

PETER VAN INWAGEN

## WHY I DON'T UNDERSTAND SUBSTITUTIONAL QUANTIFICATION

(Received, in revised form, 8 August, 1980)

### I

My difficulty with the notion of substitutional quantification is a very simple – one might almost say simple-minded – one. Yet the friends of substitutional quantification, or those I have talked to, have not been able to resolve this difficulty in conversation. With some trepidation, therefore, I venture to write down my simple problem.

Let us use 'Σ' to designate the existential – or 'particular' – substitutional quantifier. (I shall assume at the outset that my readers agree with me on one fundamental point: it is not the case that there is something called 'the existential – or particular – quantifier' of which philosophers of logic have offered two 'interpretations' or 'readings', *viz.* the objectual or referential interpretation and the substitutional interpretation. It would be better to say that there are two quantifiers, two distinct variable-binding operators: the objectual or referential and the substitutional.)

If I could understand the sentence

S         $\Sigma x x$  is a dog

then I could understand substitutional quantification. But I cannot understand this sentence. I cannot understand it because I do not know what proposition it expresses. I do not know what proposition it expresses for this reason: those whose business it is to define substitutional quantification have attempted to do this by giving 'truth-conditions' for *S* and other sentences containing the substitutional quantifiers. That is, they have attempted to do this by explaining under what conditions whatever proposition it is that is expressed by *S* (to take a particular case) is true. Now I understand these truth-conditions, at least in a rough-and-ready sort of way. My (main) problem with substitutional quantification is not one of interpreting these truth-conditions. It is rather this: I know of a proposition that has just these

*Philosophical Studies* 39 (1981) 281–285. 0031-8116/81/ 0393-0281\$00.50  
Copyright © 1981 by D. Reidel Publishing Co., Dordrecht, Holland, and Boston, U.S.A.

truth-conditions, and those whose business it is to explain substitutional quantification tell me that the proposition that *S* expresses is not *that* proposition. And nothing more will they say. Thus, my position with respect to *S* is what yours would be with respect to the sentence

John cissed Jane

if I told you just these two things and no more:

Whatever proposition is expressed by 'John cissed Jane', it is true iff John kissed Jane

'John cissed Jane' does *not* express the proposition expressed by 'John kissed Jane'

## II

I do not mean anything mysterious by 'proposition'. I use this word as a general term for the things people *assent to, reject, find doubtful, accept for the sake of argument, attempt to verify, deduce things from*, and so on. (Some of the phrases on this list take more than one sort of object. One may, for example, reject not only propositions but bribes. I hope no one is going to be difficult about this.) We have plenty of 'specialized' words for propositions in the language of everyday life, just as we have plenty of specialized words for human beings. On various occasions we call propositions 'doctrines', 'theses', 'theories', 'premises', 'conclusions', 'theorems', 'views', 'positions', 'conjectures', 'hypotheses', 'dogmas', 'beliefs', and 'heresies', just as, on various occasions we call human beings 'women', 'babies', 'thieves', 'admirals', 'Trotskyites', 'Australians', and 'Catholics'.

It is thus uncontroversial that there are propositions. The only question that could arise is: What *are* propositions? Many philosophers apparently think that propositions are sentences, since they think that sentences are what is true or false, and it is evident that those things that are true or false are just the things that are the objects of the activities and states listed above. But I can make no sense of the suggestion that propositions are sentences, and I shall not discuss it further. It is true that I am willing on occasion to speak of sentences being true or false, but this is only shorthand. When I say that a given sentence is *true*, I mean that the proposition – a non-sentence – that that sentence expresses is true. (To say that an English

sentence *expresses* a given proposition is to say, roughly, that the result of concatenating 'the proposition that' and that sentence *denotes* that proposition.) Similarly, when I say that a name is *honorable*, I mean that the individual or family that bears that name is honorable. I can no more understand the suggestion that a sentence might be true otherwise than in virtue of its expressing a true proposition that I can understand the suggestion that a name might be honorable otherwise than in virtue of its being borne by an honorable individual or family.

III

Let us call a phrase that purports to designate an object a *term*. Thus, 'Napoleon', 'Pegasus', 'the father of King Arthur', ' $2 + 2$ ', and 'the odd prime' are terms. 'All men', 'something', 'just one cat', and 'the square of  $x$ ' are not terms.

The friends of substitutional quantification tell us this much and no more about the meaning of  $S$ :

TC     ' $\Sigma x x$  is a dog' is true [i.e., whatever proposition is expressed by ' $\Sigma x x$  is a dog' is true] iff  $\exists x x$  is a term and ' $x$  is a dog' is true [expresses a true proposition].

Let us say that a sentence *has the same truth-conditions* as  $S$  if the result of replacing  $S$  with that sentence in TC is a truth. Thus — since there *is* a term that, when concatenated with 'is a dog', yields a sentence that express a truth — 'Grass is green' and ' $2 + 2 = 4$ ' have the same truth-conditions as  $S$ . I am *tempted* to say that this fact shows that what the friends of substitutional quantification have told us about  $S$  has been insufficient to rule out the possibility that  $S$  expresses the proposition that grass is green; I am *tempted* to accuse the friends of substitutional quantification of having, for this reason alone, told me nothing about what  $S$  means.

I shall resist these temptations. If I yielded to them, the friends of substitutional quantification would point out that TC is an instance of a *general* statement of truth-conditions for sentences containing ' $\Sigma$ ' and that, in virtue of this general statement, the left-hand constituent of the biconditional TC is *systematically* related to the right-hand constituent of TC by a rule that does not relate any *other* true sentence of the form ' $p$  is true' [e.g., "Grass is green" is true] to the right-hand constituent of TC.

I am not sure how effective this reply is. In fact I am not quite sure I see why it is a relevant reply to what I am tempted to say. But, since I wish to avoid this issue (for the good reason that I have nothing new to say about it), I shall steadfastly resist temptation.

I therefore do not accuse the friends of substitutional quantification of giving an explanation of 'Σx x is a dog' that fails to rule out the possibility that this sentence expresses the proposition that grass is green. But I do want to accuse them of something rather like this. Consider the proposition that  $\exists x$  x is a term and  $\lceil x$  is a dog $\rceil$  is true. Call this proposition '∃'. Suppose I use *S* to express ∃. (One way for me to do this would be for me to treat *S* as an *abbreviation* for the right-hand constituent of TC.) Am I doing anything wrong from the point of view of the friends of substitutional quantification? Should I expect them to accuse me of misusing the symbol 'Σ'? One might suppose not. After all, the proposition ∃ is true under just exactly the conditions they specify as the truth-conditions for *S*. Moreover, as is not the case with the proposition that grass is green, the fact that ∃ is true under just those conditions is not a contingent, empirical matter.

Nevertheless, the friends of substitutional quantification would deny that *S* expresses ∃. Alex Orenstein, for example, in his recent book *Existence and the Particular Quantifier*,<sup>1</sup> emphatically denies that "substitutional quantification is a metalinguistic species of objectual/referential quantification". (p. 34f.) This denial, I think, places Orenstein in the standard tradition of interpreting substitutional quantification, a tradition that can be traced to Ajdukiewicz.<sup>2</sup> I take this tradition to be sufficiently 'standard' that anyone who departs from it is proposing a *deviant* use for 'Σ'. In any case, it is Orenstein's use of the substitutional quantifier, whether it is standard or deviant, that I am unable to understand: I understand objectual quantification over terms and predicates as well as the next philosopher. It is true that some recent writers appear to conflate substitutional quantification and objectual quantification over linguistic objects. Susan Haack in her *Philosophy of Logics*<sup>3</sup> states the 'substitutional interpretation' – that this way of talking evidences a mistaken view of the relation between the substitutional and the objectual quantifier is not to the present point – of the existential quantifier in these words:

' $(\exists x)Fx$ ' is interpreted as 'At least one substitution instance of 'F...' is true'.

But I think that the most probable explanation for this statement is that Haack was expressing herself loosely.

' $\Sigma x x$  is a dog', therefore, does not express  $\exists$ . But then what proposition does it express? I do not know. No one knows. No one knows because no one has ever *said*. I know of only one proposition –  $\exists$  – that one might reasonably suppose (given TC) to be the proposition  $S$  expresses, and I am told by those who know what proposition  $S$  expresses if *anyone* knows what proposition  $S$  expresses that  $\exists$  is *not* the proposition  $S$  expresses. Thus my position with respect to  $S$  is just what your position is with respect to the sentence 'John cissed Jane' that was mentioned in Section I. That is, I do not understand  $S$ . And neither, I think, does anyone else. But then neither I nor anyone else understands substitutional quantification.<sup>4,5</sup>

*Syracuse University*

NOTES

<sup>1</sup> Philadelphia, Temple University Press: 1978.

<sup>2</sup> See Orenstein, *loc. cit.*

<sup>3</sup> Cambridge, Cambridge University Press: 1978. See p. 42.

<sup>4</sup> I have been asked why, if the arguments of this paper are correct, parallel reasoning does not show that no one understands *objectual* quantification. After all, isn't the meaning of the objectual quantifiers specified by and only by statements like this one: ' $\exists x x$  is a dog' is true iff at least one object satisfies ' $x$  is a dog'? The answer to this question is quite simply No. Our understanding of the (objectual) quantifier-variable idiom resides in our ability to translate sentences couched in it into quantificational idioms of which we have a prior grasp, *viz.* those of ordinary language. Or at least this reply will do as a first approximation. The case is complicated by the fact that the very limited resources for cross-reference in ordinary English probably render it impossible to translate any really complex sentence in the quantifier-variable idiom unambiguously into ordinary English. But, despite this *practical* difficulty, there would seem to be a difference *in principle* between the substitutional quantifier and the objectual quantifier as regards our ability to say, using idioms of which we have a prior grasp, what proposition sentences containing these operators express. At least in the case of relatively uncomplex sentences containing ' $\exists$ ', we *can* say what proposition they express. For example, ' $\exists x x$  is a dog' expresses the proposition that there is at least one dog. (Someone who knew that ' $\exists x x$  is a dog' expressed *some* proposition that was true iff at least one thing satisfied ' $x$  is a dog', but did not know *which* of the many propositions conforming to this requirement it expressed, would *not* understand objectual quantification.) But *no one* can say what proposition ' $\Sigma x x$  is a dog' expresses.

<sup>5</sup> The ideas defended in this paper bear a striking resemblance to some of those presented by William G. Lycan in his fine paper 'Semantic competence and funny functors', *The Monist* LXIV (1979), which I highly recommend to anyone interested in the general topic of the relationship between knowing what a sentence means and knowing its truth-conditions.