. p and x = the proposition that p.

Admittedly it is not true that all propositional predicate sentences follow from this sentence by universal instantiation. In fact, none do. What follow from this sentence by universal instantiation (if the nominal variable ranges only over propositions) are sentences like these:

The Axiom of Choice is true if and only if $\exists p$. p and the Axiom of Choice = the proposition that p.

The proposition that snow is white is true if and only if $\exists p$. p and the proposition that snow is white = the proposition that p.

3 'True' is, to be sure, only the *English* word that expresses this concept. But no one, I suppose, doubts that if a non-trivial definition of 'true' could be given in English, then this definition could be "translated" into German, and its translation would be a non-trivial definition of 'wahr' in German – and so for French and 'vrai', and so for every language in which there is a word that means 'true'.

Still, both these sentences are true (provided, at any rate, that sentential variables and quantifiers make sense). And the propositional predicate sentence

The proposition that snow is white is true if and only if snow is white

follows logically from the second.[4]

More generally, the sentence

x is true if and only if $\exists p \,.\, p$ and x = the proposition that p

has no false instances,[5] and every propositional predicate sentence can be deduced from it. This sentence may, therefore, be regarded as, in a loose sense, a generalization whose instances are propositional predicate sentences: it is a sort of compendium of the information they collectively provide. And it has these two important features: its left-hand constituent is 'x is true' and its right-hand constituent does not contain 'true'. We can, therefore, use this sentence to remove all mention of truth from our discourse – provided, of course, that every mention of truth that occurs in our discourse can be understood in terms of a truth-predicate that applies to propositions. Consider for example

Some of the things Alice believes are true only if the Axiom of Choice is true.

Our general sentence provides us with the resources to rewrite this sentence, preserving its content but removing all occurrences of the truth-predicate:

$\exists x$ (Alice believes x and : $\exists p \,.\, p$ and x = the proposition that p . only if $\exists p \,.\, p$ and the Axiom of Choice = the proposition that p).

The case is less clear with *sentential* predicate homophonic truth-sentences, however. Can 'is true' (understood as a predicate of sentences) be eliminated from our discourse by the application of some generalization of

'Snow is white' is true if and only if snow is white?

How should this generalization be stated? These questions will not be of any great interest to philosophers who accept the existence of propositions and who regard propositions as the "primary" bearers of truth-value. Such philosophers, having at their disposal a definition of 'is true' that applies to propositions can simply say that a sentence

4 Proof [we use obvious abbreviations and omit quotes]: *LEFT-TO RIGHT*. Assume TP(*s is w*). Our premise is TP(*s is w*) $\leftrightarrow \exists p \,.\, p$ & P(*s is w*) = P(p). Hence, we have $\exists p \,.\, p$ & P(*s is w*) = P(p). By existential instantiation – or perhaps it would be better to say "particular instantiation" – we then have q & P(*s is w*) = P(q). We now introduce a premise that would obviously be a theorem of any logic whose vocabulary contained sentential variables and the operator 'the proposition that': $\forall p\, \forall q$: P(p) = P(q) $\rightarrow$. $p \leftrightarrow q$. By universal instantiation, we have P(*s is w*) = P(q) $\rightarrow$. *s is w* $\leftrightarrow q$. From this and q & P(*s is w*) = P(q) we deduce *s is w*. *RIGHT-TO-LEFT*. Assume *s is w*. We have P(*s is w*) = P(*s is w*) and hence *s is w* & P(*s is w*) = P(*s is w*). And then by "particular generalization" we have $\exists p \,.\, p$ & P(*s is w*) = P(p). From this and our premise we have TP(*s is w*), and the proof is complete.

5 Or at least this is true if we ignore the possibility that there are propositions inexpressible by any sentence – which I propose to do.

is true just in the case that it expresses a true proposition – or is true on an occasion of utterance just in the case that the proposition it expresses on that occasion of utterance is true. But not all philosophers accept the existence of propositions. Those philosophers who see sentences as the only bearers of truth-value will wish to define 'is true' as a predicate of sentences without appealing to propositions. They will therefore be interested in the question whether this can be done by "generalizing" sentential predicate sentences – by finding a biconditional whose left-hand constituent is 'x is true' (the range of 'x' being understood to comprise sentences), which has no false instances, and from which all sentential predicate sentences can be deduced – whether by a single application of universal instantiation or by some more complicated deductive route.

What might such a sentence be? Our generalization of propositional predicate sentences does not provide us with much guidance in answering this question, for that generalization depended on the fact that the left-hand constituent of a propositional predicate sentence contains a phrase ('the proposition that') that when prefixed to a sentence yields a name of the proposition that sentence expresses. The left-hand constituent of a sentential predicate sentence, however, contains a name of a sentence, and there is no phrase that, when concatenated with a sentence, produces a name of a sentence. There is no such operator as 'the sentence that', and, even if there were, the expression 'the sentence that snow is white is true' would be a different expression from '"Snow is white" is true'. (The operator 'the quotation name of' is not such an operator, since it must be prefixed to *names* of sentences – as in 'the quotation-name of the first sentence of this paper'.) A pair of quotation-marks itself cannot be thought of as an "operator" in any useful sense. It is true that applying a pair of quotation marks to a sentence produces the quotation-name of that sentence, but quotation marks interact with sentential variables in a way that renders them useless for our purposes, for a variable does not occur (and hence does not occur free) in its own quotation-name. The sentence

x is true if and only if $\exists p \,.\, p$ and x = the proposition that p

has, perhaps, no false instances. (The phrase 'the proposition that p' is an open term, a term containing a free sentential variable.) But the sentence

x is true if and only if $\exists p \,.\, p$ and x = 'p'

has false instances whenever the value of 'x' is a true sentence, for it is extensionally equivalent to

x is true if and only if $\exists p \,.\, p$ and x = the sixteenth letter of the roman alphabet (italicized).

And

'Snow is white' is true if and only if $\exists p \,.\, p$ and 'Snow is white' = the sixteenth letter of the roman alphabet (italicized)

is false.

If we are to find a way of generalizing sentential predicate sentences, we must find some phrase to do (*mutatis mutandis*) the work that 'x = the proposition that p' does in our generalization of propositional predicate sentences. Actually, such a phrase is not hard to find. If one is willing to accept the existence of propositions, one may write such a phrase this way:

x expresses the proposition that p.

If one does not accept the existence of propositions, one will have to find a sentence that, intuitively, seems to say the same thing as this (a sentence, of course, in which 'x' and 'p' and no other variables are free), but which does not involve an open term such that, when the free sentential variable in that term is replaced by a sentence, the result represents itself as naming a proposition.

I would suggest this: 'x says that p'. Thus:

'Snow is white' says that snow is white

The first sentence of this paper says that homophonic truth-sentences are of three types.

(Both examples are intended to be true sentences. It would, of course, be no challenge to find false instances of 'x says that p'.) Those who accept the existence of propositions will say that 'x says that p' could be defined as meaning 'x expresses the proposition that p'; those who deny the existence of propositions will have to be content to take 'x says that p' as indefinable. We may then offer the following as the generalization of sentential predicate sentences we are seeking:

x is true if and only if $\exists p \,.\, p$ and x says that p.

(The nominal variable, of course, ranges over sentences.) This sentence obviously has no false instances: the first sentence of this paper is true if and only if $\exists p \,.\, p$ and it says that p – and so on.[6] Can we deduce all sentential predicate sentences from it? Can we, for example, deduce

'Snow is white' if and only if snow is white

from it? The answer to this question is a qualified Yes. The simplest deduction I have been able to construct uses two premises that are worthy of some comment. One is

6 I leave to one side the problem of "self-referential" or "liar" sentences. (See Tarski's "The Concept of Truth in Formalized Languages," in Alfred Tarski, *Logic, Semantics, Metamathematics*, translated by J. H. Woodger, second edn. edited and with an introduction by John Corcoran (Indianapolis: Hackett Publishing Co., 1983), pp. 152-287. See particularly p. 162.) I justify this omission simply by pointing out that it is not my project to defend the project of defining truth by generalizing homophonic truth-sentences. I mention "liar" difficulties at this point, rather than in connection with propositional predicate sentences, because I am convinced that these difficulties can be overcome, or evaded, in the case of propositional predicate sentences. I will not defend this conviction in the present paper. If I am wrong on this point, my mistake will not weaken the argument of the paper: again, it is not my project to defend the project of defining truth by generalizing homophonic truth-sentences.

$\forall x\, \forall p\, \forall q : x$ says that p & x says that $q \,.\! \rightarrow .\, p \leftrightarrow q$.

The other is

'Snow is white' says that snow is white.

The first premise might be justified by saying that it obviously ought to be a theorem of any 'says that' logic: any logic whose vocabulary includes nominal and sentential variables and the 'says that' connective. (This premise plays roughly the same role in the proof as the role played by '$\forall p\, \forall q : \mathrm{P}(p) = \mathrm{P}(q) \rightarrow .\, p \leftrightarrow q$' in the proof in note 4.) The second premise – which is needed both to deduce 'Snow is white' from '"Snow is white" is true' and to deduce '"Snow is white" is true' from 'Snow is white' – might be justified by the contention that the following is a reasonable rule of inference for any 'says that' logic:

> Any expression formed by writing the quotation name of a sentence and then 'says that' and then that sentence may occur as a line in a proof.

To generalize predicate homophonic truth-sentences, as we have seen, presents technical problems. These problems are due to the fact that the generalizations must contain both nominal and sentential variables. To generalize operator homophonic truth-sentences, however, presents no technical problems because the required generalization contains only sentential variables:

$\forall p$: it is true that $p \,.\! \leftrightarrow p$.

Every sentence that follows from this general sentence by universal instantiation is an operator sentence and all operator sentences follow from this general sentence by universal instantiation. The simplicity of this case suggests that those philosophers who are interested in an eliminative definition of truth ignore predicate sentences and their generalizations and concentrate their attentions on operator sentences. Whether this policy would be workable depends on just one question: Can everything we want to say by using the predicate 'is true' be equally well expressed by using the operator 'it is true that' – given, of course, that we have sentential variables at our disposal? Or put the question this way: Can the truth-predicate be eliminated in favor of the truth-operator? It is evident that this can sometimes be done. For example, one need not take the sentence 'Certain things that Monica affirmed and Bill denied are true' to have the logical structure

$\exists x$. Monica affirmed x and Bill denied x and x is true

if one has sentential variables at one's disposal. One can instead write

$\exists p$. Monica affirmed that p and Bill denied that p and it is true that p.

But the case is more difficult with sentences that contain names of propositions that do not contain sentences that express the propositions they name. I have in mind expressions like 'the Axiom of Choice', 'the special theory of relativity' and 'Erdös's first important theorem'. (Not all philosophers will be willing to call these expressions "names of propositions"; let those who reject the description characterize these expressions as they will.) Consider the sentence

The Axiom of Choice is true.

Can the content of this sentence be expressed using only the truth operator and not the truth predicate? How? As 'it is true that for every non-empty set of pairwise disjoint sets x there exists a set that contains exactly one member of each member of x and contains nothing else'? No, for no part of our original sentence expresses the content of the Axiom of Choice. (Or so I should suppose. Anyone who doubts this may substitute 'the principle that is the topic of Chapter X of Quine's *Set Theory and Its Logic*' for 'the Axiom of Choice' in the example.) With 'it is true that the Axiom of Choice holds'? No, for 'holds' is just another way of saying 'is true'. (It should be evident, by the way, that there is no problem "in the other direction." Given the truth predicate, it is easy to eliminate the truth operator: 'it is true that p' may be replaced by 'the proposition that p is true' or 'for some x, x says that p and x is true'.) I doubt whether there is any solution to this problem. I conjecture that anything we can say using the adjective 'true' can be said in a language in which this adjective occurs only in the predicate 'is true'; I conjecture that some things we can say using the adjective 'true' cannot be said in a language in which this adjective occurs only in the operator 'it is true that'. (I take it for granted that anything we can say using the noun 'truth' can be said using only the adjective 'true'.) Still, we have seen that it is at least plausible to suppose that 'is true' (whether this predicate applies to propositions or to sentences) can be eliminated from our discourse by means of what may be loosely described as "generalizations of homophonic truth-sentences" – given that we have at our disposal sentential variables. We have been assuming that sentential variables make sense. It is time to examine this assumption.

II

What are sentential variables and what are the quantifiers that bind them? How are these devices to be understood? One possibility is that they be understood *substitutionally*. But if we attempt to understand them this way, we face grave problems. Most writers explain the substitutional quantifier by providing a systematic statement of the truth-conditions for the sentences in which it occurs. These truth-conditions are standard and well known. If, for example, 'Σ' is the existential (or "particular") substitutional quantifier, the truth-condition for 'Σx x is a dog' is:

> 'Σx x is a dog' is true if and only if for some x, x is a name [not necessarily a name that has a referent] and "x is a dog" is true.[7]

7 I use bold-face double quotes to do the work of "Quine corners" or "quasi-quotation-marks." Thus

> For some x, x is a name and "x is a dog" is true

is read

> For some x, x is a name and the sentence that results from writing x and then writing 'is a dog' is true.

(The variable in this example is nominal. In cases in which 'Σ' binds a sentential variable, the right hand side will be a generalization on sentences rather than names.) But this sentence does not tell us what 'Σx x is a dog' means – it tells us only that, whatever it means, it is true just in this case: prefixing some name to 'is a dog' yields a truth. That is to say, it is true just in the case that some "substitution-instance" of 'x is a dog' is true.

What, then, *does* 'Σx x is a dog' mean? One possibility, of course, is that it means – is an abbreviation of – 'For some x, x is a name and "x is a dog" is true'. Most advocates of substitutional quantification deny that this is how substitutional quantification should be understood, but the question that should interest us is whether this interpretation enables us to understand the sentential variables that, as we saw in Part I, are essential to an eliminative definition of truth. And it would seem that the answer to this question must be No. Consider our definition of the truth-predicate (as a predicate of propositions):

> x is true if and only if $\exists p$. p and x = the proposition that p.

Suppose '∃' is understood as the substitutional particular quantifier, 'Σ', and that the substitutional particular quantifier is understood in the way suggested. Then 'Σp . p and x = the proposition that p' abbreviates

> For some y, y is a sentence and "y and x = the proposition that y" is true.

Not only does this expression contain 'true', the word the definition is supposed to eliminate, but the nominal variable 'x' does not occur free in this sentence; in fact, strictly speaking, 'x' does not occur in this expression at all – which is an abbreviation for

> For some y, y is a sentence and the sentence that results from writing y and then writing 'and x = the proposition that' and then once more writing y is true.

This expression is a closed sentence – and a false one, since its instantiation to any sentence is false. For example,

> 'Snow is white' is a sentence and the sentence that results from writing 'Snow is white' and then writing 'and x = the proposition that' and then once more writing 'Snow is white' is true

is false, since

> 'Snow is white' is a sentence and 'Snow is white and x = the proposition that snow is white' is true

is false – open sentences being neither true nor false. It seems, then, that if we wish to eliminate 'true' from our discourse, and if the way we propose to do this requires sentential variables and quantifiers, and if we propose to understand sentential quantifiers as substitutional quantifiers, we had better not understand sentential substitutional quantifiers "metalinguistically": we had better not contend that a sentence of the form 'Σp . . . . p . . .' is no more than an abbreviation for the corresponding sentence of the form

For some x, x is a sentence and "... x ..." is true.

But if we do not understand substitutional quantifiers metalinguistically, how shall we understand them? In "Why I Don't Understand Substitutional Quantification",[8] I defended the thesis that there that there is *no* way to understand substitutional quantifiers, that substitutional quantification is simply meaningless. (I explicitly considered only substitutional quantification into nominal positions, but my argument is equally applicable to substitutional quantification into sentential positions.)

I argued that this was the case because the advocates of substitutional quantification had done nothing to explain the meaning of substitutional quantifiers other than to give truth-conditions for the sentences in which they occur and to say that a sentence containing substitutional quantifiers does not have the same meaning as the statement of its truth-conditions. The argument was essentially this. Suppose I introduce some new vocabulary item, X, into our language. I set out the syntactical features of X, so that it is clear which sentences containing X are well-formed or grammatical. Let A be the (infinite) set of well-formed sentences containing X. I go on to offer a mechanical procedure that pairs each sentence *a* belonging to A with a sentence *b* that does not contain X (and which we understand), and I say two things: first, that *a* is true if and only if *b* is true; secondly, that *a* means something other than *b*. If I say only this much (and "this much" is all that stating systematic truth-conditions for sentences containing substitutional quantifiers comes to), I have not told you what X means. Consider this simple case. I introduce a unary sentence-operator '¬'. I tell you these two things and tell you nothing more: the result of prefixing '¬' to a sentence is true if and only if the negation of that sentence is true; the result of prefixing '¬' to a sentence means something different from what the negation of that sentence means. Do you now know what '¬' means? No, you do not. There are lots of sentences that have the same truth-value as the negation of 'Snow is green'. There are even lots of sentences that can be known *a priori* to have the same truth-value as the negation of 'Snow is green': the negation of 'Snow is green', the conjunction of the negation of 'Snow is green' with '2 + 2 = 4', the conjunction of the negation of 'Snow is green' with the alternation of 'Snow is green' and its negation .... If I tell you only that '¬ Snow is green' doesn't have the same meaning as the first of these, I don't tell you that it doesn't have the same meaning as the second, and I don't tell you that it doesn't have the same meaning as the third. And those two sentences have different meanings. Providing a systematic statement of truth-conditions for all sentences containing substitutional quantifiers does not, therefore, enable us to understand these sentences. Since I wrote "Why I Don't Understand Substitutional Quantification," another way of understanding substitutional quantification has been proposed. The advocates of this second approach propose to regard particular substitutional quantifications on a predicate as disjunctions – disjunctions that include as disjuncts every substitution-instance of that predicate.[9] That is, where α is any variable and F any predicate, they propose to regard "$\Sigma\, \alpha\, F\alpha$" as an abbreviation for the disjunction of all substitution-instances of "$F\alpha$". (Universal substitutional quantifications on a predicate are to be understood in the parallel way as conjunctions.) But this proposal has odd results if there are as many names as there are natural numbers – and there certainly are that many names, for the standard, arithmetical names for the natural numbers are as numerous as the natural numbers. For the sake of the example, let us suppose that the standard, arithmetical names of the natural numbers are the only names there are. Then

$\Sigma x\; x$ is odd

is equivalent, on this proposal, to

0 is odd v 1 is odd v 2 is odd v ... k is odd v ... .

This proposal, or so one might argue, *does* provide a meaning for '$\Sigma x\; x$ is odd'; it *does* tell one how to translate this sentence into language one already understands. So one might argue. But if one does so argue, one seems to be employing the premise that there are infinitely long sentences, sentences that have no end. And this is very hard to believe. And matters seem, if possible, to become even worse when we consider open sentences whose variables are bound by more than one quantifier. Suppose the dual of 'Σ' is 'A'. Consider the sentence

$\mathrm{A}x\; \Sigma y\; y > x$.

This sentence will be an abbreviation for an unending conjunction of unending disjunctions:

$0 > 0 \vee 1 > 0 \vee 2 > 0 \vee \ldots k > 0 \vee \ldots\ \&\ 0 > 1 \vee 1 > 1 \vee 2 > 1 \vee \ldots k > 1 \vee \ldots\ \&\ 0 > 2 \vee 1 > 2 \vee 2 > 2 \vee \ldots k > 2 \vee \ldots\ \&\ \ldots 0 > j \vee 1 > j \vee 2 > j \vee \ldots k > j \vee \ldots\ \&\ \ldots$.

It really is very hard to believe that there are such sentences as these. Even if one does not share my scruples about the existence of such sentences, one must admit that the advocates of the proposal we are considering must solve some difficult technical problems. Consider only nominal quantification. It is not enough to say that '$\mathrm{A}x\; \Sigma y\; y > x$' abbreviates an infinite conjunction of infinite disjunctions; one must also devise an algorithm that – in the present case – takes '$\mathrm{A}x\; \Sigma y\; y > x$' and yields a finite display (no doubt containing a lot of 'j's and 'k's and lots of '...'s) that uniquely represents the of course unwritable "doubly infinite" sentence that '$\mathrm{A}x\; \Sigma y\; y > x$' supposedly abbreviates. One would then have to devise explicit rules of inference (corresponding to UI, EG, and so on) for manipulating the finite displays the translation algorithm yields. One would, finally, have to present a proof that the display the algorithm correlates with a sentence Q in the language of quantification is a theorem of the "new" system if and

8 *Philosophical Studies* 39 (1981) pp. 281-285.

9 As far as I know, this approach to substitutional quantification was first mentioned in print by Hartry Field, in his review of Dale Gottlieb's *Ontological Economy* (*Noûs* 18, 1984, pp. 160-165). I. L. Humberstone suggested this approach to me in 1981, in a letter. A recent, very sophisticated treatment of quantification, partly based on the idea that existential or particular quantifications are disjunctions of their substitution-instances, can be found in Mark Lance, "Quantification, Substitution, and Conceptual Content," *Noûs* 30, 1996, pp. 481-507.

only if Q is a theorem of substitutional quantifier logic. Devising such a translation algorithm, such a set of rules, and such a proof, will present a considerable technical challenge. But let us suppose that this challenge can be met. (I am no logician, but my guess is that it can be.) Even if it can be, and even if there are "multiply infinite sentences" – and, of course, it would be no profound problem to find set-theoretical constructs with the properties necessary to play the role of such sentences – I do not *understand* any of them, for I can understand only finite sentences. And if the only thing that can be said about the meaning of '$\mathrm{A}x\ \Sigma y\ y > x$' is that it abbreviates a certain (specifiable) "doubly infinite" sentence, I shall therefore not understand '$\mathrm{A}x\ \Sigma y\ y > x$'. This point does not essentially involve the infinite length of the unabbreviated sentences. Essentially the same point could be made about very long finite sentences. Suppose someone proposes to use the sentence 'The natural numbers less than one thousand are even-odd' as an abbreviation for '0 is even and 1 is odd and . . . 999 is odd'. Although I believe that the unabbreviated sentence exists, and although I believe that it is true, I cannot understand it (for more or less the same reason I cannot visualize a chiliagon: I can't "get it into my mind"), and I therefore have no understanding of the sentence that abbreviates it. My contention that I cannot understand '0 is even and 1 is odd and . . . 999 is odd' should not be confused with the contention that I cannot understand '0 is even and any number less than 1000 is odd if it is the successor of an even number and even if it is the successor of an odd number'. That sentence I *can* understand. But then that sentence contains only thirty-one words: the sentence I cannot understand contains 3,999 words. I recognize, moreover, that the sentence '0 is even and 1 is odd and . . . 999 is odd' is a perfectly meaningful English sentence and that it is true. But to say that is not to say that I understand it.

III

How is sentential quantification to be understood if not as a species of substitutional quantification? In this section, I will examine the most important non-substitutionalist attempt to explain sentential quantification: that of Dorothy Grover. (What I have called sentential quantification, Grover calls propositional quantification. I will stick to my own term in the following discussion of her views.)

Recognizing that the "variables" of nominal quantification are essentially pronouns, and that a proper understanding of nominal quantification must proceed from this fact,[10] Grover proposes that the variables of sentential quantification – the '*p*'s and '*q*'s that occupy sentential positions and are bound by sentential quantifiers – be understood in an exactly parallel way: as formal versions of words or phrases of a kind she calls "prosentences," items that stand to sentences as pronouns stand to nouns. It is, however, of at most heuristic value to say, "Prosentences are items that stand to sentences as pronouns stand to nouns." Some sort of definition is required, and Grover has provided one. She has defined 'prosentence' by providing a list of three defining properties: a prosentence is any word or phrase that

(1) Is not a sentence but has the syntactical properties of a sentence

(2) Can be used to make an assertion, serve as the antecedent of a conditional assertion, and can, in general play all the linguistic roles a sentence can play. (Or can with the help of context. This is parallel to the case of pronouns and nouns: a third-person-singular pronoun – 'it' – can be used to refer to an object, but it must pick up its referent from the context in which it is used.)

(3) Can be used anaphorically – or, near enough, different occurrences of a prosentence in the same sentence can have the same antecedent.[11]

But we may ask: What reason have we to believe that anything has all the properties in the list or even that those properties are mutually consistent? We need some reason to believe that prosentences exist, or at least that the defining properties of a prosentence are consistent. (If the defining properties of a prosentence are consistent, then, even if prosentences do not in fact occur in English or other natural languages, they could easily be added to the vocabulary of a language by stipulation: one would simply pick some class of words or phrases – preferably ones that do not already occur in the language – and stipulate that its members have the defining properties of prosentences.) I, at any rate, am sufficiently sceptical about the linguistic category "prosentence" that I should want to see an argument for the conclusion that the defining properties of prosentences were consistent before I was willing to accept any theory that made essential use of this category. I know of no other way to show that the list of defining properties of prosentences is consistent than to provide an example of a prosentence. Can this be done?

Grover has endorsed Joseph Camp's suggestion that 'it is true' (or 'that is true') has the defining properties of a prosentence, and is therefore a prosentence of English.[12] Will this example do? Does it prove that there are English prosentences? I doubt whether it does. I will try to explain my doubts. I begin with an observation: even if we have a prosentence, we shall have to have some way of connecting its occurrences in a sentence with their intended antecedents if we are to be able to understand the sentences that contain arbitrarily many sentential quantifiers and arbitrarily many sentential-position variables. In *Being* and "Meta-ontology" (see note 10), I solve the analogous problem for 'it' by supplying 'it' with an indefinite stock of subscripts. (That is, 'it_x', 'it_y', and so on. Thus, the second occurrence of 'it' in the quantifier-phrase 'it is true of

10 I say "recognizing that" because I agree with this view of nominal quantification – or, as I prefer to say, quantification *tout court*, quantification full stop, quantification *period*. I defend this understanding of quantification in Chapter 1 of *Being: A Study in Ontology*, forthcoming from Oxford University Press, and, more briefly, in "Meta-ontology," *Erkenntnis* 48 (1998), pp. 233-250.

11 See "Propositional Quantifiers" in Dorothy Grover, *A Prosentential Theory of Truth* (Princeton: Princeton University Press, 1992), pp. 46-69. (This essay was originally published in the *Journal of Philosophical Logic*, in 1972.) The defining properties of a prosentence are set out on pp. 52-53.

12 See the "Introductory Essay" in *A Prosentential Theory of Truth* (pp. 3-45), p. 12.

everything that it_x is such that' is an antecedent of appropriately placed occurrences of 'it_x', but not of occurrences of 'it_y' or 'it_z'.) But the "prosententialist" who tries to use subscripts in a like manner will face a problem that I did not face: where the subscripts are to be placed. This problem arises because 'it is true' (unlike 'it') has a grammatical structure, and thus affords more than one "site" at which a subscript might be placed. There seem to be two serious candidates for the placement-site for the subscript, which we may illustrate as follows:

it_p is true

and

$(\text{it is true})_p$.

That is, we might attach the subscript to the (apparent) grammatical subject of the prosentence-candidate, or we might attach it to the prosentence-candidate "as a whole." (To attach it to the copula would seem to be simply a variant on the second option: to attach it to the copula would be a way of attaching it to 'it is true' "as a whole": after all, the copula in a subject-predicate sentence is a mere grammatical convenience, one that many languages manage very nicely without. And I can't see what could be intended by attaching a subscript to 'true'.)

Let us suppose that we have chosen the first option: we will attach our subscripts to 'it'. Are we now in a position to translate (say) '$\forall p\ (p \vee {\sim}p)$' into what Grover has called "philosopher's English"? (If we cannot succeed in this simple "one-variable" case, there will be no point in proceeding to more complex examples.) Well, we know how to translate '$p \vee {\sim}p$':

it_p is true v ~it_p is true.

But how are we to translate '$\forall p$'? How are we to translate the whole sentence '$\forall p\ (p \vee {\sim}p)$'? Someone might offer:

For all p, it_p is true v ~it_p is true.

But, to my mind, at least, this will not do. This leaves us with an untranslated variable in the quantifier-phrase. How are we to translate 'For all p'? How, that is, are we to write the quantifier-phrase so that it contains not 'p' but 'it_p is true'. (In my account of "nominal" quantification in *Being* and "Meta-ontology," it is 'it_x' and 'it_y' that occur in quantifier-phrases – not 'x' and 'y'.) I do not see any way to do this. It is, of course, easy enough to get 'it_p' into the quantifier-phrase. We could read 'for all p' as

it is true of everything that it_p is such that.

But in this phrase, the subscripted pronoun is not followed by the predicate 'is true': the supposed prosentence 'it_p is true' does not occur in this phrase. This reading of 'for all p' would seem, therefore, to leave us with good, old-fashioned nominal quantification. If we so read the quantifier-phrase, then the sentence '$\forall p\ (p \vee {\sim}p)$' reads as follows in "philosopher's English":

It is true of everything that it_p is such that it_p is true v ~ it_p is true.

The meaning of a sentence containing subscripted pronouns can hardly be supposed to depend on the particular symbols that are employed as pronoun-subscripts.[13] This sentence, therefore, is equivalent to

It is true of everything that it_x is such that it_x is true v ~ it_x is true.

That is to say:

$\forall x$ (x is true v ~x is true).

So interpreted, the language of prosentential quantification seems to be no more than a rewriting of the language of nominal quantification over sentences or propositions – for only sentences or propositions satisfy 'x is true'. And a rewriting of only a *part* of this language, for there is nothing in the language of sentential quantification that corresponds to the identity-sign. For this reason, there are things that can be said by those who are willing to combine the identity-sign and quantification over propositions that cannot be said by those who restrict themselves to that part of the language of quantification over propositions that can be translated into the language of sentential quantification. For example: 'For any contingent proposition, there is some distinct contingent proposition that it entails'.[14]

Perhaps, however, these untoward results are a consequence of the fact that we chose to attach the differentiating subscript to 'it' in 'it is true', thus (it might be argued) forcing 'it' to behave as a pronoun, contrary to the intention of the proponents of sentential quantification that 'it' should be an inseparable part of the prosentence 'it is true', a part that has no syntactical properties (as the two characters that make up 'it' have no syntactical properties). Let us now turn to the other option we mentioned. Let us suppose that the differentiating subscripts are to be attached to the prosentence-candidate "as a whole," that is, in the following fashion:

$(\text{it is true})_p$

$(\text{it is true})_q$.

(In the following discussion of this suggestion, I shall drop the round brackets; but it must be understood that throughout this discussion the 'p'- and 'q'-subscripts apply to 'it is true' and not simply to the adjective 'true'.) Then the above objections have no force against the idea of understanding sentential-position variables as prosentences. But we are still faced with the problem of how we are to formulate quantifier-phrases.

Let us pose the problem this way. Consider the formula

$\forall p\ (p \vee {\sim}p)$.

13 Cf. the principle of the equivalence of alphabetic variants ('$\forall x\ x = x$' and '$\forall z\ z = z$', for example) in formal logic.

14 I mention this only as a problem. It might be possible to solve this problem by introducing some "sentential counterpart" to '=' – something that does the work in respect of sentential quantification that '. . . and . . . are propositions & . . . = . . .' does in respect of nominal quantification and has the syntax of a sentential connective. Such a connective might be introduced as equivalent to '$\Box$ (. . . $\leftrightarrow$ . . .)' or to 'necessarily, to assert that . . . would be to assert that . . . and vice versa'.

How is the whole of this formula to be turned into English – or into a supplemented English that contains phrases like 'it is true_p' and 'it is true_q'? That is to say, with what expression of generality may we prefix

it is true_p or it is not the case that it is true_p

so as to produce something that (a) is a sentence of (supplemented) English, and (b) seems intuitively to express what is supposed to be expressed by '$\forall p\ (p \vee -p)$'?

Grover generally uses the following as a universal quantifier phrase: 'for any proposition'. (Thus, she might offer 'For any proposition, it is true or it is not the case that it is true' as an English reading of '$\forall p\ (p \vee -p)$'. Let us leave aside the charge that this *looks* a lot more like an English reading of '$\forall x$ (x is a proposition $\rightarrow$ (x is true $\vee$ $-x$ is true))' than an English reading of '$\forall p\ (p \vee -p)$'. How something looks is, after all, a matter of subjective judgment.) This quantifier-phrase will do only in the simplest cases, the one-variable cases. Since it offers no way of distinguishing one universal quantifier phrase from another or one existential quantifier phrase from another, it will not do in cases involving multiple generality. We have to be able to distinguish, say,

$\forall p\ \exists q\ (p \rightarrow q)$

from

$\forall q\ \ \exists p\ (p \rightarrow q)$,

and 'For any proposition there is a proposition' does not enable us to distinguish '$\forall p\ \exists q$' from '$\forall q\ \exists p$'. We need some way to "connect" each quantifier-phrase with a particular occurrence of a prosentence. There is, of course, always the "brute force" method:

For any proposition_p, there is a proposition_q such that if it is true_p then it is true_q.

But, really, what does this *mean*? We might try to make sense of it by supplementing English with lots of exact synonyms for 'proposition', perhaps by the ever-useful "subscript" method: 'proposition_p', 'proposition_q', and so on. Would this serve to establish an intelligible connection between quantifier-phrases and the sentential-position variables they are supposed to bind? I think not. By way of illustration, let us suppose that 'proposition' and 'thesis' are exact synonyms. And let us suppose that 'it is $\text{true}_{\text{proposition}}$' and 'it is $\text{true}_{\text{thesis}}$' are distinct prosentences. Having made these suppositions, we might try to distinguish our two sentences by providing the following readings for them:

For every proposition, there is a thesis such that if it is $\text{true}_{\text{proposition}}$ then it is $\text{true}_{\text{thesis}}$

For every thesis, there is a proposition such that if it is $\text{true}_{\text{proposition}}$ then it is $\text{true}_{\text{thesis}}$.

But I confess that these two sentences make little sense to me. (Well, to be frank, no sense at all.) 'Proposition' and 'thesis' both have meanings (even if their meanings are the same): they are not mere indices. It seems to me to be absurd to suppose that these

two sentences are sentences of a transformed, regimented English between which there is a clear difference of sense (or even an inchoate difference of sense that one might have some hope of making explicit and precise).

Let us return to the question we have posed and to which we have so far failed to find an answer: with what expression of generality may we prefix

it is true_p or it is not the case that it is true_p

so as to produce something that (*a*) is a sentence of (supplemented) English, and (*b*) seems intuitively to express what is supposed to be expressed by '$\forall p\ (p \vee -p)$'? It should be evident from the foregoing that any prefix that satisfies these conditions will have to contain 'it is true_p'. But let us make things as easy as possible for ourselves. Let us take a step backward and neglect the subscripts; let us ask this question with respect to 'it is true or it is not the case that it is true'. (If we can find no answer in this case, there will be no point in going on to consider multiple generality.) I will examine two possible answers. (In each case, I will write out the whole sentence and italicize the suggested prefix.)

1. *Under whatever circumstances it is true*, it is true or it is not the case that it is true.

But this cannot be right. For one thing, it doesn't make sense, or doesn't make sense unless 'it' (which stubbornly insists on behaving as if it were a pronoun and the subject of the verb 'is') can pick up a referent from its context. "Under whatever circumstances *what* is true?", one wants to ask. For another, if this suggestion confers truth on '$\forall p\ (p \vee -p)$', it seems to be just as happy to confer truth on '$\forall p\ p$' – for if one does manage to assert something by saying, "Under whatever circumstances it is true, it is true," what one manages to assert will certainly be true. This second problem is avoided by the following answer to our "prefix" question:

2. *Whether it is true or not*, it is true or it is not the case that it is true.

Someone who managed to assert something by saying this would no doubt say something true, and someone who said, "Whether it is true or not, it is true," would, or so I should think, say something false if 'it', in the context of utterance, referred to a false statement. But the first problem is intractable by any variant on this suggestion. The word 'it' insists on behaving like what it is: an ordinary pronoun, and, if the sentence is uttered in a context that fails to supply it with a referent, the person to whom it is addressed is going to want to ask, "Whether *what* is true or not?"

In the end, it seems that we do not have to go on to determine whether any suggestion along the present lines can be elaborated to solve the problem of multiple generality. Any such suggestion must fail even in the simplest case. The English replacement for the sentential quantifier phrase '$\forall p$' must contain 'it is true' if it is to interact in the appropriate way with the sentences to which it is prefixed (sentences in which 'it is true' occurs and allegedly functions as a prosentence). And in any such replacement, the phrase 'it is true' will stubbornly behave not like a syntactically structureless unit (as the second 'it' does in 'it is true of everything that it is such that') but like what it is: a phrase having a subject-copula-predicate structure. And any utterance of a sentence that starts with this replacement will therefore constitute an assertion only if the 'it' occurring in the replacement manages somehow to pick up a referent from the context of the utterance.

'It is true' is therefore not a prosentence – or if it is, then prosentences cannot *per se* be used to make sense of sentential quantification. (Maybe there are prosentences, hiding somewhere in the jungle of natural language, and perhaps they can be used to make sense of sentential quantification. But if there are natural-language prosentences, I have no idea what they might be.) If 'it is true' is not a prosentence, we are left without an example of a prosentence and are thus left without any reason to suppose that Grover's list of properties is consistent – without any reason to suppose that "prosentence" is a possible grammatical category. I therefore judge her attempt to explain sentential quantification to be an extremely interesting failure.

I conclude, tentatively, that it is at least extremely doubtful whether the idea of sentential quantification can be made sense of. But, as we have seen, the project discussed in Section I (finding the general theses of which homophonic truth-sentences are instances, and using these general theses to provide an eliminative definition of 'is true') depends essentially on the use of sentential quantification. It is therefore extremely doubtful whether this project can succeed.