"It Is Nonsensical to Speak of the *Total Number of Objects*"

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1.

In the *Tractatus*, Wittgenstein contends that *object* is a "pseudo-concept," and from this premise he draws several interesting conclusions: That one cannot say 'There are objects'; that one cannot say of any number that it is the number of objects; that it is nonsensical to speak of the total number of objects. Here is the core of his argument (the translation is the Pears and McGuinness translation):

> 4.1271 Every variable is the sign of a formal concept.
> For every variable represents a constant form that all its values possess, and this can be regarded as a formal property of those values.
>
> 4.1272 Thus the variable name 'x' is the proper sign for the pseudo-concept *object*.
> Wherever the word 'object' ('thing', etc.) is correctly used, it is expressed in conceptual notation by a variable name.
> For example, in the proposition, 'There are 2 objects which ...', it is expressed by '$(\exists x, y)$ ...'.
> Wherever it is used in a different way, that is as a proper concept-word, nonsensical pseudo-propositions are the result.
> So one cannot say, for example, 'There are objects', as one might say, 'There are books'. And it is just as impossible to say, 'There are 100 objects', or, 'There are $\aleph_0$ objects'.
> And it is nonsensical to speak of the *total number of objects*.[1]

My purpose in this paper is to examine and criticize the argument presented in this and in some related passages. Wittgenstein seems in the quoted passage to argue for two conclusions: that one cannot say 'There are objects' and that it is nonsensical to speak of the total number of objects. (I see no important difference between 'One cannot say of any

1. Proposition 4.1272 does not end at this point. See also 4.128 and 5.453. Proposition 4.128 reads

> Logical forms are *without* number. Hence, there are no privileged numbers in logic, and hence there is no possibility of philosophical monism or dualism, etc.

By 'monism' and 'dualism', Wittgenstein of course means the thesis that there is one object (as Spinoza held) and the thesis that there are two objects (a thesis that perhaps no one has held) – not the thesis that there is one *kind* of object (as materialists and idealists hold) and the thesis that there are two *kinds* of object (as Descartes held).

Uwe Meixner (Hg), *Metaphysik im postmetaphysischen Zeitalter / Metaphysics in the Post-Metaphysical Age.* S. 61-71.

number that it is the number of objects' and 'It is nonsensical to speak of the total number of objects'.) The former of these conclusions is stronger and perhaps more interesting than the latter, but I shall be primarily concerned to discuss Wittgenstein's theses on number. Nevertheless, the former conclusion – that one cannot say 'There are objects' – will not be neglected.

I begin by raising – only to dismiss as irrelevant to Wittgenstein's argument – three considerations that might lead one to say that one could not say how many objects there were.

(1) Suppose that one accepts the following two theses about sets. First, that the words 'set' and 'member' express, respectively, a unique attribute and a unique relation that everyone familiar with the language of set theory grasps, and that every sentence in the language of set theory, in consequence, expresses a determinate proposition – in the case of the simpler sentences, a proposition that everyone familiar with the language of set theory grasps. Secondly, that all the sentences in the language of set theory that are theorems of Zermelo-Fraenkel set theory express true propositions. One is then, as we may say, a "set-theoretical realist," and the set-theoretical realist will regard it as nonsensical to speak of the total number of objects – or, if not nonsensical at any rate demonstrably incoherent, since it is a theorem of Zermelo-Fraenkel set theory that for every number there is a set the number of whose members is greater than that number. We could state this theorem informally in these words: There are too many objects for them to be numbered. (This informal statement of the theorem presupposes that anything that is a member of a set is an "object.")

Whether these theses and their consequences are true or not, the sense in which they entail that there is no such thing as the number of objects is not the sense Wittgenstein intended to give to these words. Wittgenstein would certainly not have accepted anything remotely resembling set-theoretical realism. He would, moreover, have argued that 'There are more than 100 objects' was nonsensical for the same reason that 'There are 100 objects' was nonsensical – and the set-theoretical realist will certainly want to say that the former sentence expresses a truth. Since Wittgenstein's argument has nothing to do with set theory, I will ignore set-theoretical considerations in the body of this essay. (I will briefly return to the topic of set theory in my closing remarks.)

(2) Suppose that (as I have argued is at least possible[2]) there is such a thing as vague identity. That is, suppose that the following is possible: for some x and for some y, it is indeterminate whether $x = y$. If there were such a thing as vague identity, there would be no such thing as the total number of objects – at any rate, no number would be such that it was definitely the number of objects. Here is a simple case:

$\exists x\, \exists y$ (it is indeterminate whether $x = y$) and $\sim \exists x\, \exists y\, \exists z$ (it is indeterminate whether $x = y$ and it is indeterminate whether $y = z$ and it is indeterminate whether $x = z$) and $\sim \exists x\, \exists y\, \exists z\, (x \neq y\ \&\ y \neq z\ \&\ z \neq x)$.

2. *Material Beings* (Ithaca: Cornell University Press, 1990), § 18.

If we are asked what the number of objects is in any of the rather sparsely populated universes in which this sentence is true, we can say the following and no more than the following: it is indeterminate whether the number of objects is 1; it is indeterminate whether the number of objects is 2; for every other number (including 0), it is determinately true that that number is not the number of objects.

This point, however, is unrelated to Wittgenstein's argument. In the sequel, I will assume that when we raise the question whether there is such a thing as the total number of objects, we are asking this question about a universe in which identity is not vague. (Wittgenstein, as we shall see, denies that there is any such relation as identity. This is consistent with my assumption, for identity is vague only if there is such a relation as identity.)

(3) Suppose that "identity is always relative to a sortal term." Suppose, that is, that there is no such thing as identity *simpliciter*, but only a two-or-more-membered class of symmetrical and transitive relations each of which is expressible by a sentence of the form 'x is the same N as y', where 'N' represents the place of a sortal term (or at least that this is true of each member of the class that is expressible in English). And suppose that there are sortals M and N such that **"For some x and y, x is the same M as y and x is not the same N as y"** is true (I use bold-face double quotes as "quasi-quotation marks" – a typographically simpler replacement for "Quine corners").[3] If this is the case, then there is no such thing as counting or numbering *simpliciter*; there is only counting or numbering by Ns. Suppose for example, that there are two such relations, "is the same being as" and "is the same person as". Suppose that there exist an x, a y, and a z such that none of x, y, and z is the same person as the others, each of x, y, and z is the same person as itself [i. e., each is a person], and each of x, y, and z is the same being as the others; suppose, too, that everything has this feature: it is the same being as one of x, y, and z and it is the same person as one of x, y, and z.[4] In a universe whose population is given by this description, there is no answer to the question, 'How many objects are there?' (In fact, given that there is no such relation as identity, the question has no meaning in any universe.) The best one could do in reply to the question would be to say something like, "Well, how do you want me to count them? Counting objects by beings, there is exactly one. Counting objects by persons, there are exactly three." (There was no need to appeal to Trinitarian theology to make this point. The point could have been made in terms of 'is the same gold statue as' and 'is the same piece of gold as' – provided one was willing to say that x might be the same piece of gold as y but not the same gold statue as y.)

This point, again, is unrelated to Wittgenstein's argument. In the sequel, I will assume

3. This assumption depends on the more basic assumption that the following thesis is false: for any sortal M, **"For all x and for all y, if x is the same M as y and F...x..., then F...y..."** is true. Here 'F...x...' is a sentence in which 'y' does not occur and 'F...y...' is the result of replacing some or all of the free occurrences of 'x' in 'F...x...' with 'y'. Roughly speaking: if the analogue of the principle of the indiscernibility of identicals held for all "relative identities," one could not have a case in which relative identities did not "coincide."

4. I did not have to use English phrases like 'the others' and 'one of' to say this; all that is required to say it is the apparatus of quantifier logic, the usual sentential connectives, and the predicates '1 is the same being as 2' and '1 is the same person as 2'.

that when we raise the question whether there is such a thing as the total number of objects, we are assuming that there is no case in which x is the same M as y but not the same N; I will assume that statements of the form 'x is the same M as y' are to be understood as the corresponding statements of the form 'x is M and $x = y$' where '=' represents the classical identity-relation: the relation whose logical properties are given by these two conditions: it is reflexive; it forces indiscernibility – that is, if x is identical with y, then whatever is true of x is true of y. (Again, my assumption is consistent with Wittgenstein's thesis that there is no such relation as identity – that is to say, his assumption that the identity-sign is meaningless. If the only way to understand the expression 'x is the same M as y' is as a stylistic variant on 'x is M and $x = y$', and if the identity-sign is meaningless, all that follows is that 'x is the same M as y' is meaningless.)

So: we will assume that if there is no such thing as the number of objects, this is not because there are too many objects for them to be counted, and we will assume that identity is neither vague nor "relative to a sortal term." Given these assumptions, is there some reason to suppose that it is nonsensical to speak of the total number of objects? Let us first examine Wittgenstein's argument for this conclusion.

2.

I divide Wittgenstein's argument into two parts, the first ending with 'it is expressed by "($\exists x, y$) …".', and the second comprising the remainder of the passage I have quoted. Although I am not entirely sure I understand everything Wittgenstein says in the first part of the argument, it seems plausible to me to suppose that he is saying something very much like this:

> 'Object' is simply an unrestricted count-noun, a count-noun of maximal generality. An object is anything that can be the value of a variable, that is, anything that we can talk about using pronouns, that is, *anything*. These two points – that 'object' is an unrestricted count-noun and that an "object" is anything that can be the value of a variable can be combined in the following observation: the word 'object' is so used that any substitution-instances of the following pair of formulae are equivalent:
>
> $\forall x$ (x is an object $\rightarrow$ Fx) $\forall x$ Fx;
>
> and of the following pair:
>
> $\exists x$ (x is an object & Fx) $\exists x$ Fx.
>
> Thus, the word 'object' is a mere stylistic convenience: anything we can say using this word we can say without using it; it can be dispensed with in favor of variables – or pronouns.

If this is (more or less) what Wittgenstein is saying in the first part of the argument, then I (more or less) agree with him. But what is said in the second part of the argument does not

seem to follow from what is said in the first – nor does it seem to be true.[5] *Why* can one not say that there are objects?[6] Why not say it this way: '$\exists x\ x = x$'? And why can one not say 'There are at least two objects' like this:

$$\exists x\, \exists y \sim x = y,$$

or say 'There are exactly two objects' like this:

$$\exists x\, \exists y\, (\sim x = y\ \&\ \forall z\, (z = x \vee z = y))?$$

(Given sufficient patience and a wet Saturday afternoon, we could say 'There are at least 100 objects' and 'There are exactly 100 objects' in similar fashion.) The identity-sign would seem to express what Wittgenstein calls a formal concept, and, it would seem, it can be combined with quantifiers and variables (which express the formal concept "object") to say how many objects there are.

Wittgenstein's reply to this objection lies in his account of the identity-sign in 5.53-5.532, which in effect says that '=' means nothing and must be banished from a "correct conceptual notation." (Russell in his Introduction to the *Tractatus* (p. xvii) sees very clearly the connection between what Wittgenstein says about the total number of objects and what he says about the symbol '='.)

Let us examine what Wittgenstein says in 5.53-5.532. Proposition 5.53, on which the rest of the passage is a commentary, is

> Identity of object I express by identity of sign, and not by using a sign for identity. Difference of objects I express by difference of signs.

Now consider someone who has mastered the treatment of identity in any standard logic text of the present day, and who is reading the *Tractatus* for the first time. This reader will probably want to protest that is simply wrong to suppose that "difference of objects" *can* be expressed by "difference of signs" – or that statements apparently asserting the difference of objects can be translated into statements in which what had apparently been expressed by the negation of an identity-statement was expressed by using different signs. If we examine 5.532, which reads in part

> … I do not write '$\exists x\, \exists y\,$(Fxy & $x = y$)', but '$\exists x$ Fxx'; and not '$\exists x\, \exists y$ (Fxy & $\sim x = y$)', but '$\exists x\, \exists y$ Fxy'.

(I have replaced Wittgenstein's Principia notation with the logical notation of the present day), we shall see what troubles our imaginary reader: although the first two formulae are

5. Carnap seems to assert that the second part of the argument does not follow from the first (*The Logical Syntax of Language*, (London: Routledge and Kegan Paul, 1937), p. 295), but I am not sure I understand Carnap's point.
6. Wittgenstein says that one cannot say "'There are objects', as one might say, 'There are books'." I suppose him to be making the point that *book*, unlike *object*, is a real (not a pseudo-) concept.

logically equivalent, the second two are not. But this reaction on the part of the imaginary reader is premature, as the remainder of 5.532 shows. Wittgenstein is not proposing to replace a formula with a formula that is not logically equivalent to it; he is rather proposing an alternative logical notation. In the proposed "Tractarian notation," '$\exists x\ \exists y\ Fxy$' does not mean what it means in standard notation, but rather what '$\exists x\ \exists y\ (Fxy\ \&\ \sim x = y)$' means in standard notation. What is expressed by '$\exists x\ \exists y\ Fxy$' in standard notation is expressed in Tractarian notation by

$$\exists x\ Fxx.\ \vee\ \exists x\ \exists y\ Fxy.$$ [7]

Wittgenstein says no more than this about how Tractarian notation is to work, but it is not hard to extend the line of thought hinted at in 5.532. What is expressed in standard notation by '$\exists x\ \exists y\ \exists z\ Fxyz$' will be expressed in Tractarian notation by

$$\exists x\ Fxxx.\ \vee\ \exists x\ \exists y\ Fxyy.\ \vee\ \exists x\ \exists y\ Fxyx.\ \vee\ \exists x\ \exists y\ Fxxy.\ \vee\ \exists x\ \exists y\ \exists z\ Fxyz.$$

And what is expressed in Tractarian notation by '$\exists x\ \exists y\ \exists z\ Fxyz$' is expressed in standard notation by

$$\exists x\ \exists y\ \exists z\ (Fxyz\ \&\ \sim x = y\ \&\ \sim x = z\ \&\ \sim y = z).$$

The difference between standard and Tractarian notation has been usefully described by Hintikka in these words: the variables of standard notation are "inclusive"; the variables of Tractarian notation are "exclusive."[8]

Using exclusive variables, the sentence 'There are at least two chairs' may be rendered as

$$\exists x\ \exists y\ (x \text{ is a chair}\ \&\ y \text{ is a chair}),$$

and 'There are at most two chairs' as

$$\sim \exists x\ \exists y\ \exists z\ (x \text{ is a chair}\ \&\ y \text{ is a chair}\ \&\ z \text{ is a chair}).$$

'There are exactly two chairs' is then represented by the conjunction of these two formulae. The situation is similar with the universal quantifier. The Tractarian formula '$\forall x\ Fxx.\ \&\ \forall x\ \forall y\ Fxy$' replaces the standard '$\forall x\ \forall y\ Fxy$', and the Tractarian '$\forall x\ \forall y\ Fxy$' is equivalent to the ordinary '$\forall x\ \forall y\ (\sim x = y. \rightarrow Fxy)$'.

How far can this technique be extended? Consider the standard language of first-order logic, with the identity sign, but with no terms but variables. Can all closed sentences in this language be translated into Tractarian notation? Well, certainly not all. Not

7. In standard notation, '$\exists x\ \exists y\ Fxy$' is, of course, equivalent to $\exists x\ Fxx.\ \vee\ \exists x\ \exists y\ (Fxy\ \&\ \sim x = y)$.

8. Jaakko Hintikka, "Identity, Variables, and Impredicative Definitions," *The Journal of Symbolic Logic*, 21 (1956), pp. 225-245.

(1) $\exists x\ x = x$,

or

(2) $\exists x\ \forall y\ (Fx\ \&\ x = y)$.

But it would seem that all standard formulae that do not contain an "unpredicated" occurrence of a variable can be translated into Tractarian notation. (An occurrence of a variable is unpredicated if it occurs beside an occurrence of '=' and no occurrence of the same variable is one of the string of occurrences of variables following a predicate-letter within the scope of the quantifier that binds it. The second and third occurrences of 'x' in (1) are thus unpredicated; the second occurrence of 'y' in (2) is unpredicated. But the third occurrence of 'x' in (2) is "predicated" because another occurrence of 'x' follows the predicate-letter 'F', and that occurrence falls within the scope of the quantifier that binds the third occurrence of 'x'.) The reader is referred to the article by Hintikka cited in note 8 for systematic statement of rules for translating formulae in standard notation into Tractarian notation. Hintikka also gives systematic rules for translating formulae in Tractarian notation into standard notation. If a standard formula is translated into Tractarian notation by the first set of rules, and the resulting Tractarian formula translated into standard notation by the second set of rules, the result will not in general be the original standard formula; but it will always be provably equivalent to the original standard formula.

What is the philosophical meaning of this result? Let us call those formulae in standard notation that cannot be translated into Tractarian notation – like (1) and (2) above – "untranslatable." The standard formulae that I offered above as ways of saying 'There are objects' [$\exists x\ x = x$], and 'There are exactly two objects' [$\exists x\ \exists y\ (\sim x = y.\ \&\ \forall z\ (z = x \vee z = y))$] are untranslatable; each contains – in fact, quantifier phrases aside, contains only – unpredicated occurrences of variables.[9] But why should this trouble those who believe that the untranslatable formulae are meaningful? The fact that something cannot be translated into a particular notation does not necessarily mean that it is nonsense; it may mean only that that notation is deficient in expressive power. Wittgenstein does not argue that untranslatable formulae are meaningless simply because they cannot be translated into Tractarian notation. The general form of his argument is rather this: the identity-sign is strictly meaningless; therefore the statements in which it occurs can be meaningful only if it is a mere notational convenience, eliminable in principle. But why does Wittgenstein think the identity-sign is strictly meaningless? First, because the *Principia* definition of identity (an identity-of-indiscernibles definition: $x = y$ if and only if everything that is true of x is true of y and everything that is true of y is true of x) is inadequate, owing to the fact that it is at least *possible* for there to be two distinct things that share all their attributes (like Kant's two raindrops). Secondly,

9. 5.534 And now we see that in a correct conceptual notation pseudo-propositions like '$a = a$', '$a = b\ \&\ b = c. \rightarrow a = c$', '$\forall x\ x = x$', '$\exists x\ x = a$', etc. cannot even be written down.

(As before, I have replaced Wittgenstein's *Principia* notation with present-day notation.)

5.5303 Roughly speaking, to say of *two* things that they are identical is nonsense, and to say of *one* thing that it is identical with itself is to say nothing at all.

As to the first point, it could be debated at great length whether there could be two things that were indiscernible. I will not, however, enter into this debate. Let us suppose that Wittgenstein is right on this point. It can hardly be true that a sign that has no explicit definition is meaningless, or all signs would be meaningless. (Just as – as Wittgenstein said later in his career – explanations come to an end somewhere, so explicit definitions come to an end somewhere.) Why can we not simply define '=' as shorthand for 'is identical with' – perhaps coupled with an informal discussion of "numerical" versus "descriptive" identity? Or one might define identity as a universally reflexive relation that forces indiscernibility. Admittedly, for all anyone can say, there might be two relations that satisfied this definition; but the following is provable: if '$=_1$' expresses one of these relations, and '$=_2$' the other, then

$$\forall x\, \forall y\, (x =_1 y \leftrightarrow x =_2 y).$$

(The fact that there might be two such relations leads *me* to regard the last-mentioned definition as unacceptable. The definition does not, or at least may not, tell us what relation identity *is*. I am willing to accept the definition of '=' as 'is identical with' – a phrase we, as speakers of English, understand – and to say that "a universally reflexive relation that forces indiscernibility" embodies a correct theory of the logically valid inferences that can be made using 'is identical with'.) As to the argument of 5.5303, I think it suffices to point out that we do other things with 'is identical with' than say things like 'Tully is identical with Cicero and Tully and Cicero are two things' or 'Tully is identical with Tully'. I will not rest my case on the supposed informativeness of 'Tully is identical with Cicero', for Wittgenstein – at least this seems to me to be what he should say – will reply that the only informative proposition that this sentence could express is the proposition that something is called both 'Tully' and 'Cicero', and I do not wish to enter into the questions this reply would raise. I will point out instead that when one makes assertions using complex closed sentences that contain expressions like '$z = x$' and '$\sim y = z$', what one thereby does fits neither of the following descriptions: 'saying of two things that they are identical'; 'saying of one thing that it is identical with itself'. Whether or not sentences like '$\exists x\, \exists y\, (\sim x = y .\, \&\, \forall z\, (z = x \vee z = y))$' are meaningful, it is evident that the person who uses them can neither be said to be asserting of two things that they are identical nor said to be asserting of something that it is identical with itself.

A parenthetical remark: I am convinced that Tractarian notation can be explained only in terms of standard notation – that the concept of an exclusive variable can be grasped only by someone who has a prior and independent grasp of the concept of numerical non-identity. But any way I can think of to argue for this conclusion would be called (and probably rightly) circular, so I shall not press this point.

Let us concede, for the sake of the argument, that Wittgenstein is right, and that '=' is strictly meaningless. What follows? It does not follow that one cannot say that there are objects or cannot say how many objects there are – although it does of course follow that one cannot employ '=' in any essential way in making these assertions. Some other device will be needed. No such device can be found within the language of pure logic, for the only formulae of pure logic that can (on anyone's account) be used to make assertions contain the identity-sign: this two-place predicate is the only predicate that belongs to the language of logic, and one cannot make assertions without using predicates. (The capital roman letters that precede strings of variables in the language of pure logic are not predicates, but predicate-*letters*, place-holders for predicates. '$\forall x\, (x = x)$' is true – in my view, if not in Wittgenstein's – and can be used to make an assertion, the assertion that everything is identical with itself. '$\forall x\, Fx$' has no truth-value and cannot be used to make an assertion.)

To assert the existence of and to count "objects," we need an open sentence that everything satisfies. All such sentences that the language of pure logic affords – in fact, all sentences of any kind that the language of pure logic affords – contain '='. But the language of "impure" logic, which comprises sentences in which quantifiers bind variables that occur in expressions containing words of some natural language – afford an unlimited supply of open sentences that are satisfied by everything. And there is no rule that restricts us to the language of pure logic. Here is a proposal for representing the assertion 'There are objects' in which the formal concept "object" is expressed by means of variables:

$$\exists x\, (x \text{ is a chair} \vee \sim x \text{ is a chair}).$$

What objections can be brought against this proposal? There is, first, an aesthetic objection: the choice of the predicate 'is a chair' is entirely arbitrary, and thus the definition can hardly be put forward as a paradigm of elegance. But aesthetical deficiencies really do not bulk much larger in judging philosophical theses than they do in judging plans for rescuing a child trapped in a well: the only really important question in either case is: Will it work?

It is possible that someone will object to this proposal on the ground that 'is a chair' is a vague predicate, and that the vagueness of 'x is a chair' is inherited by 'x is a chair $\vee \sim x$ is a chair' (it may be that something that is a borderline-case of "chair" does not determinately satisfy 'x is a chair $\vee \sim x$ is a chair'; determinately to satisfy this predicate, it might be argued, is determinately to satisfy one or the other of its disjuncts). And vagueness causes problems for counting. We need not examine the merits of this objection, however, for there are predicates that are determinately satisfied by everything, whether or not 'x is a chair $\vee \sim x$ is a chair' is one of them. Here is one: 'x is evenly divisible by 3 $\vee \sim x$ is evenly divisible by 3'. (I take it that '$\sim$ Frederick the Great is evenly divisible by 3' expresses a truth. Frederick certainly does not belong to the extension of 'x is evenly divisible by 3'.) Since this "out" is available, I will assume in what follows that 'x is a chair $\vee \sim x$ is a chair' is determinately satisfied by everything. Having this predicate at our disposal, we can say, for example, 'There are exactly two objects' using only language that meets all of Wittgenstein's requirements. Using Tractarian notation (that is, exclusive variables) we render 'There are at least two objects' as follows:

$$\exists x\, \exists y\, (x \text{ is a chair} \vee \sim x \text{ is a chair}. \,\&\, .y \text{ is a chair} \vee \sim y \text{ is a chair}).$$

And we render 'There are at most two objects' as

$\sim \exists x \exists y \exists z$ (x is a chair $\vee \sim x$ is a chair. & .y is a chair $\vee \sim y$ is a chair.
& .z is a chair $\vee \sim z$ is a chair).

'There are exactly two objects' is, of course rendered as the conjunction of these two sentences.

"To say, 'Frederick the Great is a chair v ~ Frederick the Great is a chair' is to say nothing at all." (*Cf.*, "... to say of *one* thing that is identical with itself is to say nothing at all." See 5.513.) Well, it's certainly to say nothing at all controversial. ("So tell your Papa where the Yak can be got,/And if he is awfully rich,/He will buy you the creature – /Or else he will not./I cannot be positive which." – Hilaire Belloc.) But it is not to say nothing at all in the sense in which to utter any of the following sentences would be to say nothing at all:

It is now five o'clock on the sun
Boggle the main franistan
Das Nichts nichtet
Turn right, and the entrance is diagonally opposite, by the next street.[10]

Instances of the law of the excluded middle frequently appear as (obviously meaningful) lines in mathematical proofs – e.g., 'The number N is either prime or it is not prime; in the latter case, it is not the greatest prime; in the former, it is also (as we have seen) composite, which is a contradiction; Therefore, there is no greatest prime.' I conclude that there is no merit in this objection.

We could, therefore, had we but world enough, and time, write a sentence that expressed any of the propositions in the following infinite sequence:

There are no objects
There is exactly one object
There are exactly two objects
There are exactly three objects
⋮

Each of these sentences is meaningful and has a truth-value. At most one of them is true. If one of them is true, it gives the number of objects. If none of them is true, then there are infinitely many objects. In the latter case, if we allow ourselves the apparatus of transfinite numbers (if we allow ourselves, say, the language of Zermelo-Fraenkel set theory) we can say, for any number x, that the number of objects is x; we say: There is a set of objects such that x is the number of that set's members, and no number greater than x is the number of the members of any set of objects. And any such statement will be meaningful and will have a truth-value, since every set has a cardinal number and every cardinal number is either the cardinal number of a given set or it isn't. (If we do not allow ourselves the apparatus of transfinite numbers, and if all the sentences in our sequence are false, then admittedly we can't say what the number of objects is. But we can say that they are numberless: that for any number x, there are at least $x + 1$ objects. And this will be a statement that is true without qualification. But, if Wittgenstein is right, this statement is as meaningless as 'There are exactly 100 objects'.) Now if "object" is, as we have been supposing, a formal concept – so that sets and numbers are by definition objects – all such sentences will be false, owing to the fact that no distinction can be made between a "set of objects" and a set tout court, and for every set there is a set that has a greater number of members than it does. This point was conceded at the outset. But suppose we restrict the question of the number of objects to "individuals," to objects that are not sets (or, if we distinguish sets and classes, not classes).[11] Suppose – and this seems a very reasonable supposition – that the idea of the set of all individuals does not involve a logical paradox (suppose, that is, that the condition 'x is not a set' does not conceal some nasty logical surprise like the one concealed in 'x is not a member of x'). Suppose – and this, too, seems a very reasonable supposition – that there *is* a set of all individuals, that there are not "too many" individuals for them to form a set.[12] Then – we continue to assume that identity is not vague and that identity is not "always relative to a sortal term" – some number must be the number of individuals. After all, every set has a cardinality.

It seems, therefore, that if we make certain assumptions that (however questionable they may be) are irrelevant to considerations raised in Wittgenstein's argument, it follows that the conclusion of that argument is false: one can sensibly speak of the total number of objects, for there is such a thing as the total number of objects, and, by asserting its existence, I have spoken sensibly of it.[13]

10. Suppose that the second of these sentences is uttered by someone as a parody of nautical terminology. The fourth is from Stephen Potter's *Lifemanship*. It is recommended for inclusion in a letter giving instructions to a visiting team on how to find the home team's playing field. It has, according to Potter, the following annoying virtue: the fact that it is meaningless will not become evident to the visiting team till they have exhausted themselves in the attempt to find the home team's playing field. (David Chalmers has pointed out to me that Potter's sentence is meaningful if 'by' means 'beside'. Well, okay.)

11. Perhaps we had also better assume that "individuals" are not things sufficiently "like" sets to generate a universe as rich as the universe of sets – attributes and relations-in-intension and such.
12. David Lewis's "extreme modal realism" is consistent with there being too many individuals for them to form a set. (It is, in fact, consistent with there being too many pigs for them to form a set.) I know of no other metaphysic that is. Lewis has suggested to me that if there were too many individuals (or pigs) for them to form a set, there would nevertheless be an answer to the question, "How many individuals (or pigs) are there?"; the answer would be, "There are proper-class-many." Perhaps this is true. But if it is true, it is also true that there is no such thing as the number of individuals (or pigs).
13. This essay is a truncated version of the paper "The Number of Things," which I read at the Twenty-Second Wittgenstein Symposium. The larger paper contained, besides the matter of the present essay, a discussion of an argument of Hilary Putnam's (in *The Many Faces of Realism*) for the conclusion that there is no fact of the matter as to the number of objects. "The Number of Things" is a adaptation of a chapter of my forthcoming book *Being: A Study in Ontology*. The material on Wittgenstein in this chapter—more or less the present essay – was inspired by a paper read by Professor Edmund Runggaldier at a conference on analytical ontology held in Innsbruck in September 1997.