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EUREKA

THE ARCHIMEDEANS' JOURNAL

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EUREKA

THE JOURNAL OF THE ARCHIMEDEANS

(The Cambridge University Mathematical Society: Junior Branch of the University Mathematical Association.)

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Editorial

THE editing of EUREKA is done in Cambridge, but this year we have been fortunate in having a member of London University on our Committee. The Invariants Society at Oxford has sent us its ideas and its contributions, and in this issue we welcome contributions from Reading University, from King's College, London (now at Bristol), and from Princeton, U.S.A. We hope that EUREKA will find its way even further in the student world and that future issues will provide a forum for the discussion of common interests among mathematical students of all British universities. We feel that we should all benefit from such an exchange of ideas and we hope that those who are interested will send us their opinions and their contributions.

We hope also that EUREKA, though primarily a student magazine, will continue to be of interest to the senior members of our universities, whose generous support has enabled us once more to publish EUREKA and whose contributions give value to its pages. To them we would express our very great appreciation.

We would also thank the printers whose willing and valuable assistance does much to lighten the task of the editors.

What Quantum Theory is About

By Prof. P. A. M. Dirac

Classical mechanics, based on Newton's laws of motion and Maxwell's electrodynamics (with Einstein's amendments to make it conform to relativity), has the feature that it does not fix the scale on which events take place. It allows us to imagine worlds, like Gulliver met with in his travels, where everything is greater by a definite multiple, or smaller by a definite fraction, than what we know. This is because the constants of Nature that it involves are not sufficient for one to be able to construct in terms of them constants with the dimensions of distance, time, mass, or electric charge.

Now there are many phenomena in Nature for which a change in scale is impossible. There must, then, be more constants of Nature than those involved in the classical laws. The way in which these new constants enter into the description of natural phenomena forms the subject matter of quantum theory.

The easiest type of quantum law to imagine is one which says that a certain physical quantity, such as, for example, mass or electric charge, is always found in Nature in integral multiples of a certain unit or *quantum*. This quantity is then said to be *quantised*. Such a law is found experimentally to be true for electric charge, and also for mass (understood as rest-mass in the relativistic sense), if one makes the generalisation of allowing various units, corresponding to the various elementary particles, instead of having just one. However, these quantum laws are not the important ones in present-day quantum theory. In fact, they have not even been satisfactorily welded into the main body of present-day quantum theory.

The important quantum laws are concerned with the motion of electrons and other small particles, and involve a constant of Nature h having the dimensions of action. This constant was introduced by Planck in getting his law of black body radiation, and while physicists soon realised that it must be of fundamental importance for quantum theory, they had great difficulty in finding out just how to use it. The obvious idea, namely, to assume that action can occur in Nature only in integral multiples of h, is a very obscure one to work with, on account of action as ordinarily defined being a thing which varies continuously, and it was not until Bohr found a reasonable way, in the case when the motion is periodic, of defining the amount of action which is to be quantised, that the theory took definite shape and the subject of quantum mechanics was started.

Bohr's theory quickly achieved remarkable success, but soon its limitations appeared and became troublesome. It was found necessary sometimes to introduce half-quanta to get agreement with experiment, and no method could be thought of which would extend the theory to motions which are not periodic or multiply periodic.

The situation was not substantially improved until 1925, when Heisenberg discovered an entirely new scheme of dynamical equations, in which the dynamical variables are quantities which do not in general satisfy the commutative axiom of multiplication. This new scheme is a generalisation of Newton's, with Planck's constant h appearing in it in a fundamental way, expressing the extent of the non-commutation in typical cases. Newton's mechanics is the limiting form of the new mechanics when h tends to zero.

It is a surprising result that any simple and elegant generalisation of Newton's scheme should be possible at all. With the way Newton formulated his fundamental laws no such generalisation suggests itself, but subsequent mathematical developments of Newtonian theory, in particular Hamilton's formulation of the equations of motion, pave the way for the new mechanics. In fact, if one writes Hamilton's equations in the notation of Poisson brackets, the transition to the new mechanics becomes so straightforward as to make one feel that pure mathematicians ought to have discovered it as an abstract scheme long ago, like they discovered non-Euclidean geometry.

When I first learned Hamilton's equations and the related dynamical theory, I thought it all rather useless. One can solve any problem in mechanics by a direct application of Newton's laws, so why should one bother about elaborate mathematical developments which serve only to put the equations of motion that one starts from in a different form? Bohr's quantum mechanics goes a considerable way in answering this question by making great use of the Hamiltonian form of dynamics, and Heisenberg's provides the complete answer in making this work practically indispensable. It shows the genius of Hamilton and the other workers in that field, in that they set up their theory and believed it to be valuable at a time when it could easily be dispensed with and long before its essential importance became established.

The use of non-commutative dynamical variables in quantum mechanics results in its being connected with all those branches of pure mathematics that deal with non-commutative quantities, including the theory of linear operators, theory of matrices and group theory. Thus branches which previously had no physical application now acquire one.

The practical consequences of the non-commutative character of the dynamical variables are that there are severe restrictions on our ability to give them numerical values, as we have to do when we want to apply the theory to an actual problem. Two dynamical variables a and b can be given numerical values at the same time only in the very special case when ab happens to equal ba. These restrictions result in the new mechanics not having the determinacy of the older mechanics, its predictions usually giving us only the probability of a certain event occurring.

This position is satisfactory from the practical point of view, in that the experimental results to be compared with theory also involve probabilities and to just the same extent, but several physicists find it unsatisfactory from general philosophical grounds and dislike quantum mechanics accordingly. I feel that perhaps there is some justification for this dislike and that it is quite possible that the basis of quantum mechanics may get altered some time in the future, so as to make its physical interpretation rest on a more solid foundation than probabilities. However, the history of science leads one to expect that such an alteration cannot take the form of a return to classical determinism, but must rather involve a still more drastic departure from classical ideas.

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PROBLEM FOR POULTRY FARMERS

The chicken was twice as old when when the day before yesterday was to-morrow to-day was as far from Sunday as to-day will be when the day after to-morrow is yesterday as it was when when to-morrow will be to-day when the day before yesterday is to-morrow yesterday will be as far from Thursday as yesterday was when to-morrow was to-day when the day after to-morrow was yesterday. On what day was the chicken hatched out?

R. S. S.

Mathematical Tripos. Part III

In the past, students have often been unable to decide which courses they should read for Part III, their doubts have been due to the fact that many of the subjects are quite new to them and it is not obvious which courses go together to make up a logical line of study. The publishing of these notes is, therefore, very welcome and it is to be hoped that they will be found of use to those now reading for Part II, and their successors.

The pamphlet does not set out to give a detailed syllabus of the lectures delivered, but rather to show the general outline of the various subjects. It is unfortunate that little attention has been paid to the linking of the several courses of Part III and to their relation to those of Part II. This can be explained partly by the lack of any Mathematical Methods lectures dealing with the difficulties met in Part III, but the Astronomy section points out that a knowledge of quantum theory and thermodynamics is useful for the understanding of astrophysics and similar remarks could be made in other sections.

The completeness and length of the commentaries bear no relation to the importance of the subjects dealt with; the same space is allotted to the one course of Dynamics as to six on Quantum Theory; and in the latter no distinction is drawn between the symbolism of Professor Dirac and the more applied nature of the other courses. The notes have not always been written in a way calculated to help students who have not already made acquaintance of the technical terms; few Part II mathematicians have any idea of what is meant by the "Principle of Least Constraint," mentioned under Dynamics. To be of real assistance to potential Part III students, as against being of interest to those already reading for it, a fuller account written in everyday language, as in the Hydrodynamics section, is needed. Further some guidance as to useful preliminary reading suitable for the Long Vacation could be given.

On the whole these notes are an excellent innovation, but it is disappointing that no mention is made of Thermodynamics nor of the Introductory Courses, and it should be made clear where these are necessary for the comprehension of the Part III lectures. Also, some of the courses are primarily intended for research students and require too much external reading to be practicable for Part III students engaged in other studies, some note should be made of these.

Is This What You Think?

VERY few students of mathematics ever seem to have a clear idea of why they are studying the subject. Many regard it as a game with a rather elaborate set of rules, and he who has mastered the rules and had sufficient experience in applying them is considered a competent mathematician. Others, usually a little older, regard mathematics as a refuge from the cares of the outside world. To-day, under the stress of war, no mathematician can feel entirely satisfied with any of these attitudes, and the student who is shortly to leave the university for more arduous tasks has a real problem.

Professor Hardy, and several others with him, regard the applications of mathematics to war as sinister by-products, and rely on the remoteness of mathematics from ordinary human activities to keep it gentle and clean. However, the remoteness of mathematics from everyday life has never prevented it from being applied to purposes which are not necessarily gentle or clean. Galileo dedicated his work on dynamics to the cannon-foundry at Florence, and in more recent times the development of hydrodynamics has been greatly influenced by the demands of aeronautics, largely for military purposes. We must not argue from these facts that mathematics is, in itself, either good or bad, gentle and clean or ugly and repulsive.

Mathematics arose from the needs of men in agriculture, commerce and everyday life, and by abstraction from the peculiarities of individual processes, which is characteristic of scientific method, it reached its present power and generality. Mathematics is essentially the study of the quantity relations and space forms of the real world, no matter in how abstract a guise they may appear. Side by side with the development of more complex social relationships, mathematics has become more abstract, thus extending its power to deal with wider ranges of phenomena. We can see, for example, how much more powerful an instrument is the analytical geometry of Descartes and the Renaissance mathematicians than the geometry of Euclid. Yet the disciples of the Platonic school scorned the introduction of arithmetic into geometry, and it was only with the creation of algebra by a process of abstraction from arithmetic that this was possible. This interlinking of the two branches of mathematics on the basis of a higher level of abstraction has been very fruitful for the whole subsequent development of mathematics.

The ancients considered geometry as the only real mathematics. Had such an attitude been persisted in, it would have stultified the whole future development of mathematics in general and of geometry in particular. The whole of mathematics must be considered as a unity, and no one portion of it can arbitrarily be singled out as "real" mathematics. The connection between the various branches of mathematics must be recognised as a fruitful source of new ideas, enriching mathematics as a whole. In this connection the development of statistics at Cambridge is a welcome step forward. Contempt for numerical work and practical methods is unworthy of mathematicians—Gauss was a great computer—and can well act as a check on progress.

Of great importance also is the connection between mathematics and the society in which it is produced and developed. Society at a given epoch is confronted with a whole complex of problems which have a powerful influence on science and mathematics. Mathematics, because it exposes the relations existing in the real world, is, in conjunction with the other sciences, a means of increasing man's control over Nature. The eternal truth of mathematics lies precisely in the accuracy of this exposition and not in the emotional satisfaction which it may afford, however great that may be. It depends upon society whether the application of mathematics will be beneficial to the majority of society or not. We cannot say in a rigid manner that mathematics will be good or bad just according to the society in which it is produced, but we can say that the production of good mathematics and the improvement of social relations are problems which are closely related. The period following the French Revolution saw a great flowering of the sciences and mathematics which could hardly have been fortuitous.

Several mathematicians to-day wish to divorce mathematics from the real world and to regard it as a refuge. Professor Hardy seems to endorse openly this escapist attitude when he speaks of mathematics as an anodyne. But the young students cannot accept this point of view. Under present conditions, they cannot escape from hard reality, however much they may wish to do so; and for this reason, they will not accept an outlook which turns its back on the real world. They want a philosophy which will explain to them the world in which they live, which will explain to them the way forward to a better and higher civilisation, in which mathematics in common with the other sciences shall come to yet greater fruition and be used for the benefit of mankind.

W. R.
P. J. W.

. . .

British Student Congress: Leeds, 1940

EUREKA received some unexpected publicity at the British Student Congress at Leeds, when it was cited by a London student as a good example of co-operation between faculty societies in different universities.

This Annual Congress of the N.U.S. differed considerably from previous ones in that it was much larger (there were over 500 present) and included for the first time delegates from Oxford and Cambridge and the Scottish universities. The underlying idea throughout the Congress was that if we are to claim, as in fact we do, postponement of military service for all students who have successfully completed one year of study, then we should at the same time do all that lies in our power to ensure that the universities are adequately fulfilling their task. In the particular case of mathematics it was felt that, unless our studies were directly contributing to the winning of the war, or more generally contributing to culture as a whole, we had no right to evade the military service which fell so hard on the remainder of the community. In fact, we should not regard ourselves as a privileged few, but as an integral part of the community, with equal responsibilities. These facts being borne in mind, the following functions of a Faculty Society were put forward:—

(i) Lectures in the faculty subject and other closely related subjects which come outside the ordinary syllabus.

(ii) Improvement of social relations between students and staff, and amongst the students themselves.

(iii) Showing the practical significance of purely theoretical work, e.g. function theory used in statistics, which is applied to agriculture for comparison of different varieties of a crop; to psychology for systems of marking examination papers; and so on. Elasticity used in geophysics and seismology. Algebra used in quantum theory, which in turn is applied to spectroscopy, radioactivity, etc.

(iv) Discussion of faculty problems . . . lecture courses, examination syllabus, etc.

(v) Co-operation with other universities.

(vi) Arrangements for visits to laboratories and factories.

It was thought that (v) would be especially useful. Some universities still suffer from compulsory attendance at lectures and complete lack of personal supervision. Other universities perhaps have a better examination system than our own. A comparison of methods should be of benefit to all.

D. J. H.

■ ■ ■

"Almost Isosceles" Right-Angled Triangles

By P. E. Trier

The problem of determining rational right-angled triangles whose smaller sides differ by as little as possible leads to a consideration of the class of Pythagorean numbers in which the two smaller ones differ only by unity.

Any set of Pythagorean numbers is given by the well-known formula

$$m^2 - n^2,\ 2mn,\ m^2 + n^2,$$

where m, n are any positive integers such that m is greater than n. For the special class under consideration, the following relation therefore holds:

$$m^2 - n^2 - 2mn = \pm 1 \quad . \quad . \quad . \quad . \quad (1)$$

and the problem is to find integral solutions to this equation. The fundamental theorem, which will enable us to deduce any number of solutions from a single one which is known, is the following:

If m_1, n_1 satisfy equation (1), then $m_2 = 2m_1 + n_1$ and $n_2 = m_1$ will also satisfy equation (1).

In effect, we have:

$$\begin{aligned} m_2^2 - n_2^2 - 2m_2n_2 &= 4m_1^2 + 4m_1n_1 + n_1^2 - m_1^2 - 4m_1^2 - 2m_1n_1 \\ &= -(m_1^2 - n_1^2 - 2m_1n_1), \end{aligned}$$

which proves the theorem.

Similarly, $m_3 = 2m_2 + n_2 = 2m_2 + m_1$

$$n_3 = m_2$$

and more generally, $m_r = 2m_{r-1} + m_{r-2}$ . . . (2)

$$n_r = m_{r-1} \quad . \quad . \quad . \quad . \qquad (3)$$

give solutions to equation (1).

The simplest triangle of the class is that with sides 3, 4, 5, for which $m = 2$, $n = 1$.

We therefore choose $m_1 = 2$,

$$n_1 = m_o = 1,$$

and deduce other values by relations (2) and (3).

We get the following table:

r	m_r	n_r	$m_r^2 - n_r^2$	$2m_r n_r$	$m_r^2 + n_r^2$
1	2	1	3	4	5
2	5	2	21	20	29
3	12	5	119	120	169
4	29	12	697	696	985
5	70	29	4059	4060	5741
6	169	70	23661	23660	33461
7	408	169	137903	137904	195025
8	985	408	803761	803760	1136689

In this table, the general term m_r of the left hand column is obtained by solving the difference equation

$$m_r = 2m_{r-1} + m_{r-2}.$$

Suppose there is a solution of the form $m_r = x^{r+1}$

Then $$x^2 - 2x - 1 = 0$$

$$x = 1 \pm \sqrt{2}.$$

Then the general solution is $m_r = A(1 + \sqrt{2})^{r+1} + B(1 - \sqrt{2})^{r+1}$.

Also, $m_o = 1$, $m_1 = 2$.

Therefore $$1 = (A + B) + (A - B)\sqrt{2}$$

$$2 = 3(A + B) + (A - B)\,2\sqrt{2}.$$

$$A + B = 0$$

$$A - B = \frac{1}{\sqrt{2}}$$

$$A = -B = \frac{1}{2\sqrt{2}}$$

$$\therefore\ m_r = \frac{1}{2\sqrt{2}}[(1 + \sqrt{2})^{r+1} - (1 - \sqrt{2})^{r+1}].$$

On inspecting the table of values we see that the numbers in the right hand column appear to recur in larger intervals on the left hand column. This can be verified by the above expression for m_r:

We have

$$m_r^2 + n_r^2$$

$$= m_r^2 + m_{r-1}^2$$

$$= \tfrac{1}{8}[(1+\sqrt{2})^{r+1} - (1-\sqrt{2})^{r+1}]^2 + \tfrac{1}{8}[(1+\sqrt{2})^r - (1-\sqrt{2})^r]^2$$

$$= \tfrac{1}{8}[(1+\sqrt{2})^{2r+2} - 2(-1)^{r+1} + (1-\sqrt{2})^{2r+2}$$
$$+ (1+\sqrt{2})^{2r} - 2(-1)^r + (1-\sqrt{2})^{2r}]$$

$$= \tfrac{1}{8}[(1+\sqrt{2})^{2r+1}(1+\sqrt{2}-(1-\sqrt{2}))$$
$$+(1-\sqrt{2})^{2r+1}(1-\sqrt{2}-(1+\sqrt{2}))]$$

$$= \frac{1}{2\sqrt{2}}[(1+\sqrt{2})^{2r+1} - (1-\sqrt{2})^{2r+1}]$$

$$= m_{2r} \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

This relation enables us to find m_r for large values of r without a knowledge of its value for all the intermediate values of r, e.g. from the table we read

$$m_{16} = 1\ 136\ 689$$

$$m_{14} = 195\ 025$$

$$n_{16} = m_{15} = \tfrac{1}{2}(m_{16} - m_{14}) = 470\ 832$$

giving

$$m_{16}^2 - n_{16}^2 = 1\ 070\ 379\ 110\ 497$$

$$2m_{16}n_{16} = 1\ 070\ 379\ 110\ 496$$

$$m_{16}^2 + n_{16}^2 = 1\ 513\ 744\ 654\ 945.$$

The right-angled triangle whose sides are in the ratio of these numbers may be called "Almost isosceles."

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To all Geometers.—ABCD is a tetrahedron. E, F, G, H, J, K are any points on the edges AB, AC, AD, CD, DB, BC; prove that the spheres AEFG, BKJE, CHFK, DGJH have a point in common.

Prove that the 5 circumscribing spheres of the 5 tetrahedra formed by 5 planes have a point in common.

King's College (Lond.) Maths. Society

By The Secretary

In response to a request by the President of the Archimedeans, I will try to give you an outline of the activities of the above Society.

We have been very fortunate in the site of our evacuation, and Bristol University has provided us with full facilities, including the use of an excellent library. This year we have had a membership of twenty-five, which we consider very good in view of the smallness of the Mathematical Department. Meetings have been held on alternate Monday afternoons and have had good support from members and from the teaching staff. Nine lectures have been given, two by students; topics have included "Foundations of Mathematics," "The Propositional Calculus," "Affine Geometry," "Non-Euclidean Geometry" on the Pure Mathematics side; and "Quaternions," "Quantum Theory," "Minimum Theorems in Applied Mathematics" on the Applied side. During last summer, parties from the Society visited Greenwich Observatory and Imperial College, Kensington; the object of the latter visit was to inspect and try out the calculating machines there, and a very happy time was had by all! There is no ground whatever for considering these mechanisms in any way delicate! There have been a number of social activities, including a tea, concert given by members, and theatre party at the end of the Christmas and Easter terms. In addition there have been tennis and chess matches between students and staff.

Our Finals course extends over three years, and it is very regrettable that owing to the calling-up of students, the present second year people are obliged to cut short their course and take Finals this June. Not only does it mean a large amount of cramming to crowd the course into two years, but some very interesting sidelines leading outside the syllabus are lost. With regard to the course itself, I cannot be expected to reproduce a syllabus here, but I can make a few relevant remarks on the course which we have followed. Judging from the number of Cambridge questions we have answered (I should have said "been set") I imagine that our Geometry course is much on the same lines as at Cambridge. We have done no spherical trigonometry or three-dimensional differential geometry. Analysis starts with the Dedekind section and goes on to function theory, then of course, the Cauchy theory of complex variable. In Applied Maths we have dealt with Lagrange theory, Hamilton's Principle, and the theory of gyroscopes and moving axes. We had fairly extensive courses in vector analysis and potential; also a course on waves, dealing with waves in strings, water, sound and electromagnetic waves. We also have the opportunity to take two Advanced Subjects, chosen this year from Theory of Numbers, Elliptic Functions and Algebraical Geometry. Altogether we consider it a comprehensive and satisfactory course.

Before finishing, I should like to say that we are glad to have made contact with the Archimedeans and we wish EUREKA every success in its efforts to link up student mathematicians.

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What is a Mathematician?

By HAROLD JEFFREYS, M.A., D.SC., F.R.S.

PROFESSOR HARDY has defined mathematics partly by enumeration of shining examples of mathematicians, partly by reference to the whole body of mathematical knowledge with permanent aesthetic value. The criteria do not seem to be very clear. Does the permanent value depend on the results or the method? Many of the most important results of pure mathematics are now stated in ways that their original authors would find it difficult to understand, and the original proofs would often fail to pass muster by modern standards, being either non-rigorous or hopelessly long-winded. What, for instance, would Fourier make of his theorem as presented by Titchmarsh, and what would happen to a Tripos candidate who gave Fourier's original proof? Newton's *Principia* is a monumental work, but how much of it survives in anything like its original form?

It seems to me that "mathematician" is used in several different senses. To most people a mathematician is anybody that can solve a quadratic equation; in Cambridge he is perhaps anybody that has taken, or proposes to take, the Mathematical Tripos, though it becomes doubtful whether he retains the title if he takes any other Tripos too. But many people make use of mathematics and yet do not consider themselves mathematicians. To such people mathematics is a tool, and not a direct source of interest. Professor Dirac, for instance, considers himself a physicist, and says quite explicitly that he considers mathematics a tool. His work, also, makes far more appeal to experimental physicists than it does to most pure mathematicians. In spite of Professor Hardy's admission of him as a mathematician, Dirac has more in common with Rutherford than with Hardy. Now I think that this is a genuine distinction. If people must be classified as specialists, it seems to me much more important that they should be classified with regard to what they talk about than with regard to the technique they use. Dirac should be called a theoretical physicist, not an applied mathematician. The latter term would then be free to denote those who are interested in physical problems only as illustrations of mathematical methods. I mention no names, considering the description abusive.

Unfortunately the Mathematical Tripos cannot get on without applied mathematics. Theoretical physicists, whether they hope ever to make original advances of their own, or whether they would be satisfied to be able to take an intelligent interest in the work of others, must learn a certain amount of pure mathematics and the general principles and methods of theoretical physics. But problems of intrinsic physical interest usually turn out to be either bookwork or too hard to be done in the time available in an examination! The result is that ability to apply the principles can be tested in the examination only by reference to experiments invented for the purpose: rolling illustrated by a sphere inside a vertical cylinder, with no slipping at all at the point of contact, potential problems for boundaries that no experimenter could construct, and so on. That is the trouble about Parts I and II: either the questions apparently on theoretical physics will be too hard, or they will be artificial and unsatisfactory, thereby becoming applied mathematics. In Part III the difficulty is less serious, since a question that takes three hours to answer, if the candidate knows how, is possible.

In dynamics there is a special difficulty. All the technique needed for the enormous majority of actual problems was known to Laplace, with the possible exception of the modified Lagrangian function, due to Routh. For these problems these are still the best methods. The reason why dynamics to this stage sometimes appears stagnant is that it was the first branch of theoretical physics to be developed, and when Newton, d'Alembert, Lagrange, Euler and Laplace had all played their parts in improving the methods it is hardly surprising that little further remained to be done. Nevertheless gyroscopic motion and small oscillations remain interesting (though the associated mathematics is now called algebra). The more advanced development beginning with Hamilton's equations and the Hamilton-Jacobi theorem makes the harder problems easier, but it makes the easier ones harder.

De gustibus . . . . But I must dissent from Professor Hardy's remark that ballistics and aerodynamics are ugly and dull. It *is* interesting that for bodies of some shapes moving through a fluid the force is nearly perpendicular to the direction of motion; there *is* beauty in the circulation theorem of aerofoil lift and in Prandtl's theory of induced drag; also in G. I. Taylor's treatment of various problems of bodies moving at high velocity in compressible fluids. Incidentally the circulation theorem also explains why people catch crabs in rowing; and the ballistics problem is substantially the same as one that arises in the evolution of the solar system, a matter of interest though hardly of economic or military importance. The devil's possession of the good tunes need not be undisputed. Again, some of us enjoy the numerical solution of differential equations. I do myself in moderation; but where should we be without those, from Briggs and Napier to the British Association Tables Committee, who find their chief joy in numerical computation? E. W. Brown could write down the equations of the moon's motion in two minutes; it took him thirty years to solve them to the accuracy needed for comparison with observation. But when I met him his soul always seemed to be doing very well. And that is the ultimate difference between the mathematician and the theoretical physicist. The former is satisfied with a formula or even an existence theorem. The latter does not consider either an answer at all, until the work of numerical computation has been taken to a stage where comparison with observation is possible. No theoretical physicist is the worse for knowing some pure mathematics that he has never had occasion to use; but he is the worse if the characteristic outlook of the pure mathematician leads him to over-emphasize the importance of the mathematics at the expense of an understanding of why the problems are of interest.

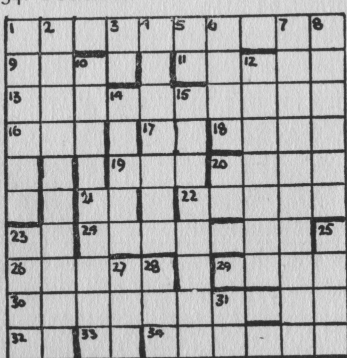
Crosswords

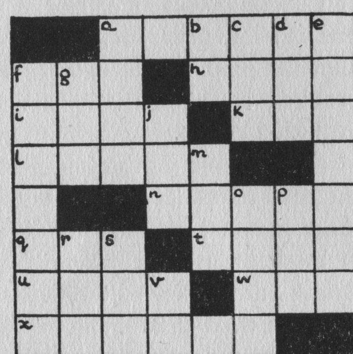
ACROSS

1. Permuting indices, we get these processes.
9. German Physicist.
11. Periodic function.
13. $\xi = \eta = o$.
16. Engaging part.
17. Roughness.
18. Closed curve.
19. Comparatively easy.
20. And I'm the Feminine Tenth.
21. Square race.
22. Letter.
23. Therefore.
24. Nazi determinant.
26. Electrostatic artifice.
29. From metal to singer.
30. These make things up.
32. Polar axis.
33. Perform.
34. Positives.

DOWN

1. Old frame of reference.
2. Odd shipping routes.
3. On.
4. Polygon.
6. To z-axis.
7. Place of this meeting determines stability.
8. Undirected.
10. Specification of axes.
12. Indeterminate quantity.
14. Condition.
15. Flowery sort of equation.
20. Twice.
23. Indication.
25. Difference.
27. Terrestrial.
28. Period.
31. Direction.





FIVE HOUR OR FIVE MINUTE PUZZLES

ACROSS

(*a*) Kilogram.
(*f*) Leap year.
(*h*) Square.
(*i*) First four figures of x (mixed).
(*k*) H.C.F. of 1484 and 1908.
(*l*) Five consecutive numbers (mixed).
(*h*) $x^3 + y^3$ where $x - y = 41$.
(*q*) $\frac{2}{\sqrt{\pi}}\int_0^1 e^{-t^2}dt$ in all probability.
(*t*) (rev.) A bale of paper.
(*u*) $11\frac{1}{2}$ square miles.
(*w*) 6 oh for the answer!
(*x*) $\underset{n\to\infty}{\mathrm{Lt}}\left(1+\frac{1}{1}+\frac{1}{2}+\,.\,.\,+\frac{1}{n}-\log_e n\right)$.

DOWN

(*a*) Expressible as $21x^3$.
(*b*) (rev.) p beheaded.
(*c*) Product of four different primes.
(*d*) Factor of 12179.
(*e*) Here's your chance, girls!
(*f*) Cube.
(*g*) Prime.
(*j*) $\int_0^3 (10x^4 + 4x^3 - 3x^2 + 2x)\,dx$.
(*m*) P/8.
(*o*) —and all that.
(*p*) Half mile.
(*r*) Product of two primes whose difference is 4.
(*s*) g mixed but still prime.
(*v*) Two kisses to finish with.

Mass Observation

By D. J. Halliday

There have been a number of questionnaires and public opinion surveys in Cambridge during the past year, and mathematicians are sure to have wondered whether their results had any true significance. It is beyond our present scope to discuss the wording of the questions; we must confine ourselves to an impartial analysis of the results.

The central problem of mass observation, as of the whole of statistical theory, is one of estimation. Time seldom permits a complete census. We have to be content with partial information, taken from a so-called *sample* of the whole *population*. The selection of this sample is a very important matter, for if our results are to have any value it must be as nearly representative of the whole population as possible. In order to fulfil this condition every one must be given an equal chance of being chosen as part of the sample. The sample is then called *random*. Unfortunately, a random sample is not to be obtained quite so easily as may at first sight be imagined. Suppose, for example, we are taking a survey of public opinion. Would it be fair, say, to go out into the street and to pick out the required number of people *at random*? Most certainly not! For:—

(*a*) The observer will (perhaps unconsciously) choose those people who look likely to answer him civilly, and avoid those who do not.

(*b*) He will also try to make his sample a random one by picking out what he himself considers to be "average" people, so that oddities have no chance of being selected.

(*c*) Some people walk about in the streets more often than others.

(*d*) The place and time will also make a difference.

Would it be better then to take a directory, open it at random, pick out a few names, and repeat this process until enough people had been chosen? Better, yes—but

(*a*) Some pages of the directory are more likely to open than others, even with a new one.

(*b*) Some names will more readily catch the eye.

It will by now be apparent, I hope, that the only fair methods of selection are those which are completely mechanical and which eliminate all human error. It is easy to devise such methods with the help of dice, roulette wheels, and so on. There have also been compiled by Mr. L. H. C. Tippett, tables of random numbers, found by experience to give unbiassed results. There is, however, a perfectly random list much nearer to hand—the directory itself (assuming, of course, that the names are placed in alphabetical order). If the directory has n pages, each containing m names, and we want a sample of N, then I can conceive no fairer method than to take every $\left[\frac{mn}{N}\right]$th person in the list.

The sample having been duly selected and the questions asked, we are faced with a heap of answers, to each question so many yes's, so many noes, so many no opinions. These are most conveniently set out in the form of percentages, especially when there are two samples of different size to be compared. What degree of accuracy can be asserted for our results? A first check is obtained by including in the questions one whose answer is known, and comparing the result of the sample with this known value, e.g. in the Public Opinion Survey conducted by the Undergraduate Council last term one of the questions was: "What subject are you reading?" The answers were as follows:—

Natural Sciences ..	15·6%	Geography	4·6%
Languages	15·6%	Divinity	0·9%
Mechanical Sciences ..	13·0%	Agriculture	0·9%
Medicine	10·2%	Moral Sciences	0·9%
History	10·6%	Architecture	0·9%
Mathematics	7·0%	General	0·9%
English	7·0%	Anthropology	0·4%
Economics	6·0%	Music	0·4%
Law	5·6%		

The agreement of these percentages with fact establishes beyond doubt the validity of the survey as a whole. Next, a rough estimate of the possible error of each question was obtained by taking two independent samples spread over a large number of colleges and comparing the results. The first question was: "Do you consider that Mr. Chamberlain should continue as Prime Minister?"

	Yes	No	No opinion
First sample (142)	68	60	14
Second sample (144)	65	62	17

The error was therefore somewhere about 3/150 = 2 per cent. either way. Comparing this with the final result—

	Yes	No	No opinion
19 Cambridge Colleges ..	48%	39%	13%
2 London Colleges	26%	62%	12%

we see that in neither case could such an error have reversed the decision. (Incidentally, the second part of the same question is a good example of how wording can influence the answer. It ran: "Would you substitute any of the following? Mr. Eden? Mr. Churchill? Sir S. Cripps? Mr. Morrison? Any other person?" Mr. Eden, whose name was read first, had 27 supporters, Mr. Churchill 21, Sir Stafford Cripps 8, and Mr. Morrison 12. No one else had more than 3 supporters!)

A great deal more might have been done in the analysis of these results, but time was very pressing, and it was only possible to state the percentages of yes's and noes to each question with a possible error of the same order as that above. Given unlimited time, standard errors might have been calculated, differences between colleges and faculties studied, the answers correlated one with another, and so on. The majority of the completed forms are still available if there is anyone sufficiently interested to carry on with the work. The procedure would be roughly as follows:—

To decide whether the opinions of London and Cambridge universities differ significantly. Let the percentages of yes's in Cambridge colleges be $x_1, x_2, \ldots x_p$, the respective numbers who expressed an opinion in these colleges being $u_1, u_2, \ldots u_p$, and the corresponding figures for London $y_1, y_2, \ldots y_q$: $w_1, w_2, \ldots w_q$. The estimated mean percentage yes's are $\frac{\bar{x} = \Sigma u_i x_i}{p}$, $\frac{\bar{y} = \Sigma w_j y_j}{q}$, and the estimated variance

$$s^2 = \frac{\Sigma u_i (x_i - \bar{x})^2 + \Sigma w_j (y_j - \bar{y})^2}{p - 1 + q - 1}.$$

The probability of obtaining a given difference $(\bar{x} - \bar{y})$ on the assumption that the true means are equal is found from the table of $t = \frac{\bar{x} - \bar{y}}{s\sqrt{\frac{1}{p} + \frac{1}{q}}}$ with $(p + q - 2)$ degrees of freedom (R. A. Fisher, *Statistical Methods for Research Workers*, Table IV). It is customary to reject the hypothesis under consideration if the probability turns out to be less than 1 in 20. An alternative method would be to use the statistic $z = \dfrac{\bar{x}-\bar{y}}{\sqrt{\dfrac{s_1^2}{p}+\dfrac{s_2^2}{q}}}$, where $s_1^2 = \dfrac{\Sigma u_i(x_i-\bar{x})^2}{p-1}$, $\dfrac{s_2^2 = \Sigma_j(y_j-\bar{y})^2}{q-1}$, but this would entail more work and the mathematical theory is somewhat doubtful.

■ ■ ■

Letter to the Editors

GENTLEMEN,—I note with regret that your examiner for the Mathematical Tripos, Part π, has omitted from question 3 the diagramming of a topologist riding a bounding cycle. This omission seems to me unfair to an active group of mathematicians who, though somewhat cyclic, are rarely circular, and never vicious.*

Faithfully yours, H. PETARD.

Princeton, New Jersey, U.S.A.

* But cf. A. W. Tucker, "Degenerate Cycles Bound," *Mat. Sbornik* (*Recueil Math.*), n.s., 3 (1938), 287–288.

[H. Pétard is the well known author of *The Mathematical Theory of Big Game Hunting* and other similar works.]

■ ■ ■

Numerical Acrostic

UPRIGHTS.

Turn your luck upside down
And just as before
Its frequent recurrence
You'll bound to deplore.

LIGHTS.

(1) The adding machine according to Johnny
Has got the grand total of everyone's money.
(2) For brevity Newton did always use me
Whenever he wanted to multiply three.
(3) Reverse this third power and take off its second.
And then its all square if you've rightly reckoned.
(4) Three little powers at a conference here
Meet altogether and leave it all square.
(5) Short of a day, this is man's earthly span
Reading it backwards (perchance) if you can.
(6) From a whole term subtract just a week
And the answer that's left is just what you seek.

On the Problem of the Sons of the Dons*

By K. Tweedie (*Reading University*)

The relevant data are as follows:—

(i) In the Mathematical Faculty of the University of Epsilon there are six dons, of whom three are pure mathematicians an analyst, a geometer, and an algebraist—and three applied mathematicians—a dynamist, a physicist, and an astronomer.

(ii) Each don has a son who is a student in the faculty.

(iii) Each son specialises in one of the six subjects taught by the fathers.

(iv) No student specialises in the same subject as his father.

(v) No two students specialise in the same subject.

(vi) There are no two students who each specialise in the same subject as the other's father.

(vii) The father of the student geometer specialises in the same subject as the son of the don who specialises in the same subject as the astronomer's son.

(viii) The geometer's son specialises in the same subject as the father of the student who specialises in the same subject as the father of the student astronomer.

(ix) The analyst's son is an applied mathematician.

(x) The student analyst is the son of an applied mathematician.

(xi) The analyst's son is taught by the father of the student physicist.

The problem is to find the subject of the father of the student algebraist.

Solution

Denote the six subjects by N, G, L, D, P and S respectively, in the order in which they are given in (i). We introduce θ such that, if x is one of these subjects, θx is the subject of the father of the student specialising in x, and $\theta^{-1}x$ is the subject of the son of the don specialising in x. The data can then be translated into this notation to form a more suitable basis for the arguments involved in the solution.

* R. Turner, *Eureka*, 2, p. 9.

(i) and (ii): Each don has his own subject and has one son in the faculty, and therefore $\theta^{-1}x$ is uniquely determined when x is given.

(ii), (iii), and (v): There is only one son to each subject, and therefore θx is uniquely determined when x is given.

From these properties of θ it follows that $\theta^n x$, where n is given, represents any one of the six different subjects according to the subject represented by x; and $\theta^n\theta^m x = \theta^{n+m}x$, where n and m are positive or negative integers and $\theta^o x = x$. If x and y are any two subjects such that $\theta^n x = y$, then $\theta^{-n}y = x$. If $\theta^p x = x$ and $\theta^n x = y$, then $\theta^p y = \theta^{n+p}x = \theta^n x = y$, so that $\theta^p y = y$ also. If x is fixed so is p. As n varies y represents n different subjects (including x) altogether, and from the data $n < 6$.

(iv): $\theta x \neq x$.

(vi): If $\theta^{-1}x = y$, then $\theta^{-1}y \neq x$.

(vii): $\theta G = \theta^{-1}\theta^{-1}\mathrm{S}$, so that $\theta^3\mathrm{G} = \mathrm{S}$.

(viii): $\theta^{-1}\mathrm{G} = \theta\theta\mathrm{S}$, so that $\theta^3\mathrm{S} = \mathrm{G}$.

(ix): $\theta^{-1}\mathrm{N} = \mathrm{D}$, P or S.

(x): $\theta\mathrm{N} = \mathrm{D}$, P or S.

(xi): $\theta^{-1}\mathrm{N} = \theta\mathrm{P}$.

The problem is to find $\theta\mathrm{L}$.

From (xi) we can easily show that $\theta^{-1}\mathrm{N} \neq \mathrm{N}$ or P, and from (ix) we then have left $\theta^{-1}\mathrm{N} = \mathrm{D}$ or S.

From (xi) $\theta\mathrm{N} = \theta^3\mathrm{P}$. Hence if $\theta\mathrm{N} = \mathrm{P}$, from (x), we have $\theta^3\mathrm{P} = \mathrm{P}$ and $\theta^3\mathrm{N} = \mathrm{N}$. If $\theta\mathrm{P} = x$, $\theta^3 x = x$, and from either (vii) or (viii) x can only be D or L.

With the remaining subjects (S, G, and either L or D) we can only form the relations $\theta\mathrm{S} = \mathrm{G}$ and $\theta\mathrm{G} = \mathrm{S}$, which violates (vi) but neither (vii) nor (viii), with either $\theta\mathrm{L} = \mathrm{L}$ or $\theta\mathrm{D} = \mathrm{D}$, which violate (iv). Consequently $\theta\mathrm{N} = \mathrm{P}$ is untenable and we must have $\theta\mathrm{N} = \mathrm{D}$ or S from (x).

If $\mathrm{D} = \theta^{-1}\mathrm{N}$, which $= \theta\mathrm{P}$, $\mathrm{D} \neq \theta\mathrm{N}$, and therefore $\mathrm{S} = \theta\mathrm{N} = \theta^3\mathrm{P}$, violating (vii). If we ignore (vii) but retain (viii) we get $\theta^3\mathrm{P} = \theta^2\mathrm{D} = \theta\mathrm{N} = \mathrm{S} = \theta^{-3}\mathrm{G}$. If x is P, D, N, S or G, we evidently have $\theta^n x \neq x$ when $\pm n = 1, 2, 3, 4, 5$ or 6, which is impossible.

We are thus left with $\mathrm{S} = \theta^{-1}\mathrm{N}$ and $\mathrm{D} = \theta\mathrm{N}$, so that $\theta^3\mathrm{P} = \theta^2\mathrm{S} = \theta\mathrm{N} = \mathrm{D}$, which $= \theta^5\mathrm{G}$ from (vii) directly and $= \theta^{-1}\mathrm{G}$ from (viii) directly. In either case, if $x = \mathrm{P}$, S, N, D or G, we have $\theta^n x \neq x$ if $n \neq \theta \pmod 6$, and consequently $\theta^6 x = x$. The sixth subject represented by x must be the remaining subject, namely, L. Hence finally we have the sequence

$$\text{G L P S N D G},$$

in which each subject is the θ of the preceding one. In particular we have, as the answer to the question propounded,

$$\theta\text{L} = \text{P},$$

i.e. the father of the student algebraist is a physicist.

Note how the data restrict the solution:

(iv): $\theta\text{L} \neq \text{L}$ and $\theta\text{D} \neq \text{D}$.

(vi): If $\theta\text{S} = \text{G}$, $\theta\text{G} \neq \text{S}$ or vice versa.

(ix) $\theta^{-1}\text{N} \neq \text{G}$ or L.

(xi) $\theta\text{N} = \text{D}$, P or S.

(iv) and (vi) appear as alternatives, as do (vii) and (viii).

The remainder can be treated as essential, except possibly for (v) which might be avoided by a slightly modified wording of (i), (ii), and (iii).

(iv), (vi), and (ix) give more information than is needed.

If (i), (ii), and (iii) were worded so that (v) became unnecessary, we could reduce the data from eleven points to eight, without spoiling the argument. Owing to the superfluity of data, the solution remains determinate for a number of different specifications selected from the data originally given. For instance, we might use (i), (ii), (iii), (v), (vii) and (x) with $\theta^{-1}\text{N} \neq \text{G}$ or L (ix),

$\theta\text{N} \neq \text{L}$ (xi),

and $\theta\text{L} \neq \text{L}$ and $\theta\text{D} \neq \text{D}$ (iv).

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JOINT COMMITTEE.—The following students were elected last term to the committee to meet the dons:—

Research students: C. A. B. Smith (Trinity); Part III, A. E. Jones (St. John's); Part II, P. E. Trier (Trinity Hall); Mays, R. Braybrook (Emmanuel); Part I, B. D. Blackwell (St. John's).

These, together with Mr. Easterfield and the President, had a discussion with Mr. Hall, Mr. Newman and Dr. Swirles, chiefly on printed lecture notes. While the dons accepted, with certain reservations, the principle of some form of printed notes, they maintained that it was a matter for the individual lecturers and that the Faculty Board would not make any move.

Syllabus of Part π

As the Archimedeans have urged and secured the publication of details of Part III, students who have taken Part π have written to say it would be helpful if they too could have a syllabus. Accordingly, we have great pleasure in bringing out this brief summary of the subjects taught.

Astronomy: A method of discovering one's future from the stars, particularly one's Integer Vitae[1] or lucky number.

Calculus: A painful disease, necessitating an operation. There is only a partial differentiation between calculus and analysis.

Geometry: The study of the countries of the world. An important result, the "5-colour theorem," shows one can divide the countries of the world into five empires, so that every nation is at war with all its neighbours. Duel: A frequent relation between planes of different countries.

Mechanics: Persons at least potentially interested in cycles.

Periodic: An acid, formula HIO_4. Its identification is known as a Fourier Analysis.

Rational: Part π is irrational.

Real Function: "A lecture . . . is the very place for provisional nonsense . . . (whose) real function is to disappear when it has served its turn".[2] Complex Functions: Those that are really imaginary.

Topology: The art of cutting trees to pleasant shapes. The so-called "figures of speech," the ellipsis, parable, hyperbole, circle, and lines at cross purposes, are greatly in favour.

Recommended Books, etc.

H. Illig: "Die Quadratur des Kreises, nach 3000 Jahren gelöst."

Prof. Moriarty: "The Dynamics of an Asteroid."

H. Pétard: "The Mathematical Theory of Big Game Hunting," *Amer. Math. Mon.*, XLV, 7, 1938.

V. E. Robson: "A Beginner's Guide to Practical Astrology."

G. G. Stokes: "On the Discontinuity of Arbitrary Constants," *Acta Math.* 26 (1902).

A. G. Walker: "Mechanics in Public 3-Space." L.M.S. (to be published).

C. A. B. S.

[1] Horace, Carm. 1.22.1.

[2] Prof. Littlewood, *Elem. of the Th. of Real Functions.* This is believed to be the only mention of real functions in the book.

Book Reviews

The Mathematical Theory of Non-Uniform Gases. By Sydney Chapman and T. G. Cowling. (Cambridge: at the University Press.) 1939. Pp. xxiii + 404. Price 30s.

Reading Dynamical Gas Theory for Part III of the Mathematical Tripos, and garnering the questions set, used to resemble falling off a log; one fell all the easier because one was confidently aware that no great depth was expected. At the same time one had—or rather, should have had—qualms about the precarious procedure one accepted in discussing transport phenomena. These qualms would have been intensified had one not been lamentably ignorant of the fact that Enskog and Chapman had separately evolved, some twenty years earlier, a firm basis for the discussion of these phenomena.

After the publication of this book, however, no excuse remains for such a state of innocence. Professor Chapman and Dr. Cowling have produced a classic account of a subject which the senior author has done much to perfect. The kernel of the book is the solution of the general integro-differential equation for the molecular velocity-distribution. The method of solution is essentially one of successive approximation. The first approximation corresponds to local uniformity of the gas, and so gives a Maxwellian distribution. The second approximation, introducing non-uniformity, naturally leads to expressions for the coefficients of viscosity, thermal conductivity and diffusion, the transport phenomena associated with non-uniform states. These general expressions are next evaluated for the collision mechanisms which correspond to particular molecular models; their dependence on temperature and density is compared at some length with experimental results. In later chapters, the effects are considered of two kinds of congestion, namely, high concentrations of classical molecules, and degeneracy for quantal particles. In the final chapter, the previous general methods are modified to deal with the forces transverse to velocity which are encountered by ions in magnetic fields.

Prospective readers, namely, all who are seriously interested in kinetic theory, should be warned that the "kernel" of the book, mentioned above, is a tough nut; but thorough mastication will be well rewarded. The vector and tensor notation to which an early chapter is devoted proves, in the course of the book, to be both useful and elegant.

N. B. S.

Mathematical Recreations and Essays. By W. ROUSE BALL, revised by H. S. M. COXETER. (Macmillan.) 10s. 6d.

It is with regret that we remark in this new edition of an old favourite the disappearance of the little "Cnossos" maze on the cover which has for so long been, as it were, a hall-mark of the work. But let us hasten to add that our regrets end there, and that between the covers the book is an unqualified success.

In his preface to the revised edition, Mr. Coxeter announces his intention of maintaining, if possible, the spirit of the book. He has done more. If anything, he has intensified it, both by the omission of material which might be judged out of keeping with the avowedly aesthetic and impractical, and by the inclusion of new material of a quite irreproachable aesthetic nature. On either hand, the alterations are all such as would have been thoroughly approved by Rouse Ball himself.

The most important omissions in the new edition are Chapter V (in the old edition) on Mechanical Recreations, Chapter VIII on Bees and their Cells, and Chapter XV on String Figures. None of these can be regarded as an essential part of the work, and in fact little of their contents has much claim to inclusion, undeniably amusing though they were. They have been replaced in this edition by a completely new chapter on Polyhedra, illustrated by numerous diagrams and two photographic plates.

The chapter on Bees and their Cells has also declined in topical interest, while the subject of String Figures has found a place elsewhere in a booklet by itself. The place of the former of these has been taken by a more extensive account of the difficult problem of map-colouring, giving as much of the solution as has hitherto been obtained, such as the seven-colour theorem.

The other chief alteration is the re-writing of the chapter on Cryptograms by Abraham Sinkov of the U.S. War Department. As the new title of Cryptography and Crypt-analysis suggests, this has been done stressing the technical aspect of the subject, rather than the historical, as had the previous version. This chapter does form an introduction to the problem of solving a cryptogram, and after reading it one feels that, even if unable to solve it oneself, one could understand someone else's efforts at a solution.

The miscellaneous problems of Chapters X and XI, together with some new material, are now placed in their appropriate chapters at the beginning of the book. As a whole, the book is much more attractively produced than previously, and it remains just as essential a part, even of a mathematical library containing the previous editions, as ever. E. M. T.

UNIVERSITY MATHEMATICAL TEXTS. (Oliver and Boyd.) 4s. 6d.

Determinants and Matrices. By A. C. AITKEN, D.Sc., F.R.S.

Viewed from the point of view of the student confronted with the Linear Algebra of Part II of the Tripos, this book has the supreme advantage of presenting the necessary material concisely and yet completely. Occasionally it seems almost too concise, as when it condenses the reduction of matrices to their normal forms into a few short paragraphs, but in no case is it really difficult to follow the course of the reasoning. One of the most noticeable omissions is that of the use of the summation convention for double suffixes. Necessarily, results, which are usually easily derived with the use of the suffix calculus depend instead upon multiplication by the special non-singular transposition matrices. This is likely to increase the reader's ability in the manipulation of matrices and provides a useful amplification of the corresponding tripos course. For those who are concerned with more than the bare Tripos course, the author has occasionally inserted short historical notes, which are of interest, and finishes with two chapters on various compound and other special matrices and determinants, which, like the earlier part, are both intelligible and interesting. A. J. F. T.

Statistical Mathematics. By A. C. AITKEN, D.Sc., F.R.S.

Until recently there has existed practically no *modern* textbook on Mathematical Statistics: this book fills the long-felt need. It covers roughly the Cambridge First Term Course in a remarkably compressed form: yet there is no loss of clarity. All essential points are covered with the elegance one would expect from Dr. Aitken, and a careful attention to matters of notation. For example, the values of constants calculated from a finite sample are always clearly distinguished from their "actual" values—a very helpful feature.

There is an error—p. 143, line 3 applies only if $\rho = 0$. One may also note that Dr. Aitken is unduly modest in not mentioning his method of solving equations. C. H. E. W.

Vector Methods. By D. E. RUTHERFORD, Dr.Math.

This is the most compact and understandable introduction to the use of vectors that I have met. After a very thorough discussion of potential theory and in particular Laplace's Equation, the applications to mechanics, electricity and hydrodynamics are investigated. This serves the very useful purpose of showing the connection between the theories of these subjects.

A chapter on Differential Geometry gives the reader an easily followed introduction to this subject.

Finally the vector notation is extended to four dimensions.

Anyone who finds vectors hard to understand would be well advised to read this book. J. T.

We have also received the following books for review:

Integration, by R. P. Gillespie, Ph.D.

Integration of Ordinary Differential Equations, by E. L. Ince, D.Sc.

Theory of Equations, by Prof. H. W. Turnbull, F.R.S.

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Notice

It is hoped that EUREKA *will continue to be published next year. All contributions and criticisms should be sent to the Editors of* EUREKA, *The Archimedeans, c/o The Maths. Faculty Library, New Museums, Cambridge. It will be of great assistance to the Editors if those who would like future copies of* EUREKA *will send their names and addresses to the Business Manager at the same address.*

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