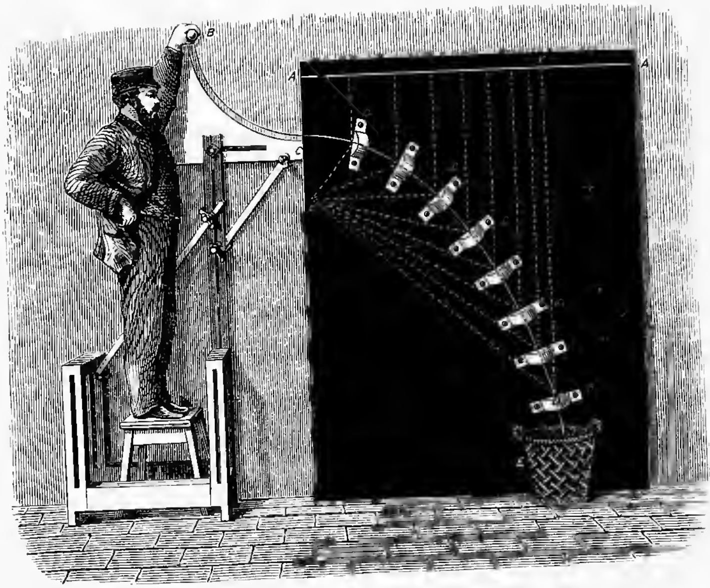
EXPERIMENTAL MECHANICS.



THE PATH OF A PROJECTILE IS A PARABOLA.

EXPERIMENTAL MECHANICS.

A COURSE OF LECTURES

DELIVERED AT THE ROYAL COLLEGE OF SCIENCE FOR IRELAND.

BY

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WITH ILLUSTRATIONS.

London and New York:

MACMILLAN AND CO.

1871.

LONDON:
R. CLAY, SONS, AND TAYLOR, PRINTERS,
BREAD STREET HILL.

PREFACE.

The Royal College of Science for Ireland was established in 1867 by the Science and Art Department of the Committee of Council on Education, for the purpose of giving instruction in science applicable to the industrial arts.

In the spring of 1870, the Author delivered at the College a special course of twenty Evening Lectures upon "Experimental Mechanics," addressed to artisans and others unable to attend the ordinary classes. These Lectures, revised and some of them rewritten, form the present volume.

It has been the aim of the Author, however fulfilled, to create in the mind of the student physical ideas corresponding to theoretical laws; and thus to produce a work which may be regarded either as a supplement or an introduction to manuals of theoretical mechanics.

To realize this design, the copious use of experimental illustrations was necessary. The apparatus used at the lectures and figured in the volume has been principally built up from Professor Willis'[1] most admirable system. It is impossible to over-estimate the number of forms which this Protean system is capable of assuming in the lecture room. It provides, on a substantial scale, the principal parts that are required for the illustration of most branches of experimental mechanics. A collection of this apparatus is in daily use in the Institution.

It is the Author's practice to allow his pupils to share in the performance of the experiments. This method of instruction, at all times desirable, is especially useful when tables of numerical results have to be constructed.

The Table of Contents will show that, in the selection of the subjects, the question of practical utility has in many cases been regarded as the one of paramount importance.

The elementary truths of mechanics are too well known to admit of novelty, but it is believed that the mode of treatment which is adopted is more or less original. This is especially the case in the Lectures

[1] Willis' "System of Apparatus for the Use of Lecturers and Experimenters in Mechanical Philosophy." London: Weale and Co.

relating to Friction (V.), to the mechanical powers (VII., IX.), to the strength of timber and structures (XI., XII., XIII.), to the laws of motion (XV.), and to the pendulum (XVIII., XIX.).

The Author thanks his friend Dr. Tarleton, F.T.C.D., for the kindness with which he undertook to read over the proof-sheets, and for many valuable suggestions.

The illustrations have been drawn from the apparatus, by Mr. Collings, under the Author's supervision. Mr. Cooper has executed the engraving.

Royal College of Science,
1871.

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EXPERIMENTAL MECHANICS.

LECTURE I.

THE COMPOSITION OF FORCES.

Introduction.—The Definition of Force.—The Measurement of Force. —Equilibrium of Two Forces.—Equilibrium of Three Forces.—A Small Force can overcome Two Larger Forces.

INTRODUCTION.

1. I SHALL endeavour in this course of lectures to prove the elementary laws of mechanics to you by means of experiments. In order to understand the subject treated in this manner, you need not possess any mathematical knowledge beyond an acquaintance with the rudiments of algebra, and a few simple geometrical terms and principles. But even to those who, having an acquaintance with mathematics, have by its means acquired a knowledge of mechanics, experimental illustrations may still be useful. By actually seeing the truth of results with which you are theoretically familiar, clearer conceptions may be produced, and perhaps new lines of thought opened up. Besides, many of the mechanical principles which lie rather beyond the scope of

elementary works on the subject are very susceptible of being treated experimentally; and to the consideration of these some of the lectures of this course will be devoted.

Many of our illustrations will be designedly drawn from very commonplace sources: by this means I would try to impress upon you that mechanics is not a science that exists in books merely, but that it is a study of those principles which are constantly in action about us. Our own bodies, our houses, our vehicles, all the implements and tools which are in daily use—in fact all objects, natural and artificial, contain illustrations of mechanical principles. Examine the action of a crane raising weights, of a canal boat descending through a lock. Notice the way a roof is made, or how it is that a bridge can sustain its load. Take some opportunity of examining the parts of a clock, of a sewing-machine, and of a lock and key; visit a saw-mill, and ascertain the action of all the machines you see there; try to familiarize yourself with the principles of the tools which are to be found in any workshop. A vast deal of interesting and useful knowledge is to be acquired in this way.

THE DEFINITION OF FORCE.

2. It is necessary to know the answer to this question, What is a force? People who have not studied mechanics occasionally reply, A push is a force, a steam-engine is a force, a horse pulling a cart is a force, gravitation is a force, a movement is a force, &c. &c. Without discussing how far these are correct, I may say at once, that not one of them conveys the precise meaning which the word has in mechanics. The true definition

of force is *that which tends to produce or destroy motion.* You may probably not fully understand this until some explanation has been given; but, at all events, put any other notion of force out of your mind. Whenever I use the word Force, do you think of the words "something which tends to produce or destroy motion," and I trust before the close of the lecture you will understand how admirably the definition conveys what force really is.

3. When a string is attached to this small weight, I can, by pulling the string, move the weight along the table. In this case, there is something transmitted from my hand along the string to the weight in consequence of which the weight moves: that something is a force. I can also move the weight by pushing it with a stick, because force is transmitted along the stick, and makes itself known by producing motion. In using a bow and arrow, when I have drawn the bow I feel the string pulling the arrow, so that when released the arrow darts off. Here motion has been produced, and the force of elasticity of the bow has produced it. Before I released the arrow there was no motion, yet still the bow was exerting force and tending to produce motion. Hence in describing force we must say "that which tends to produce motion," whether it succeed in producing it or not.

4. But forces may also be recognized by their tendency to destroy motion. Before I release the arrow I am conscious of exerting a force upon it in order to counteract the pull of the bow. Here my force is merely manifested by its destroying the motion that, if it were absent, the bow would produce. So when I hold a weight in my hand, the force of my hand destroys the motion that the weight would have were I to let it fall; and if a weight

greater than I could support were placed in my hand, my efforts to sustain it would still be properly called force, because they tended to destroy motion, though unsuccessfully. We see by these simple cases that a force may be recognized either by producing motion or trying to produce it, or by destroying motion or tending to destroy it; and hence the propriety of the definition of force must be admitted.

THE MEASUREMENT OF FORCE.

5. It is evident that forces differ in magnitude, and it becomes necessary to establish some means of measuring them. The pressure exerted by a 1 lb. weight at London is the standard with which we shall compare other forces. The piece of iron or other substance which is attracted to the earth with a force of 1 lb. in London, is attracted to the earth with a greater force at the pole and a less force at the equator; hence, in order to define the standard force, we have to mention the locality in which the pressure of the 1 lb. weight is exerted.

It is easy to conceive that the magnitude of a pushing or a pulling force may be described as equivalent to so many pounds. The force which the muscles of a man's arm can exert is measured by the weight which he can lift. If a weight be suspended from an india-rubber spring, it is evident it will stretch the spring so that the weight pulls the spring and the spring pulls up the weight; hence the number of pounds in the weight is the measure of the force the spring is exerting. In every case the magnitude of a force is described by the number of pounds expressing the weight to which it is equivalent. There is another and better mode of measuring force occasionally used in mechanics, but the simpler method will suffice for our purpose.

6. But besides knowing the magnitude of a force, it is also necessary for us to be able to express conveniently the direction in which it acts. The direction in which a force tends to make the point to which it is applied move is called the direction of the force. Let us suppose, for example, that a force of 3 lbs. is applied at the point A, Fig. 1, tending to make A move in the direction AB. A standard line C of certain length is to be taken. It is supposed that a line of this length represents a force of 1 lb. The line AB is to be measured, equal to three times C in length, and an arrow-head is to be placed upon it to show the direction in which the force acts. Hence by means of a line of certain length and direction, and having an arrow-head attached to it, we are able completely to represent a force.

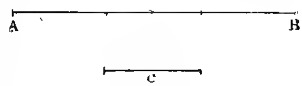

FIG. 1.

EQUILIBRIUM OF TWO FORCES.

7. In Fig. 2 we have represented two equal weights to which strings are attached; these strings, after passing over pulleys, are fastened in a knot C. C is pulled by two equal and opposite forces—I mark off parts CD, CE, to indicate the forces (Art. 6); and since there is no reason why C should move to one side more than the other, it remains at rest. Hence, then,

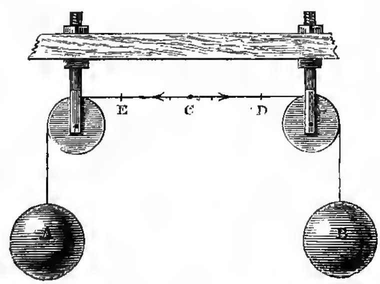

FIG. 2.

we learn that two equal and opposite forces counteract each other, and each of them may be regarded as destroying the motion which the other is striving to produce. If I make the weights unequal by placing an additional pound on one of the hooks, the knot is no longer at rest; it instantly moves off in the direction of the larger force.

8. When at rest under the action of two equal and opposite forces, a point is said to be in equilibrium. This word is used with reference to any set of forces which counteract each other. When one force acts upon a body, one more force at least must be present in order that the body should be at rest. If two forces acting on a point be not opposite, they will not be in equilibrium; this is easily shown by pulling the knot C in Fig. 2 downwards. When released, it flies back again. This proves that if two forces be in equilibrium, their directions must be opposite, for otherwise they will produce motion. We have already seen that the two forces must be equal.

EQUILIBRIUM OF THREE FORCES.

9. We now come to the important case where three forces act on a point: this is to be studied by the apparatus represented in Fig. 3. It consists essentially of two pulleys H, H, each about 2″[1] diameter, which are capable of turning very freely on their axles; the distance between

[1] We shall always, in these lectures, represent feet or inches in the manner usual among practical men—1′ is one foot, 1″ is one inch. Thus, for example, 3′ 4″ is to be read "three feet four inches." When it is necessary to use fractions we shall always employ decimals. For example, 0″·5 is the mode of expressing a length of half an inch, 3′ 1″·9 is to be read "three feet one inch and nine-tenths of an inch."

these pulleys is about 5′, and they are supported at a height of 8′ by a frame which will easily be understood from the figure. Over these pulleys passes a fine cord, 9′ or 10′ long, having a light hook at each end E, F. To the centre of this cord D another cord 2′ long is attached,

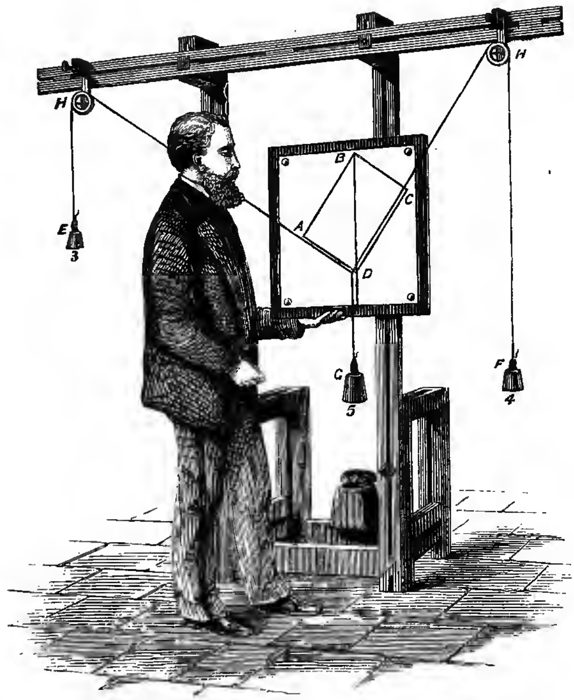

FIG. 3.

which at its free end G is also furnished with a hook. A number of iron weights, 0·5 lb., 1 lb., 2 lbs., &c., with rings at the top, are used; one or more of these can easily be suspended from the hooks as occasion may require.

10. We commence by placing one pound on each of

the hooks. The cords are seen to place themselves in a certain definite manner after a few oscillations. D is the point where the cords are united. If we move the point D to any new position, it will not, when liberated, remain there; it returns to where it was before. At this point the forces represented by the three weights are applied in directions corresponding to their respective cords. O P, O Q, O S, in Fig. 4, show the position which the cords assume. On examining these positions, we find that the three angles P O S, Q O S, P O Q, are all equal. This may very easily be proved by holding behind the cords a piece of cardboard on which three lines meeting at a point and making equal angles have been drawn; it will then be seen that the cords coincide with the three lines on the cardboard.

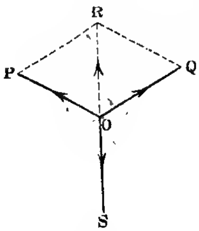

FIG. 4.

11. This might have been anticipated, because, the forces acting at O being all equal, we might have inferred that when in equilibrium they would be symmetrically arranged about the point; and the only way in which the three lines could be symmetrically arranged is when they make equal angles with each other.

12. The forces being each 1 lb., mark off along the three lines in Fig. 4 (which represent their directions) three equal parts O P, O Q, O S, and place the arrow-heads to show the direction in which each force is acting; the forces are then completely represented both in position and magnitude.

Since these forces make equilibrium, each of them may be considered to be counteracted by the other two. For example, O S is annulled by O Q and O P. But

O S could be balanced by a force O R equal and opposite to it. Hence O R is capable of producing by itself the same effect as the forces O P and O Q taken together. Therefore O R is equivalent to O P and O Q. Here we learn the important truth that two forces not in the same direction can be replaced by a single force. The process is called the *composition of forces*, and the single force is called the *resultant* of the two forces. O R is only one pound, yet it is equivalent to the forces O P and O Q together, each of which is also one pound. This is because the forces O P and O Q partly counteract each other. We shall presently learn that one force may even counteract two greater forces.

13. Draw the lines P R and Q R; then the angles P O R and Q O R are equal, because they are the supplements of the equal angles P O S and Q O S; and since the angles P O R and Q O R together make up one-third of four right angles, it follows that each of them is two-thirds of one right angle, and therefore equal to the angle of an equilateral triangle. Also O P being equal to O Q and O R common, the triangles O P R and O Q R must be equilateral. Therefore the angle P R Q is equal to the angle R O Q; thus P R is parallel to O Q: similarly Q R is parallel to O P; that is, O P R Q is a parallelogram. Hence we see that the resultant of two forces is the diagonal of a parallelogram, of which they are the two sides.

14. This remarkable property is called the parallelogram of force. Stated in its general form, the doctrine of the parallelogram of force asserts that two forces acting at a point have a resultant, and that this resultant is represented both in magnitude and direction by the diagonal of the parallelogram, two adjacent sides of which are the lines which represent the forces. H

15. The parallelogram of force may be illustrated in various ways by means of the apparatus of Fig. 3. Attach, for example, to the middle hook G 1·5 lb., and place 1 lb. on each of the end hooks E, F. Here the three weights are not equal, and symmetry will not enable us, as it did in the previous case, to foresee the condition which the cords will assume; but they will be observed to settle in a definite position, to which they will invariably return if withdrawn from it.

Let O P, O Q (Fig. 5) be the directions of the cords; O P and O Q being each of the length which corresponds to 1 lb., while O S corresponds to 1·5 lb. Here, as before, O P and O Q together may be considered to counteract O S. But O S could have been counteracted by O R equal and opposite to it. Hence O R may be regarded as the single force equivalent to O P and O Q, that is, as their resultant; and thus it is proved experimentally that these forces have a resultant. We can further verify that the resultant is the diagonal of the parallelogram of which the forces are sides. Construct a parallelogram on a piece of cardboard having its four sides equal, and one of the diagonals half as long again as one of the sides. This may be done very easily by first drawing one of the two triangles into which the diagonal divides the parallelogram. The diagonal is to be produced beyond the parallelogram in the direction O S. When the cardboard is placed close against the cords, the two cords will lie in the directions O P, O Q, while the produced diagonal will be in the vertical O S. Thus it is verified experi-

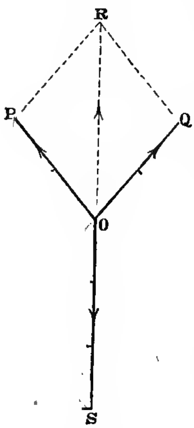

FIG. 5.

mentally that the parallelogram of force is true in this case also.

16. In the same figure the two forces O P and O S may be considered to be counterbalanced by the force O Q; in other words, O Q must be equal and opposite to a force which is the resultant of O P and O S. Here we see that two unequal forces may be compounded into one resultant.

17. Let us place on the central hook G a weight of 5 lbs., and weights of 3 lbs. and 4 lbs. on the other hooks. This is, in fact, the case shown in Fig. 3. The weights being unequal, we cannot immediately infer anything with reference to the position of the cords, but still we find, as before, that the cords assume a definite position, to which they return when temporarily displaced. Let Fig. 6 represent the positions of the cords. No two of the angles are in this case equal. Still each of the forces is counterbalanced by the other two. Each is therefore equal and opposite to the resultant of the other two. Construct the parallelogram on cardboard. This can be done by forming the triangle O P R, whose sides are 3, 4, and 5, and then drawing O Q and R Q parallel to R P and O P. Produce the diagonal O R to S. This parallelogram being placed behind the cords, you see that the directions of the cords coincide with its sides and diagonal, thus verifying the parallelogram of force in a case where all the forces are of different magnitudes.

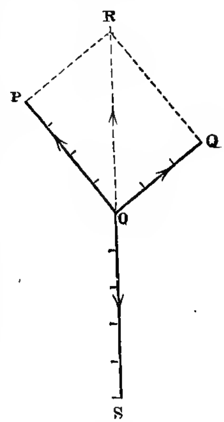

FIG. 6.

18. It is easy, by the application of a set square, to prove that in this case the cords attached to the 3 lb. and

4 lb. weights are at right angles to each other; the corner of an ordinary card, or sheet of paper, shows this very well. But we can also infer, from the parallelogram of force, that this must be the case. In Fig. 6, the sides of the triangle O P R are 3, 4, and 5 respectively. But since the square of 5 is 25, and the squares of 3 and 4 are 9 and 16, it follows that the square of one side of this triangle is equal to the sum of the squares of the two opposite sides, and therefore that this is a right-angled triangle (Euclid, i. 48). Hence since P R is parallel to O Q, the angle P O Q must also be a right angle.

A SMALL FORCE CAN OVERCOME TWO LARGER FORCES.

19. Cases might be multiplied indefinitely by placing various amounts of weight on the hooks, constructing the parallelogram on cardboard, and comparing it with the cords as before. We shall, however, confine ourselves to one more illustration, which is capable of very remarkable applications. Attach 1 lb. to each of the end hooks; the cord joining them remains straight until drawn down by placing a weight on the centre hook. A very small weight will suffice to do this. Let us put on 0·5 lb.; the position the cords then assume is indicated in Fig. 7. As before, each force is equal and opposite to the resultant of the other two. Hence a force of 0·5 lb. is the resultant of two forces each of 1 lb.; or we may say that we have a force of 0·5 lb. actually counterbalancing 2 lbs. The reason of this is, that the forces of 1lb. are very nearly opposite, and therefore to a large

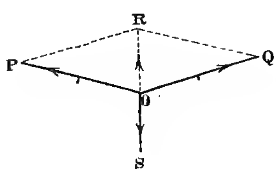

FIG. 7.

extent counteract each other. Constructing the card-board parallelogram in the manner already described, we easily see, by comparison, that the principle of the parallelogram of force holds in this case also.

20. No matter how small be the weight we place in the middle, you see that the cord is deflected; and if there be a weight in the middle, no matter how great a weight were attached to the ends, it would be impossible to straighten the cord. The cord could break, but it could not become horizontal. Look at a telegraph wire; it is never in a straight line between two consecutive poles, and its curved form is more evident the greater be the distance between the poles. But in putting up a wire great straining force is used, by means of special machines for the purpose; yet the wires cannot be straightened: this is because the weight of the wire itself acts as a force pulling it downwards. Just as the cord in our experiments cannot be straight when any force, however small, is pulling it downwards at the centre, so it is impossible by any exertion of force to straighten the long wire; the wire could be broken by the machine, but it could not be straightened. Some further illustrations of this principle will be given in our next lecture, and with one application of it the present will be concluded.

21. One of the most important practical problems in mechanics is to make a small force overcome a greater. There are a vast number of ways in which this may be accomplished for different purposes, and to the consideration of them several lectures of this course will be devoted. Perhaps, however, there is no arrangement more simple than that which is furnished by the principles we have been considering. We shall employ it

to enable us to raise a 28 lb. weight by means of a 2 lb. weight. I do not say that this particular application is of much practical use. I show it to you rather as a remarkable deduction from the parallelogram of forces than as a useful machine.

A rope is attached at one end of an upright, A (Fig. 8),

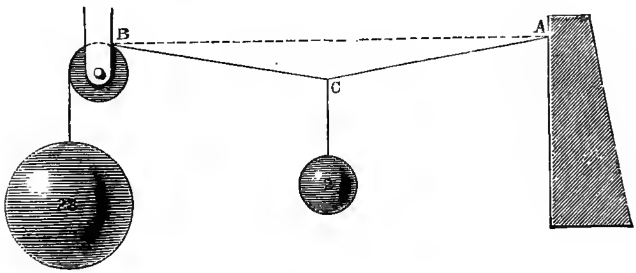

FIG. 8.

and passes over a pulley B at the same vertical height about 16′ distant. A weight of 28 lbs. is fastened to the free end of the rope, and the supports must be heavily weighted or otherwise secured from moving. The rope lies apparently horizontally, in consequence of its weight being very small compared with the strain (28 lbs.) to which it is subjected; this position is indicated in the figure by the dotted line AB. We now suspend from the middle of the rope a weight of 2 lbs. Instantly the rope moves to the position represented in the figure. But this it cannot do without at the same moment raising slightly the 28 lbs. This is evident, because, since two sides of a triangle, CB, CA, are greater than the third side, AB, more of the rope must lie between the supports when it is bent down by the 2 lb. weight than when it was horizontal. But this can only have taken place by shortening the rope between the pulley B and the 28 lb.

weight, for the rope is firmly secured at the other end. The amount by which the weight has been raised is so small that it is not visible to you at a distance. We can, however, easily show by an electrical arrangement that it is really higher.

22. When an electric current passes through this alarum you hear the bell ring, and the moment I stop the current the bell stops. A contrivance like this is used in telegraph offices when it is necessary to call the attention of the clerk to a message. I have fastened one piece of brass to the 28 lb. weight and another to the support close above it, but unless the weight be raised a little the two will not be in contact; the electricity is intended to pass from one of these pieces of brass to the other, but it cannot pass unless they are touching. When the rope is horizontal the two pieces of brass are separated, the current does not pass, and our alarum is dumb; but the moment I hang on the 2 lb. weight to the middle of the rope it raises the weight a little, brings the pieces of brass in contact, and now you all hear the alarum. On removing the 2 lbs. the current is interrupted and the noise ceases.

23. I am sure you must all have noticed that the 2 lb. weight descended through a distance of many inches, easily visible to all the room; that is to say, the small weight moved through a very considerable distance, while in so doing it only raised the larger one a very small distance. This is a point of the very greatest importance; I therefore take the first opportunity of calling your attention to it.

LECTURE II.

THE RESOLUTION OF FORCES.

Introduction.—One Force resolved into Two Forces.—Experimental Illustrations.—Sailing.—One Force resolved into Three Forces not in the same Plane.—The Jib and Tie-rod.

Fig. 9.

INTRODUCTION.

24. As the last lecture was principally concerned with discussing how one force could replace two forces, so in the present we shall examine the kindred question, How may two forces replace one force? Since the diagonal of a parallelogram is a single force equivalent to those represented by the sides, it is obvious that one force may be resolved into two others, provided it be the diagonal of the parallelogram formed by them.

25. We shall frequently employ in the present lecture, and in some of those that follow, the spring balance which is represented in Fig. 9: the weight is attached to the hook, and when the balance is suspended by the ring, a pointer indicates the number of pounds on a scale.

This balance is very convenient for showing the strain along a cord; for this purpose the balance is held by the ring while the cord is attached to the hook. It will be noticed that the balance has two rings and two corresponding hooks. The hook and ring at the top and bottom will weigh up to 300 lbs., corresponding to the scale which is seen. The hook and ring at the side correspond to another scale on the other face of the plate; this second scale weighs up to about 50 lbs., consequently for a weight under 50 lbs. the side hook and ring are employed, as they give a more accurate result than would be obtained by the top and bottom hook and ring, which are intended for larger weights. These ingenious and useful balances are very accurate, and can easily be tested by raising known weights. Besides the instrument thus described, we shall sometimes use one of a smaller size, and we shall be able with this aid to trace the existence and magnitude of forces in a most convenient manner.

ONE FORCE RESOLVED INTO TWO FORCES.

26. We shall first prove that a single force can be resolved into a pair of forces; for this purpose we shall use the arrangement shown in Fig. 10 (see next page).

The ends of a cord are fastened to two small spring balances; to the centre E of this cord a weight of 4 lbs. is attached. At A and B are pegs from which the balances can be suspended. If the distances AE, BE be each 12″, the distance AB should be about 18″. When the cord is thus placed, and the weight allowed to hang freely, each of the cords EA, EB is strained by an amount of force that is shown to be 3 lbs. by the balances. But the weight of 4 lbs. is the only weight acting; hence it must be equivalent to two forces of 3 lbs. each along the directions AE and BE. Here the two forces to which 4 lbs. is equivalent are each of them less than 4 lbs., though taken together they exceed it.

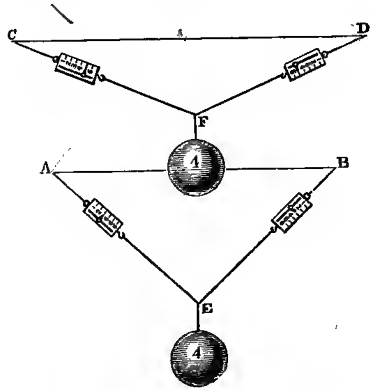

FIG. 10.

27. But remove the cords from AB and hang them on CD, the length CD being 1′ 10″, then the strains shown along FC and FD are each 5 lbs; here, therefore, one force of 4 lbs. is equivalent to two forces each of 5 lbs. In the last lecture (Art. 19) we saw that one force could balance two greater forces; here we see the analogous case of one force being changed into two greater forces. Further, we learn that the number of pairs of forces into which one force may be decomposed is unlimited, for with every different distance between the pegs different strains will be indicated by the balances.

Whenever the weight is suspended from a point halfway between the balances, the strains along the cords are equal; but by placing the weight nearer one balance than the other, a greater strain will be indicated on that scale to which the weight is nearest.

EXPERIMENTAL ILLUSTRATIONS.

28. The decomposition of one force into two forces greater than itself, is capable of being illustrated in a variety of ways, two of which will be here explained. In Fig. 11 an arrangement for this purpose is shown. A piece of stout twine AB, able to support from 20 lbs. to 30 lbs., is fastened at one end A to a fixed support, and

FIG. 11.

at the other end B to the eye of a wire-strainer. A wire-strainer consists of an iron rod, with an eye at one end and a screw and a nut at the other; it is used for tightening wires in wire fencing, and is employed in this case for the purpose of stretching the cord. When the string is tightening, the nut must be turned cautiously, otherwise the string would be broken. This being done, I take a piece of ordinary sewing-thread, which is of course weaker than the stout twine. I tie the thread to the middle of the cord at C, catch the other end in my fingers, and pull; something must break—something has broken: but what has broken? Not the slight thread, it is still whole; it is the cord which has snapped. Now this illustrates the point on which we have been dwelling. The force which I transmitted along the thread was

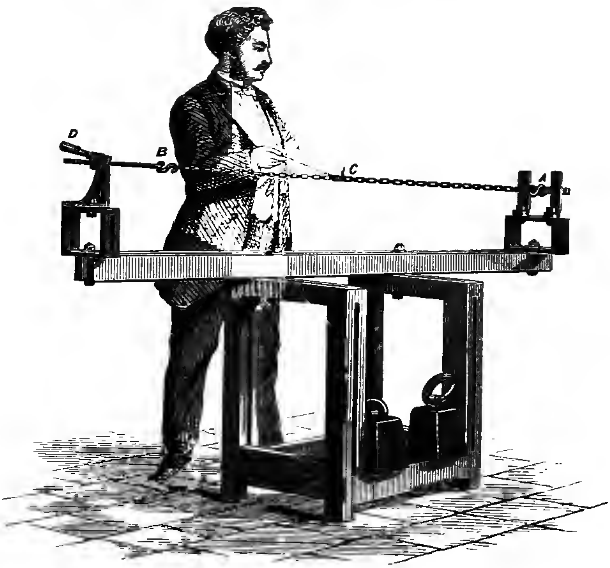

FIG. 12.

insufficient to break it; the thread transferred the force to the cord, but under such circumstances that the force was greatly magnified, and the consequence was that this magnified force was able to break the cord before the original force could break the thread. We can also see why it was necessary to stretch the cord. In Fig. 10, the strains along the cords are greater when the cords are attached at C and D, than when they are attached at A and B; that is to say, the more the cord is stretched towards a straight line, the greater are the forces into which the applied force is resolved.

29. We give a second example, in illustration of the same principle.

In Fig. 12 is shown a chain 8′ long, one end of which B is attached to a wire-strainer, while the other end is fastened to a small piece of pine A, which is 0″·5 square in section, and 5″ long between the two upright irons by which it is supported. By means of the nut of the wire-strainer I straighten the chain as I did the string of Fig. 11, and for the same reason. I then put a piece of twine round the chain and pull it gently. The strain brought to bear on the wood is so great that it breaks across. Here, then, the small force of a few pounds, transmitted to the chain by pulling the string, is magnified to upwards of a hundredweight, for less than this would not break the wood. The explanation is precisely the same as when the string was broken by the thread.

SAILING.

30. The action of the wind upon the sails of a vessel affords a very instructive and useful example of the decomposition of forces. By the parallelogram of force we are able to explain how it is that a vessel is able even to sail against the wind. A force is that which tends to produce motion, and motion generally takes place in the line of the force. In the case of the action of wind on a vessel through the medium of the sails, we have motion produced which is not necessarily in the

direction of the wind, and which may be to a certain extent opposed to it. This apparent paradox requires some elucidation.

31. Let us first suppose the wind to be blowing in a direction shown by the arrows of Fig. 13, perpendicular to the line AB in which the ship's course lies.

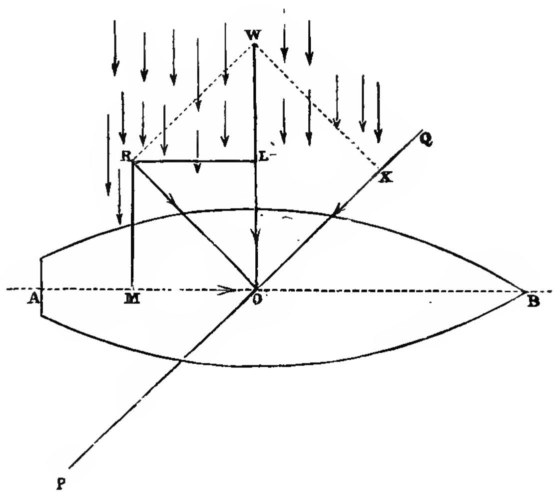

FIG. 13.

In what direction must the sail be set? It is clear that the sail must not be placed along the line AB, for then the only effect of the wind would be to blow the vessel sideways; nor could the sail be placed with its edge to the wind, that is, along the line OW, for then the wind would merely glide along the sail without producing a propelling force. Let, then, the sail be placed between the two positions, as in the direction PQ. The line OW represents the magnitude of the force of the wind pressing on the sail (Art. 6).

We shall suppose for simplicity that the sail is one of those attached to the yards of a ship, so that it extends on both sides of O. Through O draw OR perpendicular to PQ, and from W let fall the perpendicular WX on PQ, and WR on OR. By the principle of the parallelogram of force, the force OW may be decomposed into the two forces OX and OR, since these are the sides of the parallelogram of which OW, the force of the wind, is the diagonal. We may then leave OW out of consideration, and imagine the force of the wind to be replaced by the pair of forces OX and OR; but the force OX cannot produce an effect, it merely represents a force which glides along the surface of the sail, not one which pushes against it; so far as this component goes, the sail has its edge towards it, and therefore the force produces no effect. On the other hand, the sail is perpendicular to the force OR, and this is therefore the efficient component.

The force of the wind is thus measured by OR, both in magnitude and direction: this force represents the actual pressure on the mast produced by the sail, and from the mast communicated to the ship. Still OR is not in the direction in which the ship is sailing: we must again decompose the force in order to find its useful effect. This is done by drawing through R the lines RL and RM parallel to OA and OW, thus forming the parallelogram OMRL. Hence, by the parallelogram of force, the force OR is equivalent to the two forces OL and OM.

The effect of OL upon the vessel is to propel it in a direction perpendicular to that in which it is sailing. We must, therefore, endeavour to counteract this force as far as possible. For this purpose it is that a vessel has a keel, and that her form is designed so as to present the greatest possible resistance to being pushed sideways through the water: the deeper the keel the more completely is the the effect of OL annulled. Still OL would in all cases produce some effect were it not finally got rid of by means of the rudder, which, by turning the head of the vessel a little towards the wind, makes her sail in a direction sufficiently to windward to counteract the small effect of OL in driving her leeward.

Thus OL is disposed of, and the only force remaining is OM, which acts directly to push the vessel in the required direction. Here, then, we see how the wind, aided by the resistance of the water, is able to make the vessel move in a direction perpendicular to that in which the wind blows. We have seen that the sail must be set somewhere between the direction of the wind and that of the ship's motion. It can be proved that when the direction of the sail is such as to bisect the angle WOB, the magnitude of the force OM is greater than when the sail has any other position.

32. The same principles show us how a vessel is able to sail against the wind: she cannot, of course, sail straight against it, but she can sail within half a right angle of it, or perhaps even less. This can be seen from Fig. 14.

The small arrows represent the wind, as before. Let OW be the line parallel to them, which measures the force of the wind, and let the sail be placed along the line PQ; OW is decomposed into OX and OY, OX merely glides along the sail, and OY is the effective force. This is decomposed into OL and OM; OL is counteracted, as already explained, and OM is the force that propels the vessel onwards. Hence we see that there is a force acting to push the vessel onwards, even though the movement be partly against the wind.

It will be noticed in this case that the force OL acting to leewards exceeds OM pushing onwards. Hence it is that vessels with a very deep keel, and therefore opposing very great resistance to moving leewards, can sail more closely to the wind than others not so constructed; a vessel should be formed so that she shall move as freely as possible in the direction of her length, for which reason she is sharpened at the bow, and otherwise shaped for gliding through the water easily: this is in order that OM may have to overcome as little resistance as possible.

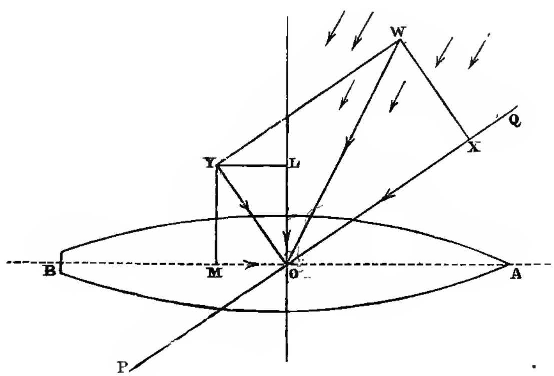

FIG. 14.

The sail PQ should bisect the angle AOW for the wind to act in the most efficient manner. Since, then, a vessel can sail towards the wind, it follows that, by taking a zigzag course, she can proceed from one port to another, even though the wind be blowing from the place to which she would go towards the place from which she comes. This well-known manœuvre is called tacking. You will understand that in a sailing-vessel the rudder has a more important part to play than in a steamer: in the latter it is only useful for changing the direction of the vessel's motion, while in the former it is not only necessary for changing the direction, but must also be used to keep the vessel to her course by counteracting the effect of leeway.

ONE FORCE RESOLVED INTO THREE FORCES NOT IN THE SAME PLANE.

33. Up to the present we have only been considering forces which lie in the same plane, but in nature we meet with forces acting in all directions, and therefore we must not be satisfied with confining our inquiries to the

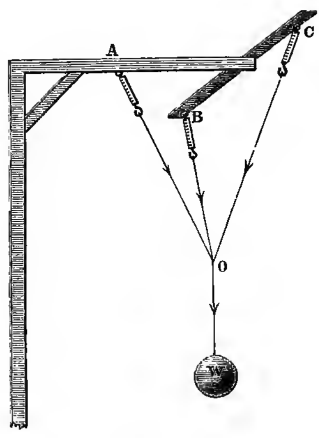

FIG. 15.

simpler case. We proceed to show, in two different ways, how a force can be decomposed into three forces not in the same plane, though passing through the same point. The first mode of doing so is as follows. To three points A, B, C (Fig. 15) three spring balances are attached; A, B, C are not in the same straight line, though they are at the same vertical height: to the spring balances cords are attached which unite in a point O, from which a weight W is suspended. This weight is supported by the three cords, and the strains along these cords are indicated by the spring balances. The greatest strain is on the shortest cord and the least strain on the longest. Here the force W lbs. produces three forces which taken together exceed its own amount. If I add a second weight W I find, as we might have anticipated, that the strains indicated by the scales are precisely double what they were before. This shows that the proportion of the force to each of the components into which it is decomposed does not depend on the actual magnitude of the force, but on the relative direction of the force and its components.

34. Another mode of showing the decomposition of one force into three forces not in the same plane is represented in Fig. 16. The tripod is formed of three strips of pine, $4' \times 0''\cdot5 \times 0''\cdot5$, secured by a piece of wire running through each at the top; one end of this wire hangs down, and carries a hook to which is attached a weight of 28 lb. This weight is supported by the wire, but the strain on the wire must be borne by the three wooden rods: hence there is a force acting downwards through the wooden rods. We cannot render this manifest by a contrivance like the spring scales, because it is a push instead of a

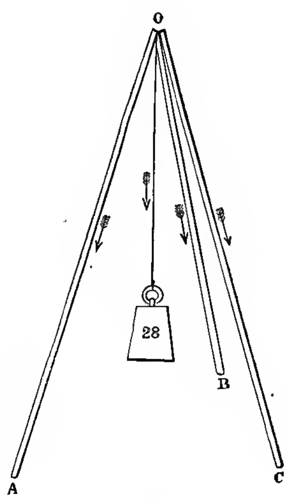

FIG. 16.

pull. However, by raising one of the legs I at once become aware that there is a force acting downwards through it. The weight is, then, decomposed into three forces, which act downwards through the legs; these three forces are not in a plane, and the three forces taken together are larger than the weight.

35. This contrivance is very well known for supporting weights; it is convenient on account of its portability, and is very steady. You may judge of its strength by the model represented in the figure, for though the legs are very slight, yet they support very securely a considerable weight. The pulleys by means of which gigantic weights are raised are often supported by colossal tripods, sometimes called shears. They possess stability and steadiness in addition to great strength. We shall have occasion to use tripods subsequently in these lectures (see Figs. 49 and 92).

36. An important point may be brought out by contrasting the arrangements of Figs. 15 and 16. In the one case three cords are used, and in the other three rods. Three rods would have answered for both, but three cords would not have done for the tripod. In one the strings are strained, and the tendency of the strain is to break the string, but in the other the nature of the force down the rods is entirely different; it does not tend to pull the rod asunder, it is trying to crush the rod, and had the weight been large enough the rods would bend and break. I hold one end of a pencil in each hand and then try to pull the pencil asunder; the pencil is in the condition of the strings of Fig. 15; but if instead of pulling I push my hands together, the pencil is like the rods in Fig. 16.

37. This distinction is of great importance in me-

chanics. A string which is in a state of tension is called a tie, while a rod in a state of compression is called a strut. Since a rod can resist both tension and compression it can serve either as a tie or a strut, but a cord or chain can only act as a tie. A pillar is always a strut, as the superincumbent load makes it to be in a state of compression. These words will very frequently be used during this course of lectures, and it is necessary that they be thoroughly understood.

THE JIB AND TIE ROD.

38. As an illustration of the nature of the tie and strut, and also for the purpose of giving a useful example of the decomposition of forces, I use the apparatus of Fig. 17 (see next page).

This represents the principle which is employed in the common lifting crane, and which has numerous applications in practical mechanics. A piece of pine B C 3′ 6″ long and 1″ × 1″ section is capable of turning round its support at the bottom B by means of a joint or hinge: this piece is called the jib; it is held up by a tie A C 3′ long, which is attached to the support exactly above the joint. A B is 1′ long. From the point C a wire descends, having a hook at the end on which a weight can be hung. The tie is attached to the spring balance, the index of which shows the strain. The spring balance is supported by a wire-strainer, by turning the nut of which the length of the wire can be shortened or lengthened as occasion requires. This is necessary because when different weights are suspended from the hook the spring is stretched more or less, and the screw is then employed to keep the entire length of the tie at 3′. The remainder of the tie consists of copper wire.

39. Suppose a weight of 20 lbs. be suspended from the hook, it endeavours to pull the top of the jib downwards; but the tie holds it back, consequently the tie is put into a state of tension, as indeed its

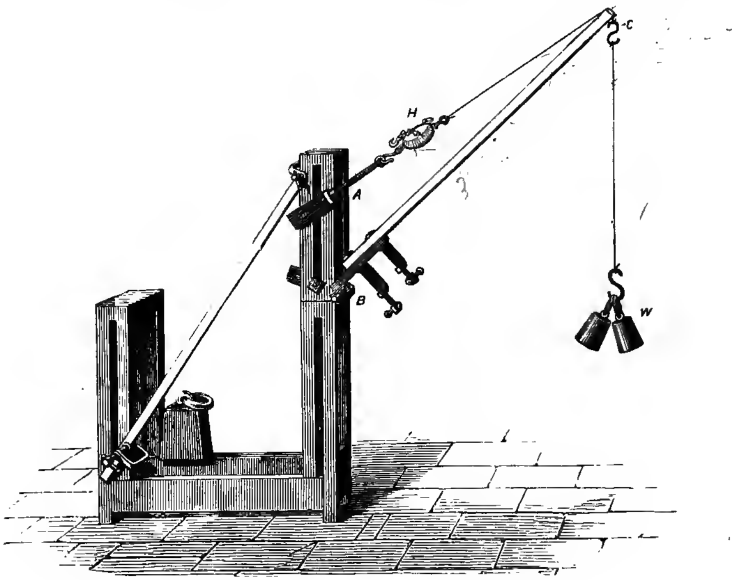

Fig. 17.

name signifies, and the magnitude of that tension is shown to be 60 lbs. by the spring balance. Here we find again what we have already so often referred to; namely, one force developing another force that is greater than itself, for the strain along the tie is three times as great as the strain in the vertical wire by which it was produced.

40. What is the condition of the jib? It is evidently being pushed downwards on its joint at B; it is there-

fore in a state of compression; it is a strut. This will be evident if we think for a moment how absurd it would be to endeavour to replace the jib by a string or chain: the whole arrangement would collapse. The weight of 20 lbs. is therefore decomposed by this contrivance into two other forces, one of which is resisted by a tie and the other by a strut.

41. We have no means of showing the magnitude of the strain along the strut, but we shall prove that it can be computed by means of the parallelogram of force; this will also explain how it is that the tie is strained by a force three times that of the weight which is used. Through C (Fig. 18) draw C P parallel to the tie A B, and P Q parallel to the strut C B, then B P is the diagonal of the

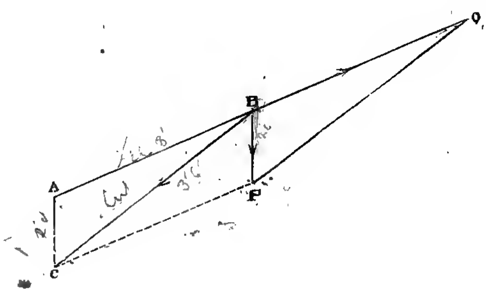

FIG. 18.

parallelogram whose sides are each equal to B C and B Q. If therefore we consider the force of 20 lbs. to be represented by B P, the two forces into which it is decomposed will be shown by B Q and B C; but A B is equal to B Q, since each of them is equal to C P; also B P is equal to A C. Hence the weight of 20 lbs. being represented by A C, the strain along the tie will be represented by the length A B, and that along the strut by the length B C. Remembering that A B is 3′ long, C B 3′ 6″, and A C 1′, it follows that the strain along the tie is 60 lbs., and along the strut 70 lbs., when the weight of 20 lbs. is suspended from the hook.

42. In every other case the strains along the tie and strut can be determined, when the suspended weight is known, by their proportionality to the sides of the triangle formed by the tie, the jib, and the upright post.

43. In this contrivance you will recognize, no doubt, the framework of the common lifting crane, but that very essential portion of the crane which provides for the raising and lowering is not shown here. To this we shall return again in a subsequent lecture (Art. 332). You will of course understand that the tie rod we have been considering is entirely different from the chain for raising.

44. It is easy to see of what importance to the engineer the information acquired by means of the decomposition of forces may become. Thus in the simple case with which we are at present engaged, suppose an engineer were required to erect a frame which was to sustain a weight of 10 tons, let us see how he would be enabled to determine the strength of the tie and jib. It is of importance in designing any structure not to make any part unnecessarily strong, as doing so involves a waste of valuable material, but it is of still more vital importance to make every part strong enough to avoid the risk of accident not only under ordinary circumstances, but also under the exceptionally great shocks and strains to which every structure is liable.

45. According to the numerical proportions we have employed for illustration, the strain along the tie rod would be 30 tons when the load was 10 tons, and therefore the

tie must at least be strong enough to bear a pull of 30 tons; but it is customary, in good engineering practice, to make the machine of about ten times the strength that would just be sufficient to sustain the ordinary load. Hence the crane must be so strong that the tie rod would only be broken by 100 tons suspended from the chain; that is, by a strain of 300 tons upon the tie rod. This large increase is necessary on account of the jerks and other occasional great strains that arise in the raising and lowering of heavy weights. For a crane intended to raise 10 tons, the engineer must therefore design a tie rod which not less than 300 tons would tear asunder. If the tie rod be composed of wrought iron rods, we can determine its size by the following considerations. It has been proved by actual trial that a rod of wrought iron of average quality one square inch in section, requires twenty tons to tear it asunder. Hence fifteen such rods, or one rod the section of which was equal to fifteen square inches, would require 300 tons to pull it asunder, and this is therefore the proper size for the tie rod of the crane we have been considering.

46. In the same way we ascertain the actual strain down the jib; it amounts to 35 tons, and the jib must be ten times as strong as a strut which would collapse under a strain of 35 tons.

47. It is necessary that the upright support AB (Fig. 17) be secured very firmly. It is easy to see from the figure that the tie rod is pulling the upright, and tending, in fact, to make it snap off near B; a crane post must be firmly imbedded in masonry, or otherwise secured, to resist the pull of the tie.

LECTURE III.

PARALLEL FORCES.

Introduction.—Pressure of a Loaded Beam on its Supports.—Equilibrium of a Bar supported on a Knife-edge.—The Composition of Parallel Forces.—Parallel Forces acting in opposite directions.—The Couple.—The Weighing Scales.

INTRODUCTION.

48. THE parallelogram of force enables us to find the resultant of two forces which intersect: but since parallel forces do not intersect, we are unable to apply the construction to determine the resultant of two parallel forces. We can, however, find this resultant very simply by other means; to explain the method of doing so, we shall approach the subject by means of some experimental arrangements, which appear to lead most naturally to the desired end.

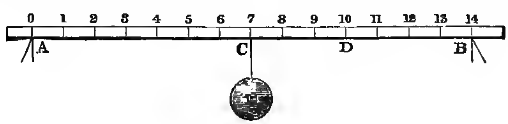

FIG. 19.

49. Fig. 19 represents a wooden rod 4′ long, sustained by resting on two supports A B, and having the length A B divided into 14 equal parts. Let a weight of

14 lbs. be hung on the rod at its middle point C; this weight must be borne by the supports, and it is evident that they will bear it equally, for since the weight is at the middle of the rod, there is no reason why one end should be differently circumstanced from the other. Hence the total pressure on each of the supports will be 7 lbs., together with half the weight of the wooden bar.

50. If a weight of 14 lbs. be placed at D, it is not then so easy to see in what proportion the weight is divided between the supports. We can easily understand that the support near the weight must bear more than the remote one, but how much more? When we are able to answer this question, we shall see that it will lead us to a knowledge of the composition of parallel forces.

PRESSURE OF A LOADED BEAM ON ITS SUPPORTS.

51. We shall employ the apparatus shown in Fig. 20. An iron bar 5′ 6″ long, weighing 10 lbs., rests in the hooks of the spring balances A, C, in the manner shown in the figure. These hooks are exactly five feet apart, so that the bar projects 3″ beyond each end. The space between the hooks is divided into twenty equal portions, each of course 3″ long. The bar is sufficiently strong to bear the weight B of 20 lbs. suspended from it by an S hook, without appreciable deflection. Before the weight of 20 lbs. is suspended, the spring scales each show a strain of 5 lbs. We would expect this, for it is evident that the whole weight of 10 lbs. should be borne equally by the two supports.

52. When I place the weight in the middle, 10 divisions from each end, I find the balances each indicate 15 lbs.

But 5 lbs. is due to the weight of the bar. Hence the 20 lbs. is divided equally, as we have already stated that it should be. But let the 20 lbs. be moved to a position which is 4 divisions from the right, and 16 divisions from the left; then the right-hand scale reads 21 lbs., and the left-hand reads 9 lbs. To get rid of the weight of the bar itself, we must subtract 5 lbs. from each.

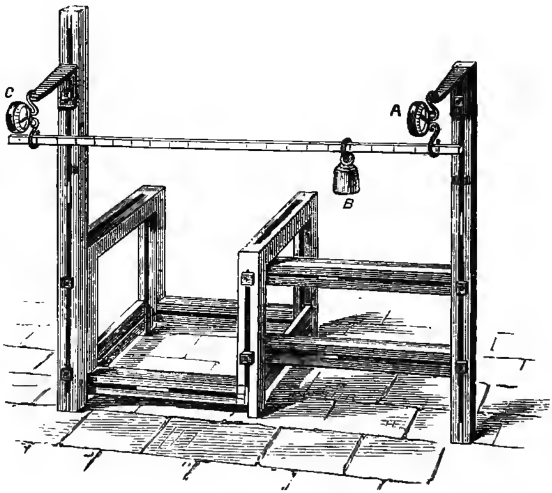

FIG. 20.

We learn therefore that the 20 lbs. pull the right-hand spring scale with a strain of 16 lbs., and the left with a strain of 4 lbs. Observe this closely; you see the number of divisions in the bar is equal to the number of pounds weight suspended from it, and here we see that when the weight is 16 divisions from the left, the strain of 16 lbs. is shown on the right. At the same time the weight is 4 divisions from the right, and 4 lbs. is the strain shown on the left.

53. I will state the law a little more generally, and we shall find that the bar will prove it to be true in all cases. The law is this, divide the bar into as many equal parts as there are pounds in the weight, then the pressure in pounds on one end is the number of divisions that the weight is distant from the other.

54. For example, suppose I place the weight 2 divisions from one end: I read by the scale at that end 23 lbs.; subtracting 5 lbs. I find that the pressure is 18 lbs., but the weight is then exactly 18 divisions distant from the other end. We can easily verify this rule whatever be the position which the 20 lbs. occupies.

55. If the weight be placed between two divisions, instead of being, as we have hitherto supposed, exactly at one of the marks on the bar, the result is also readily ascertained. If the weight were, for example, 3·5 divisions from one end, the strain on the other would be 3·5 lbs., and in like manner for other cases.

56. We have then proved by actual experiment this very curious and beautiful law of nature; the same result could be inferred, by reasoning from the parallelogram of force, but the purely experimental proof is more in accordance with our scheme. This is one of the most important truths of mechanics, and we shall have many occasions to employ it in this and subsequent lectures.

57. Returning now to Fig. 19, with which we commenced, the rule we have acquired will enable us to see how the weight is distributed. We divide the length of the bar between the supports into 14 equal parts because the weight is 14 lbs.: if, then, the weight be at D, 10 divisions from one end A, and 4 from the other B, the pressure at the corresponding ends will be 4 and 10. If the weights were 2·5 divisions from one end, and therefore

11·5 from the other, the corresponding pressures would be 11·5 lbs. and 2·5 lbs. These are the pressures produced by the 14 lb. weight, but the actual weight supported at each end is 6 ounces greater if the wooden bar which weighs 12 ounces be taken into account.

58. Let us suspend a second weight from another point of the bar. We must then find the pressures which each separately would produce according to the rule, and these are to be added together, and to half the weight of the bar to find the total pressure. Thus, if one weight of 14 lbs. were in the middle, and another at a distance of 11 divisions from one end, the middle weight would produce 7 lbs. at each end and the other 3 lbs. and 11 lbs., and therefore the total pressures produced by the weights would be 10 lbs. and 18 lbs. The same principles will evidently apply, if there be several weights: the application of the rule is more simple when all the weights are equal, for then the same divisions will answer for finding the effect of each weight.

59. The principles involved in these calculations are of the very greatest importance. We shall further examine them by a different method which, however, leads to a similar result.

EQUILIBRIUM OF A BAR SUPPORTED ON A KNIFE-EDGE.

60. The weight of the bar has hitherto somewhat complicated our calculations; our results would appear more satisfactorily if we could avoid this weight, but since we want a strong bar, its weight is not so small that we could afford to overlook it altogether. By means of the arrangement of Fig. 21, we can, however, counterpoise the weight of the bar. To the centre of A B a cord is

attached, which passing over a pulley D attached to the framework carries a hook. The bar being a pine rod, 4 feet long and 1 inch square, weighs about 12 ounces;

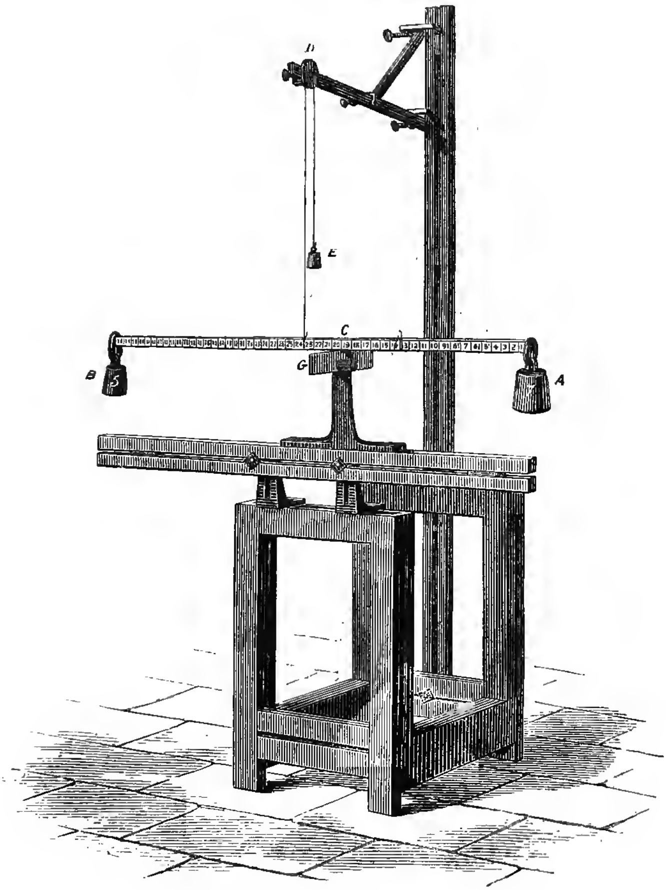

FIG. 21.

consequently if this weight be, as it is in the figure, suspended from the hook, the bar will be counterpoised, and will remain at whatever height it is placed.

61. A B is divided by lines drawn along it at distances of 1″ apart; there are thus 48 of these divisions. It carries at one end a small pin driven into it, from which a weight may be hung while at the other end, which is intended for larger weights; the ring of the weight is slipped on the bar itself.

62. Underneath the bar lies an important portion of the arrangement; namely, the knife-edge G. This is a blunt edge of steel firmly fastened to the support which carries it. This support can be moved along underneath the bar so that the knife-edge can be placed under any of the divisions required. It is shown in the figure at the division C. The bar being counterpoised, though still unloaded with weights, may be brought down till it just touches the knife-edge; it will then remain horizontal, and it will retain this position whether the knife-edge be at either end of the bar, or in any intermediate position. I shall hang weights at the extremities of the rod, and we shall find that there is for each pair of weights just one position at which, if the knife-edge be placed, it will sustain the rod horizontally. We shall then examine the relations between these distances and the weights that have been attached, and we shall trace the connection between the results of this method and those of the arrangement that we used in Art. 51.

63. Supposing that 6 lbs. be hung at each end of the rod, we might easily foresee that the knife-edge should be placed in the middle, and we find our anticipations verified. When the edge is exactly at the middle, the rod remains horizontal; but if it be moved, even by a very small amount, to either side, the rod instantly descends on the other. The edge is then 24 inches distant from each end; and if I multiply this number by 6, the

number of pounds I find 144 for the product, and this number is the same of course for both sides. The importance of this remark will be seen directly.

64. The weight of the bar being counterpoised in the manner already explained, we may omit every thought of its weight; the total weight then to be supported by the knife-edge is 12 lbs.

65. If I remove one of the 6 lb. weights and replace it by 2 lbs., leaving the other and the knife-edge unaltered, the bar instantly descends on the side of the heavy weight; but, by slipping the knife-edge along the bar, I find that when I have moved it to within a distance of 12 inches from the 6 lb., and therefore 36 inches from the 2 lb., the bar will remain horizontal. The edge must be at the right place; a quarter of an inch to one side or the other would upset the bar. The whole strain borne by the knife-edge is of course 8 lbs., being the sum of the weights. If we multiply 2, the number of pounds at one end, by 36, the distance of that end from the knife-edge, we find the product 72; and we would have found precisely the same number by multiplying 6, the number of pounds in the other weight, by 12, its distance from the knife-edge. To express this result concisely we shall introduce the word "moment," a term of frequent use in mechanics. The 2 lb. weight is a force tending to pull its end of the bar downwards by making the bar turn round the knife-edge. The magnitude of this force, multiplied into its distance from the knife-edge, is called the moment of the force. We can then express the result at which we have arrived by saying that, when the knife-edge has been placed so that the bar remains horizontal, the moments of the forces produced by the weights about the knife-edge are equal.

66. This may be illustrated by hanging 7 lbs. and 5 lbs. from the ends; it is found that the knife-edge must be placed 20 inches from the larger weight, and, therefore, 28 inches from the smaller, but 5 × 28 = 140, and 7 × 20 = 140, thus verifying the law. The apparatus will verify the law in every case, provided the weights be not too heavy for the bar.

THE COMPOSITION OF PARALLEL FORCES.

67. Having now examined these cases experimentally, we proceed to investigate what may be learned from the results we have proved.

The weight of the bar in the first case being allowed for in the way we have explained by subtracting 5 lbs. from each of the strains indicated by the spring balance, we may omit it from consideration. The balances being pulled downwards by the bar when it is loaded, they must conversely pull the bar upwards. This will be evident if we look at a weight—say 14 lbs.—suspended from one of these scales: it hangs at rest; therefore its weight, which is constantly urging it downwards, must be counteracted by an equal force pulling it upwards. The scale of course shows 14 lbs.; thus the spring exerts in an upward pull a force which is precisely equal to the force with which it is itself pulled downwards.

68. Hence the springs are exerting forces at the ends of the bar in pulling them upwards, and the scales indicate the magnitude of these forces. The bar is thus subject to three forces, viz.: the weight of 20 lbs. which is hanging from it, and which acts vertically downwards, and the two other forces which act vertically upwards, and the united action of the three make equilibrium.

69. Let lines be drawn, representing the forces in the manner already explained (Art. 6). We have then three parallel forces AP, BQ, CR acting on a rod in equilibrium (Fig. 22). The two forces AP and BQ may be considered as balanced by the force CR in the position shown in the figure, but the force CR would be balanced by the equal and opposite force CS, represented by the dotted line. Hence this last force is precisely equivalent to AP and BQ. In other words, it must be their resultant. Here then we learn that a pair of parallel forces, acting in the same direction, can be compounded into a single resultant.

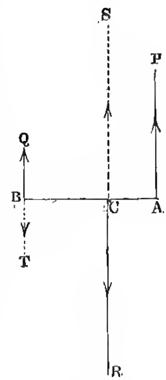

FIG. 22.

70. We also see that the magnitude of the resultant is equal to the sum of the magnitude of the forces, and further we find the position of the resultant by this rule. Add the two forces together; divide the distance between them into as many equal parts as is contained in the sum, measure off from the greater of these two forces as many parts as there are pounds in the magnitude of the smaller force, and that is the point required: this rule is very easily inferred from that which we were taught by the experiments in Art. 51.

PARALLEL FORCES ACTING IN OPPOSITE DIRECTIONS.

71. Since the forces AP, BQ, CR (Fig. 22) are in equilibrium, it follows that we may look on B Q as balancing in the position which it occupies the two forces of AP and CR in their positions. This may remind us of the numerous instances we have already met with, where

our force balanced two greater forces: in the present case AP and CR are acting in opposite directions, and the force BQ which balances them is equal to their difference. A force BT equal and opposite to BQ must then be the resultant of CR and AP, since it is able to produce the same effect. Notice that in this case the resultant of the two forces is not between them, but that it lies on the side of the larger. When the forces act in the same direction, the resultant is always between them.

72. The actual position which the resultant of the opposite parallel forces occupies is to be found by the following rule. Divide the distance between the forces into as many equal parts as there are pounds in their difference, then measure from the point of application of the larger force as many of these parts as there are pounds in the smaller; the point thus found determines the position of the resultant. Thus, if the forces be 14 and 20, the difference between them is 6, and therefore the distance between their directions is divided into six parts; from the point of application of the force of 20, 14 parts are measured off, and thus the position of the resultant is determined. Hence we have the means of compounding two parallel forces in all cases.

THE COUPLE.

73. In one case, however, two parallel forces have no resultant; this occurs when the two forces are equal, and in opposite directions. A pair of forces of this kind is called a couple; there is no single force which could balance a couple,—it can only be counterbalanced by another couple acting in an opposite manner. This

remarkable case, as well as others, may be studied by the arrangement of Fig. 23.

A wooden rod, A B 48″ × 0″·5 × 0″·5, has strings attached to it at points A D, one foot distant. The string at D passes over a pulley E, and to the end of each a hook P Q is attached for the purpose of receiving weights; the weight of the rod itself, which only amounts to three ounces, may be neglected, as it is very small compared with the weights which will be used.

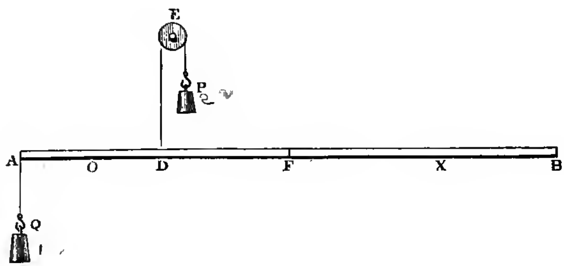

FIG. 23.

74. Supposing 2 lbs. to be placed at P, and 1 lb. at Q, we have two parallel forces acting in opposite directions; since their difference is 1 lb., the line A D is not to be divided, and the point F where D F is equal to A D is the point where the resultant is applied. You see that this is easily verified, for by placing my finger over the rod at F, it remains horizontal and in equilibrium; whereas, when I move my finger to one side or the other, equilibrium is impossible. If I move it nearer to B, the end A ascends. If I move it towards A, the end B ascends.

75. For the case when the two forces are equal, 2 lbs. is to be placed on each of the hooks P and Q. It will then be found that the finger placed in any position along the rod will not keep it in equilibrium; that is to

say, no single force can counteract the two forces which form the couple. Let O be the point midway between A and D. The forces evidently tend to raise O B and turn the part O A downwards; but if I try to restrain O B by holding a rod firmly above it so that it presses against it as at the point X, instantly the rod begins to bend round X and the part from A to X descends. I find similarly that any attempt to prevent O A from going down by holding a rod under it fails equally to produce equilibrium. But if I press the rod downwards at one point, and at the same time upwards at another with suitable force, I can produce equilibrium; in this case the two pressures form a couple, and it is this couple which neutralizes the couple produced by the weights. We learn then, that a couple can be balanced by a couple, and by a couple only.

76. We have already defined a moment. You see from Fig. 20 a confirmation of the property shown in Art. 65. The moment of the force 16 at A around the point B is equal to the moment of the force 4 at C about B, since each of them is the product of 4 and 16. This will indicate the connection between the results represented in Fig. 20 and the arrangement of Fig. 21. There we found that the moments of the forces at each end of the rod about the knife-edge were equal.

THE WEIGHING SCALES.

77. Another apparatus by which the nature of parallel forces may be investigated is shown in Fig. 24; this consists of a slight frame of wood A B C, 4′ long. At E, a pair of steel knife-edges is clamped to the frame. The knife-edges rest on two pieces of steel, one of which

is shown at O F. When the knife-edges are suitably placed, the frame balances itself very delicately; in fact, a small piece of paper laid at A will instantly cause that side to descend. Indeed, it is found that some slight counterpoise must always be added to one side or the other, in order to compensate for the inevitable slight difference in weight, which even by careful construction of the frame cannot be avoided.

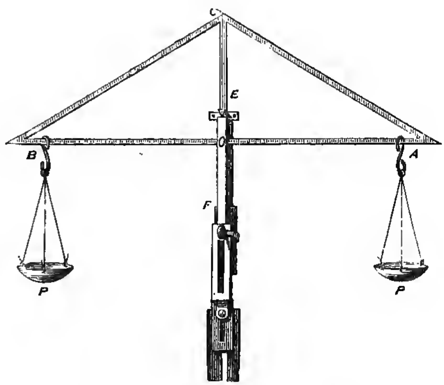

FIG. 24.

78. We attach two small hooks A and B: these are made of fine wire and weigh but little. The frame being exactly balanced, its weight may be left out of consideration. With this apparatus we can easily verify the principle of equality of moments: for example, if I place the hook A at a distance of 9″ from O and load it with 1 lb., I find that when B is laden with 0·5 lb. it must be at a distance of 18″ from O in order to counterbalance A; the moment in the one case is 9 × 1, in the other 18 × 0·5, and these are obviously equal.

79. Let a weight of 1 lb. be placed on each side of the centre, the frame will only be in equilibrium when

the weights are at precisely the same distance from the centre. This is the principle of the ordinary weighing scales; the frame which is in this case called a *beam* is sustained by two knife-edges, smaller, however, than those represented in the figure. The pans P,P are suspended from the extremities of the beam, and should be at equal distances from its centre. These scale-pans must be of equal weight, and then, when equal weights are placed in them, the beam will remain horizontal. If the weight in one slightly exceed that in the other, the pan containing the heavier weight will of course descend.

80. That a pair of scales should weigh accurately, it is necessary that the weights be correct; but even with correct weights, a balance of defective construction will give an inaccurate result. The error frequently arises from a slight inequality in the lengths of the arms of the beam. When this is the case, the two weights which will balance are not equal. Supposing, for instance, that with an imperfect balance I endeavour to weigh a pound of shot. If I put the weight on the short side, then the quantity of shot balanced is less than 1 lb.; while if the 1 lb. weight be placed at the long side, it will require more than 1 lb. of shot to balance it. The mode of testing a pair of scales is then evident. Let weights be placed in the pans which balance each other; if then the weights be interchanged and the balance still remains horizontal, it is correct.

81. Suppose, for example, that the two arms be 10 inches and 11 inches long, then, if 1 lb. weight be placed in the pan of the 10-inch end, its moment is 10; and if $\frac{10}{11}$ of 1 lb. be placed in the pan belonging to the 11-inch end, its moment is also 10: hence 1 lb. at the short end balances $\frac{10}{11}$ of 1 lb. at the long end; and, therefore, if

the shopkeeper placed his weight in the short arm, his customers would lose $\frac{1}{11}$ part of each pound for which they paid; on the other hand, if the shopkeeper placed his 1 lb. weight on the long arm, then this would require $\frac{11}{10}$ lb. in the pan belonging to the short arm to balance it. Hence in this case the customer would get $\frac{1}{10}$ lb. too much. It follows, therefore, that if a shopman placed the weights alternately in the one scale and the other he would be a loser on the whole; because, though every alternate customer gets $\frac{1}{11}$ lb. less than he ought, yet the others get $\frac{1}{10}$ lb. more than they have paid for.

LECTURE IV.

THE FORCE OF GRAVITY.

Introduction.—Specific Gravity.—The Plummet and Spirit Level.—The Centre of Gravity.—Stable and Unstable Equilibrium.—Property of the Centre of Gravity in a Revolving Wheel.

INTRODUCTION.

82. In the last three lectures we have been occupied with forces in the abstract; we have seen how they are to be represented, how compounded together and decomposed into others; we have explained what is meant by forces being in equilibrium, and we have shown instances where the forces lie in the same plane or in different planes, and where they intersect or are parallel to each other. These subjects are the elements of mechanics; they form the skeleton which in this and subsequent lectures we shall try to clothe in a more attractive garb. We shall commence by studying the most remarkable force in nature, a force constantly in action, and one to which all bodies are subject, a force which distance cannot annihilate, and one the properties of which have led to the most sublime discoveries of human intellect. This is the force of gravity.

83. If I drop a stone from my hand, it falls to the ground. Now that which produces motion is a force: hence the stone must have been acted upon by a force

which drew it to the ground. On every part of the earth's surface experience shows that a body tends to fall. This fact will prove that there is an attractive force in the earth tending to draw all bodies towards it.

84. Let ABCD (Fig. 25) be points from which stones are let fall, and let the circle represent the section of the earth; let PQRS be the points on the surface of the earth

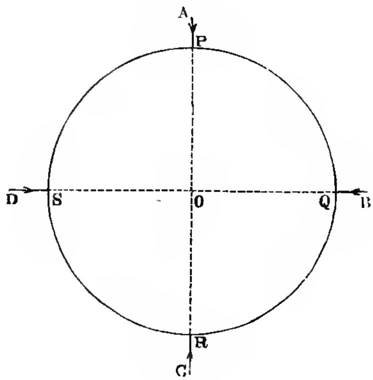

FIG. 25.

on which the stones will drop when allowed to do so. The four stones will move in the directions of the arrows: from A to P the stone moves in an opposite direction to the motion from C to R; from B to Q it moves from right to left, while from D to S it moves from left to right. The movements are in different directions; but if I produce these directions, as indicated by the dotted lines, they each pass through the centre O.

85. Hence each stone in falling moves towards the centre of the earth, and the force actuating each stone acts towards the centre of the earth. We therefore assert that the earth has an attraction for the stone, in consequence of which the stone tries to get as near its

centre as possible, and this attraction is called the force of gravitation.

86. We are so excessively familiar with the falling of a body that it does not excite in us any astonishment, and rarely even provokes our curiosity. A clap of thunder, which every one notices, though much less frequent, is not really more remarkable. We all look with attention on the attraction of a piece of iron by a magnet, and justly so, for the phenomenon is very curious, and yet the falling of a stone is produced by a far grander and more important force than the force of magnetism.

87. It is gravity which causes the weight of bodies. I hold a piece of lead in my hand: gravity tends to pull it downwards, and it produces a pressure on my hand which I call weight. Gravity acts with slightly different force at different parts of the earth's surface. This is due to two distinct causes, one of which may be mentioned here, while the other will be subsequently referred to. The earth is not perfectly spherical, it is flattened a little at the poles; consequently at the pole a body is nearer the general mass of the earth than it is at the equator; therefore it is more attracted at the pole, and therefore weighs more. A mass which weighs 200 lbs. at the equator would weigh one pound more at the pole: about one-third of this increase is due to the cause here pointed out. (See Lecture XVII.)

88. Gravity is a force which attracts every particle of matter; it acts not merely on those parts of a body which are on the surface, but it equally affects those in the interior. This is proved by observing that a body weighs the same amount, however its shape be altered: for example, suppose I take a ball of putty which weighs 1 lb., I shall find that its weight remains unchanged when

the ball is flattened into a thin plate, though in the latter case the surface, and therefore the number of superficial particles, is larger than it was in the former.

SPECIFIC GRAVITY.

89. Gravity produces different effects upon different bodies. This is commonly expressed by saying that some substances are heavier than others; for example, I have here a piece of wood and a piece of lead of equal bulk. The lead is drawn to the earth with a greater force than the wood. Bodies are usually termed heavy when they sink in water, and light when they float upon it. But a body sinks in water if it weigh more than an equal bulk of water, and floats if it weigh less. Hence it is natural to take water as a standard with which the weights of other bodies may be compared.

90. I take a certain volume, say a cubic inch of cast iron such as this I hold in my hand, and which has been accurately shaped for the purpose. This cube is heavier than one cubic inch of water, but I shall find that a certain quantity of water is equal to it in weight; that is to say, a certain number of cubic inches of water, and it may be fractional parts of a cubic inch, are precisely of the same weight. This number is called the specific gravity of cast iron.

91. It would be impossible to counterpoise water with the iron without holding the water in a vessel, and the weight of the vessel must then be allowed for. I adopt the following plan. I have here a number of inch cubes of wood (Fig. 26), which alone are of course lighter than cubic inches of water, but I have weighted them by placing grains of shot into holes bored for the purpose. The weight of each cube has been accurately adjusted to

be equal to that of a cubic inch of water. This may be tested by actual weighing. I weigh one of the cubes and find it to be 252 grains, which is well known to be the weight of a cubic inch of water.

92. But the cubes may be shown to be identical in weight with the same bulk of water by a simpler method. One of them placed in water should have no tendency to sink, since it is not heavier than water, nor on the other hand,

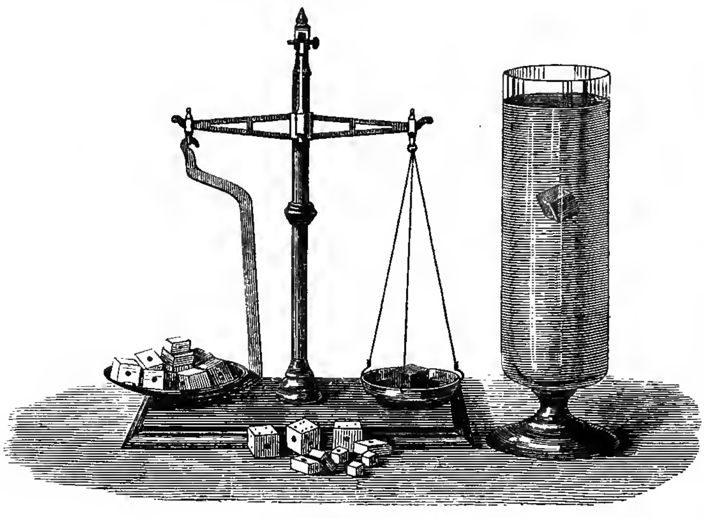

FIG. 26.

since it is not lighter, should it have any tendency to float. It should then remain in the water in whatever position it may be placed. It is very difficult to prepare one of these cubes so accurately that this result should be attained, and it is impossible to ensure its continuance for any time owing to changes of temperature and the absorption of water by the wood. We can, however, by a slight modification, show you that one of these cubes is at all events nearly equal in weight to the same bulk of water. In Fig. 26 is shown a tall jar which is filled with fluid; its appearance is that of a vessel filled with water, but I have arranged it in the following manner. I first poured into the jar a very weak solution of salt and water which partially filled it, I then poured gently upon this a little pure water, and finally filled up the jar with water containing a little spirits of wine: the salt and water is a little heavier than pure water, while the spirit and water is a little lighter. I take one of the cubes and drop it gently into the glass; it falls through the spirit and water, and after making a few oscillations settles itself at rest in the stratum shown in the figure. This shows us that our prepared cube is a little heavier than spirit and water, and a little lighter than salt and water, and hence we infer that it must at all events be very near the weight of pure water which lies between the two. We have also a number of half cubes, quarter cubes, and half quarter cubes, which have been similarly prepared to be of equal weight with an equal bulk of water.

93. We shall now be able to measure the specific gravity of a substance. In one pan of the scales I place the inch cube of cast iron, and I find that $7\frac{1}{4}$ of the wooden cubes, which we may call cubes of water, will balance it. We therefore say that the specific gravity of iron is rather over 7. The exact number found by more accurate methods is 7·2. It is often convenient to remember that 23 cubic inches of cast iron weigh 6 lbs., and that therefore one cubic inch weighs very nearly $\frac{1}{4}$ lb.

94. I have also cubes of brass, lead, and ivory; by counterpoising them with the cubes of water, we can easily find their specific gravities; they are shown

together with that of cast iron in the following table ;—

Substance.	Specific Gravity.
Cast Iron	7·2
Brass	8·1
Lead	11·3
Ivory	1·8

95. The mode here adopted of finding specific gravities is entirely different from the far more accurate methods which are actually used, but the latter are complicated, and depend on more difficult principles than we have been considering. The method we have used is intended more as an explanation of the nature of specific gravity than as a good means of determining it, though, as we have seen, it gives a result which is sufficiently near the truth for many purposes.

THE PLUMMET AND SPIRIT-LEVEL.

96. The tendency of the earth to draw all bodies towards it is well illustrated by the useful line and plummet. This consists merely of a string to one end of which a leaden weight is attached. The string when at rest hangs vertically; if the weight be drawn to one side, it will, when released, swing backwards and forwards, until it finally settles again in the vertical: the reason of this is, that when the string is vertical the weight is nearer the earth than in any other position.

97. The surface of water in equilibrium is a horizontal plane; this is also a consequence of gravity. All the particles of water try to get as near the earth as possible, and therefore, if any portion of the water were higher than the rest, it would immediately spread, as by doing so it could get lower.

98. Hence the surface of a fluid at rest enables us to find a perfectly horizontal plane, while the plummet gives us a perfectly vertical line: both these consequences of gravity are of the utmost importance.

99. The spirit-level is another common and very useful instrument which depends on gravity. It consists of a glass tube slightly curved, with its convex surface upwards, and attached to a plate. This tube is nearly filled with spirit, but a bubble of air is allowed to remain. The tube is permanently adjusted so that when the plate is laid on a perfectly horizontal surface, the bubble will rise to the top: this gives a means of ascertaining whether a surface is level, for unless it be so, the bubble will not rest at the top.

THE CENTRE OF GRAVITY.

100. We proceed to an experiment which will give us an insight into a curious property of gravity. I have here a plate of sheet iron; it has the irregular shape shown in Fig. 27. Five small holes A B C D E are punched at different positions on the margin. Attached to the framework is a small pin from which I can suspend the iron plate by one of its holes A: the plate is not supported in any other way; it hangs freely from the pin, around which it can be easily turned. I find that there is one position, and one only, in which the plate will rest; if I withdraw it from that position, it returns to it after a few oscillations. In order to mark this position, I suspend a line and plum-

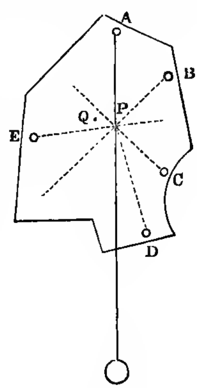

FIG. 27.

met from the pin, having rubbed the line with chalk. I allow the line to come to rest in front of the plate. I then carefully flip the string against the plate, and thus produce a chalked mark: this of course traces out a vertical line AD on the plate.

I now remove the plummet and suspend the plate from another of its holes B, and repeat the process, thus drawing a second chalked line BP across the plate, and so on with the other holes: I thus obtain five lines across the plate, represented by dotted lines in the figure. It is a very remarkable circumstance that these five lines all intersect, in the same point P; and if additional holes were bored in the plate, whether in the margin or not, and the chalk line drawn from each of them in the manner described, they would one and all pass through the same point. This remarkable point is called the centre of gravity of the plate, and the result at which we have arrived may be expressed by saying that from whatever point the plate be suspended the vertical line through it passes through the centre of gravity.

101. At the centre of gravity P a hole has been bored, and when I place the supporting pin through this hole you see that the plate will rest indifferently in all positions: this is a curious property of the centre of gravity. The centre of gravity may in this respect be contrasted with another hole Q, which is only an inch distant: when I support the plate by this hole, it has only one position of rest, viz. when the centre of gravity P is vertically beneath Q. Thus the centre of gravity differs remarkably from any other point in the plate.

102. We may conceive the force of gravity on the plate to act as a force applied at P. It will then be easily seen why this point remains vertically underneath the

point of suspension when the body is at rest. If I attached a string to the plate and pulled it, the plate would evidently place itself so that the direction of the string would pass through the point of suspension; in like manner gravity so places the plate that the direction of its force passes through the point of suspension.

103. We have learned, then, that a plate of any form has in it one point possessing very remarkable properties, and we may state in general that in every body, no matter what its shape be, there is a point called the centre of gravity, such that if the body be suspended from this point it will remain in equilibrium indifferently in any position, and that if the body be suspended from any other point then it will be in equilibrium, when the centre of gravity is directly underneath the point of suspension. In general, it will of course be impossible to support a body exactly at its centre of gravity, as this point is in the mass of the body, and it may also sometimes happen that the centre does not lie in the body at all, as for example in a ring, in which case the centre of gravity is at the centre of the ring. We need not, however, dwell on these exceptional cases, as sufficient illustrations of the truth of the laws mentioned will present themselves subsequently.

STABLE AND UNSTABLE EQUILIBRIUM.

104. An iron rod A B, capable of revolving round an axis passing through its centre P, is shown in Fig. 28.

The centre of gravity is at the axis, and consequently, as is easily seen, the rod will remain at rest in whatever position it be placed. But let a weight R be attached to the rod by means of a binding screw. The centre of gravity of the whole is no longer at the centre of the rod; it has moved to a point S nearer the weight; we may easily ascertain its position by removing the rod from its axle and then ascertaining the point about which it will balance. This may be done by placing the bar on a knife-edge, and moving it to and fro until the right position be secured; mark this position on the rod, and return it to its axle, the weight being still attached. We do not now find that the rod will balance in every position. You see it will rest if the point S be directly underneath the axis, but not if it lie to one side or the other. But if S be directly over the axis, as in the figure, the rod is in a curious condition. It will, when carefully placed, remain at rest; but if it receive the slightest displacement, it will tumble over. The rod is in equilibrium in this position, but it is what is called unstable equilibrium. If the centre of gravity be vertically below the point of suspension, the rod will return again if moved away: this position is therefore called one of stable equilibrium. It is very important to notice the distinction between these two kinds of equilibrium.

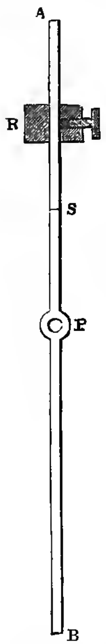

FIG. 28.

105. Another way of stating the case is as follows. A body is in stable equilibrium when its centre of gravity is at the lowest point; unstable when it is at the highest. This may be very simply illustrated by an ellipse, which I hold in my hand. The centre of gravity of this figure is at its centre. Now the ellipse, when resting on its side, is in a position of stable equilibrium; its centre

of gravity is then clearly at its lowest point. But I can also balance the ellipse on its narrow end, though if I do so the smallest touch suffices to overturn it. The ellipse is then in unstable equilibrium; in this case, obviously, the centre of gravity is at the highest point.

106. I have here a sphere, the centre of gravity of which is at its centre; in whatever way the sphere is placed on the plane, its centre is at the same height, and therefore cannot be said to have any highest or lowest point; in such a case as this the equilibrium is neutral. If the body be displaced, it will not return to its old position, as it would have done had that been a position of stable equilibrium, nor will it deviate further therefrom as if the equilibrium had been unstable: it will simply remain in the new position to which it is brought.

107. An iron ring about 6″ diameter is shown in Fig. 29.

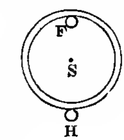

FIG. 29.

I try to balance it upon the end of a stick H, but I cannot succeed in doing so. This is because its centre of gravity S is above the point of support; but if I place the stick at F, the ring is in stable equilibrium, for now the centre of gravity is below the point of support.

PROPERTY OF THE CENTRE OF GRAVITY IN A REVOLVING WHEEL.

108. There are many other very curious consequences which follow from the properties of the centre of gravity, and we shall conclude by illustrating one of the most remarkable, which is at the same time of the utmost importance in machinery.

109. It is necessary that a machine should work

as steadily as possible, and that undue vibration and shaking of the framework should be avoided: this is particularly the case when any parts of the machine move with a great velocity, as, if these be heavy, very great vibration will be produced when the proper adjustments are not made. The connection between this and the centre of gravity will be understood by reference to the

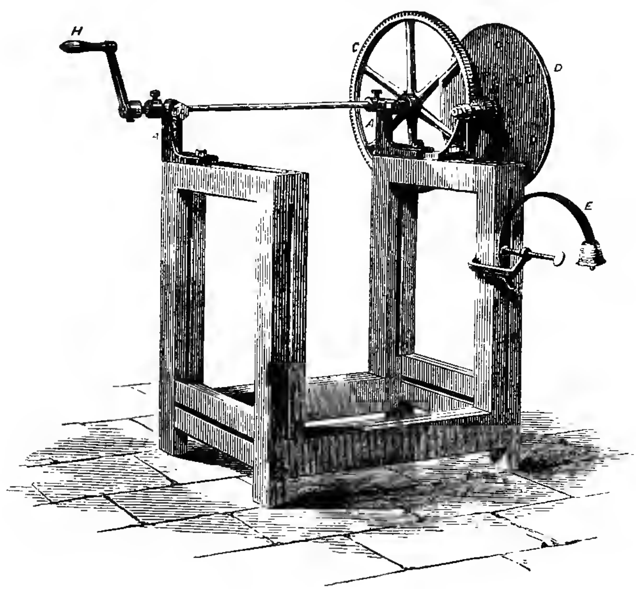

FIG. 30.

accompanying figure (Fig. 30). In this we have an arrangement consisting of a large cog wheel C working into a small one B, whereby, when the handle H is turned, a velocity of rotation can be given to the iron disk D, which weighs 14 lbs., and is 18″ in diameter. This disk being uniform, and being attached to the axis at its centre, it follows that its centre of gravity is also the centre of rotation. The wheels are attached to a stand, which, though massive, is still unconnected with the floor. By turning the handle I can rotate the disk very rapidly, even as much as twelve times in a second. Still the stand remains quite steady, and the shutter bell attached to it at E is silent.

110. Through one of the holes in the disk, I fasten a small iron bolt and a few washers, altogether weighing about 1 lb.; that is, only one-fourteenth of the weight of the disk. When I turn the handle very slowly, the machine works as smoothly as before; but as I increase the speed up to one revolution every two seconds, the bell begins to ring violently, and when I increase it still more, the stand quite shakes about on the floor. What is the reason of this? By adding the bolt, I slightly altered the position of the centre of gravity of the disk, but I made no change of the axis about which the disk rotated, and consequently the disk was not on this occasion turning round its centre of gravity: this it was which caused the vibration. It is absolutely necessary that the centre of gravity of any heavy piece, rotating rapidly about an axis, should lie in the axis of rotation. The amount of vibration produced by a high velocity is quite out of proportion to the very small size of the mass which produces it.

111. But in order that the machine may work smoothly again, it is not necessary to remove the bolt from the hole. If by any means I bring back the centre of gravity to the axis, the same end will be attained. This is very simply effected by placing a second bolt of the same size at the opposite side of the disk, the two being at equal distances from the axis; on turning the handle, the machine is seen to work as smoothly as it did in the first instance.

112. The most common rotating pieces in machines are wheels of various kinds; and in these the centre of gravity is evidently identical with the centre of rotation; but if from any cause a wheel, which is to turn rapidly, has an extra weight attached to one part, this weight must be counterpoised by one or more on other portions of the wheel, in order to keep the centre of gravity of the whole in its proper place. The cause of the vibration will be understood after the lecture on centrifugal force (Lect. XVII.)

LECTURE V.

THE FORCE OF FRICTION.

Introduction.—The Mode of Experimenting.—The Coefficient of Friction.—A more accurate Law of Friction.—Effect of the Extent of the Experiments.—The Angle of Friction.—Another Law of Friction.—Concluding Remarks.

INTRODUCTION.

113. A DISCUSSION of the force of friction is a necessary preliminary to the study of the mechanical powers which we shall presently commence. Friction renders the inquiry into the mechanical powers more difficult than it would be if this force were absent; but it is too important in its effects to be overlooked.

114. The nature of friction may be understood by Fig. 31: this represents a section of the top of a smooth

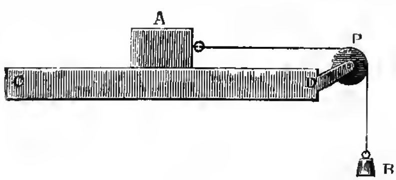

FIG. 31.

table levelled so that C D is a horizontal line; on this rests a block of wood or any other material A, its surface in contact with the table being also smooth. To A a cord is attached, which, passing over a pulley P, is attached to another weight B. If B exceed a certain weight, A is pulled along the table; but if B be small, both A and B remain at rest. What supports B when at rest? It is the friction between A and the table; there is a certain amount of coherence between the two surfaces which the weight of B cannot overcome. Friction is a force, because it prevents the motion of B. It is generally manifested as a force by destroying motion, though sometimes indirectly producing it.

115. The true cause of the force is roughness of the surfaces in contact, which the utmost care in polishing cannot wholly efface. The minute asperities on one surface are detained in corresponding hollows in the other, and consequently force must be exerted to make one surface slide upon the other. By care in polishing the surfaces the amount of friction may be diminished, but it can only be decreased to a certain limit, beyond which no amount of polishing produces any perceptible difference.

116. The law of friction between smooth surfaces must, then, be inquired into, in order that we may make allowance for it when its effect is of importance. We shall find in this inquiry that some interesting laws of nature will appear, but the discussion of the experiments is sometimes a little difficult, and the truths arrived at are principally numerical.

THE MODE OF EXPERIMENTING.

117. Friction is present between every pair of surfaces which are in contact: there is friction between two pieces of wood, and between a piece of wood and a piece of iron; but the amount of the force depends upon the character of the surfaces. We shall confine ourselves to

the friction of wood upon wood, as more will be learned by a careful study of a special case than by a less minute examination of a number of pairs of different substances.

118. The apparatus used is shown in Fig. 32. A plank of pine 6′ × 11″ × 2″ is planed on its upper surface, levelled by a spirit-level, and firmly secured to the framework at a height of about 4′ from the ground. On it is a pine slide 9″ × 9″, the grain of which is crosswise to that of the plank; upon the slide the load A is placed. A rope is attached to the slide, which passes over a very freely mounted cast iron pulley C, 14″ diameter, and carries at the other end a hook weighing one pound, to which weights B can be attached.

119. The mode of experimenting consists in placing a certain load on A, and then ascertaining what weight applied to B will draw the loaded slide along the plane. As several trials are generally necessary to determine the power, a rope is attached at the back of the slide, and passes over the two pulleys D; this makes it easy for the experimenter, when applying the weights at B, to draw back the slide to the end of the plane by pulling the ring E: this rope is of course left quite slack during the process of the experiment, since the slide must not be retarded. The loads used at A during the series of experiments ranged from one stone up to eight stone. These weights include the weight of the slide, which is under 1 lb. A number of weights with rings were used for the hook B; they consisted of 0·1, 0·5, 1, 2, 7, 14 lbs. A slight amount of friction has to be overcome in the pulley C, but the pulley being large its friction is very small, and can easily be allowed for on principles which will be explained in Art. 130.

120. An example of the experiments tried is thus described. A weight of 56 lbs. is placed on the slide, and it is found on trial that 29 lbs. on B, including

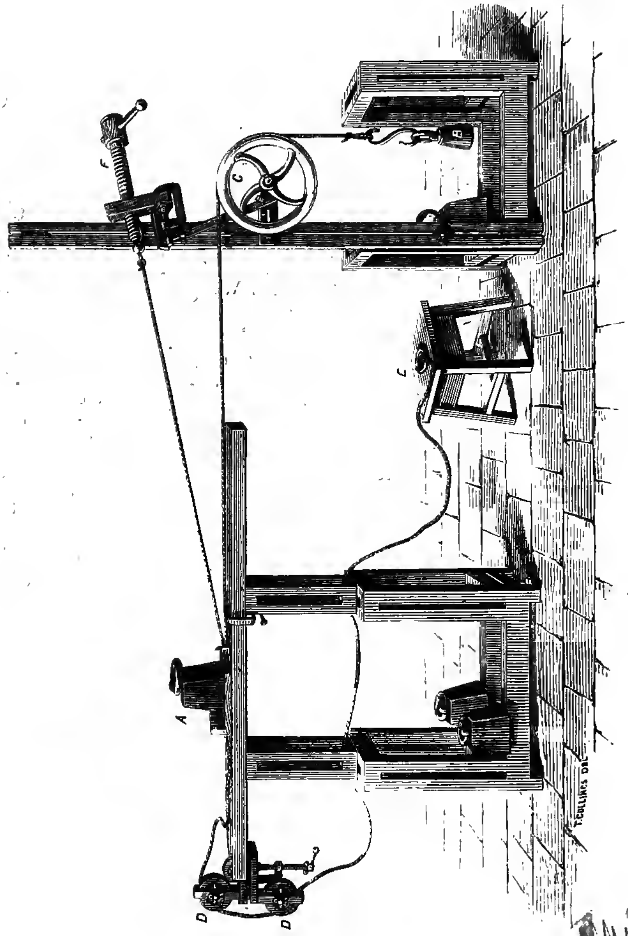

FIG. 32.

the weight of the hook itself, is sufficient to start the slide; the weight is placed on the hook pound by pound, care being taken to avoid a sudden jerk.

121. These experiments were tried when the weights on A were successively increased, and the results are recorded in Table I.

TABLE I.—FRICTION.

Smooth horizontal surface of pine 72″ × 11″; slide also of pine 9″ × 9″; grain crosswise; slide is not started; force acting on slide is gradually increased until motion commences.

Number of Experiment.	Load on slide in lbs., including weight of slide.	Force necessary to move slide. 1st Series.	Force necessary to move slide. 2nd Series.	Mean values.
1	14	5	8	6·5
2	28	15	16	15·5
3	42	20	15	17·5
4	56	29	24	26·5
5	70	33	31	32·0
6	84	43	33	38·0
7	98	42	38	40·0
8	112	50	33	41·5

In the first column a number is given to each experiment for convenience of reference. In the second column the load on the slide is stated in lbs. In the third column is found the force necessary to overcome the friction. In the fourth column is a second series of experiments performed in the same manner as the first series; while in the last column the means of the results will be found.

122. The first remark to be made upon this table is, that the results do not appear satisfactory or concordant. Thus from 6 and 7 of the 1st series it would appear that the friction of 84 lbs. was 43 lbs., while that of 98 lbs. was 42 lbs., so that here the greater weight appears to have the less friction, which is evidently contrary to the whole tenor of the results, as a glance will show. Moreover the results in the 1st and the 2nd series do not agree,

being generally greater in the former than in the latter, the discordance being especially noticeable in experiment 8, where the results were 50 lbs. and 33 lbs. In the column of mean results these irregularities do not appear so strongly marked: this column certainly shows that the friction increases with the weight, but it is sufficient to observe that while the difference of 1 and 2 is 9 lbs., and that of 2 and 3 is only 2 lbs., it is hopeless to get much accurate information from these results.

123. But is friction so capricious that it is amenable to no better law than these experiments appear to indicate? We must look a little more closely into the matter. When two pieces of wood have remained in contact and at rest for some time, a second force besides friction resists their separation: the wood is compressible, the surfaces come closely into contact, and the coherence due to this cause must be overcome before motion commences. The initial coherence is uncertain; it depends probably on a multitude of minute circumstances which it is impossible to estimate, and its presence has vitiated the results which we have found so unsatisfactory.

124. These difficulties we can avoid by starting the slide in the first instance. This may be conveniently effected by the screw shown at F in Fig. 32; a string attached to its end is fastened to the slide, and by giving the handle of the screw a few turns the slide is set in motion. A body once set in motion will continue to move with the same velocity unless acted upon by a force; hence the weight at B just overcomes the friction when the slide moves along uniformly after receiving a start: this velocity was in one case of average speed measured to be 16″ per minute.

125. Indeed in no case can the slide *commence* to

move unless the force exceed the friction. The amount of this excess is quite indeterminate. It is certainly greater between wooden surfaces than between less compressible surfaces like those of metals. In the latter case, when the force exceeds the friction by a small amount, the slide starts off with an excessively slow motion, while with wood the force must exceed the friction by a larger amount before the slide commences to move, but when it does move the motion is rapid.

126. If the power be too small, the load either does not continue moving after the start, or it stops irregularly. If the power be too great, the load is drawn with an accelerated velocity. The correct amount is easily recognized by the uniformity of the movement, and even when the slide is heavily laden, a few tenths of a pound on the power hook make a great difference.

127. The accuracy with which the friction can be measured may be appreciated by inspecting Table II.

TABLE II.—FRICTION.

Smooth horizontal surface of pine 72″ × 11″; slide also of pine 9″ × 9″; grain crosswise; slide started; force applied is sufficient to maintain uniform motion of the slide.

Number of Experiment.	Load on slide in lbs., including weight of slide.	Power necessary to maintain motion. 1st Series.	Power necessary to maintain motion. 2nd Series.	Mean values.
1	14	4·9	4·9	4·9
2	28	8·5	8·6	8·5
3	42	12·6	12·4	12·5
4	56	16·3	16·2	16·2
5	70	19·7	20·0	19·8
6	84	23·7	23·0	23·4
7	98	26·5	26·1	26·3
8	112	29·7	29·9	29·8

128. Two series of experiments to determine the power necessary to maintain the motion have been recorded. Thus, in experiment 7, the load on the slide being 98 lbs., it was found that 26·5 lbs. was sufficient to draw the slide along, and a second trial being made quite independently, the power found was 26·1 lbs.: a mean of the two values, 26·3 lbs., is adopted as being near the truth. The greatest difference between the two series, amounting to 0·7 lb., is found in experiment 6; a third value was therefore obtained for the friction of 84 lbs.: this amounted to 23·5 lbs., which is intermediate between the two former results, and 23·4 lbs., a mean of the three, is adopted as the final result.

129. The close concordance of the experiments in this table shows that the means of the fifth column are probably very near the true values of the friction for the corresponding loads upon the slide.

130. The mean values must, however, be slightly diminished before we can assert that they represent only the friction of the wood upon the wood. The pulley over which the rope passes turns round its axle with a small amount of friction which must be overcome by the power. The mode of estimating this amount, which in these experiments never exceeds 0·5 lb., may be gathered from Art. 160, but need not be dwelt on further. The corrected values are shown in the third column of Table III. Thus, for example, 4·9 of experiment 1 consists of 4·7, the true friction of the wood, and 0·2, which is the friction of the pulley; and 26·3 of experiment 7 is similarly composed of 25·8 and 0·5. It is the corrected values which will be employed in our subsequent calculations.

THE COEFFICIENT OF FRICTION.

131. Having ascertained the values of the force of friction for eight different weights, we proceed to inquire what law may be founded on our results. It is evident that the friction increases with the load, of which it is always greater than a fourth, and less than a third. It is then natural to surmise that the friction is really a constant fraction of the load—in other words, that $F = k\ R$, where k is a constant number.

132. To test this supposition we must try to determine k; this may be ascertained by dividing any value of F by the corresponding value of R. If this be done, we shall find that each of the experiments yields a different quotient; the first gives 0·336, and the last 0·262, while the other experiments give results between these extreme values. These numbers are tolerably close together, but there is still sufficient discrepancy to show that it is not strictly true to assert that the friction is proportional to the load.

133. But the law as thus stated is still approximately true, and sufficiently so for many purposes of calculation, and the question then arises, which of the different values of k shall we adopt? or can we adopt any of them? By a method which is described in the Appendix we can determine a value for k which, while it does not represent any one of the experiments precisely, yet represents them collectively better than it is possible for any other value to do. The number thus found is 0·27. It is intermediate between the two values already stated as extremes. The character of this result is determined by an inspection of Table III.

TABLE III.—FRICTION.

Friction of pine upon pine; the mean values of the friction given in Table II. (corrected for the friction of the pulley) compared with the formula $F = 0{\cdot}27\ R$.

Number of Experiment.	R. Total load on slide in lbs.	Corrected mean value of friction.	F. Calculated value of friction.	Difference of the observed and calculated values.
1	14	4·7	3·8	− 0·9
2	28	8·2	7·6	− 0·6
3	42	12·2	11·3	− 0·9
4	56	15·8	15·1	− 0·7
5	70	19·4	18·9	− 0·5
6	84	23·0	22·7	− 0·3
7	98	25·8	26·5	+ 0·7
8	112	29·3	30·2	+ 0·9

The fourth column of this table has been calculated from the formula $F = 0{\cdot}27\ R$. Thus, for example, in experiment 5 the friction of a load of 70 lbs. is 19·4 lbs., and the product of 70 and 0·27 is 18·9, which is 0·5 lbs. less than the true amount. In the last column of this table the differences between the observed and calculated values are recorded, for facility of comparison. It will be observed that the greatest difference is under 1 lb.

134. Hence the law $F = 0{\cdot}27\ R$ represents the experiments with a tolerable amount of accuracy; 0·27 is called the coefficient of friction. We may apply this law to ascertain the friction in any case where the load lies between 14 lbs. and 112 lbs.; for example, if the load be 63 lbs., the friction is $63 \times 0{\cdot}27 = 17{\cdot}0$.

135. The coefficient of friction would have been slightly different had the grain of the slide been parallel to that of the plank; and it of course varies with the nature of the surfaces. Experimenters have given tables

of the coefficients of friction of various substances, wood, stone, metals, &c. The use of these coefficients depends upon the assumption of the ordinary law of friction, namely, that the friction is proportional to the pressure: this law is accurate enough for most purposes, especially when used for loads that lie between the extreme weights employed in calculating the value of the coefficient which is employed.

A MORE ACCURATE LAW OF FRICTION.

136. In performing one of these experiments with care, it is unusual to make an error amounting to more than a few tenths of 1 lb., and it is hardly possible that any of the mean values we have found should be in error to so great an extent as 0·5 lb. But with the value of the coefficient of friction which is used in Table III., the differences amount sometimes to 0·9 lb. With any other coefficient than that adopted, the differences would have been greater. Now these differences are too great to be attributed to errors of experiment, and hence we infer that the law of friction which has been assumed is not strictly true. The signs of the differences indicate that this law gives values which are too small for small loads, and for large loads are too great.

137. We are therefore led to inquire whether some other relation between F and R may not represent the experiments with greater fidelity than the common law of friction. If we diminished the coefficient by a small amount, and then added a constant quantity to the product of the coefficient and the load, the effect of this change would be that for small loads the calculated values would be increased, while for large loads they

would be diminished. This is the kind of change which we have indicated as necessary in order to reconcile the observed and calculated values.

138. We infer therefore that some relation of the form $F = x + y R$ will probably be a more correct law, and we must find x and y. By substituting a value of R and the corresponding value of F, one equation between x and y is obtained, and a second equation is found by taking another pair of corresponding values. From these two equations values of x and y may be deduced by the well-known process, but the formula thus obtained will not represent the whole series of experiments well. For this reason the method described in the Appendix must be used, which, founded on all the experiments together, gives a formula representing them collectively. The formula thus found is

$$F = 1{\cdot}44 + 0{\cdot}252\ R.$$

This formula is compared with the experiments in Table IV.

TABLE IV.—FRICTION.

Friction of pine upon pine; the mean values of the friction given in Table II. (corrected for the friction of the pulley) compared with the formula $F = 1{\cdot}44 + 0{\cdot}252\ R$.

Number of Experiment.	R. Total load on slide in lbs.	Corrected mean value of friction.	F. Calculated value of friction.	Difference of the observed and calculated values.
1	14	4·7	5·0	+ 0·3
2	28	8·2	8·5	+ 0·3
3	42	12·2	12·0	− 0·2
4	56	15·8	15·6	− 0·2
5	70	19·4	19·1	− 0·3
6	84	23·0	22·6	− 0·4
7	98	25·8	26·1	+ 0·3
8	112	29·3	29·7	+ 0·4

The fourth column contains the calculated values: thus, for example, in experiment 4, where the load is 56 lbs. the calculated value is $1{\cdot}44 + 0{\cdot}252 \times 56 = 15{\cdot}6$; the difference 0·2 between this and the observed value 15·8 is shown in the last column.

139. It will be observed that the greatest difference in this table is 0·4 lbs., and that therefore the formula represents the experiments with considerable accuracy. It is undoubtedly nearer the truth than the former law (Art. 133); in fact, the differences are now such as might really belong to errors unavoidable in making the experiments.

140. This formula may be used for calculating the friction for any load between 14 lbs. and 112 lbs. Thus, for example, if the load be 63 lbs., the friction is $1{\cdot}44 + 0{\cdot}252 \times 63 = 17{\cdot}3$ lbs., which does not differ much from 17·0 lbs., the value found by the former law. We must, however, be cautious not to apply this formula to weights which do not lie between the indicated limits: for example, to take an extreme case, if $R = 0$, the formula would indicate that the friction was 1·44, which is evidently absurd; here the formula errs in excess, while if the load were extremely large it is certain it would err in defect.

EFFECT OF THE EXTENT OF THE EXPERIMENTS.

141. In a subsequent lecture we shall employ as an inclined plane the plank we have been examining, and we shall require to use the knowledge of its friction which we are now acquiring. The weights which we shall then employ range from 7 lbs. to 56 lbs. Now, assuming the ordinary law of friction, we have found

that 0·27 is the best value of its coefficient when the loads range between 14 lbs. and 112 lbs. Suppose we only consider loads up to 56 lbs., we find that the coefficient 0·288 will best represent the experiments within this range, though for 112 lbs. it would give an error of nearly 3 lbs. The results calculated by the formula $F = 0·288\ R$ are shown in Table V., where the greatest difference is 0·7 lb.

TABLE V.—FRICTION.

Friction of pine upon pine; the mean values of the friction given in Table II. (corrected for the friction of the pulley) compared with the formula $F = 0·288\ R$.

Number of Experiment.	R Total load on slide in lbs.	Corrected mean value of friction.	F. Calculated value of friction.	Difference of the observed and calculated values.
1	14	4·7	4·0	− 0·7
2	28	8·2	8·1	− 0·1
3	42	12·2	12·1	− 0·1
4	56	15·8	16·1	+ 0·3

142. But we can replace the common law of friction by the more accurate law of Art. 138, and the formula computed so as to give the best account of the experiments up to 56 lbs., disregarding all others, is $F = 0·9 + 0·266\ R$. The formula is obtained by the method referred to in Art. 138. We find that it represents the experiments better than the formula used in Table V. Between the limits named, this formula is also more accurate than that of Table IV. It is compared with the experiments in Table VI., and it will be noticed that it represents them with great precision, as the difference does not exceed 0·1.

TABLE VI.—FRICTION.

Friction of pine upon pine; the mean values of the friction given in Table II. (corrected for the friction of the pulley) compared with the formula $F = 0{\cdot}9 + 0{\cdot}266\ R$.

Number of Experiment.	R. Total load on slide in lbs.	Corrected mean value of friction.	F. Calculated value of friction.	Difference of the observed and calculated values.
1	14	4·7	4·6	− 0·1
2	28	8·2	8·3	+ 0·1
3	42	12·2	12·1	− 0·1
4	56	15·8	15·8	0·0

THE ANGLE OF FRICTION.

143. There is another mode of examining the action of friction besides that we have been considering. The apparatus for this purpose is shown in Fig. 33. B C represents the plank of pine which we have already used, it is mounted so as to be capable of turning about one end B; the end C is attached to the hook of the chain from the epicycloidal pulley-block E (Art. 224). This block is very convenient for the purpose. By its means the plank can be raised or lowered with the greatest nicety, as the raising chain G is held in one hand and the lowering chain F in the other. Another great convenience is that the plank does not run down, but is firmly held when both the chains are left free. The plank is simply clamped on to the hinge about which it turns, so that its surface is not injured by holes. The frames by which both the hinge and the block are supported are weighted in order to secure steadiness. The inclination of the plane is easily measured by ascertaining the difference in height of its two ends above the floor,

which divided by the length of the plane, 6′, is the sine of the inclination. The starting-screw D, whose use has

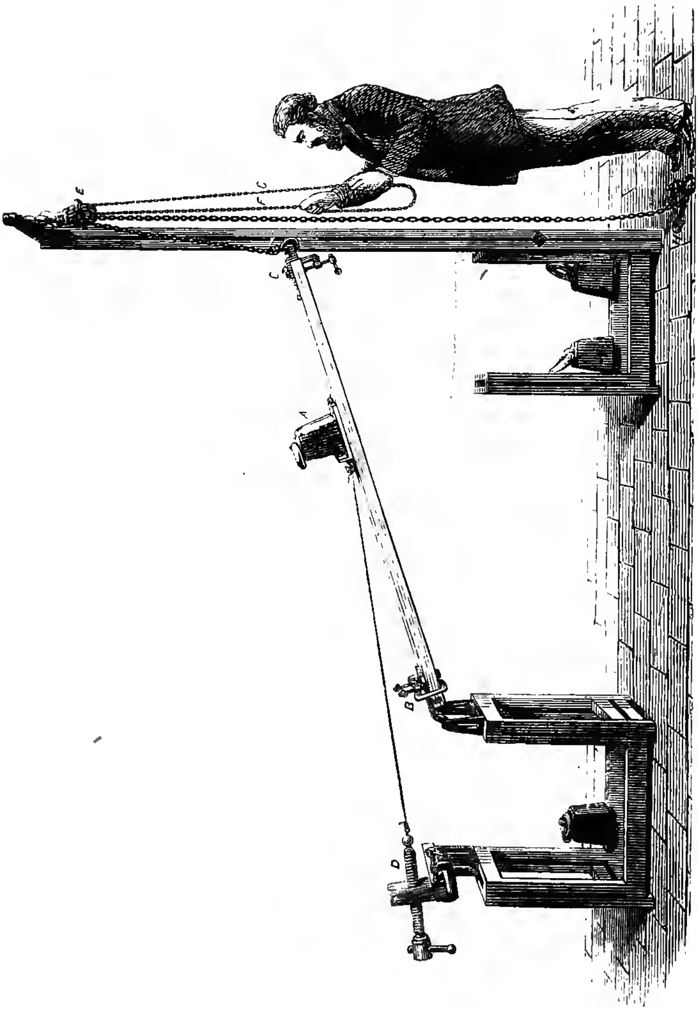

FIG. 33.

been already mentioned, is also fastened to the framework in the position shown in the figure.

144. Suppose the slide A be weighted and placed upon the inclined plane B C, if the end C be only slightly elevated, the slide remains at rest; the reason being that the friction between the slide and the plane neutralizes the force of gravity. But suppose, by means of the pulley-block, C be gradually raised, an elevation is at last reached at which the slide starts off, and runs with an accelerating velocity to the bottom of the plane. The angle of elevation of the plane when this occurs is called the angle of friction.

145. The weights with which the slide was laden in these experiments were 14 lbs., 56 lbs., and 112 lbs., and the results are given in Table VII.

TABLE VII.—ANGLE OF FRICTION.

A smooth plane of pine 72″ × 11″ carries a loaded slide of pine 9″ × 9″; one end of the plane is gradually elevated until the slide starts off.

Number of Experiment.	Total load on the slide in lbs.	Angle of elevation. 1st Series.	Angle of elevation. 2nd Series.	Mean values of the angles.
1	14	19°·5	—	19°·5
2	56	20°·1	17°·2	18°·6
3	112	20°·3	18°·9	19°·6

We see that 56 lbs. started when the plane reached an angle of 20°·1 in the first series, and in the second series of 17°·2, the mean value 18°·6 being given in the fifth column. The mean values of the angles for the three different weights agree very closely, so that we may assert the remarkable law that the angle of friction does not depend upon the magnitude of the load.

146. We might, however, proceed differently in determining the angle of friction, by giving the slide a start,

and ascertaining if the motion would continue. This requires the aid of an assistant who must continually start the slide with the help of the screw, while the elevation of the plane is being slowly increased. The result of these experiments is given in Table VIII.

TABLE VIII.—ANGLE OF FRICTION.

A smooth plane of pine 72″ × 11″ carries a loaded slide of pine 9″ × 9″; one end of the plane is gradually elevated until the slide, having received a start, moves off uniformly.

Number of Experiment.	Total load on the slide in lbs.	Angle of elevation.
1	14	14°·3
2	56	13°·0
3	112	13°·0

We see from this table also that the angle of friction is independent of the magnitude of the weight, but the amount of the angle is less by 5° or 6° than when the slide is not started.

147. It is commonly stated that the coefficient of friction is the tangent of the angle of friction, and this can easily be proved to be true when the ordinary law of friction is assumed. But as we have seen that the law of friction is only approximately correct, we need not expect to find this other law completely verified.

148. When the slide is started, the mean value of the angle of friction is 13°·4. The tangent of this angle is 0·24: this is about 11 per cent. less than the coefficient of friction 0·27, which we have already determined. The mean value of the angle of friction when the slide is not

started is 19°·2, and its tangent is 0·35. The experiments of Table I. are, as already pointed out, essentially uncertain, but it is necessary to refer to them here in order to show that in no sense is the coefficient of friction exactly equal to the tangent of the angle of friction. If we adopt the mean values given in the last column of Table I., the best coefficient of friction which can be deduced from them is 0·41. Whether, therefore, the slide be started or not started, the tangent of the angle of friction is smaller than the corresponding coefficient of friction. When the slide is started, the tangent is about 11 per cent. less than the coefficient; and when the slide is not started, it is about 14 per cent. less. There are doubtless many cases in which these differences are sufficiently small to be neglected, and in which, therefore, the law may be received as true.

ANOTHER LAW OF FRICTION.

149. The area of the wooden slide is $9'' \times 9''$, but we should have found that the friction was the same whatever were the area of the slide, so long as the nature of its surface remained unaltered. This follows as a consequence of the approximate law that the friction is proportional to the pressure. Suppose that the weight were 100 lbs., and the area of the slide 100 inches, there would then be a pressure of 1 lb. per square inch over the surface of the slide, and therefore the friction to be overcome on each square inch would be 0·27 lb., or for the whole slide 27 lbs. If, however, the slide had only an area of 50 square inches, the load would produce a pressure of 2 lbs. per square inch; the friction would therefore be $2 \times 0·27 = 0·54$ lb. for each square inch, and the total

friction would be 50 × 0·54 = 27 lbs., the same as before: hence the total friction is independent of the extent of surface. This would be equally true even though the weight were not, as we have supposed, uniformly distributed over the surface of the slide.

CONCLUDING REMARKS.

150. The importance of friction in mechanics arises from its universal presence. We often recognize it as a destroyer or impeder of motion, as a waster of our energy, and as a source of loss and inconvenience. But, on the other hand, friction is often indirectly the means of producing motion, and of this we have a splendid example in the locomotive engine. The engine being very heavy, the wheels are pressed closely to the rails; there is friction enough to prevent the wheels slipping, consequently when the engines force the wheels to turn round they must roll onwards. The coefficient of friction of wrought iron upon wrought iron is about 0·2. Suppose a locomotive weigh 30 tons, and the share of this weight borne by the driving wheels be 10 tons, the friction between the driving wheels and the rails is 2 tons. This is the greatest force the engine can exert on a level line. A force of 10 lbs. for every ton weight of the train is known to be sufficient to sustain the motion, consequently the engine we have been considering should draw along the level a load of 448 tons.

151. But we need not to go to the steam-engine to learn the use of friction. We could not exist without it. In the first place we could not move about, for walking is only possible on account of the friction between the soles of our boots and the ground; nor if we were once in

motion could we stop without coming into collision with some other object, or grasping something to hold on by. Objects could only be handled with difficulty, nails would not remain in wood, and screws would be equally useless. Buildings could not be erected, nay, even hills and mountains would gradually disappear, and finally dry land would be immersed beneath the level of the sea. Friction is, so far as we are concerned, quite as essential a law of nature as the law of gravitation. We must not seek to evade it in our mechanical discussions because it makes them a little more difficult. Friction obeys laws; its action is not vague or uncertain. When inconvenient it can be diminished, when useful it can be increased; and in our lectures on the mechanical powers, to which we now proceed, we shall have opportunities of describing machines which have been devised in obedience to its laws.

LECTURE VI.

THE PULLEY.

Introduction.—Friction between a Rope and an Iron Bar.—The Use of the Pulley.—Large and Small Pulleys.—The Law of Friction in the Pulley.—Wheels.—Energy.

INTRODUCTION.

152. The pulley forms a good introduction to the very important subject of the mechanical powers. But before entering on the discussion of the mechanical powers, it will be necessary for us to explain what is meant in mechanics by "work," or "energy," as it is more appropriately called, and we shall therefore include a short outline of this subject in the present lecture.

153. The pulley is a machine which is employed for the purpose of changing the direction of a force. We frequently wish to apply a force in a different direction from that in which it is convenient to exert it, and the pulley enables us to do so. We are not now speaking of the arrangements for increasing power in which pulleys play an important part; these will be considered in the next lecture: we refer only to change of direction. In fact, as we shall presently see, a small amount of force is lost when the single pulley is used, so that this machine cannot be called a mechanical power.

154. The occasions upon which a single pulley is used are very numerous and familiar. Let us suppose a sack of corn has to be elevated from the lower to one of the upper stories of a building. It may of course be raised by a man who carries it, but he has to carry his own weight in addition to that of the sack, and therefore the quantity of exertion used is greater than absolutely necessary. But supposing there be a pulley at the top of the building over which a rope passes; then, if a man attach one end of the rope to the sack and pull the other, he raises the sack without raising his own weight. The pulley has thus provided the means by which the downward force has been changed in direction to an upward force.

155. The weights, ropes, and pulleys which are used in our windows for counterpoising the weight of the sash afford a very familiar instance of how a pulley changes the direction of a force. Here the downward force of the weight is changed by means of the pulley into an upward force, which nearly counterbalances the weight of the sash.

FRICTION BETWEEN A ROPE AND AN IRON BAR.

156. You are doubtless familiar with the ordinary form of the pulley; it consists of a wheel capable of turning very freely on its axle, and it has a groove in its circumference in which the rope lies. But why is it necessary to give the pulley this form? Why could not the direction of the rope be changed by simply passing it over a bar, as well as by the more complicated pulley? We shall best answer this question by actually trying the experiment, which we can do by means of the apparatus

of Fig. 34 (see page 91). In this are shown two iron studs, G, H, 0″·6 diameter, and about 8″ apart; over these passes a rope which has a hook at each end. If I suspend a weight of 14 lbs. from one hook A, and pull the hook B, I can by exerting sufficient force raise the weight on A, but with this arrangement I am conscious of having to exert a very much larger force than would have been necessary to raise 14 lbs. by merely lifting it.

157. In order to study the question exactly, we shall ascertain what weight suspended from the hook B will suffice to raise A. I find that in order to raise 14 lbs. on A no less than 47 lbs. is necessary on B, consequently there is an enormous loss of force: more than two-thirds of the force which is exerted is expended uselessly. If instead of the 14 lbs. weight I substitute any other weight, I find the same result, viz. that more than three times its amount is necessary to raise it by means of the rope passing over the studs. If the man, in raising a sack, were to pass the rope over two bars such as these, for every stone the sack weighed he would have to exert a force of more than three stones, and therefore there would be a very extravagant loss of power.

158. Whence arises this loss? The rope in moving slides over the surface of the iron studs. Although these are quite smooth and polished, yet when there is a strain on the rope it presses closely upon them, and there is a certain amount of force necessary to make the rope slide along the iron. In other words, when I am trying to raise up 14 lbs. with this contrivance, I not only have its weight opposed to me, but also another force due to the sliding of the rope on the iron: this force is friction (Lecture V.). Were it not for friction, a force of 14 lbs. on one hook would exactly balance 14 lbs. on the other, and

the slightest addition to either weight would make it descend and raise the other. If, then, we are obliged to change the direction of a force, we must devise some means of doing so which does not require so great a sacrifice as the arrangement with the two bars.

THE USE OF THE PULLEY.

159. We shall next inquire how it is that we are enabled to obviate friction by means of a pulley. It is evident we must provide an arrangement in which the rope shall not be required to slide upon an iron surface. This end is attained by the pulley, of which we may take I, Fig. 34, as an example. This represents a cast iron wheel 14″ in diameter, with a V-shaped groove in its circumference to receive the rope : this wheel turns on a $\frac{5}{8}$-inch wrought iron axle, which is well oiled. The rope used is about 0″·25 in diameter.

160. From the hooks E F at each end of the rope a 14 lb. weight is suspended. These equal weights balance each other. According to our former experiment with the studs, it would be necessary for me to treble the weight on one of these hooks in order to raise the other, but here I find that an additional 0·5 lb. placed on either hook causes it to descend and make the other ascend. This is a great improvement ; 0·5 lb. now accomplishes what 33 lbs. was before required for. We have avoided a great deal of friction, but we have not got rid of it altogether, for 0·25 lb. is incompetent, when added to either weight, to make that weight descend.

161. To what is the improvement due ? When the weight descends the rope does not slide upon the wheel, but it causes the wheel to revolve with it, consequently

there is little or no friction at the circumference of the pulley; the friction is transferred to the axle. We still have some resistance to overcome, but for smooth oiled iron axles the friction is very small, hence the advantage of the pulley.

There is in every pulley a small loss of power from the necessity of bending the rope; this need not concern us at present, for with the very pliable plaited rope that we have employed the effect is inappreciable, but with large strong ropes the loss becomes of importance. The amount of loss in different kinds of ropes has been determined by careful experiments.

LARGE AND SMALL PULLEYS.

162. There is a considerable advantage obtained by using large rather than small pulleys. The amount of force necessary to overcome friction varies inversely as the size of the pulley. We shall be able to demonstrate this by actual experiment with the apparatus of Fig. 34. A small pulley K is attached to the large pulley I; they are in fact one piece, and turn together on the same axle. Hence if we first determine the friction with the rope over the large pulley, and then with the rope over the small pulley, any difference can only be due to the difference in size, as all the other circumstances are the same.

163. In making the experiments we must attend to the following point. The pulleys and the socket on which they are mounted weigh several pounds, and consequently there is friction on the axle arising from the weight of the pulleys, quite independently of any weights that may be placed on the hooks. We must then, if possible,

evade the friction of the pulley itself, so that the amount of friction which is observed will be entirely due to the weights raised. This can be easily done. The rope and hooks being on the large pulley I, I find that 0·16 lb. attached to one of the hooks is sufficient to overcome the

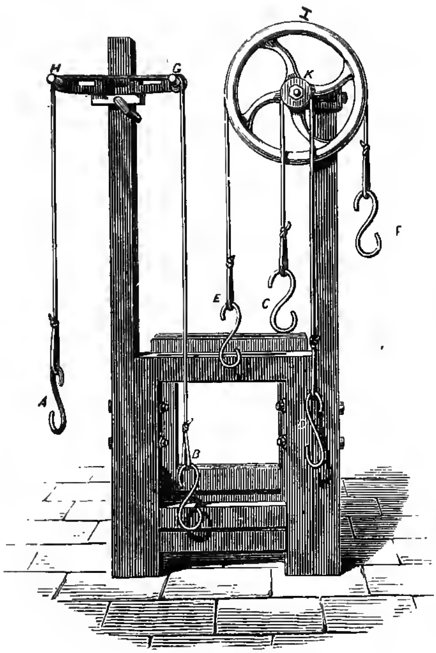

FIG. 34.

friction of the pulley, and to make the hook E descend and raise F. If therefore we leave 0·16 lb. on E, we may consider the friction due to the weight of the pulley, rope, and hooks as neutralized.

164. I now place a stone weight on each of the hooks E and F. The amount necessary to make the hook E and its load descend, is 0·28 lb. This does not of course include the weight of 0·16 lb. already referred to. We see

therefore that with the large pulley the amount of friction to be overcome in raising one stone is 0·28 lb.

165. Let us now perform precisely the same experiment with the small pulley. I transfer the same rope and hooks to K, and I find that 0·16 lb. is not now sufficient to overcome the friction of the pulley, but I add on weights until C will just descend, which occurs when the load reaches 0·95 lb. This weight is to be left on C as a counterpoise, for the reasons already pointed out. I place a stone weight on C and on D, and you see that C will descend when it receives an additional load of 1·35 lbs.; this is therefore the amount of friction to be overcome when a stone weight is raised over the pulley K.

166. Let us compare these results with the dimensions of the pulleys. The proper way to measure the effective circumference of a pulley when carrying a certain rope, is to measure the length of that rope which will just embrace it. The length measured in this way will of course depend to a certain extent upon the size of the rope. I find that the circumferences of the two pulleys are 43″·0 and 9″·5. The ratio of these is 4·5; the corresponding amounts of friction we have seen to be 0·28 lb. and 1·35 lbs. The larger of these quantities is 4·8 times the smaller. This number is very close to 4·5; we must, as already explained (Art. 136), not expect perfect accuracy in experiments in friction. In the present case the agreement is within the 1-16th of the whole, and we may regard it as a proof that the friction of a pulley is inversely proportional to the circumference of the pulley.

167. It is easy to see the reason why friction should diminish when the size of the pulley is increased. The friction acts at the circumference of the axle about which the wheel turns; it is there present as a force tending

to retard motion. Now the larger the wheel the greater will be the distance from the axis at which the force acts which overcomes the friction, and therefore the less need be the magnitude of the force. You will perhaps understand this better after the principle of the lever has been discussed (Art. 237).

168. We may deduce from these considerations the practical maxim that large pulleys are economical of power. This rule is well known to engineers; large pulleys should be used, not only for diminishing friction, but also to avoid loss of power by excessive bending of the rope. A rope is bent gradually around the circumference of a large pulley with far less force than is necessary to accommodate it to a smaller pulley: the rope also is apt to become injured by excessive bending. In coal pits the trucks laden with coal are hoisted to the surface, or as miners say, "to bank," by means of wire ropes which pass from the pit over a pulley into the engine-house: this pulley is of very large dimensions, for the reasons we have pointed out.

THE LAW OF FRICTION IN THE PULLEY.

169. I have here a wooden pulley 3″·5 in diameter; the boss is lined with brass, and turns very freely on an iron spindle. I place the rope and hooks upon the groove. Brass rubbing on iron has but little friction, and when 7 lbs. is placed on each hook, 0·5 lb. added to either will make it descend and raise up the other. Let 14 lbs. be placed on each hook, 0·5 lb. is no longer sufficient; 1 lb. is required: hence when the weight is doubled the friction is also doubled. Repeating the experiment with 21 lbs. and 28 lbs. on each side, the corresponding weights

necessary to overcome friction are 1·5 lb. and 2 lb. In the four experiments the weights used are in the proportion 1, 2, 3, 4; and the forces necessary to overcome friction, 0·5 lb., 1 lb., 1·5 lb., and 2 lb., are in the same proportion. Hence the friction is proportional to the load.

WHEELS.

170. The wheel is one of the most simple and effective devices for overcoming friction. A sleigh is a very admirable vehicle on a smooth surface such as ice, but it is totally unadapted for use on common roads; the reason being that the amount of friction between the sleigh and the road is so great that to move the sleigh the horse would have to exert a force which would be very great compared with the load he was drawing. But a vehicle properly mounted on wheels moves with the greatest ease along the road, for the circumference of the wheel does not slide, and consequently there is no friction between the wheel and the road; the wheel however turns on its axle, therefore there is sliding, and consequently friction, at the axle, but the axle and the wheel are very perfectly fitted to each other, and the surfaces are lubricated with oil, so that the friction is extremely small.

171. With large wheels the amount of friction on the axle is less than with small wheels; other advantages of large wheels are that they do not sink much into depressions in the roads, and that they have also an increased facility in surmounting the innumerable small obstacles from which even the best road is not free.

172. When it is desired to make a pulley turn with extremely small friction, its axle, instead of revolving in fixed bearings, is mounted upon what are called friction

wheels. A set of friction wheels is shown in the apparatus of Fig. 66: when the axle revolves, the friction between the axles and the wheels causes the latter to turn round with a comparatively slow motion; thus all the friction is transferred to the axles of the four friction wheels, which, as they move in their bearings with extreme slowness, cause the pulley to be but little affected by friction. The amount of friction may be understood from the following experiment. A silk cord is placed on the pulley, and 1 lb. weight is attached to each of its ends: these of course balance. A number of fine wire hooks, each weighing 0·001 lb., are prepared, and it is found that when a weight of 0·004 lb. is attached to either side it is sufficient to overcome friction and set the weights in motion.

ENERGY.

173. In connection with the subject of friction, and also as introductory to the mechanical powers, the notion of "work," or as it is more properly called "energy," is of great importance. The meaning of this word as employed in mechanics will require a little consideration.

174. In ordinary language, whatever a man does that can cause fatigue, whether of body or mind, is called work. If the man be carrying up hods of mortar, or breaking stones, or digging or rowing, or pushing a laden wheelbarrow, or forging hot iron, or engaged in any other occupation which induces bodily fatigue, he is said to be doing work; or if a man be engaged in any intellectual occupation, such as studying or writing a book, or making a speech, he experiences mental fatigue, and perhaps bodily fatigue as well, and is justly said to be

doing work. In mechanics, however, we mean by energy the particular kind of work which is equivalent to raising weights.

175. Suppose a weight to be on the floor and a stool beside it: if a man raise the weight and place it upon the stool, the exertion that he expends is energy in the sense in which the word is used in mechanics. The amount of exertion necessary to place the weight upon the stool depends upon two things, the magnitude of the weight, and the height of the stool. It is clear that both these things must be taken into account, for although we know the weight which is raised, we cannot tell the amount of exertion that will be required until we know the height through which it is to be raised; and if we know the height, we cannot appreciate the quantity of exertion until we know the weight.

176. The following plan has been adopted for expressing quantities of energy. The small amount of exertion necessary to raise 1 lb. avoirdupois through one British foot is taken as a standard, compared with which all other quantities of energy are estimated. This quantity of exertion is called in mechanics the unit of energy, and sometimes also the foot-pound.

177. If a weight of 1 lb. has to be raised through a height of 2 feet, or a weight of 2 lbs. through a height of 1 foot, it will be necessary to expend twice as much energy as would have raised a weight of 1 lb. through 1 foot, that is 2 foot-pounds.

If a weight of 5 lbs. had to be raised from the floor up to a stool 3 feet high, how many units of energy would be required? To raise 5 lbs. through 1 foot requires 5 foot-pounds, and the process must be again repeated twice before the weight arrive at the top of the stool.

For the whole operation 15 foot-pounds will therefore be necessary.

If 100 lbs. be raised through 20 feet, 100 foot-pounds of energy is required for the first foot, the same for the second, third, &c., up to the twentieth, making a total of 2,000 foot-pounds.

Here is a practical question for the sake of illustration. Which would it be preferable to carry, a trunk weighing 40 lbs. to a height of 20 feet, or a trunk weighing 50 lbs. to a height of 15 feet? We shall find how much energy would be necessary in each case: 40 times 20 is 800; therefore in the first case the energy would be 800 foot-pounds. But 50 times 15 is 750; therefore the amount of work, in the second case, is only 750 lbs. Hence it is less exertion to carry 50 lbs. up 15 feet than 40 lbs. up 20 feet.

178. Every source of energy, whether it be the muscles of men or other animals, water-wheels, steam-engines, or other prime movers, is to be measured by foot-pounds.

The power of a steam-engine is spoken of as so many horse-power. By this it is meant that a steam-engine, for example, of 3 horse-power, could, when working for an hour, do as much work as 3 horses could when working for the same time; but as the power of a horse is an uncertain quantity, differing in different animals and perhaps not quite uniform in one, the selection of this measure for the efficiency of the steam-engine is inconvenient. We replace it by a standard horse-power which is, I believe, somewhat larger than the actual energy of any horse. A horse-power in the steam-engine is a power capable of exerting 33,000 foot-pounds per minute.

179. To illustrate this by an example: if a mine be 1,000 feet deep, how much water per minute would a 50 horse-power engine be capable of raising from the bottom? The engine would yield 50 × 33,000 units of work per minute, but the weight has to be raised 1,000 feet, consequently the number of pounds of water raised is

$$\frac{50 \times 33{,}000}{1{,}000} = 1{,}650.$$

180. We shall apply the principle of work to the consideration of the pulley already described (Art. 169). In order to raise a weight of 14 lbs., it is necessary that the rope to which the power is applied should be pulled downwards by a force of 15 lbs., the extra pound being on account of the friction. To fix our ideas, we shall suppose the 14 lbs. to be raised 1 foot; to lift this load directly, without the intervention of the pulley, 14 foot-pounds would be necessary, but when it is raised by means of the pulley, 15 foot-pounds are necessary. Hence there is an absolute loss of 1 foot-pound of energy when the pulley is used. If a steam-engine of one horse-power were employed in raising weights by a rope passing over a pulley similar to that on which we have experimented, only $\frac{14}{15}$ths of the work would be employed, but

$$33{,}000 \times \frac{14}{15} = 30{,}800.$$

The engine would therefore usefully perform 30,800 foot-pounds per minute.

181. The effect of friction on a pulley, or on any other machine, is always to waste energy. To perform a piece of work directly requires a certain number of foot-pounds, while to do it by the machine requires more, on account of loss by friction. This may at first sight

appear somewhat paradoxical, as it is well known that by levers, pulleys, &c., an enormous mechanical advantage may be gained. This subject will be fully explained in the next and following lectures, which relate to the mechanical powers.

182. We shall conclude with a few observations on a point of the greatest importance. We have seen a case where 15 foot-pounds of energy only accomplished 14 foot-pounds of work, and thus 1 foot-pound appeared to be lost. We say that this was expended upon the friction; but what is the friction? The axle is gradually worn away by rubbing in its bearings, and, if it be not properly oiled, it becomes heated. The unit of energy that is lost to us usefully is expended in grinding down the axle, and it may be in heating it; the energy is not lost, but produces its effect in a way we do not want, and is rather injurious than otherwise. We know that energy cannot be lost, however it may be transformed; if it disappear in one shape, it is only to reappear in another. A loss by friction merely means a transference of work to some other object rather than that which we wish to accomplish. It has long been known that matter is indestructible: it is equally certain that energy is indestructible.

LECTURE VII.

THE PULLEY-BLOCK.

Introduction. — The Single Moveable Pulley. — The Three-sheave Pulley-block.—The Differential Pulley-block.—The Epicycloidal Pulley-block.

INTRODUCTION.

183. In the first lecture I showed how a large weight could be raised by a smaller weight (Art. 21), and I stated that this subject would again occupy our attention during the course. I now commence to fulfil this promise. The question to be discussed is this, how can we by means of a small force overcome a greater force? This is a subject of practical importance. A man of average strength is not able to raise more than 1 cwt. without great exertion, yet the weights which it is necessary to move about often weigh many hundredweights, or even tons. It is not always practicable to employ numerous hands for the purpose, nor is a steam-engine or other great source of power at all times available. But what are called the mechanical powers enable the forces at our disposal to be greatly increased. One man, by their aid, can exert as much force as several could without such assistance; and when they are employed to augment the power of several men or of a steam-engine, gigantic weights amounting to sixty tons or more can be managed with facility.

184. In the various arts we find innumerable cases where great resistances have to be overcome; we also find a corresponding number and variety of devices contrived by human skill to conquer them. The girders of an iron bridge have to be adjusted upon their piers; the boilers and engines of an ocean steamer have to be placed in position; a great casting has to be lifted from its mould; a railway locomotive has to be placed on the deck of a vessel for transit; a weighty anchor has to be lifted from the bottom of the sea; an iron plate has to be rolled or cut or punched: for all of these cases suitable arrangements must be devised in order that the requisite power may be obtained.

185. We are ignorant of the means which the ancients employed in raising the vast stones of those buildings which travellers in the East have described to us. It is sometimes thought that by a large number of men these stones could have been transported without the aid of appliances which we would now use for a similar purpose. But it is more likely that some of the mechanical powers were used, as, with a multitude of men, it is difficult to ensure the proper application of their united strength. In Easter Island, hundreds of miles distant from civilized land, and now inhabited by savages, vast idols of stone have been found in the hills, which must have been raised by human labour. It is curious to speculate on the extinct race by whom this work was achieved, and on the means which they must have employed.

186. The mechanical powers are usually enumerated as follows:—The pulley, the lever, the wheel and axle, the wedge, the inclined plane, the screw. These different powers are so frequently used in combination that the distinctions cannot be always maintained. The classifica-

tion will, however, suffice to give a general notion of the subject at the commencement.

187. Many of the most valuable mechanical powers are machines in which cords or chains play an important part. Pulleys are employed wherever it is necessary to change the direction of a cord which is transmitting power. In the present lecture we shall examine into the most important mechanical powers that are produced by the combination of a rope with pulleys.

THE SINGLE MOVEABLE PULLEY.

188. We commence with the most simple case, that of the single moveable pulley (Fig. 35). The rope is firmly secured at one end A; it then passes down under the moveable pulley B, and upwards over a fixed pulley. To the free end C, which depends from the fixed pulley, the power C is applied while the load D to be raised is suspended from the moveable pulley. We shall first study the relation between the power and the load in a simple way, and then we shall describe the more careful and exact experiments.

189. When the load is raised the moveable pulley B must of course be raised up with it, and part of the power is expended for this purpose. But we can get rid of the weight of B by first attaching to the power end of the rope a weight just sufficient in itself to lift up the moveable pulley when not carrying a load. The weight necessary for doing this is easily found by trial to be a little over 1·5 lbs, weight. This is to be permanently attached to the power rope, and also a hook for the reception of the power weights.

190. Let us suspend 14 lbs. from the load hook, and

ascertain what power will raise the load. We leave the weight of the pulley and 1·5 lbs. at C out of consideration, since they mutually destroy. I find by experiment that 7 lbs. on the power hook is not sufficient to raise the load, but if one pound be added, the power descends, and the load is raised. Here, then, is a remarkable result; a

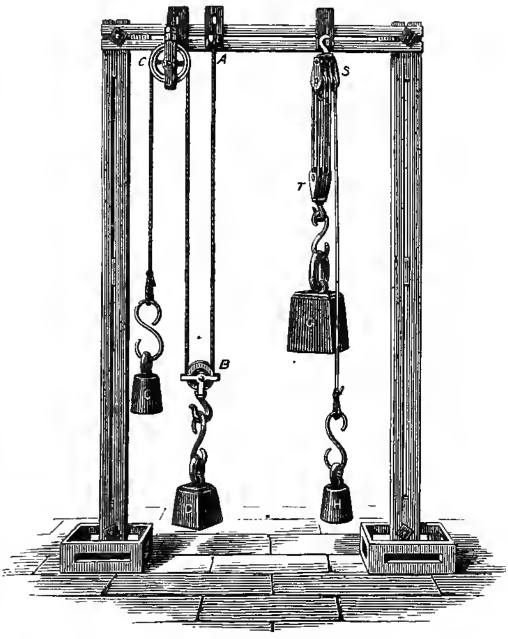

Fig. 35.

weight of 8 lbs. has overcome 14 lbs. In this we have the first application of the mechanical powers to increase our available forces.

191. We shall examine the reason of this mechanical advantage. If the load be raised one foot, the power must descend two feet: this is apparent, for in order to raise the load the two parts of the rope descending

from A and C to B must each be shortened one foot, and this can only be done by the power descending two feet. Hence when the load of 14 lbs. is placed on the load hook, for every foot it is raised the power must descend two feet: this, though a simple point, is one of the greatest importance, as upon it the action depends. In all the mechanical powers it is essential to examine into the number of feet through which the power must act in order to raise the load one foot: this number we shall always call the *velocity ratio.*

192. To raise 14 lbs. through one foot requires 14 foot-pounds. Hence, were there no such thing as friction, 7 lbs. on the power hook would be sufficient to raise the load; because 7 lbs. descending through two feet yields 14 foot-pounds. But there is a loss of energy on account of friction, and a power of 7 lbs. is not sufficient: 8 lbs. are necessary. 8 lbs. in descending two feet performs 16 foot-pounds; of these only 14 are utilized on the load, the remainder being the quantity of energy that has been absorbed by friction. We learn, then, that in the moveable pulley the quantity of energy employed is really greater than that which would lift the weight directly, but that the actual power which has to be exerted is less.

193. Suppose that 28 lbs. be placed on the load hook, a few trials assure us that a power of 16 lbs. (but not less) will be sufficient to raise it; that is to say, when the load is doubled, we find, as we might have expected, that the power must be doubled also. It is easily seen that the loss of energy by friction amounts to 4 foot-pounds. We thus verify, in the case of the moveable pulley, the remarkable law of friction already referred to as approximately true (Art. 135).

194. By means of a moveable pulley a man is able to raise a weight nearly double as great as he could lift directly. By experiments carefully made, it has been found that when a man is employed in the particular exertion necessary for raising weights over a pulley, he is able to work most efficiently when the pull he is required to make is about 40 lbs. A man could, of course, exert greater power than this, but in an ordinary day's work it is found that he is able to perform more foot-pounds when the pull is 40 lbs. than when it is larger or smaller. If therefore the weights to be lifted amount to about 80 lbs., energy may be economized by the use of the single moveable pulley, although by so doing a greater quantity of energy would be actually expended than would have been necessary to raise the weights directly.

195. Some experiments on larger weights, made with care, have been tried with the moveable pulley we have just described; their results are recorded in Table IX.

TABLE IX.—SINGLE MOVEABLE PULLEY.

Moveable pulley of cast iron 3″·25 diameter, groove 0″·6 wide, wrought iron axle 0″·6 diameter; fixed pulley of cast iron 5″ diameter, groove 0″·4 wide, wrought iron axle 0″·6 diameter, axles oiled; flexible plaited rope 0″·25 diameter; velocity ratio 2, mechanical efficiency 1·8, useful effect 90 per cent.; formula $P = 2·21 + 0·5453\,R$.

Number of Experiment.	*R.* Load in lbs.	Observed power in lbs.	*P.* Calculated power in lbs.	Difference of the observed and calculated values.
1	28	17·5	17·5	0·0
2	57	33·5	33·3	− 0·2
3	85	48·5	48·6	+ 0·1
4	113	64·0	63·8	− 0·2
5	142	80·0	79·6	− 0·4
6	170	94·5	94·9	+ 0·4
7	198	110·5	110·2	− 0·3
8	226	125·5	125·5	0·0

The dimensions of the pulley are stated in the table because, for pulleys of different construction, the results would not necessarily be the same. An attentive study of this table will, however, show the general character of the relation existing between the power and the resistance in all the arrangements of this class.

The table consists of five columns. The first contains merely the numbers of the experiments for convenience of reference. In the second column, headed *R*, the weights, expressed in pounds, which are raised in each experiment, are given; that is, the weight attached to the hook, not including the weight of the lower pulley. The weight of this pulley is not counterpoised in these experiments. In the third column the weights are recorded, which were found to be of sufficient power to raise the corresponding weights in the second column. Thus, in experiment 7, a weight of 198 lbs. being attached to the moveable pulley, it is found that 110·5 lbs. applied as a power will be sufficient to raise it. The third column has been determined by actual trial in the manner described in Art. 190.

196. By an examination of the columns of the power and the load, we see that the power always amounts to more than half the load. The excess is partly due to a small portion of the power (about 1·5 lbs.) being employed in raising the lower block, and partly to friction. For example, in experiment 7, if there had been no friction and if the lower block were without weight, a power of 99 lbs. would have been sufficient; but, owing to the presence of these disturbing causes, 110·5 lbs. are necessary: of this amount 1·5 lbs. is due to the weight of the pulley, 10 lbs. is the force of friction, and the remaining 99 lbs. raises the load.

197. By a careful examination of this table we can ascertain a certain relation between the power and the load; it is found that they are connected together by a rule, which may be enunciated as follows.

The power is found by multiplying the weight of the load by 0·5453, and adding 2·2 to the product. Calling P the power, and R the load, we may express the relation thus: $P = 2{\cdot}21 + 0{\cdot}5453\,R$. For example, in experiment 5, the product of 142 and 0·5453 is 77·43, and to which, when 2·21 is added, we find for P 79·64, very nearly the same as 80 lbs., the observed value of the power.

In the fourth column the values of P calculated by means of this rule are given, and in the last column we find the difference between the observed and the calculated values shown for the sake of comparison. It will be seen that the difference in no case amounts to 0·5 lb., consequently the rule expresses the experiments very well. The mode of deducing this rule from the experiments is given in the Appendix.

198. The quantity 2·21 is partly that portion of the power due to the weight of the moveable pulley, and partly due to friction.

199. We can readily ascertain from the rule how much power is necessary to raise a given weight; for example, suppose 200 lbs. be attached to the moveable pulley, we find that 111 lbs. must be applied as the power. But in order to raise 200 lbs. one foot, the power exerted must act over two feet; hence the number of foot-pounds required is $2 \times 111 = 222$. The quantity of energy that is lost is 22 foot-pounds. Out of every 222 foot-pounds applied, 200 are usefully employed; that is to say, about 90 per cent. of the applied energy is utilized, while the remaining 10 per cent. is lost.

THE THREE-SHEAVE PULLEY-BLOCK.

200. The next arrangement we shall employ is a pair of pulley-blocks S T, Fig. 35, each containing three sheaves, as the small wheels are termed. A rope is fastened to the upper block, S; it then passes down to the lower block T under one sheave, up again to the upper block and over a sheave, and so on, as shown in the figure. To the end of the rope from the last of the upper sheaves the power H is applied, and the load G is suspended from the hook attached to the lower block. When the rope is pulled, it gradually raises the lower block; and to raise the load one foot, each of the six parts of the rope from the upper block to the lower block must be shortened one foot, and therefore the power must have pulled out six feet of rope. Hence for every foot that the load is raised the power must have acted through six feet; that is to say, the *velocity ratio* is 6.

201. If there were no friction, the power would only be one-sixth of the load. This follows at once from the principles already explained. Suppose the load be 60 lbs., then to raise it one foot would require 60 foot-pounds, and the power must therefore exert 60 foot-pounds; but the power moves over six feet, therefore a power of 10 lbs. would be sufficient. Owing, however, to friction, some energy is lost, and we must have recourse to experiment in order to test the real efficiency of the machine. The single moveable pulley nearly doubled our power; we shall prove that the three-sheave pulley-block will quadruple it. In this case we deal with large weights of 1 cwt. and 2 cwt., so with reference to them we may leave the weight of the lower block out of consideration.

202. Let us first attach 1 cwt. to the load hook; we find that 29 lbs. placed on the power hook is the smallest weight that will raise it: this is almost exactly one-quarter of the load; 28 lbs. would be precisely so. If 2 cwt. be placed on the hook, we find that 56 lbs. will just raise it: this time it is exactly one-quarter. The experiment has been tried of placing 4 cwt. on the hook; it is then found that 109 lbs. will raise it, which is only 3 lbs. short of 1 cwt. These experiments demonstrate that for a three-sheave pulley-block of this construction we may safely apply the rule, that the power is one-quarter of the load.

203. We are thus enabled to see how much of our exertion in raising weights must be expended in merely overcoming friction, and how much may be utilized. Suppose for example that we have a weight of 100 lbs. to raise one foot by means of the pulley-block; the power we must apply is 25 lbs., and six feet of rope must be drawn out from between the pulleys: therefore the power exerts 150 foot-pounds of energy. Of these only 100 foot-pounds are usefully employed, and thus 50 foot-pounds, one-third of the whole, have been expended on friction. Here we see what occurs in all the mechanical powers, that notwithstanding a small force overcomes a large one, there is an actual loss of energy in the machine. The real advantage consists in this, that by the pulley-block I can raise a greater weight than I could move without assistance, but I do not create energy; I merely modify it, and lose by the process.

204. The result of a series of experiments made with this pair of pulley-blocks is given in Table X.

TABLE X.—THREE-SHEAVE PULLEY-BLOCKS.

Sheaves cast iron 2″·5 diameter; plaited rope 0″·25 diameter; velocity ratio 6; mechanical advantage 4; useful effect 67 per cent.; formula $P = 2·36 + 0·238\, R$.

Number of Experiment.	*R.* Load in lbs.	Observed power in lbs.	*P.* Calculated power in lbs.	Difference of the observed and calculated values.
1	57	15·5	15·9	+ 0·4
2	114	29·5	29·5	0·0
3	171	43·5	43·1	– 0·4
4	228	56·0	56·6	+ 0·6
5	281	70·0	69·2	– 0·8
6	338	83·0	82·8	– 0·2
7	395	97·0	96·4	– 0·6
8	452	109·0	109·9	+ 0·9

205. This table contains five columns; the weights raised (shown in the second column) range up to somewhat over 4 cwt. The observed values of the power are given in the third column; each of these is generally about one-quarter of the corresponding value of the load. There is, however, a more accurate rule for finding the power; it is as follows.

206. To find the power necessary to raise a given load, multiply the loads in lbs. by 0·238, and add 2·36 lbs. to the product. We may express the rule by the formula $P = 2·36 + 0·238\, R$.

207. Thus to find the power which would raise 228 lbs.: the product of 228 and 0·238 is 54·26; adding 2·36, we find 56·6 lbs. for the power required; the actual observed power is 56 lbs., so that the rule is accurate to within about half a pound. In the fourth column will be found the values of P calculated by means of this rule. In the

fifth column, the differences between the observed and the calculated values of the powers are given, and it will be seen that the difference in no case reaches 1 lb.

208. I will next perform an experiment with the three-sheave pulley-block, which will give us an insight into the exact amount of friction without calculation by the help of the velocity ratio. We can first counterpoise the weight of the lower block by attaching weights to the power. It is found that about 1·6 lbs. is sufficient for this purpose. I attach a 56 lb. weight as a load, and find that 13·1 lbs. is sufficient power to raise it. This amount is partly composed of the force necessary to raise the load if there were no friction, and the rest is due to the friction. I next remove the power weights: when I have taken off a pound, you see the power and the resistance balance each other; but when I reduce the power to 5·5 lbs. (not including the counterpoise), the load is sufficient to overhaul the power, and raise it. We have therefore proved that a power of 13·1 lbs. or greater raises 56 lbs., that any power between 13·1 lbs. and 5·5 lbs. balances 56 lbs., and that any power less than 5·5 lbs. is raised by 56 lbs.

When the power is raised, the force of friction, together with the power, must be overcome by the load. Let us call X the real power that would be necessary to balance 56 lbs. in a perfectly frictionless machine, and Y the force of friction. We shall be able to determine X and Y by the experiments just performed. When the load is raised a power equal to $X + Y$ must be applied, and therefore $X + Y = 13{\cdot}1$. On the other hand, when the power is raised, the force X is just sufficient to overcome both the friction Y and the weight 5·5; therefore $X = Y + 5{\cdot}5$.

Solving this pair of equations, we find that $X = 9\cdot3$ and $Y = 3\cdot8$. Hence we infer that the power in the frictionless machine would be 9·3; but this is exactly what would have been deduced from the velocity ratio, for $56 \div 6 = 9\cdot3$ lbs. In this result we find a perfect accordance between theory and experiment.

THE DIFFERENTIAL PULLEY-BLOCK.

209. By increasing the number of sheaves in a pair of pulley-blocks the power may be increased; but the length of rope (or chain) requisite for several sheaves becomes a practical inconvenience. There are also other reasons which make the differential pulley-block, which we shall now consider, more convenient for many purposes than the common pulley-blocks when a considerable augmentation of power is required.

210. The principle of the differential pulley is very ancient, but it is only recently that it has been embodied in a machine of practical utility. In designing any mechanical power the object to be aimed at is this, that while the power moves over a considerable distance, the load shall only be raised a short distance. When this object is attained, we then know by the principle of energy that we have gained an increase of power.

211. Let us consider the means by which this is effected in that ingenious contrivance, Weston's differential pulley-block. The principle of this machine will be understood from Fig. 36 and Fig. 37.

It consists of three parts,—an upper pulley-block, a moveable pulley, and an endless chain. We shall briefly describe them. The upper block P is furnished with a hook for attachment to a support. The sheave it contains resembles two sheaves, one a little smaller than the other, fastened together: they are in fact one piece. The grooves are furnished with ridges, which prevent the chain from slipping round them. The lower pulley Q consists of one sheave, which is also furnished with a groove; it carries a hook, to which the load is attached. The endless chain performs a part that will be understood by the arrow-heads attached to it in the figure. The chain passes from the hand at A up to L over the larger groove in the upper pulley, then downwards at B, under the lower pulley, up again at C, over the smaller groove in the upper pulley at A, and then back again by D to the hand at A. When the hand pulls the chain downwards, the two grooves of the upper pulley begin to turn together in the direction shown by the arrows on the chain. The large groove is therefore winding up the chain, while the smaller groove is lowering.

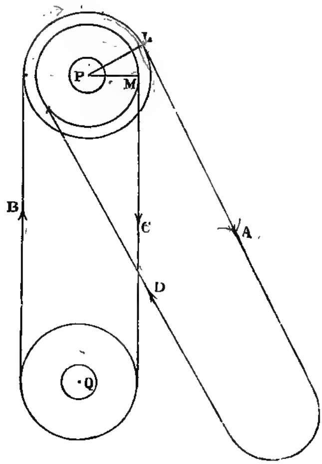

FIG. 36.

212. In the pulley which has been employed in the experiments to be described, the effective circumference of the large groove is found to be 11″·84, while that of the small groove is 10″·36. When the upper pulley has made one revolution, the large groove must have drawn up 11″·84 of chain, since the chain cannot slip on account of the ridges; but in the same time the small groove has lowered 10″·36 of chain: hence when the upper pulley has revolved once, the chain between the two must have

been shortened by the difference between 11″·84 and 10″·36, that is by 1″·48, but this can only have taken place by raising the moveable pulley through half 1″·48, that is through a space 0″·74. The power has then acted through 11″·84, and has raised the resistance 0″·74. The power has therefore moved through a space 16 times greater than that through which the load moves. In fact, it is very easy to verify by actual trial that the power must be moved through 16 feet in order that the load may be raised 1 foot. We express this by saying that the *velocity ratio* is 16.

213. By applying power to the chain at D proceeding from the smaller groove, the chain is lowered by the large groove faster than it is raised by the small one, and the lower pulley descends. The load is thus raised or lowered with great facility by simply pulling one chain A or the other D.

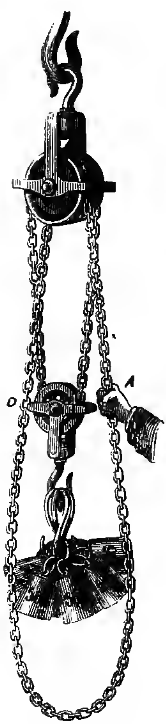

FIG. 37.

214. We shall next consider the mechanical efficiency of the differential pulley-block. The block (Fig. 37) which we shall use is intended to be worked by one man, and will raise any weight not exceeding a quarter of a ton.

We have already learned that for the load to be raised one foot the power must act through sixteen feet. Hence, were it not for friction, we should infer that the power need only be the sixteenth part of the load. A few trials will show us that the real efficiency is not so large, and that in fact more than half the power exerted is merely expended upon overcoming friction. This will lead afterwards to a result of considerable practical importance.

215. Placing upon the load-hook a weight of 200 lbs., I find that 38 lbs. attached to a hook fastened on the power-chain is sufficient to raise the load; that is to say, the power is about one-sixth of the load. If I make the load 400 lbs. I find the requisite power to be 64 lbs., which is only about 3 lbs. less than one sixth of 400 lbs. We may safely adopt the practical rule, that with a differential pulley-block of this class a man would be able to raise a weight six times greater than he could raise without such assistance.

216. A series of experiments carefully tried with different loads have given the results shown in Table XI.

TABLE XI.—THE DIFFERENTIAL PULLEY-BLOCK.

Circumference of large groove 11″·84, of small groove 10″·36; velocity ratio 16; mechanical efficiency 6·07; useful effect 38 per cent.; formula $P = 3{\cdot}87 + 0{\cdot}1508\,R$.

Number of Experiment.	R. Load in lbs.	Observed power in lbs.	P. Calculated power in lbs.	Difference of the observed and calculated values.
1	56	10	12·3	+ 2·3
2	112	20	20·8	+ 0·8
3	168	31	29·2	− 1·8
4	224	38	37·7	− 0·3
5	280	48	46·1	− 1·9
6	336	54	54·6	+ 0·6
7	392	64	63·1	− 0·9
8	448	72	71·5	− 0·5
9	504	80	80·0	0·0
10	560	86	88·4	+ 2·4

The first column contains the numbers of the experiments, the second the weights raised, the third the observed values of the corresponding powers. From these the following rule for finding the power has been obtained:—

217. To find the power, multiply the load by 0·1508, and add 3·87 lbs. to the product; this rule may be expressed by the formula $P = 3{\cdot}87 + 0{\cdot}1508\ R$. (See Appendix.)

218. The calculated values of the powers are given in the fourth column, and the differences between the observed and calculated values in the last column. The differences do not in any case amount to 2·5 lbs., and considering the size of the loads raised (up to a quarter of a ton), the formula represents the experiments with satisfactory precision.

219. Suppose for example 280 lbs. is to be raised; the product of 280 and 0·1508 is 42·22, to which, when 3·87 is added, we find 46·09 to be the requisite power. The mechanical efficiency found by dividing 46·09 into 280 is 6·07.

220. To raise 280 lbs. one foot 280 foot-pounds of energy would be necessary, but in the differential pulley-block 46·09 lbs. must be exerted for a distance of 16 feet in order to accomplish this object. The product of 46·09 and 16 is 737·4. Hence the differential pulley-block requires 737·4 foot-pounds of energy to be applied to it in order to produce 280 foot-pounds; but 280 is only 38 per cent. of 734·4, and therefore with a load of 280 lbs. only 38 per cent. of the energy applied to a differential pulley-block is utilized. In general, we may state that not more than about 40 per cent. is profitably used, and that the remainder is employed in overcoming friction.

221. It is a very remarkable and useful property of the differential pulley, that a weight which has been

hoisted by it will remain suspended without any tendency to run down: this is a source of great practical convenience. In the pulleys we have previously considered this property does not exist. The weight raised by the three-sheave pulley-block, for example, will run down unless the free end of the rope be properly secured. The difference in this respect between these two mechanical powers is not a consequence of any special mechanism; it is simply caused by the excessive friction in the differential pulley-block.

222. The reason why the load does not run down in the differential pulley may be thus explained. Let us suppose that a weight of 400 lbs. is to be raised one foot by the differential pulley-block; 400 units of work are necessary, and therefore 1,000 units of work must be applied to the power-chain to produce the 400 units (since only 40 per cent. is utilized). The friction will thus have consumed 600 units of work when the load has been raised one foot. If the power-weight be removed, the pressure supported by the upper pulley-block is diminished. In fact, since the power-weight is about $\frac{1}{6}$th of the load, the pressure on the axle when the power-weight has been removed is only $\frac{6}{7}$ths of its previous value. The friction is produced by the pressure of the pulleys on their axles, and is nearly proportional to that pressure: hence when the power has been removed the friction on the upper axle is $\frac{6}{7}$ths of its previous value, while the friction on the lower pulley remains unaltered.

We may therefore assume that the total friction is at least $\frac{6}{7}$ths of what it was before the power-weight was removed. Will friction allow the load to descend? 600 foot-pounds of work were required to overcome the friction in the ascent: at least $\frac{6}{7} \times 600 = 514$ foot-pounds would be necessary to overcome friction in the descent. But where is this energy to come from? The load in its descent could only yield 400 units, and thus descent by the mere weight of the load is impossible. To enable the load to descend we have actually to aid the movement by pulling the chain D (Figs. 36 and 37), which proceeds from the small groove in the upper pulley.

223. The principle which we have here established extends to other mechanical powers, and may be stated generally. Whenever rather more than half of the applied energy is uselessly consumed by friction, the load will remain suspended without overhauling.

THE EPICYCLOIDAL PULLEY-BLOCK.

224. We shall conclude this lecture with some experiments upon a mechanical power which has been recently introduced by Mr. Eade under the name of the epicycloidal pulley-block. It is shown in Fig. 49, and also in Fig. 33. In this machine there are two chains: one a slight endless chain to which the power is applied; the other a stout chain which has a hook at each end, from either of which the load may be suspended. Each of these chains passes over a sheave in the block: these sheaves are connected by an ingenious piece of mechanism which we cannot describe here. This mechanism is so contrived that, when the power causes the sheave to revolve over which the slight chain passes, the sheave which carries the large chain is also made to revolve, but very slowly.

225. By actual trial it is ascertained that the power must be exerted through twelve feet and a half in order to raise the load one foot; the velocity ratio of the machine is therefore 12·5.

226. The mechanical efficiency of the machine would, if it were frictionless, be of course equal to its velocity ratio; owing to the friction the mechanical efficiency is less than the velocity ratio, and it will therefore be necessary to make experiments. I attach to the load-hook a weight of 280 lbs., and insert a few small hooks into the links of the power chain in order to receive weights: 56 lbs. is sufficient to produce motion, hence the mechanical efficiency is 5. Had there been no friction a power of 56 lbs. would have been capable of overcoming a load of $12{\cdot}5 \times 56 = 700$ lbs. Thus 700 units of energy must be applied to the machine in order to perform 280 units of work. In other words, only 40 per cent. of the applied energy is utilized.

227. An extended series of experiments upon the epicycloidal pulley-block is recorded in Table XII.

TABLE XII.—THE EPICYCLOIDAL PULLEY-BLOCK.

Size adapted for lifting weights up to 5 cwt.; velocity ratio 12·5; mechanical efficiency 5; useful effect 40 per cent.; formula $P = 5{\cdot}8 + 0{\cdot}185\,R$.

Number of Experiment.	*R.* Load in lbs.	Observed power in lbs.	*P.* Calculated power in lbs.	Difference of the observed and calculated values.
1	56	15	16·2	+ 1·2
2	112	27	26·5	− 0·5
3	168	40	36·9	− 3·1
4	224	47	47·2	+ 0·2
5	280	56	57·6	+ 1·6
6	336	66	68·0	+ 2·0
7	392	78	78·3	+ 0·3
8	448	88	88·6	+ 0·6
9	504	100	99·0	− 1·0
10	560	110	109·4	− 0·6

The fourth column shows the calculated values of the powers derived from the formula. It will be seen by the last column that the formula represents the experiments with but little error.

228. Since 60 per cent. of energy is consumed by friction, this machine, like the differential pulley-block, sustains its load when the chains are free. The differential pulley-block gives a mechanical efficiency of 6, while the epicycloidal pulley-block has only a mechanical efficiency of 5, and so far the former machine has the advantage; on the other hand, that the epicycloidal pulley contains but one block, and that its lifting chain has two hooks, are practical conveniences strongly in its favour.

LECTURE VIII.

THE LEVER.

The Lever of the First Order.—The Lever of the Second Order.—The Shears.—The Lever of the Third Order.

THE LEVER OF THE FIRST ORDER.

229. THERE are many cases in which a machine for increasing power is necessary where pulleys would be quite inapplicable. To meet these various demands a correspondingly various number of mechanical powers has been devised. Amongst these the lever in several different forms holds an important place.

230. The lever of the first order will be understood by reference to Fig. 38. It consists of a straight rod M N, to one end of which the power is applied by means of the weight C. At another point B the load is raised, while at A the rod is supported by what is called the fulcrum. In the case represented in the figure the rod is of iron, 1″ × 1″ in section and 6′ long; it weighs 19 lbs. The power is a 56 lb. weight: the fulcrum consists of a moderately sharp steel edge firmly secured to the framework. The load in this case is not a weight but a spring balance H, and the hook of the balance is attached to the frame. The spring is strained by the power of the lever, and the index records the magnitude of the strain pro-

duced by the power. This is the lever with which we shall commence our experiments.

231. In examining the relation between the power and the load, the question is a little complicated by the weight of the lever itself (19 lbs.), but we shall be able to

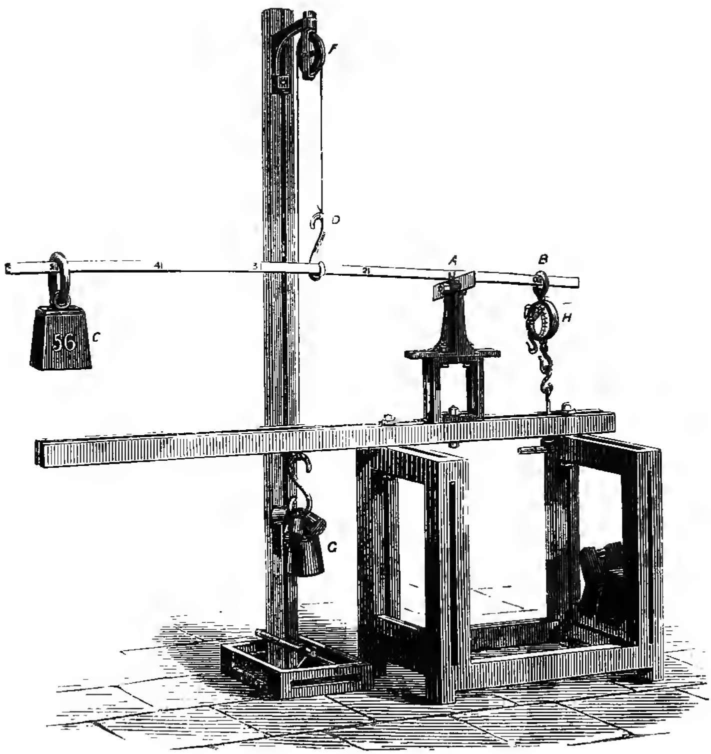

FIG. 38

evade the difficulty by means similar to those employed on a former occasion (Art. 60) ; we can counterpoise the weight of the iron bar. This is easily done by attaching a rope to the middle of the bar at D, carrying this rope over a pulley F, and suspending a weight G of 19 lbs. from its free extremity. The bar is balanced, and we may leave its weight out of consideration.

232. We might also adopt another plan analogous to that of Art. 51, which is not, however, so convenient. The weight of the bar produces a certain strain upon the spring balance. I may first read off the strain produced by the bar alone, and then apply the weight C and read again. The observed strain is due both to the weight C and to the weight of the bar. If I subtract the known effect of the bar, the remainder is the effect of C. It is, however, less complicated to counterpoise the bar, and then the strains indicated by the balance are entirely due to the power.

233. The lever is 6′ long; the point B is 6″ from the end, and B C is 5′ long. B C is divided into 5 equal portions of 1′; A is at one of these divisions, 1′ distant from B, and C is 5′ distant from B in the figure; but C is capable of being placed at any position, by simply sliding its ring along the bar.

234. The mode of experimenting is as follows:—The weight is placed on the bar at the position C; a strain is immediately produced upon H; the spring stretches a little, and the bar becomes inclined. It may be noticed that the hook of the spring balance passes through the eye of a wire-strainer, so that by a turn or two of the nut upon the strainer the lever can be restored to the horizontal position.

235. The power of 56 lbs. being 4′ from the fulcrum, while the load is 1′ from the fulcrum, it is found that the strain indicated by the balance is 224 lbs.; that is, four times the amount of the power. If the weight be moved, so as to be 3′ from the fulcrum, the strain is observed to be 168 lbs.; and whatever be the distance of

the power from the fulcrum, we find that the strain produced is obtained by multiplying the magnitude of the power in pounds by the distance expressed in feet, and it may be fractional parts of a foot. This law may be expressed more generally by stating that the power is to the load as the distance of the load from the fulcrum is to the distance of the power from the fulcrum.

236. We can verify this law under varied circumstances. I move the steel edge which forms the fulcrum of the lever until the edge is 2′ from B, and secure it in that position. I place the weight C at a distance of 3′ from the fulcrum. I now find that the strain on the balance is 84 lbs.; but 84 is to 56 as 3 is to 2, and therefore the law is also verified in this instance.

237. There is another aspect in which we may express the relation between the power and the load. The law in this form is thus stated: "The power multiplied by its distance from the fulcrum is equal to the product of the load and its distance from the fulcrum."

Thus, in the case we have just considered, the product of 56 and 3 is 168, and this is equal to the product of 84 and 2. This simple law gives a very convenient method of calculating the load, when we know the power and the distances of the power and the load from the fulcrum. These distances are commonly called the arms of the lever, and the rule is expressed more concisely by stating that "The power multiplied into its arm is equal to the load multiplied into its arm:" hence the load may be found by dividing the product of the power and the power arm by the load arm.

238. When the power arm is longer than the load arm, the load is greater than the power; but when the

power arm is shorter than the load arm, the power is greater than the load.

We may regard the strain on the balance as a power which supports the weight, just as we regard the weight to be a power producing the strain on the balance. We see, then, that for the lever of the first order to be efficient as a mechanical power it is necessary that the power arm be longer than the load arm.

239. The lever is an extremely simple mechanical power; it has only one moving part. Friction produces but little effect upon it, so that the laws which we have given may be actually applied in practice, without making any allowance for friction. In this we notice a very marked difference between the lever and the pulley-blocks already described.

240. In the lever of the first order we find an excellent machine for augmenting power. The pressure of 14 lbs. by means of this lever can produce a strain of 1 cwt., if the power be eight times as far from the fulcrum as the load is from the fulcrum. This principle it is which gives utility to the crowbar. The extremity of the bar is placed under a heavy stone, which it is required to raise; a support near that end serves as a fulcrum, and then a comparatively small force exerted at the power end will suffice to elevate the stone.

241. The applications of the lever are innumerable. It is used not only for increasing power, but for modifying and transforming it in various ways. The lever is also used in weighing-machines, the principles of which will be readily understood, for they are consequences of the law we have explained. Into these various appliances it is not our intention to enter at present; the great majority of them may, when met with, be easily under-

stood by any who are familiar with the principle we have laid down.

THE LEVER OF THE SECOND ORDER.

242. In the lever of the second order, the power is at one end, the fulcrum at the other end, and the load lies between the two: this lever therefore differs from the lever of the first order, in which the fulcrum lies between the two forces. The relation between the power and the load in the lever of the second order may be studied by the arrangement in Fig. 39.

243. The bar A C is a rod of iron $72'' \times 1'' \times 1''$, as before mentioned. The fulcrum A is a steel edge on which the bar rests; the power consists of a spring balance H, in the hook of which the end C of the bar rests; the spring balance is sustained by a wire-strainer, by turning the nut of which the bar may be adjusted horizontally. The part of the bar between the fulcrum A and the power C is divided into five portions, each $1'$ long, and the points A and C are each $6''$ distant from the extremities of the bar. The load employed is 56 lbs.; through the ring of this weight the bar passes, and thus the bar supports the load. The bar is counterpoised by the weight of 19 lbs. at G, in the manner already explained (Art. 231).

244. The mode of experimenting is as follows:—Let the weight B be placed $1'$ from the fulcrum; the strain shown by the spring balance is about 11 lbs. If we calculate the value of the power by the rule already given, we should have found it to be almost the same. The product of the load by its distance from the fulcrum is 56, the distance of the power from the fulcrum is 5; hence the value of the power should be $56 \div 5 = 11 \cdot 2$.

245. If the weight be placed 2′ from the fulcrum, the strain is about 22·5 lbs., and it is easy to ascertain that this is the same amount as would have been found by the application of the rule. A similar result would have been obtained if the 56 lb. weight had been placed upon

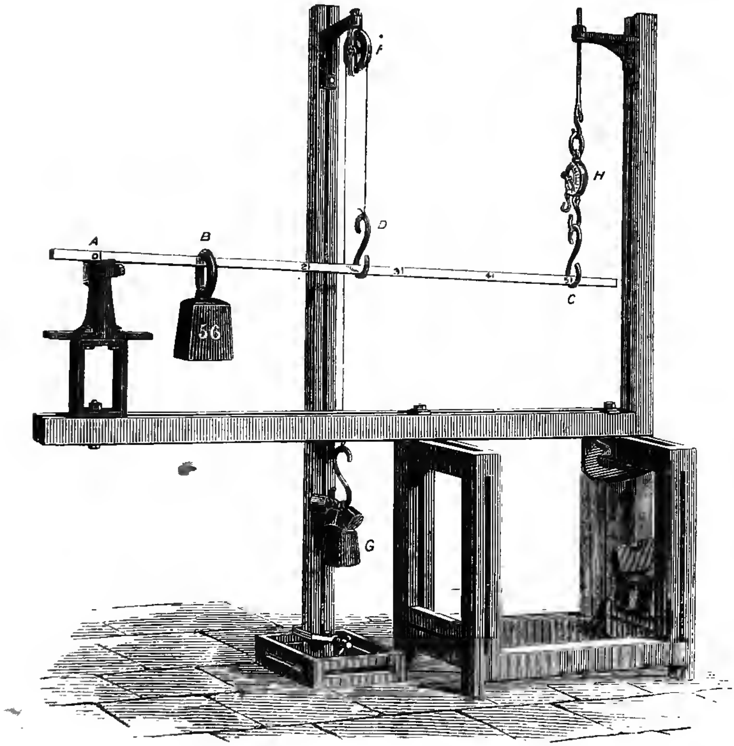

FIG. 39.

any other part of the bar; and hence we may regard the rule (Art. 237) proved for the lever of the second order as well as for the lever of the first order. In the present case, the load is uniformly 56 lbs., while the power by which it is sustained is always less than 56 lbs.

246. The lever of the second order, like that of the first order, is frequently applied to practical purposes; one of the most instructive of these applications is illustrated in the shears shown in Fig. 40.

These shears consist of two levers of the second order, which by their united action enable a man to exert a very large force, sufficient, for example, to cut with ease a rod of iron 0″·25 square. The mode of action is simple.

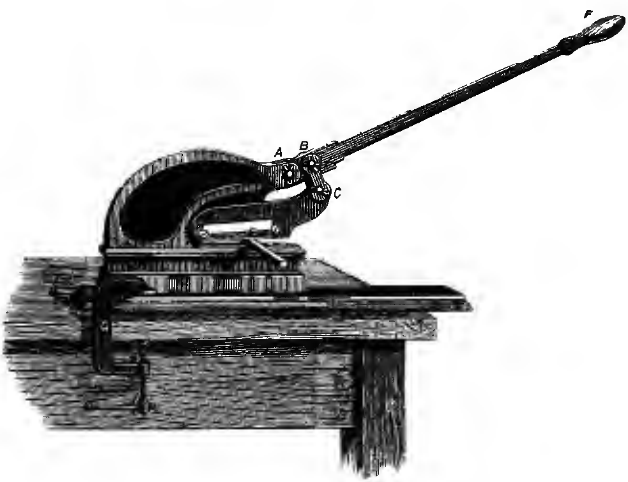

FIG. 40.

The first lever A F has a handle at one end F, which is 22″ distant from the other end A, where the fulcrum is placed. At a point B on this lever, 1″·8 distant from the fulcrum A, a short link B C is attached; the end of the link C is jointed to a second lever C D: this second lever is 8″ long; it forms one edge of the cutting shears, the other edge being fixed to the framework.

247. I place a rod of iron 0″·25 square between the jaws of the shears in the position E, the distance D E being

3″·5, and proceed to cut the iron by applying pressure to the handle. Let us calculate the amount by which the levers increase the power exerted upon F. Suppose for example that I press downwards on the handle F with a force of 10 lbs., what is the magnitude of the pressure upon the piece of iron? The effect of each lever is to be calculated separately. We may ascertain the power exerted at B by the rule of Art. 237; the product of the power and its arm is 22 × 10 = 220: this divided by the number of inches, 1·8 in the line A B, gives a quotient 122, and this quotient is the number of pounds pressure which is exerted by means of the link upon the second lever. We proceed in the same manner to find the magnitude of the pressure upon the iron at E. The product of 122 and 8 is 976. This is divided by 3·5, and the quotient found is 279. Hence the exertion of a pressure of 10 lbs. at F produces a pressure of 279 lbs. at E. In round numbers, we may say that the pressure is magnified 28-fold by means of this combination of levers of the second order.

248. A pressure of 10 lbs. is not sufficient to shear across the bar of iron, even though it be magnified to 279 lbs. I therefore suspend weights from F, and gradually increase the load until the bar is cut. I find at the first trial that 112 lbs. is sufficient, and a second trial with the same bar gives 114 lbs.; 113 lbs., the mean between these results, may be considered an adequate force. This is the load on F; the real pressure on the bar is 113 × 27·9 = 3,153 lbs.: thus the actual pressure which was necessary to cut the bar amounted to more than a ton.

249. We can calculate from this experiment the amount of force necessary to shear across a bar one square inch in section. We may reasonably suppose that the necessary power is proportional to the section, and therefore the

power will bear to 279 lbs. the proportion which a square of one inch bears to the square of a quarter inch; but this ratio is 16: hence the force is 16 × 3,153 lbs., equal to about 22·5 tons.

250. It is remarkable that 22·5 tons is nearly the strain (Art. 45) which would suffice to tear the bar in sunder by actual tension. We shall subsequently return to the subject of shearing iron in the lecture upon Inertia (Lecture XVI.)

THE LEVER OF THE THIRD ORDER.

251. The lever of the third order may be easily understood from Fig. 39, of which we have already made use. In the lever of the third order the fulcrum is at one end, the load is at the other end, while the power lies between the two. In this case, then, the power is represented by the 56 lb. weight, while the load is indicated by the spring balance. The power always exceeds the load, and consequently this lever is never employed when it is required to gain power. Thus, for example, when the power, 56 lbs., is 2′ distant from the fulcrum, the load indicated by the spring balance is about 23 lbs.

252. There are, however, numerous cases in which this lever is of use: for example, the treadle of a lathe or grindstone is a lever of the third order. The fulcrum is at one end, the foot applies the power, and the load is at the other end: the convenience of the arrangement consists in this, that the foot has only to move through a small space.

253. The principles which have been discussed in Lecture III. with respect to parallel forces, explain the rules which have been laid down for levers of different

orders; and will also enable us to express these rules more concisely.

254. A comparison of Figs. 39 and 20 shows that the only real difference between the arrangements is that in Fig. 20 we have a spring balance C in the same place as the steel edge A in Fig. 39. We may in Fig. 20 regard one spring balance as the power, the other as the fulcrum, and the weight as the load. Nor is there much difference between the apparatus of Fig. 38 and that of Fig. 20. In Fig. 38 the bar is pulled down by a force at each end, one a weight, the other a spring balance, while it is supported by the upward pressure of the steel edge. In Fig. 20 the bar is being pulled upwards by a force at each end, and downwards by the weight. The two cases are substantially the same. In each of them we find a bar acted upon by a pair of parallel forces applied at its extremities, and retained in equilibrium by a third parallel force acting between them.

255. We may therefore apply to the lever the principles of parallel forces already explained. We showed that two parallel forces acting upon a bar could be compounded into a resultant, applied at a certain point of the bar. We have defined the moment of a force, and proved that the moments of two parallel forces about the point of application of their resultant are equal (Art. 65).

256. In the lever of the first order there are two parallel forces, one at each end; these are compounded into a resultant, and it is necessary that this resultant be applied to the bar exactly over the steel edge or fulcrum in order that the bar may be supported. In the levers of the second and third orders, the power and the load are two parallel forces acting in opposite directions; their resultant, therefore, does not lie between the forces, but is

applied on the side of the greater, and at the point where the steel edge supports the bar. In all cases the moment of one of the forces about the fulcrum must be equal to that of the other. From the equality of moments it follows that the product of the power by the distance of the power from the fulcrum equals the product of the load, and the distance of the load from the fulcrum: from this principle the rules already given are immediately inferred.

257. The principle of the lever may be deduced from the principle of work; the load, if nearer than the power to the fulcrum, is moved through a smaller distance than the power. Thus, for example, in the lever of the first order: if the load be 12 times farther than the power from the fulcrum, then for every inch the load moves it will be easily seen that the power must move 12 inches. The number of units of work applied at one end of a machine is equal to the number yielded at the other, always excepting the loss due to friction, which is, however, so small in the lever that it may be omitted. If then a power of 1 lb. be applied to move the power end through 12 inches, one unit of work will have been put into the machine. Hence one unit of work must be done by the load, but the load only moves through $\frac{1}{12}$ of a foot, and therefore it must exert a force of 12 lbs.: this is the same result as would be given by the rule (Art. 237).

258. To conclude: we have first by actual experiment determined the relation between the power and the load in the lever; we have seen that the law thus obtained harmonizes with the principle of the composition of parallel forces; and, finally, we have shown how the same result could also be deduced from the fertile and important principle of work.

LECTURE IX.

THE INCLINED PLANE AND THE SCREW.

The Inclined Plane without Friction.—The Inclined Plane with Friction.—The Screw.—The Screw-jack.—The Bolt and Nut.

THE INCLINED PLANE WITHOUT FRICTION.

259. THE mechanical powers now to be considered are often used for other purposes beside those of raising great weights. For example: the parts of a structure have to be forcibly drawn together, a force of compression has to be exerted, or the particles of a mass have to be driven asunder, as in splitting. For purposes of this kind the inclined plane in its various forms, and the screw, are of the greatest use. The screw also, in the form of the screw-jack, is sometimes used in raising weights. It is principally convenient when the weight is enormously great, and the distance through which it has to be raised comparatively small.

260. We shall commence with the study of the inclined plane. The apparatus used is shown in Fig. 41. A B is a plate of glass 4′ long, mounted on a frame and turning round a hinge at A; B D is an arc, whose centre is at A, to which the frame may be clamped; D C is a vertical rod, to which the pulley C is clamped. This pulley can be moved up and down, to be accommodated to the

position of A B; the pulley is made of brass, and turns very freely. A little truck R is adapted to run on the plane of glass. The truck is laden to weigh 1 lb., and this weight is constant throughout the experiments; the wheels being very free, the truck runs with but little friction on the glass plane.

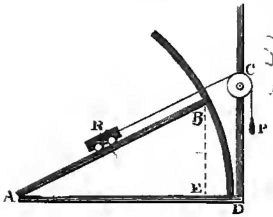

FIG. 41.

261. But the friction, though small, is appreciable, and it will be necessary to ascertain the amount of the friction in order to counteract the effect upon the motion. The silk cord attached to the truck is very fine, and its weight is neglected. A series of weights is provided; they are made of brass wire, and S-shaped, and weigh 0·1 lb. and 0·01 lb.: these can easily be hooked into the loop on the cord at P. We first make the plane A B horizontal, and bring down the pulley C so that the cord shall be parallel to the plane; a certain amount of weight must be applied at P in order to draw the truck along the plane: this weight is of course the friction, and when it is applied at P the friction may be said to be counterbalanced. But we cannot be sure that the friction is the same when the plane is horizontal as when the plane is inclined. We must therefore examine into this question by a method analogous to that used in Art. 208.

262. Let the plane be elevated until B E, the elevation of B above A D, is 20″; let C be properly adjusted: it is found that when P is 0·45 lb. R is just pulled up; and on the other hand, when P is only 0·40 lb. R descends and raises P; and when P has any value intermediate between these two, the truck remains in equilibrium. We call the force of gravity acting down the plane R,

and it follows that R must be 0·425 lb., and the friction 0·025 lb. For when P raises R, P must overcome R together with friction; therefore the power must be 0·025 + 0·425 = 0·45. On the other hand, when R raises P, R must also overcome the friction 0·025, and therefore P can only be 0·425 − 0·025 = 0·40; R is thus found to be a mean between the greatest and least values of P consistent with equilibrium. If the plane be raised so that the height B E is 33″, the greatest and least values of P are 0·66 and 0·71; therefore R is 0·685 and the friction 0·025, the same as before. Finally, making the height B E 2″, the friction is ascertained to be 0·020, almost the same as the previous determinations. This inquiry shows us that we may consider the friction constant at different inclinations of the plane, at all events to the degree of delicacy at which we are aiming. As in the experiments R is always raised, we may place 0·025 lb. permanently at P; this will just counteract the friction, which we may therefore dismiss from consideration. It is hardly necessary to remark that, in afterwards recording the weights placed at P, this counterpoise is not included.

263. We have now the means of studying the relation between the power and the load in the frictionless inclined plane. The plane being raised to different elevations, we shall observe the force necessary to raise the constant load of 1 lb. Our course will be guided by first examining into the subject with the aid of the principle of energy. Suppose B E to be 2′; when the truck has been moved from the bottom of the plane to the top, it will have been raised vertically through a space of 2′, and two units of work must have been consumed. But the plane being 4′ long, the force which urges it up the plane need only be 0·5 lb., for 0·5 lb. acting over 4′ produces

two units of work. In general, if l be the length of the plane and h its height, R the load, and P the power, the number of units necessary to raise the weight is $R h$, and the number of units expended in pulling it up the plane is Pl: hence $R h = Pl$, and consequently $P:h::R:l$; that is, the power is to the height of the plane as the load is to its length. In the present case $R = 1$ lb., $l = 48''$; therefore $P = 0{\cdot}0208\ h$, where h is the height of the plane, and P the power expressed in pounds.

264. We compare, then, the values of the powers calculated by this formula with the actual observed values: the result is given in Table XIII.

TABLE XIII.—INCLINED PLANE.

Glass plane 48″ long, truck 1 lb. in weight, friction counterpoised; formula $P = 0{\cdot}0208 \times h''$.

Number of Experiment.	Height of plane.	Observed power in lbs.	*P.* Calculated power in lbs.	Difference of the observed and calculated powers.
1	2″	0·04	0·04	0·00
2	4″	0·08	0·08	0·00
3	6″	0·13	0·12	− 0·01
4	8″	0·16	0·17	+ 0·01
5	10″	0·21	0·21	0·00
6	15″	0·31	0·31	0·00
7	20″	0·42	0·42	0·00
8	33″	0·71	0·69	− 0·02

Thus for example, in experiment 6, where the height is 15″, it is found that the power necessary to draw up the truck is 0·31 lb. The truck is placed in the middle of the plane, and the power is adjusted so as to be able to draw the truck to the top of the plane with certainty; the necessary power as calculated by the formula is also 0·31 lb., so that the formula is verified in this case.

265. The fifth column of the table shows the difference between the observed and calculated powers. The very slight differences, in no case exceeding the fiftieth part of a pound, may undoubtedly be referred to the inevitable errors of experiment.

THE INCLINED PLANE WITH FRICTION.

266. The friction of the truck upon the glass plate is very small in amount, and is shown to be practically constant for the inclinations of the plane which were used. But when the friction is large, we shall not be justified in considering it constant at different elevations, and we must adopt more rigorous methods. For this inquiry we shall use the pine plank and slide already described in Art. 118. We do not in this case seek to diminish friction by the aid of wheels, and consequently it will be of considerable amount.

267. In another respect also the experiments of Table XIII. contrast with those now to be described. In the former the load was constant, while the elevation was changed. In the latter the elevation is to remain constant while the load is changed. We shall find in this experiment also that when the proper allowance is made for friction, the law connecting the power and the load is fully borne out.

268. The apparatus used is shown in Fig. 33; the plane is, however, secured in one position, and the pulley shown in Fig. 32 is attached to the framework, so that the rope from the pulley to the slide is parallel to the incline. The elevation of the plane in the position adopted is 17°·2, so that its length, base, and height are in the proportions of the numbers 1, 0·955, and 0·296. Weights ranging

from 7 lbs. to 56 lbs. are placed upon the slide, and the power is found which, when the slide is started by the screw, will draw it steadily up the plane. The requisite power consists of two parts, that which is necessary to overcome gravity acting down the plane, and that which is necessary to overcome friction.

269. The forces are shown in Fig. 42. R G, the force of gravity, is resolved into R L and R M; R L is evidently the force acting down the plane, and R M the pressure against the plane; the triangle G L R is similar to A B C, hence if R be the load, the force R L acting down the plane must be 0·296 R, and the pressure upon the plane 0·955 R.

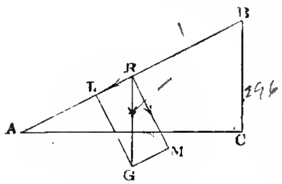

FIG. 42.

270. We shall first suppose the ordinary law that the friction is proportional to the pressure to be true. The pressure upon the plane A B, to which the friction is proportional, is not the weight of the load. The pressure is that component (R M) of the load which is perpendicular to the plane A B. When the weights do not extend beyond 56 lbs., the best value for the coefficient of friction is 0·288 (Art. 141): hence the amount of friction upon the plane is

$$0{\cdot}288 \times 0{\cdot}955\, R = 0{\cdot}275\, R.$$

This force must be overcome in addition to 0·296 R (the component of gravity acting along the plane): hence the value of the power is

$$0{\cdot}275\, R + 0{\cdot}296\, R = 0{\cdot}571\, R.$$

271. The values of the powers which have been observed compared with the powers calculated by this formula are shown in Table XIV.

TABLE XIV.—INCLINED PLANE.

Smooth plane of pine 72″ × 11″; angle of inclination 17°·2; slide of pine, grain crosswise; slide started; formula $P = 0{\cdot}571\ R$.

Number of Experiment.	*R*. Total load on slide in lbs.	Power in lbs. which just draws up slide.	*P*. Calculated value of the power.	Difference of the observed and calculated powers.
1	7	4·6	4·0	– 0·6
2	14	8·3	8·0	– 0·3
3	21	12·3	12·0	– 0·3
4	28	16·5	16·0	– 0·5
5	35	20·0	20·0	0·0
6	42	24·2	24·0	– 0·2
7	49	28·0	28·0	0·0
8	56	31·8	32·0	+ 0·2

272. Thus for example, in experiment 6, 42 lbs. when started was raised by a force of 24·2 lbs., while the calculated value is 24·0 lbs.; the difference, 0·2 lbs., is shown in the last column.

273. The calculated values are found to agree tolerably well with the observed values, but the presence of so large a difference as 0·6 lb. leads us to inquire whether by employing the more accurate law of friction (Art. 142) a better result may not be obtained.

In Table VI. we have seen that the friction for weights less than 56 lbs. is best expressed by the formula $F = 0{\cdot}9 + 0{\cdot}266 \times$ pressure, but the pressure is in this case $= 0{\cdot}955\ R$, and hence the friction is

$$0{\cdot}9 + 0{\cdot}254\ R.$$

To this must be added $0{\cdot}296\ R$, the component of the force of gravity which must be overcome, and hence the total force is

$$0{\cdot}9 + 0{\cdot}55\ R.$$

The powers calculated by this formula are compared with those actually observed in Table XV.

TABLE XV.—INCLINED PLANE.

Smooth plane of pine 72″ × 11″; angle of inclination 17°·2; slide of pine, grain crosswise; slide started; formula $P = 0·9 + 0·55\,R$.

Number of Experiment.	*R.* Total load on slide in lbs.	Power in lbs. which just draws up slide.	*P.* Calculated value of the power.	Difference of the observed and calculated powers.
1	7	4·6	4·7	+ 0·1
2	14	8·3	8·6	+ 0·3
3	21	12·3	12·5	+ 0·2
4	28	16·5	16·3	− 0·2
5	35	20·0	20·1	+ 0·1
6	42	24·2	24·0	− 0·2
7	49	28·0	27·8	− 0·2
8	56	31·8	31·7	− 0·1

For example: in experiment 5, 35 lbs. is seen to be raised by a force of 20·0 lbs., while the calculated power is 0·9 + 0·55 × 35 = 20·1 lbs.

274. The calculated values of the powers are seen in this table to agree extremely well with the observed values, the greatest difference being only 0·3 lb. Hence there can be no doubt that the principles on which the formula has been calculated are correct. This table may therefore be regarded as verifying both the law of friction, and the rule laid down for the relation between the power and the weight in the inclined plane.

275. The inclined plane is one of the mechanical powers; if the weight on the slide be 30 lbs., we calculate by the formula that 17·4 lbs. would be sufficient to raise it, so that, notwithstanding the loss by friction, we have here a smaller weight raising a larger one, which

is the essential feature of a mechanical power. The mechanical efficiency is 30 ÷ 17·4 = 1·72.

276. The velocity ratio in the inclined plane is the proportion of the distance through which the power moves to the vertical height through which the weight is raised, 1 ÷ 0·296 = 3·38. To raise 30 lbs. one foot, a force of 17·4 lbs. must therefore be exerted through 3·38 feet; that is, 58·8 units of work must be expended, though only 30 units are accomplished. Hence 51 per cent. of the energy applied is utilized, and the rest is expended in overcoming friction.

277. We have pointed out in Art. 223 that a machine will overhaul when more than half the energy is utilized: this is the case in the present instance; hence the weight will run down the plane if allowed to do so. This agrees with what we have ascertained regarding the angle of friction (Art. 148), for it was there that at about 13°·4, and *a fortiori* at any greater inclination, the slide would descend when started.

THE SCREW.

278. The inclined plane as a mechanical power is generally disguised under the form of a wedge or a screw. A wedge is an inclined plane which is forced under the load; it is usually moved by means of a hammer, so that the efficiency of the wedge as a mechanical power is augmented by the effect of a blow.

279. The screw is the most useful mechanical power which we possess. A screw may be formed by wrapping a wedge-shaped piece of paper around a cylinder, and then cutting a groove in the cylinder along the spiral line indicated by the margin of the paper. Such a

groove is a screw. In order that the screw may be used, a nut is necessary; the nut consists of a hollow cylinder, the internal diameter of which is equal to the diameter of the cylinder from which the screw is made. The nut contains a spiral ridge, which fits into the corresponding groove in the screw; when the nut is turned round, it moves backwards or forwards according to the direction of the rotation. Large screws of the better class, such as those with which we shall first be occupied, are always turned in a lathe, and are thus made with extreme accuracy. Small screws are made in a simpler manner by means of what is called a screw plate.

280. It is important to understand what is meant by the word "pitch." A screw is said to have 10 threads to the inch when it requires 10 revolutions of the nut in order to move it one inch. A screw of this kind is said to have a pitch of 10 threads to the inch. The shape of the section of the thread is also to be noticed; the thread may be square or triangular, or, as is generally the case in small screws, of a rounded form.

281. There is so much friction in the screw that experiments are necessary to give us any insight into the law connecting the power and the load.

282. We shall commence with an examination of the screw shown in Fig. 43.

The nut A is mounted upon a stout frame; to the end of the screw hooks are attached, in order to receive the load, which does not exceed 224 lbs.; at the top of the screw is an arm E by which the screw is turned; to the end of the arm a rope is attached, which passing over a pulley D, carries a hook for receiving the power C.

283. We first apply the principle of work to this screw. The diameter of the circle described by the end of the

arm is 20″·5; its circumference is therefore 64″·4. The screw contains three threads to the inch, hence in order to raise the load 1″ the power must travel through $3 \times 64''\cdot 4 = 193''$; therefore the velocity ratio is 193, and were the screw capable of working without friction, 193 would represent the mechanical efficiency. In actually performing the experiments the arm E is placed at right

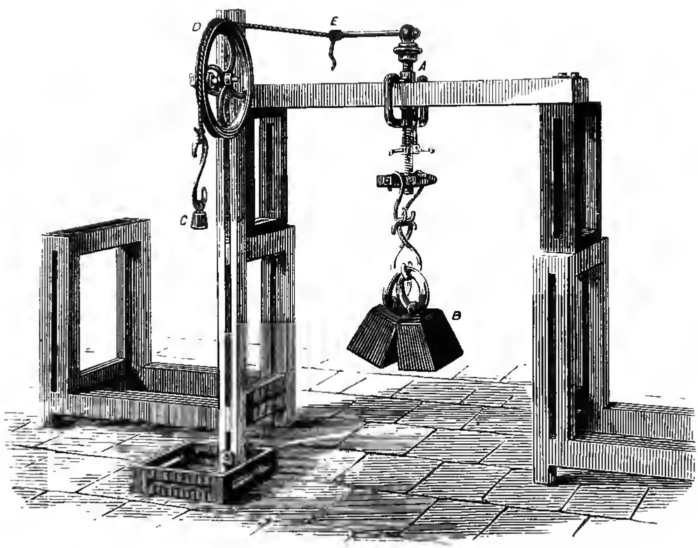

FIG. 43.

angles to the rope leading to the pulley, and the power hook is weighted until, with a slight start, the arm is drawn towards the pulley. The power can never draw the arm more than a few inches, as when the cord ceases to be perpendicular to the arm the power acts with diminished efficiency; consequently the load is only raised in each experiment through a small fraction of an inch, perhaps about one-twentieth.

TABLE XVI.—THE SCREW.

Wrought iron screw, square thread, diameter 1″·25, pitch 3 threads to the inch, arm 10″·25; nut cast iron, bearing surfaces oiled, velocity ratio 193, useful effect 36 per cent., mechanical efficiency 70; formula $P = 0{\cdot}0143\,R$.

Number of Experiment.	*R.* Load in lbs.	Observed power in lbs.	*P.* Calculated power in lbs.	Difference of the observed and calculated powers.
1	28	0·4	0·4	0·0
2	56	0·8	0·8	0·0
3	84	1·2	1·2	0·0
4	112	1·6	1·6	0·0
5	140	2·0	2·0	0·0
6	168	2·4	2·4	0·0
7	196	2·7	2·8	+ 0·1
8	224	3·3	3·2	− 0·1

284. These experiments are shown in Table XVI. If the motion had not been aided by a start the results would have been different. Thus in experiment 6, 2·4 lbs. is the power with a start, when without a start 3·2 lbs. was found to be necessary. The experiments have all been aided by a start, and the results recorded have been corrected for the friction of the pulley over which the rope passes: this correction is very small, in no case exceeding 0·2 lb. The fourth column contains the values of the powers computed by the formula $P = 0{\cdot}0143\,R$. This formula has been deduced from the observations in the manner described in the Appendix. The fifth column proves that the experiments are truly represented by the formula: in each of the experiments 7 and 8, the difference between the calculated and observed values amounts to 0·1 lb., and this is quite inconsiderable in comparison with the size of the weights we are employing.

285. In order to lift 100 lbs. the formula shows that 1·43 lbs. would be necessary: hence the mechanical efficiency of the screw is 100÷1·43=70. Thus this screw is vastly more powerful than any of the pulley systems which we have discussed. A machine so powerful, so compact, and so cheap is invaluable.

286. It is evident, however, that the distance through which the screw can raise a weight must be limited by the length of the screw itself.

287. We have seen that the velocity ratio is 193; therefore, in order to raise 100 lbs. 1 foot, 1·43 × 193 = 276 units of work must be consumed: of this quantity only 100 units, or 36 per cent., is usefully employed; the rest being consumed in overcoming the friction of the screw. Thus about two-thirds of the energy applied to such a screw is lost. Hence we find that the screw does not overhaul, since less than 50 per cent. of the applied energy is usefully employed. This is one of the most valuable properties which the screw possesses.

288. We may contrast the screw with the pulley block (Art. 200). They are both powerful machines: the latter is bulky and economical of power, the former is compact and wasteful of power; the latter is adapted for raising weights through considerable distances, and the former for exerting pressures through short distances.

THE SCREW-JACK.

289. The importance of the screw as a mechanical power justifies us in examining one of its most useful forms, the screw-jack. This machine is used for exerting great pressures, such for example as starting a ship which is reluctant to be launched, or replacing a locomotive upon

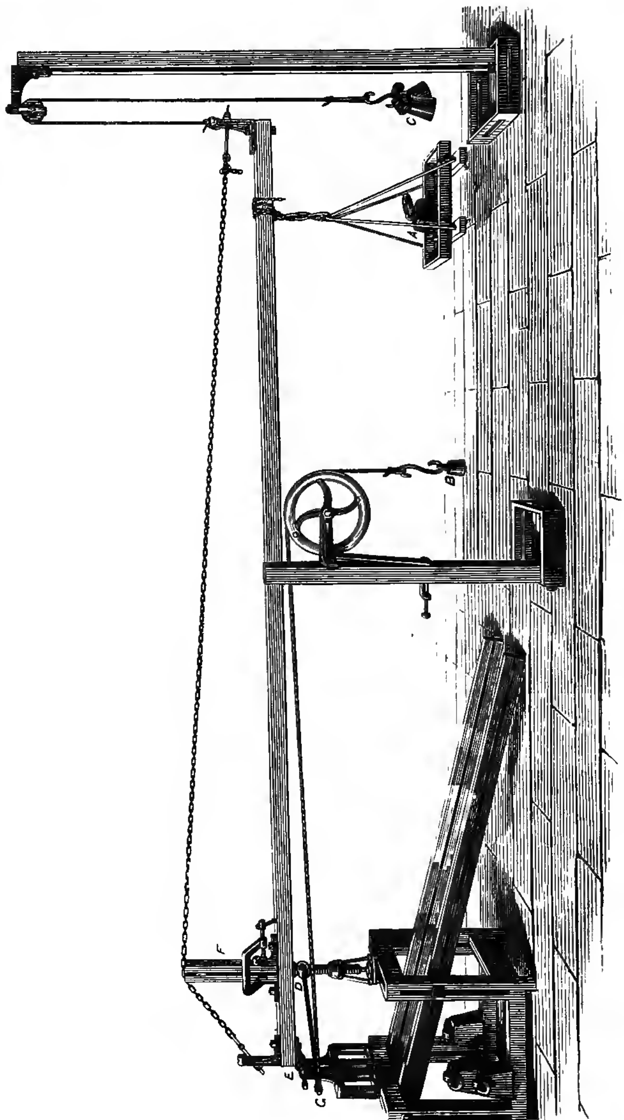

FIG. 44.

the line from which its wheels have slipped. These machines vary slightly in form, as well as in the weights for which they are adapted; one of them is shown at D in Fig. 44, and a description of its details is given in Table XVII. We shall determine the powers to be applied to this machine for overcoming pressures not exceeding half a ton.

290. To employ weights so large as half a ton would be inconvenient under any circumstances, and impossible in the lecture-room, but the required pressures can be produced by means of a lever. In Fig. 44 is shown a stout wooden bar 16′ long. It is prevented from bending by means of a chain; at E the lever is attached to a hinge, about which it turns freely; at A a tray is placed for the purpose of receiving weights. The screw-jack is 2′ distant from E, consequently the bar is a lever of the second order, and any weight placed in the tray exerts a pressure eightfold greater upon the top of the screw-jack. Thus each stone in the tray produces a pressure of 1 cwt. at the point D. The weight of the lever and the tray is counterpoised by the weight C, so that until the tray receives a load there is no pressure upon the top of the screw-jack, and thus we may omit the lever itself from consideration. The screw-jack is furnished with an arm DG; at the extremity G of this arm a rope is attached, which passes over a pulley and supports the power B.

291. The velocity ratio for this screw-jack with an arm of 33″, is found to be 414, by the method already described (Art. 283).

292. To determine its mechanical efficiency we must resort to experiment. The result is given in Table XVII.

TABLE XVII.—THE SCREW-JACK.

Wrought iron screw, square thread, diameter 2″, pitch 2 threads to the inch, arm 33″; nut brass, bearing surfaces oiled; velocity ratio 414; useful effect, 28 per cent.; mechanical efficiency 116; formula $P = 0{\cdot}66 + 0{\cdot}0075\,R$.

Number of Experiment.	*R.* Load in lbs.	Observed power in lbs.	*P.* Calculated power in lbs.	Difference of the observed and calculated powers.
1	112	1·4	1·5	+ 0·1
2	224	2·2	2·3	+ 0·1
3	336	3·3	3·2	– 0·1
4	448	4·1	4·0	– 0·1
5	560	5·0	4·9	– 0·1
6	672	5·7	5·7	0·0
7	784	6·5	6·5	0·0
8	896	7·4	7·4	0·0
9	1008	8·1	8·2	+ 0·1
10	1120	9·0	9·1	+ 0·1

293. It may be seen from the column of differences how closely the experiments are represented by the formula. The power which is required to raise a given weight, say 600 lbs., may be calculated by this formula; it is $0{\cdot}66 + 0{\cdot}0075 \times 600 = 5{\cdot}16$. Hence the mechanical efficiency of the screw-jack is $600 \div 5{\cdot}16 = 116$. Thus the screw is very powerful, increasing the force applied to it more than a hundredfold. In order to raise 600 lbs. one foot, a quantity of work represented by $5{\cdot}16 \times 414 = 2136$ units must be expended; of this only 600, or 28 per cent., is utilized, so that nearly three-quarters of the energy applied is expended upon friction.

294. This screw does not overhaul, since less than 50 per cent. is utilized, and in order to lower the weight the lever has actually to be pressed backwards.

295. The details of an experiment on this subject will be instructive, and afford a confirmation of the principles laid down. In experiment 10 we find that 9·0 lbs. suffice to raise 1,120 lbs.; now by moving the pulley to the other side of the lever, and placing the rope perpendicularly to the lever, I find that to produce motion the other way—that is, of course, to lower the screw—a force of 3·4 lbs. must be applied. Hence, even with the assistance of the load, a force of 3·4 lbs. is necessary to overcome friction. This will enable us to determine the amount of friction in the same manner as we determined the friction in the pulley-block (Art. 208). Let X be the force usefully employed in raising, and Y the force of friction; then to raise the load the power applied must be sufficient to overcome both X and Y, and therefore we have $X + Y = 9·0$. When the weight is to be lowered the force X of course aids in the lowering, but X alone is not sufficient to overcome the friction; it requires the addition of 3·4 lbs., and we have therefore $X + 3·4 = Y$, and hence $X = 2·8$, $Y = 6·2$.

That is, 2·8 is the amount of force which with a frictionless screw would have been sufficient to raise half a ton. But in the frictionless screw the power is found by dividing the load by the velocity ratio. In this case $1120 \div 414 = 2·7$, which is within 0·1 lb. of the value of X. The agreement of these results is satisfactory.

THE SCREW BOLT AND NUT.

296. The most useful application of the screw is met with in the common bolt and nut, shown in Fig. 45. It consists of a wrought iron rod with a head at one end and a screw on the other, upon which the nut works. Bolts

in many different sizes and forms represent the stitches by which machines and frames are most readily united. There are several reasons why the bolt is so convenient. It draws the parts into close contact with tremendous force; it is itself so strong that the parts united practically form one piece. It can be adjusted quickly, and removed as readily. The same bolt by the use of washers can be applied to pieces of very different sizes. No skilled hand is required to use the simple tool that turns the nut. Adding to this that bolts are cheap and durable, we shall easily understand why they are so extensively used.

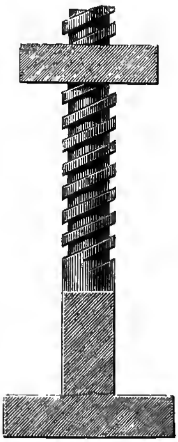

FIG. 45.

297. We must remark, in conclusion, that the bolt owes its utility to friction; the screw does not overhaul, hence when the nut is screwed home it does not recoil. If it were not that more than half the power applied to a screw is consumed in friction, the bolt and the nut would either be rendered useless, or at least would require to be furnished with some complicated apparatus for preventing the motion of the nut.

LECTURE X.

THE WHEEL AND AXLE.

Introduction.—Experiments upon the Wheel and Axle.—Friction upon the Axle.—The Wheel and Barrel.—The Wheel and Pinion.—The Crane.—Conclusion.

INTRODUCTION.

298. The mechanical powers discussed in these lectures may be grouped into two classes,—the first where ropes or chains are used, and the second where ropes or chains are absent. Belonging to that class in which ropes are not employed, we have the screw discussed in the last lecture, and the lever discussed in Lecture VIII.; while among those machines in which ropes or chains form an essential part of the apparatus, the pulley and the wheel and axle hold a prominent place. We have already examined several forms of the pulley, and we now proceed to the not less important subject of the wheel and axle.

299. Where great resistances have to be overcome, but where the distance through which the resistance must be urged is short, the lever or the screw is generally found to be the most appropriate means of increasing power. When, however, the resistance has to be moved a considerable distance, the aid of the pulley, or the wheel and

axle, or sometimes of both combined, is called in. The wheel and axle is the form of mechanical power which is generally used when the distance is considerable through which a weight must be raised, or through which some resistance must be overcome.

300. The wheel and axle assumes very many forms corresponding to the various purposes to which it is applied. The general form of the arrangement will be

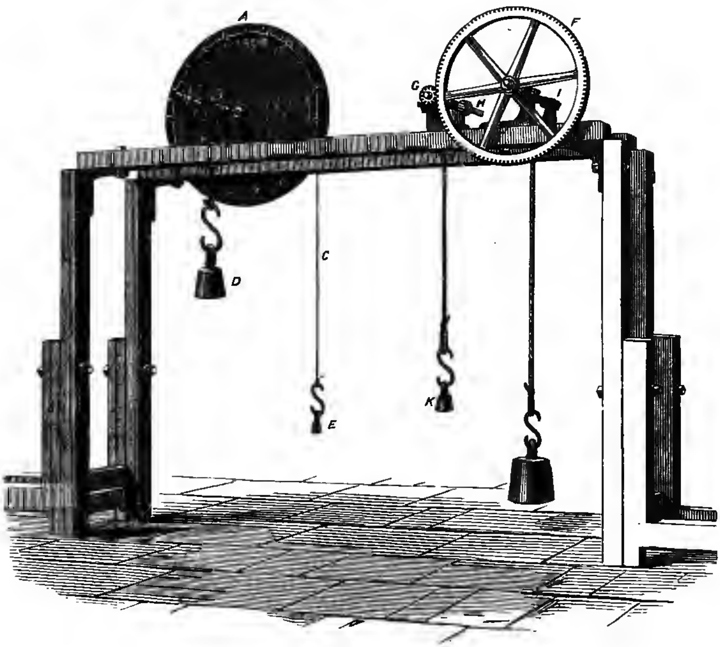

Fig. 46.

understood from Fig. 46. It consists of an iron axle B, mounted in bearings, so as to be capable of turning freely; to this axle a rope is fastened, and at the extremity of the rope is a weight D, which is gradually raised as the axle revolves. Attached to the axle, and turning with it, is a wheel A with hooks in its circumference, upon which lies a rope; one end of this rope is attached to the circumference of the wheel, and the other supports a weight E. This latter weight may be called the power, while the weight D suspended from the axle is the load. When the power is sufficiently large, E descends, making the wheel to revolve; the wheel causes the axle to revolve, and thus the rope is wound up and the load D is raised.

301. When compared with the differential pulley as a means of raising a weight, this arrangement appears rather bulky and otherwise inconvenient, but, as we shall presently learn, it is a far more economical means of applying energy. In its practical application, moreover, the arrangement is simplified in various ways, two of which may be mentioned.

302. The capstan is essentially a wheel and axle; the power is not in this case applied by means of a rope, but by direct pressure on the part of the men working it; nor is there actually a wheel employed, for the pressure is applied to what would be the extremities of the spokes of the wheel if a wheel existed.

303. In the ordinary winch, the power of the labourer is directly applied to the handle which moves round in the circumference of a circle.

304. There are innumerable other applications of the principle which are constantly met with, and which can be easily understood with a little attention. These we shall not stop to describe, but we pass on at once to the important question of the relation between the power and the load.

EXPERIMENTS UPON THE WHEEL AND AXLE.

305. We shall commence a series of experiments upon the wheel A and axle B of Fig. 46. We shall first determine the velocity ratio, and then ascertain the mechanical efficiency by actual experiment. The wheel is of wood; it is about 30″ in diameter. The string to which the power is attached is coiled round a series of hooks, placed near the margin of the wheel; the effective circumference is thus a little less than the real circumference. I measure a single coil of the string, and find the length to be 88″·5. This length, therefore, we shall adopt for the effective circumference of the wheel. The axle is 0″·75 in diameter, but its effective circumference is larger than the circle of which this length is the diameter.

306. The proper mode of finding the effective circumference of the axle in a case where the rope bears a considerable proportion to the axle is as follows. Attach a weight to the extremity of the rope sufficient to stretch it thoroughly. Make the wheel and axle revolve suppose 20 times, and measure the height through which the weight is lifted; then the one-twentieth part of that height is the effective circumference of the axle. By this means I have found the circumference of the axle we are using to be 2″·87.

307. Let us ascertain the velocity ratio in this machine. When the wheel and axle have made one complete revolution the power has been lowered through a distance of 88″·5, and the load has been raised through 2″·87. This is evident because the wheel and axle are attached together, and therefore each completes one revolution in the same time; hence the ratio of the distance which the

power moves over to that through which the load is raised is $88''\cdot5 \div 2''\cdot87 = 31$ very nearly. We shall therefore suppose the velocity ratio to be 31. Thus this wheel and axle has a far higher velocity ratio than any of the pulleys which we have been considering.

308. Were friction absent the velocity ratio, 31, would also express the mechanical efficiency of this wheel and axle; but owing to the presence of friction the real efficiency is less than this—how much less, we must ascertain by experiment. I attach a load of 56 lbs. to the hook which is borne by the rope descending from the axle: this load is shown at D in Fig. 46. I find that a power of 2·6 lbs. applied at E is just sufficient to raise D. We infer from this result that the mechanical efficiency of this machine is $56 \div 2\cdot6 = 21\cdot5$. I add a second 56 lb. weight to the load, and I find that a power of 5·0 lbs. raises the load of 112 lbs. The mechanical efficiency in this case is $112 \div 5 = 22\cdot5$. We adopt the mean value 22. Hence the mechanical efficiency is reduced by friction from 31 to 22.

309. We may compute from this result the number of units of energy which are utilized out of every 100 units applied. Let us suppose a load of 100 lbs. is to be raised one foot; a force of $100 \div 22 = 4\cdot6$ lbs. will suffice to raise this load. This force must be exerted through a space of 31′, and consequently $31 \times 4\cdot6 = 143$ units of energy must be expended; of this amount 100 units are usefully employed, and therefore the percentage of energy utilized is $100 \div 143 \times 100 = 70$. It follows that 30 per cent. of the applied energy is consumed in overcoming friction.

310. We can see the reason why the wheel and axle overhauls—that is, runs down of its own accord—when

allowed to do so ; it is because less than half the applied energy is expended upon friction (Art. 223).

311. A series of experiments which have been carefully made with this wheel and axle are recorded in Table XVIII.

TABLE XVIII.—WHEEL AND AXLE.

Wheel of wood ; axle of iron, in oiled brass bearings ; weight of wheel and axle together, 16·5 lbs. ; effective circumference of wheel, 88″·5 ; effective circumference of axle, 2″·87 ; velocity ratio, 31 ; mechanical efficiency, 22 ; useful effect 70 per cent. ; formula, $P = 0·204 + 0·0426\ R$.

Number of Experiment.	*R.* Load in lbs.	Observed power in lbs.	*P.* Calculated power in lbs.	Difference of the observed and calculated values.
1	28	1·4	1·4	0·0
2	42	2·0	2·0	0·0
3	56	2·6	2·6	0·0
4	70	3·2	3·2	0·0
5	84	3·7	3·8	+ 0·1
6	98	4·4	4·4	0·0
7	112	5·0	5·0	0·0

By the method of the Appendix a relation connecting the power and the load has been determined ; it is expressed in the form—

$$P = 0·204 + 0·0426\ R.$$

312. Thus for example in experiment 5 a load of 84 lbs. was found to be raised by a power of 3·7 lbs. The value calculated by the formula is $0·204 + 0·0426 \quad 84 = 3·8$. The calculated value only differs from the observed value by 0·1 lb., which is shown in the fifth column. It will be seen from this column that the values calculated from the formula represent the experiments with great fidelity.

313. We have deduced the relation between the power and the load from the principle of energy, but we might

have obtained it from the principle of the lever. The wheel and axle both revolve about the centre of the axle; we may therefore regard the centre as the fulcrum of a lever, and the points where the cords meet the wheel and axle as the points of application of the power and the load respectively.

314. By the principle of the lever of the first order (Art. 237), the power is to the load in the inverse proportion of the arms; in this case, therefore, the power is to the load in the inverse proportion of the radii of the wheel and the axle. But the circumferences of circles are in proportion to their radii, and therefore the power must be to the load as the circumference of the axle is to the circumference of the wheel.

315. This mode of arriving at the result is a little artificial; it is more natural to deduce the law directly from the principle of energy. In a mechanical power of any complexity it would be difficult to trace exactly the transmission of power from one part to the next, but the principle of energy evades this difficulty; no matter what be the mechanical arrangement, simple or complex, of few parts or of many, we have only to ascertain by trial how many feet the power must traverse in order to raise the load one foot; the number thus obtained is the theoretical efficiency of the machine.

FRICTION UPON THE AXLE.

316. In the wheel and axle upon which we have been experimenting, we have found that about 30 per cent. of the power is consumed by friction. We shall be able to ascertain to what this loss is due, and then in some degree to remove its cause. From the experiments

of Art. 166 we learned that the friction of a small pulley was very much greater than that of a large pulley—in fact, the friction is inversely proportional to the diameter of the pulley. We infer from this that by winding the rope upon a barrel instead of upon the axle, the friction may be diminished.

317. We can examine experimentally the effect of friction on the axle by the apparatus of Fig. 47. B is a shaft

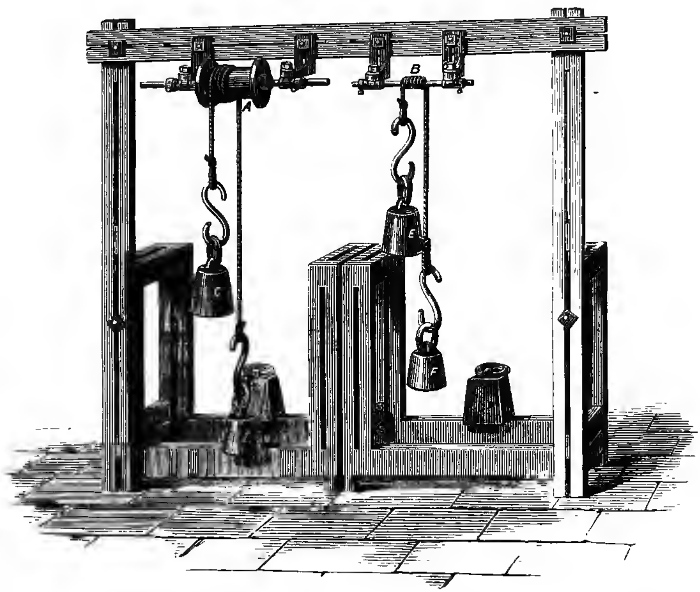

FIG. 47.

0″·75 diameter, about which a rope is coiled several times; the ends of this rope hang down freely, and to each of them hooks E, F are attached. This shaft revolves in brass bearings, which are oiled. In order to investigate the amount of power lost by winding the rope upon an axle of this size, I shall place a certain weight—suppose

56 lbs.—upon one hook F, and then I shall ascertain what amount of power hung upon the other hook E will be sufficient to raise F. There is here no mechanical advantage, so that the excess of load which E must receive in order to raise F is the true measure of the friction.

318. I add on weights at E until the power reaches 85 lbs., when E descends. We thus see that to raise 56 lbs. an excess of 29 lbs. was necessary to overcome the friction. We may roughly enunciate the result by stating that to raise a load in this way, half as much again is required for the power. This law is verified by suspending 28 lbs. at F, when it is found that a power of 43 lbs. at E is required to lift it. Had the power been 42 lbs., it would have been exactly half as much again as the load.

319. Hence in raising F upon this axle, about one-third of the power which must be applied at the circumference of the axle is wasted. This experiment teaches us where the loss lies in the wheel and axle of Art. 305, and explains how it is that about a third of its efficiency is lost. 85 lbs. was only able to raise two-thirds of its own weight, owing to the friction; and hence we should expect to find, as we actually have found, that the power applied at the circumference of the wheel has an effect which is only two-thirds of its theoretical efficiency.

320. From this experiment we should infer that the proper mode of avoiding the loss by friction is to wind the rope upon a barrel of considerable diameter rather than upon the axle itself. I place upon a similar axle to that on which we have been already experimenting a barrel of about 15″ circumference. I coil the rope two or three times about the barrel, and let the ends hang down as before. I then attach to each end 56 lbs. weight, and I find that 10 lbs. added to either of the weights is suffi-

cient to overcome friction, to make it descend, and raise the other weight. The apparatus is shown in Fig. 47. A is the barrel, C and D are the weights. In this arrangement 10 lbs. is sufficient to overcome the friction which required 29 lbs. when the rope was simply coiled around the axle. In other words, by the barrel the friction is reduced to one-third of its amount.

THE WHEEL AND BARREL.

321. We next place the barrel upon the axis already experimented upon and shown in Fig. 46 at B. The circumference of the wheel is 88″·5 ; the circumference of the barrel is 14″·9. The proper mode of finding the circumference of the barrel is to suspend a weight from the rope, then raise this weight by making one revolution of the wheel, and the distance through which the weight is raised is the effective circumference of the barrel. The velocity ratio of the wheel and barrel is then found, by dividing 14·9 into 88·5, to be 5·94.

322. The mechanical efficiency of this machine is determined by experiment. I suspend a weight of 56 lbs. from the hook, and apply power to the wheel. I find that 10·1 lbs. is just sufficient to raise the load.

323. The mechanical efficiency is to be found by dividing 10·1 into 56 ; the quotient thus obtained is 5·54. The mechanical efficiency does not differ much from 5·94, the velocity ratio ; and consequently in this machine but little power is expended upon friction.

324. We can ascertain the loss by computing the percentage of applied energy which is utilized. Let us suppose a weight of 100 lbs. has to be raised one foot : for this purpose a force of $100 \div 5{\cdot}54 = 18{\cdot}1$ lbs. must be applied.

This is evident from the definition of the mechanical efficiency; but since the load has to be raised one foot, it is clear from the meaning of the velocity ratio that the power must move over 5′·94: hence the number of units of work to be applied is to be measured by the product of 5·94 and 18·1, that is, by 107·5; in order therefore to accomplish 100 units of work, 107·5 units of work must be applied. The percentage of energy usefully employed is $100 \div 107 \cdot 5 \times 100 = 93$. This is far more than 70, which is the percentage utilized when the axle was used without the barrel (Art. 309).

325. A series of experiments made with care upon the wheel and barrel are recorded in Table XIX.

TABLE XIX.—THE WHEEL AND BARREL.

Wheel of wood, 88″·5 in circumference, on the same axle as a cast-iron barrel of 14″·9 circumference; axle is of wrought iron, 0″·75 in diameter, mounted in oiled brass bearings; power is applied to the circumference of the wheel, load raised by rope round barrel; velocity ratio, 5·94; mechanical efficiency, 5·54; useful effect, 93 per cent.; formula, $P = 0{\cdot}5 + 0{\cdot}169\,R$.

Number of Experiment.	*R.* Load in lbs.	Observed power in lbs.	*P.* Calculated power in lbs.	Difference of the observed and calculated values.
1	14	2·7	2·9	+ 0·2
2	28	5·3	5·2	− 0·1
3	42	7·7	7·6	− 0·1
4	56	10·1	10·0	− 0·1
5	70	12·4	12·4	0·0
6	84	14·7	14·7	0·0
7	98	17·1	17·1	0·0
8	112	19·4	19·5	+ 0·1

The formula which represents the experiments with the greatest amount of accuracy is $P = 0{\cdot}5 + 0{\cdot}169R$. This formula is compared with the experiments, and the column of differences shows that the calculated and the observed values agree very closely. The constant part 0·5 is partly due to the constant friction of the heavy barrel and wheel, and partly, it may be, to small irregularities which have prevented the centre of gravity of the whole mass from being strictly in the axle.

326. Though this machine is more economical of power than the wheel and axle of Art. 305, yet it is less powerful; in fact, the mechanical efficiency, 5·54, is only about one-fourth of that of the wheel and axle. It is therefore necessary to inquire whether we cannot devise some method by which to secure the advantages of but little friction, and at the same time have a large mechanical efficiency: this we shall proceed to investigate.

THE WHEEL AND PINION.

327. By means of what are called cog-wheels or toothed-wheels, we are enabled to combine two or more wheels and axles together, and thus greatly to increase the power which can be produced by a single wheel and axle. Toothed-wheels are used for a great variety of purposes in mechanics; we have already had some illustration of their use during these lectures (Fig. 30). The wheels which we shall employ are those often used in lathes and other small machines; they are what are called 10-pitch wheels,—that is to say, a wheel of this class contains ten times as many teeth in its circumference as there are inches in its diameter. I have here a wheel 20″ diameter, and consequently it has 200 teeth; here is another which is 2·5″ diameter, and which consequently contains 25 teeth. We shall mount these wheels upon two parallel shafts, so that they work one

into the other in the manner shown in Fig. 46 : F is the large wheel of 200 teeth, and G the pinion of 25 teeth. The axles are 0″·75 diameter; around each of them a rope is wound, from which a hook is suspended.

328. A comparatively small weight at K is sufficient to raise a much larger weight at L; but before inquiring into the mechanical efficiency of this arrangement, we shall as usual ascertain the velocity ratio. The wheel contains eight times as many teeth as the pinion; it is therefore evident that when the wheel has made one revolution, the pinion will have made eight revolutions conversely, and the pinion must turn round eight times to turn the wheel round once : hence the power which is turning the pinion round must be lowered through eight times the circumference of the axle, while the load is raised through a length equal to the circumference of the axle. It is therefore evident that the velocity ratio of the machine is 8.

329. We determine the mechanical efficiency by trial. Attaching a load of 56 lbs. at L, it is easily seen that a power of 13·7 lbs. at K will be sufficient to raise the load; the mechanical efficiency of the machine is therefore about 4·1, which is almost exactly half the velocity ratio. You see the machine is barely able to overhaul; from this we might have inferred, by the principles already explained (Art. 223), that about half the power is expended on friction, and that therefore the mechanical efficiency is about half the velocity ratio. The actual percentage of energy that is utilized in this machine is about 51. If I suspend 112 lbs. from the hook L, 26 lbs. is just enough to raise the load; the mechanical efficiency that would be deduced from this result is $112 \div 26 = 4·3$, which is slightly in excess of the amount

obtained in the former experiment. It is found to be generally a property of the mechanical powers, that as the load increases the mechanical efficiency slightly increases.

330. In Table XX. will be found a series of experiments upon the relation between the power and the load in the wheel and pinion; the table will sufficiently explain itself, after the description of similar tables already given (Arts. 312, 325).

TABLE XX.—THE WHEEL AND PINION.

Wheel (10-pitch), 200 teeth; pinion, 25 teeth; axles equal, effective circumference of each being 2″·87; oiled brass bearings; velocity ratio, 8; mechanical efficiency, 4·1; useful effect, 51 per cent.; formula, $P = 2{\cdot}46 + 0{\cdot}21\ R$.

Number of Experiment.	*R.* Load in lbs.	Observed power in lbs.	*P.* Calculated power in lbs.	Difference of the observed and calculated powers.
1	14	5·4	5·4	0·0
2	28	8·7	8·3	− 0·4
3	42	11·0	11·3	+ 0·3
4	56	13·7	14·2	+ 0·5
5	70	17·5	17·2	− 0·3
6	84	20·0	20·1	+ 0·1
7	98	23·0	23·0	0·0
8	112	26·0	26·0	0·0

331. The large amount of friction present in this contrivance is chiefly due to the cause already pointed out (Art. 319); it is a consequence of winding the rope directly upon the axle instead of upon a barrel. We might place barrels upon these axles and test the truth of this anticipation; but we shall not delay to do so, as we can see the effect sufficiently in the machines which we shall next describe.

THE CRANE.

332. We have already explained (Art. 38) the construction of the lifting crane, so far as its framework is concerned. We now examine the mechanism by which the weight is raised. We shall employ for this purpose the model which is represented in Fig. 48. The jib is supported by a tie as in Fig. 17, and the crane is counterpoised by means of the weights placed at H: this counterpoise is necessary, because, when a load is suspended from the cord passing over the pulley, the crane would have a tendency to turn over if not counterpoised.

333. The load F is carried by a rope or chain which passes over the pulley E and thence to the barrel D, upon which the rope is to be wound. This barrel receives its motion from a large wheel A, which contains 200 teeth.

The wheel A is turned by the pinion B: this pinion contains 25 teeth. In the actual use of the crane, the axle which carries this pinion would be turned round by means of a handle; but in order to make experiments upon the relation of the power and the load, the handle would be inconvenient, and therefore we have placed upon the axle of the pinion a wheel C containing a groove in its circumference. Around this groove a string is wrapped, so that when a weight G is suspended from the string it will cause the wheel to revolve. This weight G will constitute the power by which the load F may be raised.

334. Let us compute the velocity ratio of this machine before commencing experiments upon its mechanical efficiency. The effective circumference of the barrel E is found by trial to be 14″·9. Since there are 200 teeth on A and 25 on B, it follows that the pinion B must

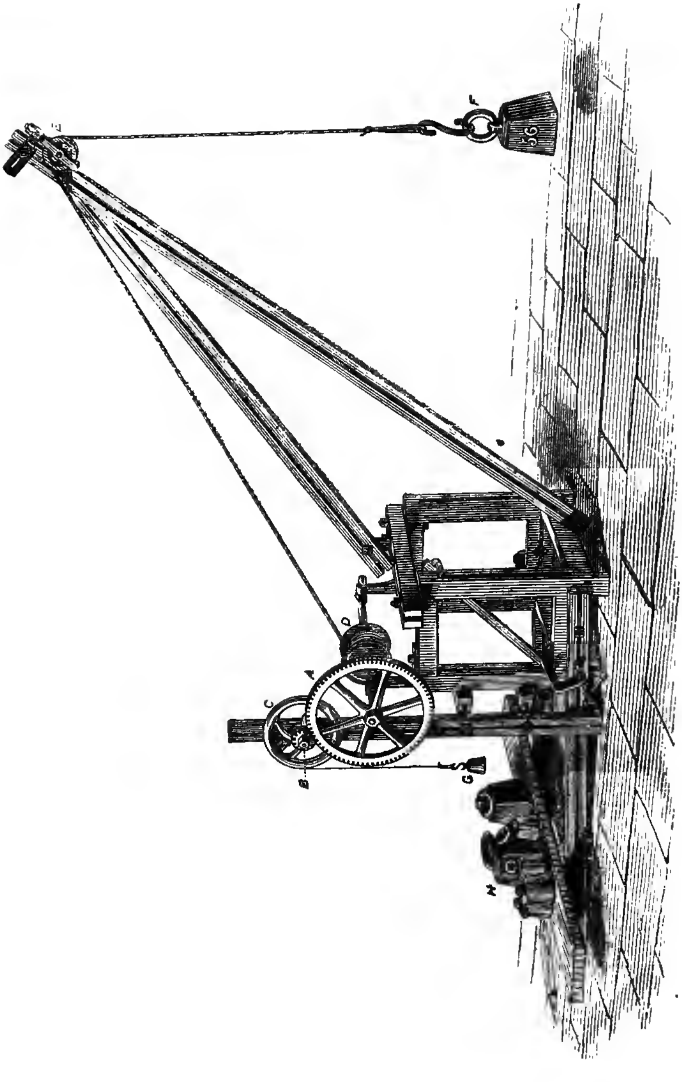

FIG. 48.

revolve eight times to produce one revolution of the barrel. Hence the wheel C to the circumference of which the power is applied must also revolve eight times for one revolution of the barrel. The effective circumference of C is 43″; the power must therefore have been applied through 8 × 43″ = 344″, in order to raise the load 15″·9. The velocity ratio is 344 ÷ 14″·9 = 23 very nearly. We can easily verify this value of the velocity ratio by actually raising the load 1′, when it is seen that the number of revolutions of the wheel B is such that the power must have moved over 23′.

335. The mechanical efficiency is to be found as usual by trial. 56 lbs. placed at F is raised by 3·1 lbs. at G; hence the mechanical efficiency deduced from this experiment is 56 ÷ 3·1 = 18. The percentage of useful effect is easily obtained, as in Art. 324. It is found to be about 78. Here, then, we have a machine possessing very considerable efficiency, and being at the same time economical of energy.

TABLE XXI.—THE CRANE.

Circumference of wheel to which the power is applied, 43″; train of wheels, 25 ÷ 200; circumference of drum on which rope is wound, 14″·9; velocity ratio, 23; mechanical efficiency, 18; useful effect, 78 per cent.; formula, $P = 0{\cdot}0556\,R$.

Number of Experiment.	*R.* Load in lbs.	Observed power in lbs.	*P.* Calculated power in lbs.	Difference of the observed and calculated values.
1	14	0·9	0·8	− 0·1
2	28	1·6	1·6	0·0
3	42	2·4	2·3	− 0·1
4	56	3·1	3·1	0·0
5	70	3·8	3·9	+ 0·1
6	84	4·5	4·7	+ 0·2
7	98	5·3	5·5	+ 0·2
8	112	6·2	6·2	+ 0·0

336. A series of experiments made with care is recorded in Table XXI., and a comparison of the calculated and observed values will show that the formula $P = 0{\cdot}0556\, R$ represents the experiments with considerable accuracy.

337. It may be noticed that in this formula there is no constant term, as is usual in the expression of the relation between the power and the load. The probable explanation is to be found in the fact that some minute irregularity in the form of the barrel or of the wheel has been constantly acting as a small weight in favour of the power. The power is always started from the same position of the wheels in each experiment, and hence any irregularity will be constantly acting in favour of the power or against it; here the former appears to have been the case. In other cases doubtless the latter has occurred; the difference is, however, of extremely small amount. The other cause of the presence of the constant term is the friction of the machine itself when carrying no load; it has happened in the present case that this friction has been almost exactly overcome by the influence of the other cause referred to.

338. It is usual in cranes to have the power of adding a second train of wheels, when the load is of large amount. The power is applied to an axle which carries a pinion of 25 teeth: this pinion works into a wheel of 200 teeth; on the axle of the wheel with 200 teeth is a pinion of 30 teeth, which works into a wheel of 180 teeth; the barrel is on the axle of the last wheel. A series of experiments upon this arrangement is shown in Table XXII.

TABLE XXII.—THE CRANE.

Circumference of wheel to which power is applied, 43″; train of wheels, 30 ÷ 180 × 25 ÷ 200; circumference of drum on which rope is wound, 14″·9; velocity ratio, 137; mechanical efficiency, 87; useful effect, 63 per cent.; formula, $P = 0{\cdot}185 + 0{\cdot}00782\,R$.

Number of Experiment.	R. Load in lbs.	Observed power in lbs.	P. Calculated power in lbs.	Difference of the observed and calculated values.
1	14	0·30	0·29	− 0·01
2	28	0·40	0·40	0·00
3	42	0·50	0·51	+ 0·01
4	56	0·60	0·62	+ 0·02
5	70	0·75	0·73	− 0·02
6	84	0·85	0·84	− 0·01
7	98	0·95	0·95	0·00
8	112	1·05	1·06	+ 0·01

The velocity ratio is now 137, and the mechanical efficiency is 87; one man could therefore, by applying a power of 26 lbs. to a crane of this kind, raise one ton with ease.

CONCLUSION.

339. It will be useful to contrast the wheel and axle on which we have experimented (Art. 305) with the differential pulley (Art. 210). The velocity ratio of the former machine is nearly double that of the latter, and its mechanical efficiency is almost four times greater. Less than half the applied power is wasted in the wheel and axle, while more than half is wasted in the differential pulley. This makes the wheel and axle both a more powerful machine, and a more economical machine than the differential pulley. On the other hand, the greater

compactness of the differential pulley, its facility of application, and the practical conveniences arising from the property which it possesses of not overhauling, nearly, if not quite, compensate for the superior mechanical advantage of the wheel and axle.

340. We may also contrast the wheel and axle with the screw (Art. 293). The screw is remarkable among the mechanical powers for a very high velocity ratio, and for excessive friction. Thus we have seen in Art. 291 how the velocity ratio of a screw-jack with an arm attached exceeded 400, while its mechanical efficiency was little more than a fourth of the amount. No single wheel and axle could conveniently be made to give a mechanical efficiency of 100; but we have described (Art. 338) a combination of different wheels and axles which gives an efficiency nearly of this amount. The friction in the wheel and axle is very much less than in the screw, and consequently a great saving of power is obtained by the use of the former machine.

341. In practice, however, it generally happens that economy of energy does not weigh much in the selection of a mechanical power for any purpose, as there are always other considerations of far greater consequence.

342. For example, let us take the case of a lifting crane employed in loading or unloading a vessel, and let us inquire why it is that a train of wheels is used for the purpose of producing the requisite power rather than a screw. The answer is simple, the train of wheels is convenient, for by their aid any length of chain can be wound upon the barrel; whereas if a screw were used, we should require a screw as long as the greatest height of lift. This screw would be inconvenient, and be utterly impracticable. This is the reason why a train of wheels

is used, and the additional circumstance that a train of wheels is more economical of energy than a screw is, comparatively speaking, of no consequence.

343. On the other hand, suppose that a very heavy load has to be overcome for a short distance, for example that of starting a ship launch, a screw-jack is evidently the proper machine to employ; it is portable and easily applied, and though to use the screw-jack requires a little more energy than would be necessary if a windlass were employed, yet no engineer would erect the latter elaborate machine for the purpose of making a single effort, when the object can be simply accomplished otherwise.

LECTURE XI.

THE MECHANICAL PROPERTIES OF TIMBER.

Introduction.—The General Properties of Timber.—Resistance to Extension.—Resistance to Compression.—Condition of a Beam strained by a Transverse Force.

INTRODUCTION.

344. In the lectures on the mechanical powers which have been just completed, we have seen how great weights may be raised or other large resistances overcome. Of not less importance than the mechanical powers is the application of mechanical principles to structures. These are fixtures, while machines are adapted for motion; a roof or a bridge is a structure, but a crane or a screw-jack is a machine. Structures are employed for supporting weights, and the mechanical powers give the means of raising them.

345. A structure has to support both its own weight and also any load that may be placed upon it. Thus a railway bridge must at all times sustain what is called the permanent load, and frequently, of course, the weight of one or more trains. The problem which the engineer solves is to design a bridge which shall be sufficiently strong, and, at the same time, economical; his skill is shown by the manner in which he can attain these two ends in the same structure.

346. In the four lectures of the course which will be devoted to this subject it will only be possible to give a slight sketch, and therefore few details can be entered upon. An extended account of the properties of different materials used in structures would be beyond our scope, but there are some general principles relating to the strength of materials which may be discussed. Timber, as a building material, has, in modern times, been replaced to a great extent by iron in large structures, but timber is more capable than iron of being experimented upon in the lecture-room. The elementary laws also, which we shall demonstrate with reference to the strength of timber, are substantially the same as the corresponding laws for the strength of iron or any other material. Hence we shall commence the study of structures by two lectures on timber. The laws which we shall prove experimentally will afterwards be applied to a few simple cases of bridges and other actual structures.

THE GENERAL PROPERTIES OF TIMBER.

347. The uses of timber in the arts are as various as its qualities. Some wood is useful for its beauty, and other kinds for their strength or durability under different circumstances. We shall only employ pine in our experiments upon timber. This wood is selected because it is so well known and so much used. A knowledge of the properties of pine would probably be more useful than a knowledge of the properties of any other wood, and at the same time it must be remembered that the laws which we shall establish by means of slips of pine may be generally applied.

348. A transverse section of a tree shows a number of rings, each of which represents the growth of wood in one year. The age of the tree may sometimes be approximately found by counting the number of rings. The outer rings are the newer portions of the wood.

349. When a tree is felled it contains a large quantity of sap, which must be allowed to evaporate before the wood is fit for use. With this object the timber is stored in suitable yards for two or more years according to the purposes for which it is intended; sometimes the process of seasoning, as it is called, is hastened by other means. Wood, when seasoning, contracts; hence blocks of timber are often found split from the circumference to the centre, for the outer rings, being newer and containing more sap, contract more than the inner rings. For the same reason a plank is found to warp when the wood is not thoroughly seasoned. The side of the plank which was farthest from the centre of the tree contracts more than the other side, and becomes concave. This can be easily verified by looking at the edge of the plank, for we there see the rings of which the plank is composed.

350. Timber may be softened by steaming. I have here an ordinary rod of pine, $24'' \times 0''\cdot5 \times 0''\cdot5$, and here a second rod cut from the same piece and of the same size, which has been exposed to steam of boiling water for more than an hour: securing these at one end to a firm stand, I bend them down together, and you see the dry rod breaks very soon, while the steamed rod can be bent much farther before it breaks. This property of wood is used in shaping the timbers of wooden ships. We shall be able to understand the reason by considering the nature of wood. Wood is composed of a number of fibres ranged side by side and united together.

A rope is composed of a number of fibres laid together and twisted, but the fibres are not coherent as they are in wood. This is the reason why a rod of wood is stiff, while a rope is flexible. The steam finds its way into the interstices between the fibres of the wood ; it softens their connections, and perhaps increases the flexibility of the fibres themselves, and thus, when strained, the fibres are better able to modify themselves than those of a rod which has not been thus treated.

351. The structure of wood is also shown by the following simple experiment :—Here are two pieces of pine, each 9″ × 1″ × 1″. One of them I can easily snap across with a blow, while I am totally unable to break the other. Why is this ? Because one of these pieces is cut against the grain, while the other is with it. In the first case I merely tear asunder the connection between the fibres, which is quite easy. In the other case I would have to tear asunder the fibres themselves, which is vastly more difficult. To a certain extent the distinction of grain is also found in wrought iron, but the contrast between the strength of iron with the grain and against the grain is not so marked as it is in wood.

RESISTANCE TO EXTENSION.

352. It will be necessary to explain a little more definitely what is meant by the strength of timber. We may conceive a rod to be broken in three different ways. In the first place the rod may be taken by a force at each end and torn asunder by pulling, as a thread may be broken. To do this requires a very great force, and the strength of the beam with reference to such a mode of destroying it is called its resistance to extension. In the

second place, it may be broken by actual pressure upon each end, as a pillar may be crushed by the superincumbent weight being too large; the strength that relates to this form of force is called resistance to compression: finally, the rod may be broken by a force applied transversely (see Art. 362). The strength of pine with reference to these different forms of force will be considered successively. The rods that are used have been cut from the same piece of timber, which has been selected on account of its straightness of grain and freedom from knots. The rods are of different sections, $1'' \times 0''{\cdot}5$ and $0''{\cdot}5 \times 0''{\cdot}5$ being generally used, but sometimes $1'' \times 1''$ is employed.

353. With reference to the strength of timber in its capacity to resist extension, we can do but little in the lecture room. I have here a pine rod A B, Fig. 49, of dimensions $48'' \times 0''{\cdot}5 \times 0''{\cdot}5$. Each end of this rod is firmly secured between two pieces of iron, which are bolted together; by means of these irons, the rod is suspended from the hook of the epicycloidal pulley-block (Art. 224), which is itself supported by a tripod; a number of hooks are attached to the rod for the purpose of carrying weights. By placing 3 cwt. on these hooks and pulling the hand chain of the pulley-block, I find that I can raise the weight safely, and therefore the rod will support at all events a tension of 3 cwt. From experiments which have been made on the subject, it is ascertained that nearly a ton would be necessary to tear such a rod asunder; hence we see that pine is enormously strong in resisting a strain of extension. The tensile strength of the rod does not depend upon its length, but upon the area of its section. The section of the rod we have used is one-fourth of a square inch, and the breaking weight

of a rod one square inch in section is therefore about four tons.

354. A rod of any material generally elongates more or less under the action of a straining weight; we can ascertain whether this occurs perceptibly in wood. Before

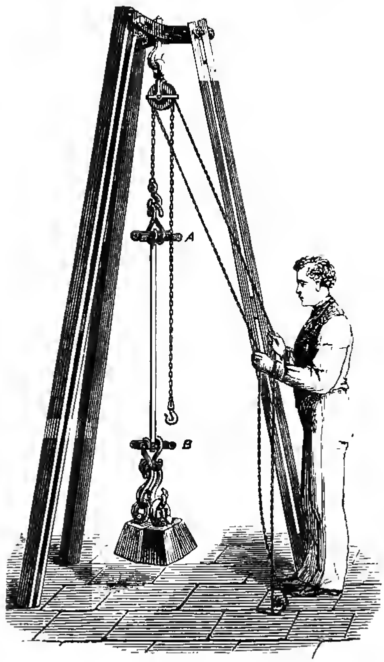

FIG. 49.

the rod was strained I had marked two points upon it exactly 2 feet apart. Now when the rod supports 3 cwt. I find that the distance between the two points remains perceptibly the same. By more delicate measurement I have no doubt we should find that the distance had slightly elongated, but to an insignificant extent.

355. Let us contrast the resistance of a rod of timber to extension with the effect upon a rope under the same

circumstances. I have here a rope about 0″·25 diameter; it is suspended from a point, and bears a 14 lb. weight in order to stretch the rope completely. I mark points upon the rope 2′ apart. I now change the stone weight for a weight of 1 cwt., and on measurement I find that the two points which before were 2′ apart, are now 2′ 2″; thus the rope has stretched nearly at the rate of an inch per foot for a strain of 1 cwt., while the timber did not stretch perceptibly for a strain of 3 cwt.

356. We have already (Art. 37) explained the meaning of the words "tie" and "strut;" a tie is used to resist forces of extension, and thus from what we have seen timber is admirably adapted to form ties: an enormous strain is required to rend a beam asunder, and it does not stretch to any appreciable extent. We shall subsequently see the manner in which these useful properties of timber are utilized in the arts of construction.

RESISTANCE TO COMPRESSION.

357. We proceed to examine into the capability of timber to resist forces of compression, either as a pillar or in any other form of "strut." A rope may compete with timber as a tie, but is wholly inapplicable as a strut. The use of timber as a strut depends in a great degree upon the coherence of the fibres to each other, as well as upon their actual rigidity. The action of timber in resisting forces of compression is thus very different from its action when resisting forces of extension; we can examine, by actual experiment, the strength of timber under the former conditions, as the weights which it will be necessary to employ are within the capabilities of our lecture-room apparatus.

358. The apparatus is shown in Fig. 50. It consists of a lever of the second order, 10′ long, the mechanical power of which is threefold; the resistance of the bar D E to be broken is the load to be overcome, and the power consists of weights, to receive which the tray B is used; every pound placed in the tray produces a compressive strain of 3 lbs. on the rod at D. The fulcrum is at A and guides at G. The lever and the tray are heavy.

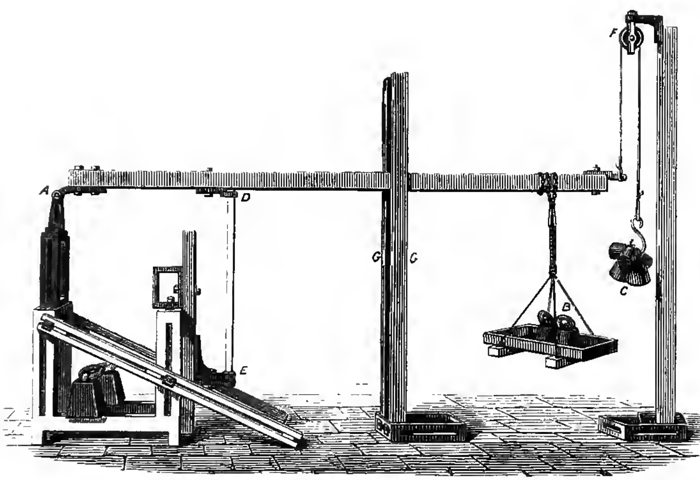

FIG. 50.

Their weight would complicate our calculations if it were not counterpoised. A cord attached to the extremity of the lever passes over a pulley F; at the other end of this cord, sufficient weights C are attached to neutralize the weight of the lever. In fact, the lever and tray now swing as if they had no weight, and we may therefore leave them out of consideration. The rod D E to be experimented upon is fitted at its lower end E into a hole in a cast-iron bracket: this bracket can be adjusted so as to take in rods of different lengths; the other end D of the rod passes through a hole in a second piece of cast-iron, which is bolted to the lever: thus the rod is secured at each end, and risk of slipping is avoided. The stands are heavily weighted to secure the stability of the arrangement.

359. The first experiment we shall make with this apparatus is upon a pine rod 40″ long and 0″·5 square; the lower bracket is so placed that the lever is horizontal when just resting upon the top of the rod. I begin to place weights in the tray: these will produce a threefold pressure, the effect of which will first be to bend the rod, and, when the deflection has reached a certain amount, to break it across. I place 28 lbs. in the tray: this produces a pressure of 84 lbs. upon the rod, but the rod still remains perfectly straight, so that it bears this pressure easily. When the pressure is increased to 96 lbs. a very slight amount of deflection may be seen. When the strain reaches 114 lbs. the rod begins to bend into a curved form, but with this pressure the amount of deviation of the middle of the rod is still less than 0″2·5. Gradually augmenting the pressure, I find that when it reaches 132 lbs. the deviation has reached 0″·5; and finally, when 48 lbs. is placed in the scale, that is, when the rod is subjected to 144 lbs., it breaks across the middle. Hence we see that this rod bore about 100 lbs. without sensibly bending, but that it was broken when the weight was increased about half as much again. Another experiment with a similar rod gave a slightly less value (132 lbs.) for the breaking weight. If I add these results together, and divide the sum by 2, I find 138 lbs. as the mean value of the breaking weight, and it is probably near the truth.

360. Let us next try the resistance of a shorter rod of

the same section. I place a piece of pine 20″ long and 0″·5 square in the apparatus, firmly securing each end, as in the former case. The lower bracket is adjusted so as to make the lever horizontal; the counterpoise, of course, remains the same, and weights are placed in the tray as before. No deflection is noticed when the rod supports 126 lbs.; a very slight amount of bending is noticeable with 186 lbs.; with 228 lbs., the amount by which the centre of the rod has deviated laterally from its original position is about 0″·2; and finally, when the load reaches 294 lbs., the rod breaks. Fracture first occurs in the middle, but is immediately followed by other fractures at the points where the ends of the rod are secured.

361. Hence the breaking load of a rod of 20″ is more than double the breaking load of a rod of 40″ of the same section; from this we learn that the sections being equal, short pillars are stronger than long pillars. The weight that would tear either rod asunder by extension as a tie, is very much greater than that which would suffice to crush it as a strut. It has been ascertained by experiment that the strength of a square pillar to resist compression is proportional to the square of its section. Hence a rod of pine, 40″ long and 1″ square, having four times the section of the 40″ rod we have experimented on, would be sixteen times as strong, and consequently its breaking weight would amount to nearly a ton.

CONDITION OF A BEAM STRAINED BY A TRANSVERSE FORCE.

362. We next come to the very important subject of the strength of timber when supporting a transverse strain; that is, when used as a beam. What is meant

by a transverse strain will be understood from Fig. 51, which represents a small beam, strained by a load at its centre. Fig. 52 shows two supports 40″ apart, across which a rod of pine 48″ × 1″ × 1″ is laid; at the middle of this rod a hook is placed, from which a tray for the reception of weights is suspended. A rod thus supported, and bearing weights, is said to be strained transversely. This form of strain is as commonly met with as the tensile

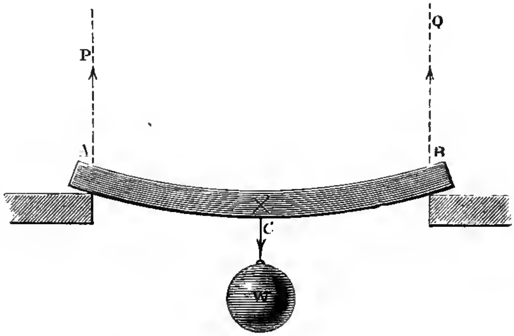

FIG. 51.

and compressive strains already considered. A rafter of a roof, the flooring of a room, a gangway, many forms of bridge, and innumerable other examples, might be given of beams strained in this manner. This subject is so important that the remainder of this lecture and the whole of the next will be devoted to it.

363. The first point to be noticed is the deflection of the beam when a weight is suspended from it. The beam is at first horizontal; but as the weight in the tray is augmented, the beam gradually curves downwards until, when the weight reaches a certain amount, the beam breaks across in the middle and the tray falls.

For convenience in recording the experiments the tray chain and hooks have been adjusted to weigh exactly 14 lbs. A B is a cord which is kept horizontal by the weights D: this cord gives a rough measure of the deflection of the beam from its horizontal position when strained by a load in the tray. In order to observe the deflection accurately an instrument is used called the cathetometer (G, Fig. 52). It consists of a small telescope,

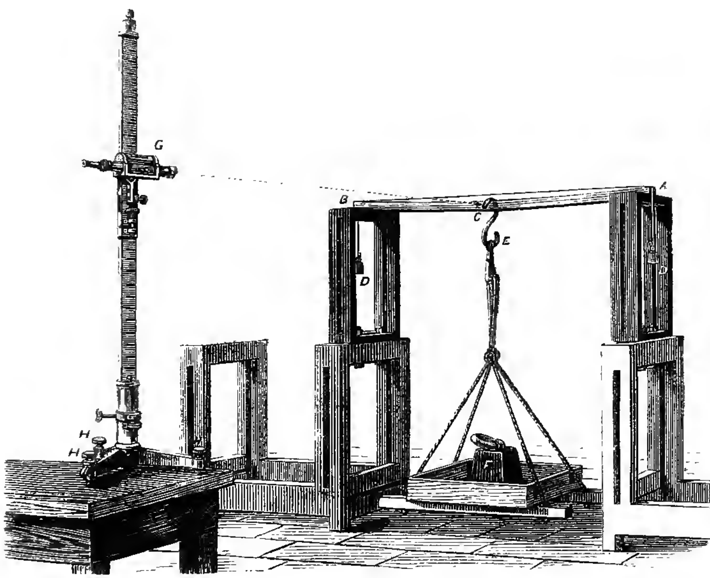

FIG. 52.

which is always directed horizontally, though capable of sliding up and down a vertical triangular rod; on one of the sides of the rod a scale is engraved, so that the height of the telescope in any position can be accurately determined. The cathetometer is levelled by means of the screws H H, so that the triangular rod on which the telescope slides is accurately vertical: the dotted line shows the direction of the visual ray when the centre C of the beam is seen by the observer through the telescope.

Inside the telescope and at its focus a line of spider's web is fixed horizontally; on the bar to be observed, and near its middle point C, a cross of two fine lines is marked. The tray being removed, the beam becomes horizontal; the telescope of the cathetometer is then directed towards the beam, so that the lines marked upon it can be seen distinctly. By means of a screw the telescope may be raised or lowered until the spider's web inside the telescope is observed to pass through the image of the intersection of the lines. The scale then indicates precisely how high the telescope is along its rod.

364. While I look through the telescope my assistant suspends the tray from the beam. Instantly I see the cross descend in the field of view. I lower the telescope until the spider's web again passes through the image of the intersection of the lines, and then by looking at the scale I see that the telescope has been moved down $0''\cdot19$, that is, about one-fifth of an inch: this is, therefore, the distance by which the cross lines on the beam, and therefore the centre of the beam itself, must have descended. Without this apparatus it would be difficult to measure the amount of deflection with any degree of precision. By placing successively one stone after another upon the tray, the beam is seen to deflect more and more, so that without the telescope you can easily see the beam has deviated from the horizontal.

365. By observing, however, with the telescope, and measuring in the way already described, the deflections shown in Table XXIII. were determined. The scale along the vertical rod was read after the spider's web had been adjusted for each increase in the weight. The difference between each reading and the reading before the tray was suspended is recorded as the deflection for each load.

TABLE XXIII.—DEFLECTION OF A BEAM.

A beam of pine 48″ × 1″ × 1″; resting freely on supports 40″ apart; and laden in the middle.

Number of Experiment.	Magnitude of load.	Deflection.
1	14	0″·19
2	28	0″·37
3	42	0″·55
4	56	0″·74
5	70	0″·94
6	84	1″·13
7	98	1″·35
8	112	1″·61
9	126	1″·95
10	140	2″·37

366. The first column records the number of the experiment. The second represents the load, and the third contains the corresponding deflections. It will be seen that up to 98 lbs. the deflection is about 0″·2 for every stone weight, but afterwards the deflection increases more rapidly. When the weight reaches 140 lbs. the deflection at first indicated is 2″·37; but gradually the cross lines are seen to descend in the field of the telescope, showing that the beam is yielding and finally it breaks across. This experiment teaches us that a beam is at first deflected by an amount proportional to the weight it supports; but that when two-thirds of the breaking weight is reached, the beam is deflected more rapidly.

367. It is a question of the utmost importance to ascertain the greatest load a beam can sustain without injury to its strength. This subject is to be studied by examining the effect of different deflections upon the fibres of a beam.

A beam is always deflected whatever be the load it supports; thus by looking through the telescope of the cathetometer I can detect an increase of deflection when a single pound is placed in the tray: hence whenever a beam is used we must have deflection, it cannot be avoided. An experiment will, however, show what amount of deflection does not produce an injurious effect.

368. A pine rod $40'' \times 1'' \times 1''$ is freely supported at each end, the distances between the supports being $38''$, and the tray is suspended from its middle point. A fine pair of cross lines is marked upon the beam, and the telescope of the cathetometer is adjusted so that the spider's line exactly passes through the image of the intersection. 14 lbs. being placed in the tray, the cross is seen to descend; the weight being removed, the cross returns precisely to its original position with reference to the spider's line: hence, after this amount of deflection, the beam has clearly returned to its initial condition, and is evidently just as good as it was before. The tray next received 56 lbs.; the beam was, of course, considerably deflected, but when the weight was removed the cross again returned,—at all events, to within $0''\cdot01$ of where the spider's line was left to indicate its former position. We may consider that the beam is in this case also restored to its original condition, even though it has borne a strain which, including the tray, amounted to 70 lbs. But when the beam has been made to carry 84 lbs. for a few seconds, the cross does not completely return on the removal of the load from the tray, but it shows that the beam has now received a permanent deflection of $0''\cdot03$. This is still more apparent after the beam has carried 98 lbs., for when this load is removed the centre of the beam is permanently deflected by $0''\cdot13$.

Here, then, we may infer that the fibres of the beam are beginning to be strained beyond their powers of resistance, and this is verified when we find that 28 additional pounds in the tray break the beam.

369. Reasoning from this experiment, we might infer that the elasticity of a beam is not affected by a weight which is less than half that which would break it, and that, therefore, it may bear without injury a weight not exceeding this amount. As, however, in our experiments the weight was only applied once, and then but for a short time, we cannot be sure that a longer-continued or more frequent application of the same strain might not prove injurious; hence, to be on the safe side, we assume one-third of the breaking weight of a beam is the greatest strain it should be made to bear in any structure.

370. We next consider the condition of the fibres of a beam when strained by a transverse force. It is evident that since the fracture commences at the lower surface of the beam, the fibres there must be in a state of tension, while those at the concave upper surface of the beam are compressed together. This condition of the fibres may be proved by the following experiment.

371. I take two pine rods, each $48'' \times 1'' \times 1''$, perfectly similar in all respects, cut from the same piece of timber, and therefore probably of very nearly identical strength. With a fine tenon saw I cut each of the rods half through at its middle point. I now place one of these beams on the supports 40″ apart, with the cut side of the beam upwards. I suspend from it the tray, which I gradually load with weights until the beam breaks, which it does when the total weight is 81 lbs.

If I were to place the second beam on the same supports with the cut upwards, then there can be no doubt that it would require as nearly as possible the same weight to break it. I place it, however, with the cut downwards, I suspend the tray, and find that the beam breaks with a load of 31 lbs. This is less than half the weight that would doubtless have been required if the cut had been upwards.

372. What is the cause of this difference? The fibres being compressed together on the upper surface, a cut has no tendency to open there; and if the cut could be made with an extremely fine saw, so as to remove but little material, the beam would be substantially the same as if it had not been tampered with. On the other band, the fibres at the lower surface are in a state of tension; therefore when the cut is below it yawns open, and the beam is greatly weakened. It is, in fact, no stronger than a beam of 48″ × 0″·5 × 1″, placed with its shortest dimension vertical. If we remember that an entire beam of the same size required about 140 lbs. to break it (Art. 366), we see that the strength of a beam is reduced to one-fourth by being cut half-way through and having the cut underneath.

373. We may learn from this the practical consequence that the sounder side of a beam should always be placed downwards. Any flaw on the lower surface will seriously weaken the beam: thus a knot in the wood should certainly be placed uppermost, if there be a choice. But if a portion of the actual substance of a beam be removed—for example, if a notch be cut out of it—this will be almost equally injurious on either side of the beam.

374. This may be illustrated by a simple experiment. I make two cuts 0″·5 deep in the middle of a pine rod 48″ × 1″ × 1″. These cuts are 0″·5 apart, and slightly

inclined; the piece between them being removed, a wedge is shaped to fit tightly into the space; the wedge is long enough to project a little on one side. If the wedge be uppermost when the beam is placed on the supports, the beam will be in the same condition as if it had two fine cuts on the upper surface. I now load the beam with the tray in the usual manner, and I find it to bear 70 lbs. securely. On examining the beam, which has curved down considerably, I find that the wedge is held in very tightly by the pressure of the fibres upon it, but, by a sharp tap at the end, I knock out the wedge, and instantly the load of 70 lbs. breaks the beam; the reason is simple—the piece being removed, there is no longer any resistance to the compressive strain of the upper fibres, and consequently the beam gives way.

375. The collapse of a beam by a transverse strain commences by fracture of the fibres on the lower surface, followed by a rupture of all fibres up to a considerable depth. Here, then, we see that by a transverse force the fibres in a beam of $48'' \times 1'' \times 1''$ are broken with a strain of 140 lbs. (Art. 366); but we have already seen (Art. 353) that to tear such a rod across by a direct pull at each end a force of about four tons is necessary. Now, the breaking strain of the fibres must be a certain definite quantity, yet we find that to overcome it in one way four tons is necessary, while by another mode of applying the strain 140 lbs. is sufficient.

376. To understand this we may refer to the experiment of Art. 28, wherein a piece of string was broken by the transverse pull of a piece of thread. This was shown to be due to the fact that one force may be resolved into two others, each of them very much greater than itself. This is what occurs also in the transverse deflection of

the beam: the force of 140 lbs. is changed into two other forces enormously greater and sufficient to rupture the fibres. We need not suppose that the force thus developed is so great as four tons, because that is the amount required to tear across a square inch of fibres simultaneously, whereas in the transverse fracture the fibres appear to be broken row after row; the fracture is thus only gradual, nor does it extend through the entire depth of the beam.

377. We shall conclude this lecture with one more remark, on the condition of a beam when strained by a transverse force. We have seen that the fibres on the upper surface are compressed, while those on the lower surface are extended; but what is the condition of the fibres in the interior? There can be no doubt that the following is the state of the case:—The fibres immediately beneath the upper surface are in compression; at a greater depth the amount of compression diminishes until at the middle of the beam the fibres are in their natural condition; on approaching the lower surface the fibres commence to be strained in extension, and the amount of the extension gradually increases until it reaches a maximum at the lower surface.

LECTURE XII.

THE STRENGTH OF A BEAM.

A Beam free at the Ends and loaded in the Middle.—A Beam uniformly loaded.—A Beam loaded in the Middle, whose Ends are secured.—A Beam supported at one end and loaded at the other.

A BEAM FREE AT THE ENDS AND LOADED IN THE MIDDLE.

378. In the preceding lecture we have examined some general circumstances in connection with the condition of a beam acted on by a transverse strain; we proceed in the present to inquire more particularly into the strength under these conditions. We shall, as before, use for our experiments rods of pine only as we wish rather to illustrate the general laws than to determine the strength of different materials. The strength of a beam depends upon its length, breadth, and thickness; we must endeavour to distinguish the effects of each of these elements.

We shall only employ beams of rectangular section; this being generally the form in which beams of wood are used. Beams of iron, when large, are usually not rectangular, as the material can be more effectively disposed in sections of a different form. It is important to distinguish between the stiffness of a beam in its capacity to resist flexure, and the strength of a beam in its capacity to resist fracture. Thus the stiffest beam which can be made from the cylindrical trunk of a tree 1′ in diameter is 6″ broad and 10″·5 deep, while the strongest beam is 7″ broad and 9″·75 deep. We shall consider the strength (not the stiffness) of beams.

379. We shall commence the inquiry by making a number of experiments: these we shall record in a table, and then we shall endeavour to see what we can learn from an examination of this table. I have here ten pieces of pine, of lengths varying from 1′ to 4′, and of three different sections, viz. 1″ × 1″ 1″ × 0″·5, and 0″·5 × 0″·5. I have arranged four different stands, on which we can break these pieces: on the first stand the distance between the points of support is 40″, and on the other stands the distances are 30″, 20″, and 10″ respectively; the pieces being 4′, 3′, 2′, and 1′ long, will just be conveniently held on the supports.

380. The mode of breaking is as follows:—The beam being laid upon the supports, an S hook is placed at its middle point, and from this S hook the tray is suspended. Weights are then carefully added to the tray until the beam breaks; the load in the tray, together with the weight of the tray, is recorded in the table as the breaking load.

381. In order to guard as much as possible against error, I have here another set of ten pieces of pine, duplicates of the former. I shall also break these; and whenever I find any difference between the breaking loads of two similar beams, I shall record in the table the mean between the two loads. The results are shown in Table XXIV.

TABLE XXIV.—STRENGTH OF A BEAM.

Slips of pine cut from the same piece supported freely at each end; the length recorded is the distance between the points of support; the load is suspended from the centre of the beam, and gradually increased until the beam breaks;

$$\text{Formula, } P = 6080 \frac{\text{area of section} \times \text{depth}}{\text{length}}.$$

Number of Experiment.	Dimensions.			Mean of the observations of the breaking load in lbs.	P. Calculated breaking load in lbs.	Difference of the observed and calculated values.
	Length.	Breadth.	Depth.			
1	40″·0	1″·0	1″·0	152	152	0·0
2	40″·0	0″·5	1″·0	77	76	− 1·0
3	40″·0	1″·0	0″·5	38	38	0·0
4	40″·0	0″·5	0″·5	19	19	0·0
5	30″·0	1″·0	0″·5	59	51	− 8·0
6	30″·0	0″·5	0″·5	25	25	0·0
7	20″·0	1″·0	0″·5	74	76	+ 2·0
8	20″·0	0″·5	0″·5	36	38	+ 2·0
9	10″·0	1″·0	0″·5	154	152	− 2·0
10	10″·0	0″·5	0″·5	68	76	+ 8·0

382. In the first column is a series of figures for convenience of reference. The next three columns are occupied with the sizes of the beams. By length, is meant the distance between the points of support; the real length is of course greater: the depth is that dimension of the beam which is vertical. The fifth column gives the mean of two observations of the breaking load. Thus for example, in experiment No. 5, the two beams used were each 36″ × 1″ × 0″·5, they were placed on points of support 30″ distant, so the length recorded is 30″: one of the beams was broken by a load of 58 lbs., and the second by a load of 60 lbs.; the mean between the two, 59 lbs., is recorded as the mean breaking load.

In this manner the column of breaking loads has been found. The meaning of the two last columns of the table will be explained presently.

383. We shall endeavour to elicit from these observations the laws which connect the breaking load with the length, breadth, and depth of the beam.

384. Let us first examine the effect of the length; for this purpose we bring together the observations upon beams of the same section, but of different lengths. Sections of 0″·5 × 0″·5 will be convenient for this purpose; Nos. 4, 6, 8 and 10 are experiments upon beams of this section. Let us first compare 4 and 8. Here we have two beams of the same section, and the length of one (40″) is double that of the other (20″). When we examine the breaking weights we find that they are 19 lbs. and 36 lbs.; the former of these numbers is rather more than half of the latter. In fact, had the breaking load of 40″ been $\frac{3}{4}$ lb. less, 18·25 lbs., and had that of 20″ been $\frac{1}{2}$ lb. more, 36·5 lbs., one of the breaking loads would have been exactly half the other.

385. You must not look for perfect numerical accuracy in these experiments; we must only expect to meet with approximation, because the laws for which we are in search are in reality only approximate laws. Wood itself is variable in quality, even when cut from the same piece: parts near the circumference are different in strength from those nearer the centre; in a young tree they are generally weaker, and in an old tree generally stronger. Minute differences in the grain, greater or less perfectness in the seasoning, these are some of the numerous circumstances which prevent one piece of timber from being identical with another. We shall, however, generally find that the effect of these differ-

ences is small, but occasionally this is not the case, and in trying many experiments upon the breaking of timber, discrepancies occasionally appear for which it is difficult to account.

386. But you will find, I think, that, making reasonable allowances for such difficulties as do occur, the laws on the whole represent the experiments very closely.

387. We shall, then, assume that the breaking weight of a bar of 40″ is half that of a bar of 20″ of the same section, and we ask, Is this generally true? is it true that the breaking weight is inversely proportional to the length? In order to test this hypothesis, we can calculate the breaking weight of a bar of 30″ (No. 6), and then compare the result with the observed value; if the supposition be true, the breaking weight should be given by the proportion—

30″ : 40″ : : 19 : Answer.

The answer is 25·3 lbs.; on reference to the table we find 25 lbs. to be the observed value, hence our hypothesis is verified for this bar.

388. Let us test the law also for the 10″ bar, No. 10—

10″ : 40″ : : 19 : Answer.

The answer in this case is 76, whereas the observed value is 68, or 8 lbs. less; this does not agree very well with the theory, but still the difference, though 8 lbs., is only about 11 or 12 per cent. of the whole, and we shall still retain the law, for certainly there is no other that can express the result as well.

389. But the table will supply another verification. In experiment No. 3, a 40″ bar, 1″ broad, and 0″·5 deep, broke with 38 lbs.; and in experiment No. 7, a 20″ bar of the same section broke with 74 lbs.; but 37, the half of 74, is almost identical with the breaking weight of the

40″ bar. We shall, therefore, adopt the approximate law, that for a given section the breaking load varies inversely as the length of the beam.

390. We next inquire, what is the effect of the breadth of the beam upon its strength? For this purpose we compare experiments Nos. 3 and 4: we there find that a bar 40″ × 1″ × 0″·5 is broken by a load of 38 lbs., while a bar just half the breadth is broken by 19 lbs. We might have anticipated this result, for it is evident that the bar of No. 3 must have the same strength as two bars similar to that of No. 4 placed side by side.

391. This view is confirmed by a comparison of Nos. 78, and where we find that a 20″ bar takes twice the load to break it that is required for a bar of half its breadth. The law is also verified to a certain extent by Nos. 5 and 6, though half the breaking weight of No. 5, namely 29·5 lbs., is a little more than 25, the observed breaking weight of No. 6: a similar remark may be made about Nos. 9 and 10.

392. Supposing we had a bar 40″ long, 2″ broad, and 0″·5 deep, we can easily see that it is equivalent to two bars like that of No. 3 placed side by side; and we infer generally that the strength of a bar is proportional to its breadth; or to speak more definitely, if I take two bars of the same length and depth, the ratio of their breaking loads is the same as the ratio of their breadths.

393. We next examine the effect of the depth of a beam upon its strength. In experimenting upon a beam placed edgewise, a precaution must be observed, which would not be necessary if the same beam were to be broken flatwise. When the tray is suspended, the beam, if merely placed edgewise on the supports, would almost certainly turn over; it is therefore necessary to have its

extremities in recesses in the supports, which will obviate the possibility of this occurrence ; at the same time the ends must not be firmly secured, for we are at present discussing a beam free at each end, and the case where the ends are not free will be presently considered.

394. Let us first compare together experiments Nos. 2 and 3 ; here we have two bars of the same size, the section in each being 1″·0 × 0″·5, but the first bar is broken edgewise, and the second flatwise. The first breaks with 77 lbs., and the second with 38 lbs. ; hence the same bar is twice as strong placed edgewise as flatwise when one dimension of the section is twice as great as the other We may generalize this law, and assert that the strength of a beam broken edgewise is to the strength of a similar beam broken flatwise, as the greater dimension of its section is to the lesser dimension.

395. The strength of a beam 40″ × 0″·5 × ″1 is four times as great as the strength of 40″ × 0″·5 × 0″·5, though the quantity of wood is only twice as great in one as in the other. We have seen that the strength of 40″ × 1″ × 0·″5 placed flatwise is only as strong as two beams 40″ × 0″·5 × 0″·5 side by side, but the same is not true of a beam placed edgewise : thus, for example, two beams of experiment No. 4, placed one on the top of the other, would break with about 40 lbs., whereas if the same rods were in one piece, the breaking weight would be nearly 80 lbs.

396. This may be illustrated in a different manner. I have here two beams of 40″ × 1″ × 0″·5 laid one on the other ; they form one beam, equivalent to that of No. 1 in bulk, but I find that they break with 80 lbs., thus showing that in reality the two are only twice as strong as one of them.

397. I take two similar bars, and, instead of laying them loosely one on the other, I clamp them together with clamps of Fig. 56. I now find that the bars thus fastened together require 104 lbs. to break them. What is the cause of this increase of strength? The moment the rods begin to bend under the action of the weight, the surfaces which are in contact move slightly one upon the other in order to accommodate themselves to the change of form. By clamping I render this motion difficult, hence the beams deflect less, and require a greater load to break them; the case is therefore to some extent approximated to the state of things when the two rods form one solid piece, in which case it would require 152 lbs. to produce fracture.

398. We shall be able by a little consideration to understand the reason why a bar is stronger edgewise than flatwise. Suppose I try to break a rod across my knee by pulling the ends held one in each hand, what is it that resists the breaking? It is chiefly the tenacity of the fibres on the convex surface of the bar. If the bar be edgewise, these fibres are further away from the centre of the bar, and act therefore at a greater leverage than if the bar be flatwise: nor is the case different when the bar is supported at each end, and the load placed in the centre; for the reactions of the supports are exactly similar to the forces with which I pulled the ends of the bar in the former case.

399. We shall now be able to calculate the strength of any beam of pine when we know its dimensions.

Let us suppose a beam 12′ long, 5″ broad, and 7″ deep. This is five times as strong as a beam 1″ broad and 7″ deep—in fact, we may conceive the original beam to consist of 5 of these beams placed side by side; the beam

1″ broad and 7″ deep, is 7 times as strong as a beam 7″ broad, 1″ deep (Art. 394). Hence the original beam must be 35 times as strong as a beam 7″ broad, 1″ deep; but the beam 7″ broad and 1″ deep is seven times stronger than a beam the section of which is 1″ × 1″, hence the original beam is 245 times as strong as a beam 12′ long and 1″ × 1″ in section, but the strength of the last beam is found by the proportion—

$$144'' : 40'' :: 152 : \text{Answer}.$$

The answer is 42·2 lbs., and thus the strength of the original beam is 42·2 × 245 = 10339.

400. We shall find it very useful to determine a general expression by which we can calculate the weight at once; we shall therefore find the strength of a beam which is l'' long, b'' broad, and d'' deep.

Let us suppose a rod l'' long, and 1″ × 1″ in section. The breaking strength of this rod is thus found—

$$l : 40 :: 152 : \text{Answer};$$

hence the breaking strength is $\frac{6080}{l}$. A beam which is d'' broad, l'' long, and 1″ deep, would be just as strong as d of the bars $l'' \times 1'' \times 1''$ placed side by side; its strength would therefore be—

$$\frac{6080}{l} \times d.$$

If this beam, instead of being flatwise, were placed edgewise, its strength would be increased in the ratio of its depth to its breadth—that is, it would be increased d-fold—and would therefore be

$$\frac{6080}{l} \times d^2.$$

This, therefore, is the strength of a beam 1″ broad, d'' deep, and l'' long. Now, the strength of b of these bars placed side by side, would be the same as the strength of one bar b'' broad, d'' deep, and l'' long, which would therefore be

$$\frac{6080}{l} \times d^2 \times b.$$

Since $b\,d$ is the area of the section, we can express this result conveniently by saying that the breaking weight of a bar expressed in lbs. is

$$6080 \times \frac{\text{area of section} \times \text{depth}}{\text{length}};$$

the depth and length being expressed in inches linear measure, and the section in square inches.

401. In order to verify this rule, we have calculated by its help the strength of all the ten bars given in the table, and the result is recorded in the sixth column. The difference between the amount calculated in this way and the observed mean values is recorded in the last column.

402. Thus, for example, in experiment No. 7 the length is 20″, breadth 1″, depth 0″·5; the formula gives, since the area is 0″·5,

$$P = 6080 \frac{0{\cdot}5 \times 0{\cdot}5}{20} = 76.$$

This agrees very nearly with 74 lbs., which is the mean of the two observed values.

403. With the exceptions of Nos. 5 and 10, the differences are very small, and even in the excepted cases the differences are not sufficient to make us doubt that the law is really what it professes to be, namely an approximation.

404. We have already pointed out that a beam begins to sustain permanent injury when it carries a load which is greater than half that which would break it (Art. 368), and we have shown that it is not in general safe to load

a beam which is part of a permanent structure with more than about a third or a fourth of the breaking weight. Hence if we wanted to calculate a fair working load for a beam of pine, we might employ the rule that the load in lbs. was

$$1500 \times \frac{\text{area of section} \times \text{depth}}{\text{length}}.$$

405. What we have said hitherto relates to pine; had we adopted any other kind of wood we should have found a similar expression for the breaking weight, the only difference being in the number which forms part of the product. Thus, for example, had I taken oak, I should have found that the number 6080 must be replaced by one a little larger.

A BEAM UNIFORMLY LOADED.

406. We have up to the present only considered the case where the load is suspended from the centre of the beam. But in the actual employment of beams the load is not generally applied in this manner. Thus in the rafters which support a roof the weight of the roof is not applied only at the middle point, but every part in the entire length has its own burden to support. The beams which support a floor have to carry their load in whatever manner it may be placed on the floor: sometimes, as for example in a corn-store, the pressure will be tolerably uniform along the beams, while if the weights be irregularly distributed on the floor, there will be corresponding inequalities in the mode in which the loads are carried by the beams. In order, therefore, to complete our study of the strength of beams, it will be necessary to examine the strength of a beam when its load is

applied otherwise than in the manner we have already considered.

407. We shall employ, in the first place, a beam 40″ long, 0″·5 broad, and 1″ deep; and we shall break this by applying a load at two points of the beam instead of at one point: this may be done in the manner shown in the diagram, Fig. 53. A B is the beam resting on two supports; C and D are the points of trisection of the beam; from these loops descend, in which rests an iron bar P Q; at the centre R of the bar P Q the load W is suspended.

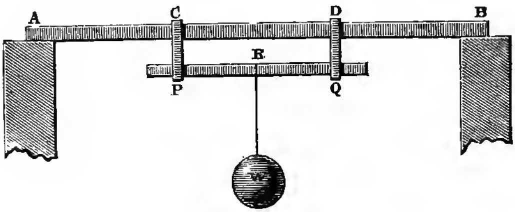

FIG. 53.

The load is thus applied equally at the two points C and D, and we may regard A B as a beam loaded at its two points of trisection. The tray is used which is shown in Fig. 58.

408. I proceed to break this bar. Adding weights to the tray, I find that it yields with 117 lbs., and cracks across between C and D. On reference to Table XXIV. we see by experiment No. 2 that a bar precisely similar required 77 lbs. at the centre; now $\frac{3}{2} \times 77 = 115{\cdot}5$; hence we may state that as nearly as possible the bar is half as strong again when the load is suspended from the two points of trisection as it is when the load is suspended at the centre. It is remarkable that in breaking the beam

in this manner the fracture is equally likely to occur at any point between C and D.

409. A beam uniformly loaded requires twice the weight to break it that would be necessary if the load were merely suspended from its centre. The mode of applying a load uniformly is shown in Fig. 54.

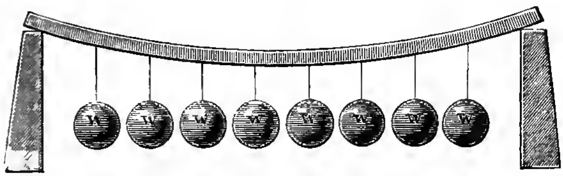

FIG. 54.

A beam similar to that just described, 40″ × 0″·5 × 1″, bears 10 stone, ranged along it in this manner, without breaking; one or two stone more would, however, doubtless produce fracture.

410. We infer from these considerations that beams in the manner in which they are usually employed are stronger than would apparently be indicated by Table XXIV.; this is because the loads are most commonly not applied at the centre.

EFFECT OF SECURING THE ENDS OF A BEAM UPON ITS STRENGTH.

411. You must have noticed that when weights were suspended from a beam and the beam began to bend, the ends curved upwards from the supports. This bending of the ends is shown in Fig. 54. If we restrain the beam from bending up in this manner, we shall add very considerably to its strength. I can do this by clamping the two ends down to the supports.

412. Let us try this upon a beam $40'' \times 1'' \times 1''$. I clamp each of its ends and then break it by a weight suspended from the middle. I find that it requires 238 lbs. to accomplish fracture. This is a little more than half as much again as 152 lbs., which we find, from Table XXIV., was the weight required to break this bar when its ends were free. In general we may say that the strength of a beam is increased upwards of fifty per cent. by having its extremities firmly secured.

413. When the beam breaks under these circumstances, there is not only a fracture in the centre, but there is also a fracture of the beam at each of the points of support; the necessity for three fractures instead of one explains the increase of strength which is obtained by clamping the ends.

414. In structures the beams are generally more or less secured at each end, and are therefore more capable of bearing resistance than would be indicated by Table XXIV. From the consideration of Arts. 409 and 412, we can infer that a beam secured at each end and uniformly loaded would require fully three times the weight to break it that would be necessary if its ends were free and if the load were applied at its centre.

BEAMS SECURED AT ONE END AND LOADED AT THE OTHER.

415. A beam, one end of which is firmly imbedded in masonry or otherwise secured, is occasionally called upon to support a weight suspended from its extremity. Such a case is shown in Fig. 55.

A B is a beam which is firmly imbedded at A, and the weight W is suspended from B. In the case which we shall examine, A B is a bar 20″ × 0″·5 × 0″·5, and you see that, when W reaches 10 lbs., the bar breaks. In experiment No. 8, Table XXIV., a similar bar required 36 lbs.; hence we see that the bar is broken in the manner of Fig. 55 by one-fourth of the load which would have been required if the beam had been supported at each end and laden in the middle. The same law may be observed by trial with other beams.

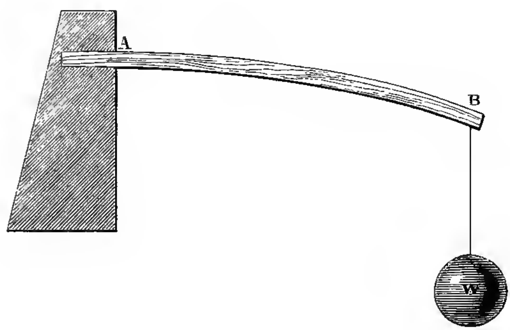

FIG. 55.

In our next lecture we shall have occasion to apply some of the results we have obtained.

LECTURE XIII.

THE PRINCIPLES OF FRAMEWORK.

Introduction.—Weight sustained by Tie and Strut.—Bridge with Two Struts.—Bridge with Four Struts.—Bridge with Two Ties.—Simple Form of Trussed Bridge.

INTRODUCTION.

416. In this lecture and the next we shall give a slight sketch of the arts of construction. We shall employ slips of pine 0″·5 × 0″·5 in section for the purpose of making models of simple constructions: these slips can be attached to each other by means of the small clamps, about 3″ long, shown in Fig. 56, and the general appearance of the models thus produced may be seen from Figs. 58 and 62.

Fig. 56.

417. The following experiment shows the tenacity with which these clamps hold. Two slips of pine, each 12″× 0″·5 × 0″·5, are clamped together, so that they overlap about 2″, thus forming a length of 22″: this rod is raised as in Fig. 49, and it is found that weights amounting to 2 cwt. can be suspended from it. Thus the clamped rods bear a direct strain of 2 cwt. This property of the clamps depends principally upon friction, aided doubtless by a slight crushing of the wood, which brings the surfaces into perfect contact.

418. Hence the models thus united by the clamps are possessed of strength quite sufficient for the experiments which will be made upon them. They possess the great advantage of being erected, varied or pulled down, with the utmost facility.

We have learned that the compressive strength, and, still more, the tensile strength of timber, is much greater than its transverse strength. This principle is largely used in the arts of construction. We endeavour by means of suitable combinations to turn transverse strains into strains of tension or compression, and thus strengthen our constructions. It is most important to bear this principle in mind. We shall illustrate it by simple forms of framework.

WEIGHT SUSTAINED BY TIE AND STRUT.

419. We shall begin by a very simple case, and one which is in extensive use; it is represented by Fig. 57.

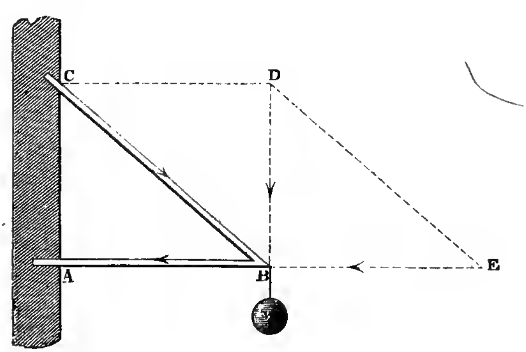

Fig. 57.

A B is a rod of pine 20″ long. In the diagram it is represented, for simplicity, imbedded at the end A in the support. In reality, however, it is clamped to the support, and the same remark may be made about the other diagrams referred to in this lecture. Were A B unsupported except at its end A, it would break when a weight of 10 lbs. was suspended at B, as we have already seen in Art. 415.

420. We must ascertain whether the transverse strain on A B cannot be changed into strains of tension and compression. The tie B C is attached by means of clamps; A B is sustained by this tie; it cannot bend downwards under the action of the weight W, because we should then require to have on the same base and on the same side of it two triangles, having their conterminous sides equal, but this we know from Euclid (I. 7) is impossible. Hence B is supported, and we find on trial that 112 lbs. is easily borne at W, so that the strength is enormously increased. In fact the transverse strain is now changed into a compressive strain on A B, and a tensile strain on B C.

421. The amount of these strains can be computed. Draw the parallelogram C D E B; then if B D represent the weight W, it may be resolved into two forces,—one, B C, a force of extension on the tie; the other, B E, a compressive force on A B, which is therefore a strut. Hence the forces are proportional to the sides of the triangle, A B C. In the present case

$$A B = 20'', A C = 18'', B C = 27'';$$

therefore, when W is 112 lbs., it is easy to see that the strain on A B is 124 lbs., and on C B 168 lbs. A B would require about 300 lbs. to crush it, and C B about 2,000 lbs. to tear it asunder, consequently the tie and strut can support 1 cwt. with ease. If, however, W were increased to about 270 lbs., the strain on A B would become too great, and the construction would fail by the collapse of this strut.

422. When a structure is loaded up to the breaking point of one part, it is proper that all the other parts should be so designed that they shall be as near as possible to their breaking points. In fact, since nothing is stronger than its weakest part, any additional strength which the remaining parts may possess adds no strength to the whole, and is only so much material wasted. Hence the structure would be just as strong, and would be more properly designed if the section of B C were reduced to one-fifth, for the tie would then break when the strain upon it amounted to 400 lbs. When W is 270 lbs. the strain on A B amounts to 300 lbs., and on BC to about 400 lbs., so that both tie and strut attain their breaking strain together. In large structures where economy of material is of importance, this principle is carefully attended to by the designer. We shall not, however, refer to it again.

A BRIDGE WITH TWO STRUTS.

423. We shall next examine the structure of the bridge, which is shown in Fig. 58.

It consists of two beams, A B, 4′ long, placed parallel to each other at a distance of 3″·5, and supported at each end; they are firmly clamped to the supports, and a roadway of short pieces is laid upon them. At the points of trisection of the beams C, D, struts C F, D E are clamped, their lower ends being supported by the framework: these struts are 2′ long, and there are two of them supporting each of the beams. The tray G is attached by a chain to a stout piece of wood, which rests upon the roadway at the centre of the bridge.

424. We shall first determine the strength of this bridge by actual experiment, and then we shall endeavour to explain the results by what we have already learned. We can observe the deflection of the bridge by the cathetometer in the manner already described (Art. 363).

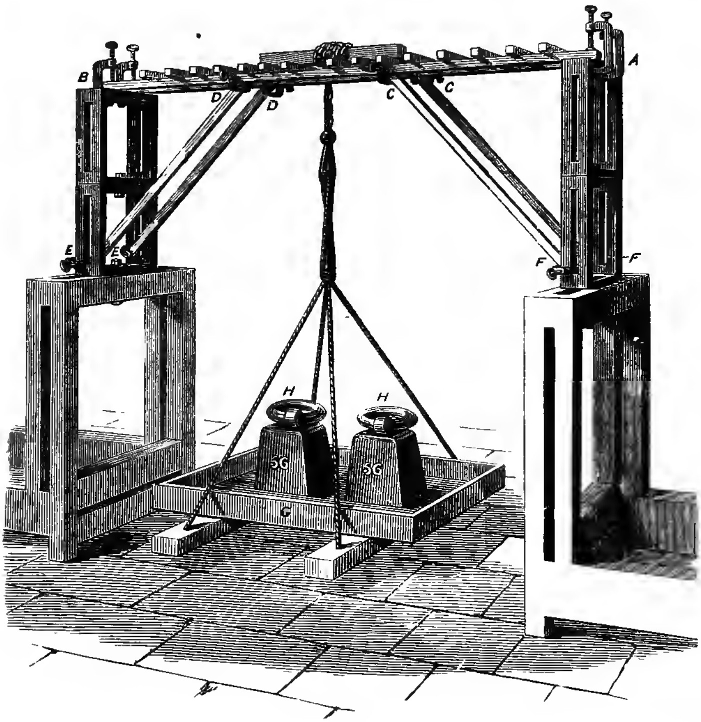

FIG. 58.

By this means we shall ascertain whether the load has permanently injured the elasticity of the bridge (Art. 368). We shall first test the strength when a load is distributed uniformly, just as the weights are disposed in the case of Fig. 62. A cross is marked upon one of the beams, and is viewed in the cathetometer. I arrange 11 stone weights along the bridge, and the cathetometer shows that the deflection is only 0″·09: the elasticity of the bridge remains unaltered, for when the weights are removed the cross on the beam returns to its original position; hence the bridge is well able to bear this load.

425. I remove the row of weights from the bridge and suspend the tray from the roadway. I take my place at the cathetometer to note the deflection, while my assistant places weights H H on the tray. 1 cwt. being the load, I see that the deflection amounts to 0″·2; with 2 cwt. the deflection reaches 0·43″; and the bridge breaks with 238 lbs.

426. Let us endeavour to calculate the strength which the struts have really imparted to the bridge. By Table XXIV. we see that a rod 40″ × 0″·5 × 0″·5 is broken by a load of 19 lbs.: hence the beams of the bridge would have been broken by a load of 38 lbs. This load is for beams which are free at the ends, while the beams of the bridge were secured at the ends. Securing the ends according to the principle of Art. 412 doubles the strength, but about 80 lbs. would certainly have broken the bridge had it not been sustained by the struts. The strength is, therefore, increased about threefold by the struts, for a load of 238 lbs. was required to produce fracture.

427. We might have anticipated this result, because the points C and D being supported by the struts may be considered as almost fixed points; in fact, the point C cannot descend, because the triangle A C F is unalterable, and for a similar reason D cannot descend: the beam breaks between C and D, and the force required must therefore be sufficient to break a beam supported at the

points C and D, whose ends are secured. But C D is one-third of A B, and we have already seen that the strength of a beam is inversely as its length (Art. 389); hence the force required to break the beam when supported by the struts is three times as large as would have been necessary to break the unsupported beam. Thus the strength of the bridge is explained.

428. As a load of 238 lbs. applied at the centre is necessary to break this bridge, it follows from the principle of Art. 409 that a load of double this amount, or nearly 500 lbs., must be placed uniformly on the bridge before it succumbs; we can, therefore, understand how a load of 11 stone was borne (Art. 424) without permanent injury to the elasticity of the bridge. If we take the factor of safety as 3, we see that a bridge of the form we have been considering may carry, as its ordinary working strain, a load which would have crushed the bridge if unsupported by the struts.

429. The strength of the bridge in Fig. 58 is greater in some parts than in others. At the points C and D a very great strain could be borne; in fact the clamps would slip before fracture could occur: the weakest places on the bridge are in the middle points of the segments A C, C D, and D B. The load we applied by the tray was principally borne at the middle of D C, but owing to the piece of wood which sustained the chain having some length, the pressure was slightly distributed.

The exact strain upon the struts is difficult to find. The strain upon C F must, however, be less than if the part C D were removed, and half the load were suspended from C. The strain in this case can be found (see Arts. 419—421).

A BRIDGE WITH FOUR STRUTS.

430. The same principles that we have employed in the construction of the bridge of Fig. 58 may be extended further, as shown in the diagram of Fig. 59.

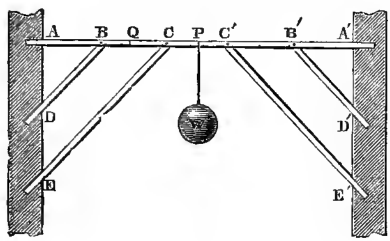

FIG. 59.

We have here two horizontal rods, 48″ × 0″·5 × 0″·5, each end being secured to the supports; one of these rods is shown in the figure. It is divided into five equal parts in the points B, C, C′, B′. We support the rod in these four points by struts, the other extremities of which are fastened to the framework. Now B, C, C′, B′ are fixed points, as they are sustained by the struts: hence a weight suspended from P, which is to break the bridge, must be sufficiently strong to break a piece C C′, which is secured at the ends; the rod A A′ would have been broken with 38 lbs., hence 190 lbs. would be necessary to break C C′. There is a similar beam on the other side of the bridge, and therefore to break the bridge 380 lbs. would be necessary, but this force must be applied exactly at the centre of C C′; and if the weight be so applied that it is distributed over any considerable length, a heavier load will be necessary. If I distribute the load over the whole of C C′, it appears from Art. 409 that 760 lbs. would be necessary to produce fracture.

431. This bridge is extremely strong. I place 18 stone upon it ranged uniformly, and the cathetometer tells me that the bridge only deflects 0″·1, and that its elasticity is not injured. Placing the tray in position, and loading the bridge by this means, I find with a weight of 2 cwt. that there is a deflection of 0″·15; with 4 cwt., however, the deflection amounts to 0″·72. We therefore infer that the bridge is beginning to yield, and it collapses when the load is increased to 500 lbs.

A BRIDGE WITH TWO TIES.

432. It often happens that circumstances may not make it convenient to obtain points of support below the bridge on which to erect the struts. In such a case, if suitable positions for ties can be obtained, a bridge of the form represented in Fig. 60 may be used.

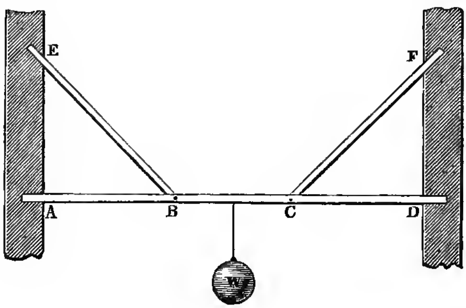

FIG. 60.

A D is a horizontal rod of pine 40″ × 0″·5 × 0″·5; it is trisected in the points B and C, from which points the ties B E and C F are secured to the upper parts of the framework. A D is then supported in the points B and C, which may therefore be regarded as fixed points. Hence, in the manner we have already explained, the strength of the bridge should be increased nearly threefold. It would require about 70 lbs. or 80 lbs. to break it without the ties, and therefore we might expect that it would require over 200 lbs. when supported by the ties. I perform the experiment, and you see the bridge yields when the load reaches 194 lbs.: this is somewhat less than the amount we had calculated; the reason being, I think, that one of the clamps slipped before fracture. The clamps do not answer as well for ties as for struts.

A SIMPLE FORM OF TRUSS.

433. It is often not convenient, or even possible, to sustain a bridge by the methods we have been considering. It is desirable therefore to inquire whether we cannot arrange some plan of strengthening, which shall not depend upon external support.

434. We shall only be able to describe here some very simple methods for doing this. Superb examples are to be found in railway bridges all over the country, but the full investigation of these complex structures is a problem of no little difficulty, and one into which it would be quite beyond our province to enter. We shall, however, be able to show how the transverse strains can be changed into strains of tension or compression, and it is on this principle that the most complex lattice bridge is based.

435. Let A B (Fig. 61) be a rod of pine $40'' \times 0''{\cdot}5 \times 0''{\cdot}5$, secured at each end. We shall suppose that the load is applied at the two points G and H, in the manner shown in the figure. The load which a bridge must bear when a train passes over it is distributed over a space

equal to the length of the train, and the weight of the bridge is of course distributed along the length of the bridge; hence the load which a bridge bears is at all times more or less distributed, and never all concentrated at the centre in the manner we have been considering. In the present experiment we shall apply the breaking load at the two points G and H, as this will be a variation from the mode we have previously used. E F is an iron

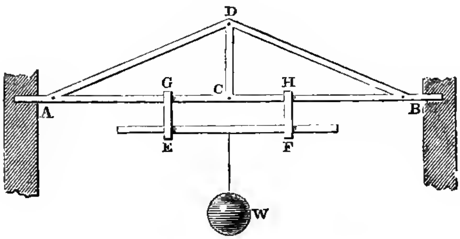

FIG. 61.

bar supported in the loops E G and F H. Let us first try what weight will break the beam. Suspending the tray from E F, I find that a load of 48 lbs. is sufficient; about 30 lbs. would have been enough had not the ends been clamped. The strength is due to the causes we have already pointed out (see Arts. 407 and 412).

436. You observed that the beam, as usual, deflected before it broke; if we could prevent deflection we might fairly expect to increase the strength. If I could support the centre of the beam C, deflection would be prevented. Now this can be done very simply. I clamp the pieces D A, D B, D C on a new beam, and it is evident that C cannot descend so long as the joints at A, B, D, C remain firmly secured. We now find that even with a weight of 112 lbs. in the tray, the bar is unbroken. An arrangement of this kind is called a truss, and we see

that the truss bears securely more than double the load which is sufficient to break the unsupported beam.

437. Two trusses of this kind, with a roadway laid between them, would form a bridge, or if the trusses were turned upside down they would answer equally well, but a better arrangement for a bridge will be next described.

THE TRUSSED BRIDGE.

438. A splendid example of the trussed bridge was erected by the late Sir I. Brunel over the Wye, for the purpose of carrying a railway. The essential parts of the bridge are shown in Fig. 62, which is made up of slips of pine clamped together in the manner already explained.

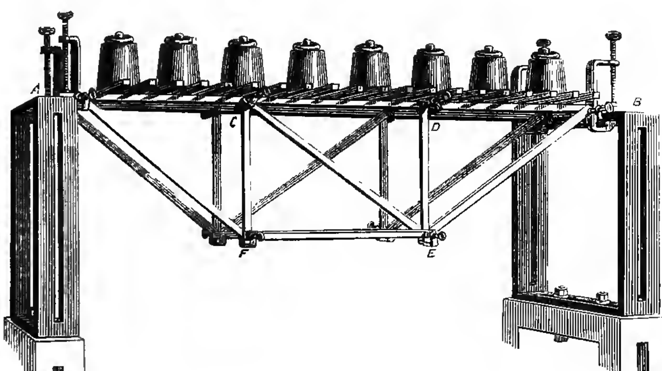

FIG. 62.

439. The model is composed of two similar trusses, one of which we shall describe. A B is a rod of pine 48″ × 0″·5 × 0″·5, supported at each extremity. This rod is sustained at its points of trisection D, C by the uprights D E and C F, while E and F are supported by the rods

B E, F E, and A F; the rectangle D E F C is stiffened by the piece C E, and it would be desirable, though not essential, to have a piece connecting D and F, but it has not been introduced into the model.

440. We shall understand the use of the stiffening-piece by an inspection of Fig. 63. Suppose A B C D be a quadrilateral, formed of four pieces of wood hinged at the corners. It is evident that this quadrilateral can be deformed by pressing A and C together, or by pulling them asunder; even if there were actual joints at the corners, it would be almost impossible to make the quadrilateral stiff by the strength of the joints. You see this by the quadrilateral of wood which I hold in my hand; the pieces are clamped together at the corners, and no matter how tightly I compress the clamps, I am able with the slightest exertion to deform the quadrilateral.

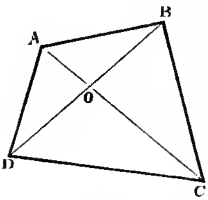

FIG. 63.

441. We must therefore look for some method of stiffening the figure. I have here a triangle of three pieces, which have been simply clamped together at the corners: this triangle is unalterable in form; in fact, since it is impossible to form two different triangles with the same three sides, it is evident the triangle cannot be altered. This points to the mode of evading the difficulty. The quadrilateral is not stiff because innumerable different quadrilaterals can be made with the same four sides. But if I draw the diagonal A C of the quadrilateral I divide it into two triangles, and hence if I attach to the quadrilateral, which has been clamped at the four corners, an additional piece in the direction of one of the diagonals, it becomes unalterable in shape.

442. In Fig. 63 we have drawn the two diagonals A C and B D: one would be theoretically sufficient, but it is desirable to have both, and for the following reason. If I pull A and C apart, I stretch the diagonal A C and compress B D. If I compress A and C together, I compress the line A C and extend B D; hence in one of these cases A C is a tie, and in the other it is a strut. It is therefore easy to see that one of the diagonals is always a tie, and the other always a strut. If then we have only one diagonal, it is called upon to perform alternately the functions of a tie and of a strut. This is not desirable, because it is evident that a piece which may act perfectly as a tie would be very unsuitable for a strut, and *vice versâ*. But if we insert both diagonals we may make both of them ties, or both of them struts, and the frame must be rigid. Thus for example, I might make A C and B D slender bars of wrought iron, which form admirable ties, though quite incapable of acting as struts.

443. What we have said with reference to the necessity for dividing a quadrilateral figure into triangles applies still more to a polygon of a large number of sides, and we may lay down the general principle that every piece of rigid framework must be composed of triangles.

444. Returning to Fig. 62, we see the reason why the rectangle E D C F must have one or both of its diagonals introduced. A load placed, for example, at D would tend to depress the piece D E, and thus deform the rectangle, but when the diagonals are introduced this deformation is impossible.

445. Hence one of these trusses is almost as strong as a beam supported at the points C and D, and therefore,

from the principle of Art. 389, its strength is three times greater than that of an unsupported beam.

446. The two trusses placed side by side and carrying a roadway form an admirable bridge, quite independent of all external support with the exception of the piers upon which the extremities of the trusses rest. It would be proper to connect the trusses together by means of braces, which are not, however, shown in the figure. The model is represented as carrying a uniform load in contradistinction to Fig. 58, where the load is applied at a single point.

447. With a load of eight stone ranged along it, the bridge of Fig. 62 did not indicate an appreciable deflection.

LECTURE XIV.

THE MECHANICS OF A BRIDGE.

Introduction.—The Girder.—The Tubular Bridge.—The Suspension Bridge.

INTRODUCTION.

448. PERHAPS you may have thought that the structures we have been considering are not those which are most universally used, and that the bridges which are generally referred to as monuments of engineering skill are of quite a different construction. Every one is familiar with the arch, and at all events, by name, with suspension bridges, and tubular bridges. We must therefore allude further to some of these structures, and this we propose to do in the present lecture. It will only be possible to do so to a very small extent, for whole treatises have been written on these subjects.

We shall first give a brief account of the use of iron in the arts of construction. We shall then explain simply the principle of the tubular bridge, and also of the suspension bridge. The more complex forms are beyond our scope.

THE GIRDER.

449. A beam which is intended to be supported at each end, and to carry its load between the ends, is called a

girder. Those rods upon which we have performed experiments, the results of which have been given in Table XXIV., are small girders; but the term "girder" is generally understood to relate to structures of iron, as beams on a large scale are often made of bars or plates of iron riveted together.

450. We shall first consider the application of cast iron to girders, and show what form they should assume.

451. A beam of cast iron, supposing its section to be rectangular, has its strength determined by the same laws as the rods of pine. Thus, supposing the section of the beams to be the same, their strengths are inversely proportional to their lengths, and the strength of a beam placed edgewise is to its strength placed flatwise, in the proportion of the greater dimension of its section to the less dimension. These laws determine the strength of every beam of cast iron when the strength of one beam is known, and we must perform an experiment in order to find the strength of one beam.

452. I take here a beam of cast iron, which is 2′ long, and 0″·5 × 0″·5 in section. I support this beam at each end upon a frame; the distance between the supports is 20″. I attach the tray to the centre of the beam and load it with weights. The ends of the beam rest freely upon the supports, but I have taken the precaution of tying each end by a piece of wire, so that they may not fly when the fracture occurs. Loading the tray, I find that 280 lbs. breaks the rod of iron.

453. Let us compare this result with No. 8 of Table XXIV. There we find that a piece of pine, the same size as the cast iron, was broken with 36 lbs.: the ratio of 280 to 36 is nearly 8, so that the beam of cast iron is about 8 times as strong as the piece of pine of the same size.

This result is a little larger than I would have expected, from an examination of tables of the strength of large bars of cast iron; the reason may be that a very small casting, such as this bar, is stronger in proportion than a larger casting, owing to the iron not being so uniform throughout the larger mass.

454. I hold here a bar of cast iron 12″ long and 1″ × 1″ in section. I have not sufficient weights at hand to break it, but we shall easily be able to compute how much would be necessary by our former experiment.

455. In the first place a bar 12″ long, and 0″·5 × 0″·5 of section, would require $20 \times 280 \div 12 = 467$ lbs. by the law that the strength is inversely as the length. We also know that a beam 12″·1 × 1″ × 1″ is just the same as two beams 12″ × 1″ × 0″·5, each placed edgewise; each of these latter beams is twice as strong as 12″ × 1″ × 0″·5 placed flatwise, because the strength when placed edgewise is to the strength when placed flatwise, as the depth to the breadth, that is as 2 to 1: hence the original beam is four times as strong as one beam 12″ × 1″ × 0″·5 placed flatwise; but this last beam is twice as strong as a beam 12″ × 0″·5 × 0″·5, and hence we see that a beam 12″ × 1″ × 1″ is really 8 times as strong as a beam of 12″ × 0″·5 × 0″·5, but this last beam would require a load of 467 lbs. to break it, and hence the beam of 12″ × 1″ × 1″ would require $467 \times 8 = 3736$ lbs. to produce fracture. This amounts to about a ton and a half.

456. It is a rule sometimes useful to practical men that a bar of iron one foot long by one inch square would break with about a ton weight. If the iron be of the same quality as that which we have used, this result is too small, but the error is on the safe side; the real strength will then be generally a little greater than the

strength as calculated by this rule. Of course what we have said with reference to the factor of safety in bars of wood applies also to cast iron. The strain which the beam has to bear in ordinary practice should only be a small fraction of the load which would break the beam.

457. In making a girder of cast iron it is desirable for the sake of economy that as little material as possible be uselessly employed. It will of course be remembered that a girder has to support its own weight, besides whatever may be placed upon it; and if the girder be massive, its own weight is a serious item. Of two girders, each capable of bearing the same total load, the lighter, besides employing less material, will be able to bear a greater weight placed upon it. It is therefore for a double reason desirable to diminish the weight. This remark applies especially to a material such as cast iron, which can at once be cast into the form in which it shall be capable of offering the greatest resistance.

458. The principles which will guide us in ascertaining the proper form to give a cast iron girder, are easily deduced from what we have laid down in Lectures XI. and XII. We have seen that depth is very desirable for a strong beam. If therefore we strive to attain great depth in a light beam, the beam must be very thin. Now an extremely narrow beam will not answer. In the first place it would not be stiff, but would be liable to move sideways; and, in the second place, there is a still more fatal difficulty. We have shown that when a beam of wood is supporting a weight, the fibres at the bottom of the beam are strained, the tendency being to tear them across. The fibres on the top of the beam are compressed, while the centre of the beam is in its natural condition. The condition of strain of a cast-iron beam is precisely

similar; the bottom portions are in a state of extension, while the top is compressed. If therefore a beam be very thin, and not inconveniently deep, the material at the lower part may not be sufficient to withstand the strain, and fracture is produced. The way to obviate this, is to strengthen the bottom of the beam by placing extra material upon it. Thus we are led to the idea of a thin beam with an excess of iron at the bottom.

459. E F (Fig. 64) is the thin iron beam along the bottom of which is the stout flange shown at C D; rupture cannot commence at the bottom unless this flange be torn asunder; and unless the bottom be torn across, it is clear that the strain cannot rupture the beam at the part F above the flange.

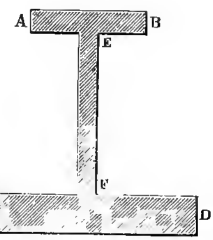

FIG. 64.

460. But the beam is in a state of compression along its upper side, just as in the wooden beams which we have already considered. If therefore the upper parts were not powerful enough to resist this compression, they would be crushed, and the beam would give way. The remedy for this source of weakness is obvious; a second flange runs along the top of the beam, as shown at A B. If this be strong enough to resist the compression, the stability of the beam is ensured.

461. It will be noticed that the upper flange is very much smaller than the lower flange; the reason for this depends upon a property of cast iron. This metal is more capable of resisting forces of compression than forces of extension, and it is only necessary to have one-sixth of the iron on the upper flange that is required for the lower flange. When the flanges have this proportion, the beam

is equally strong at both top and bottom; adding material to either flange without strengthening the other, will not strengthen the beam, but will rather prove a source of weakness, by increasing the weight which has to be supported.

462. I have here a small girder of common tin-plate, which has been made of the shape shown in Fig. 64. It is 12″ long. I support it at each end, and you see it bears two hundred weight without apparent deflection.

THE TUBULAR BRIDGE.

463. I shall commence the description of the principle of the tube by performing some experiments upon the tube, which I hold in my hand. It is made of what we are familiar with under the name of "tin," but which is really sheet iron thinly covered over with tin; the tube is square, 1′ × 1″ in section, and 38″ long. It weighs a little less than a pound.

464. Here is a solid rod of iron which is of the same length as the tube, but which contains more iron. This is easily verified by weighing the tube and the rod one against the other. I shall regard the rod and the tube as two girders, and experiment upon their strength, and we shall find that, though the tube contains less substance than the rod, it is much stronger.

465. I place the rod on a pair of supports about 3′ apart; I then attach the tray to the middle of the rod: 14 lbs. produces a deflection of 0″·51, and 42 lbs. bends down the rod through 3″·18. This is a very large deflection; and when I remove the load, the rod only returns through 1″·78, thus showing that a permanent deflection of 1″·40 is produced. This considerable permanent

deflection shows us that the bar is weakened, and very little more would doubtless break it.

466. But we place the tube upon the same supports, and load it in the same manner. A load of 56 lbs. only produces a deflection of 0″·09, and, when this load is removed, the tube returns to its original position : this I see by the cathetometer, for a cross is marked on the tube, and I bring the image of it on the horizontal wire of the telescope before the load of 56 lbs. is placed in the tray. When the load is removed, I see that the cross returns exactly to where it was before, thus proving that the elasticity of the tube is unimpaired. When I double the load, thus placing 1 cwt. in the tray, the deflection only reaches 0″·26, and, when the load is removed, the tube is found to be permanently deflected by a quantity, at all events not greater than 0″·004 ; hence we learn that the tube bears easily, and without injury, a load more than double as great as that which completely destroyed a rod of wrought iron, containing more iron than the tube. I load the tube still further by placing additional weights in the tray, and with 140 lbs. the tube breaks: this is, however, accidental; the fracture has occurred at a joint which was soldered, and the real breaking strength of the tube is doubtless far greater. Enough, however, has been borne to show how great is the increase of strength obtained by the tubular form.

467. Let us inquire into the reason of this remarkable result. We shall be able to understand it by means of Fig. 64. If the thin portion of the girder E F be made of two parts placed side by side, the strength will not be altered. If we then imagine the flange A B widened to the width of C D, and the two parts which form E F

opened out so as to form a tube, the strength of the girder is still retained in its modified form.

468. A tube of rectangular section has the advantage of greater depth than a solid rod of the same weight; and if the bottom of the tube be strong enough to resist the extension, and the top strong enough to resist the compression, the girder will be stiff and strong.

469. In the Menai Tubular Bridge, where a gigantic tube supported at each end bridges over a space of four hundred and sixty feet, special arrangements have been made for strengthening the top. It is formed of cells, as wrought iron disposed in this way is more effective in resisting compression than where it is in solid masses.

470. We have only spoken of rectangular tubes, but it is equally true for tubes of circular or other section that they are always stronger than the same quantity of material, if made into a solid rod.

471. We find this principle in nature; bones and quills are frequently hollow in order to combine lightness with strength, and the stalks of many plants are hollow for the same reason.

THE SUSPENSION BRIDGE.

472. Where a great span is required, the suspension bridge possesses many advantages. It is lighter than a girder bridge of the same span, and consequently cheaper, while its singular elegance contrasts very favourably with the appearance of more solid structures. On the other hand, a suspension bridge is unable to carry railway traffic, as it does not possess the steadiness which is necessary for safety.

473. The mechanical character of this bridge is very

simple. If we suppose a chain to be suspended from two points to which its ends are attached, the chain forms a certain curve called the catenary. It resembles an arc of a circle to a certain extent, though still a distinct curve. It would not be possible to make the chain lie in a straight line between the two points of support, for reasons pointed out in Art. 20. No matter how much the chain be strained, it will still be concave. When the chain is strained so much that the amount of depression in the middle is small compared with the distance between the points of support, the curve in which the chain lies, though still really a catenary, becomes indistinguishable from the parabola.

474. In Fig. 65 is shown a model of a suspension bridge. The chains are fixed at the points E and F; they then pass over the piers A, D, and form a span of nine feet. The line B C shows the amount by which the chain has deflected from the horizontal A D. When the deflection of the middle of the chain is about one-tenth part of A D, the curve A C D is quite indistinguishable from the parabola. Since the chains hang in a curve, it would be impossible to attach the roadway to the chains; the roadway is therefore suspended from the chains, the lengths of the suspension bars being so regulated as to make the roadway as nearly horizontal as possible.

475. The roadway in the model is laden with 8 stone weights, placed side by side. We have distributed this load along the roadway in order to represent the permanent load which a suspension bridge has to carry. The hundredweight thus arranged is substantially the same as if it were actually distributed uniformly along the length of the bridge. In a real suspension bridge

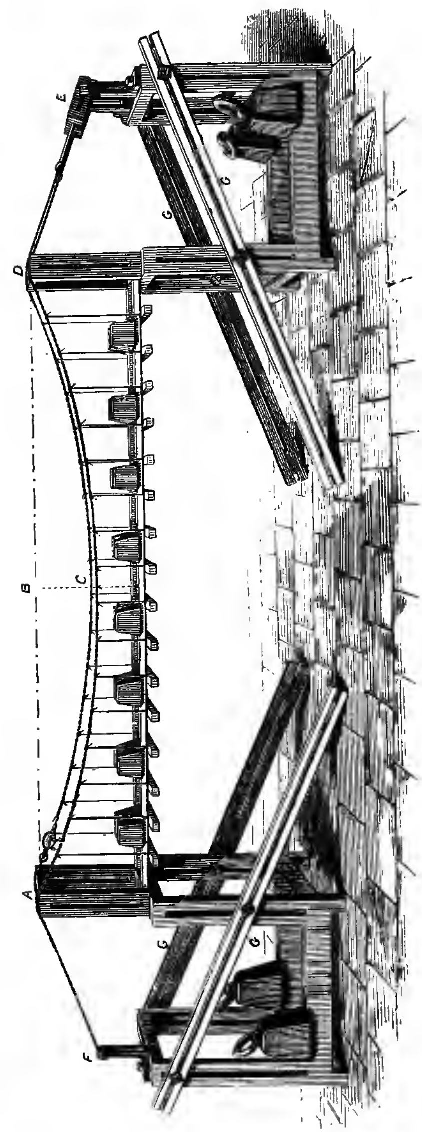

Fig. 65.

the weight of the roadway produces a very considerable strain along the chains.

476. We assume that the chain hangs in the form of a parabola, and that the load is uniformly ranged along the bridge. The strain upon the chains is greatest at their highest points, and least at their lowest points, though the difference is small. The amount of the strain can be calculated when the load, span, and deflection are known. We cannot give the steps of the calculation, but we shall enunciate the result.

477. The magnitude of the strain in pounds at the lowest point C of each chain is found by multiplying the total weight (including chains, suspension rods, and roadway) by the span, and dividing the product by sixteen times the deflection.

The strain upon the chain at the highest point A exceeds the strain at the lowest point C, by a number of pounds, which is found by multiplying the total load by the deflection, and dividing the product by twice the span.

478. The total weight of roadway, chains, and load in the model is 120 lbs.; the deflection is 10″, the span 108″; the product of the weight and span is 12,960; sixteen times the deflection is 160; and, therefore, the strain at the point C is found, by dividing 12,960 by 160, to be 81 lbs.

To find the strain at the point A, we multiply 120 by 10, and divide the product by 216; the quotient found is 6. This added to 81 lbs. gives 87 lbs. for the strain on the chain at A.

479. One chain of the model is attached to a spring-balance at A; by reference to the scale we see the strain indicated is 90 lbs.: this is very close to the calculated strain of 87 lbs.

480. A large suspension bridge has its chains strained by an enormous force. It is therefore necessary that the ends of these chains be very firmly secured in the ground. A good point of attachment is sometimes obtained by anchoring the chain to a large mass of iron imbedded in solid rock.

481. In Art. 45 we have pointed out how the dimensions of the tie rod could be determined when the strain was known. Similar considerations will enable us to calculate the size of the chain necessary for a suspension bridge when we have ascertained the strain to which it will be subjected.

482. We can easily determine by trial what effect is produced on the tension of the chain, by placing a weight upon the bridge in addition to the permanent load. I place another stone weight in the centre, and we see that the tension of the spring-scale is now 100 lbs.; of course the tension of the other chain is the same: and thus we find that a weight of 14 lbs. has produced additional strains of 10 lbs. each in the two chains. A weight of 28 lbs. is found to give a strain of 110 lbs.

483. The additional weights may be regarded as analogous to the occasional loads which the suspension bridge is required to carry. In a large suspension bridge the tension produced by the occasional load is usually only a small fraction of that produced by the permanent load.

LECTURE XV.

THE MOTION OF A FALLING BODY.

Introduction.—The First Law of Motion.—The Experiment of Galileo from the Tower of Pisa.—The Space is proportional to the Square of the Time.—A Body falls 16′ in the First Second.—The Action of Gravity is independent of the Motion of the Body.—How the Force of Gravity is measured.—The Path of a Projectile is a Parabola.

INTRODUCTION.

484. The branch of mechanics which treats of motion and the forces producing it is called dynamics, and is rather more difficult than statics, with which subject we have been hitherto occupied; the difficulty arises from the introduction of a new element, time, into our calculations. The principles of dynamics were unknown to the ancients. Galileo discovered some of its truths in the seventeenth century; and, since his time, the science has grown rapidly. The motion of a falling body was first correctly described by Galileo; with this subject we can appropriately commence the lectures on dynamics.

THE FIRST LAW OF MOTION.

485. Velocity, in ordinary language, is supposed to convey a notion of rapid motion. Such is not pre-

cisely the meaning of the word in mechanics. By velocity is meant the rate at which a body moves, whether the rate be fast or slow. This rate is most conveniently measured by the number of feet moved over in one second. Hence, when it is said the velocity of a body is 25, it is meant that if the body continued to move for one second with its velocity unaltered, it would in that time have moved over 25 feet.

486. The first law of motion may be stated thus. If no force act upon a body, it will, if at rest, remain for ever at rest; or if in motion, it will continue for ever to move with a uniform velocity. We know this law to be true, and yet no one has ever seen it to be true for the simple reason that we cannot realize the condition which it requires. We cannot place a body in the condition of being unacted upon by any forces. But we may convince ourselves of the truth of the law by some such reasoning as the following. If a stone be thrown along the road, it soon comes to rest. The stone leaves the hand with a certain velocity and receives no more force from the hand. Hence, if no other force acted upon it, we should expect, if the first law be true, that it would continue to run on for ever with the velocity it had at the moment of leaving the hand. But other forces do act upon the stone; the attraction of the earth pulls it down; and, when it begins to bound and roll upon the ground, friction commences to act, deprives it of its velocity, and finally brings it to rest. But let the stone be thrown upon a surface of smooth ice; when it begins to slide, the force of gravity is counteracted by the reaction of the ice: there is no other force acting upon the stone except friction, which is small. Hence we find that the stone will run on for an enormous

distance. It requires but little effort of the imagination to suppose a lake whose surface is an infinite plane, perfectly smooth, and that the stone is perfectly smooth also. In such a case as this the first law of motion amounts to the assertion that the stone would never stop.

487. We may, in the lecture room, see the truth of this law verified to a certain extent by Atwood's machine (Fig. 66). This machine has been devised for the purpose of investigating the laws of motion by actual experiment. It consists principally of a pulley C, which is mounted so that its axle rests upon two pairs of wheels, as shown in the figure; the object of this contrivance is to get rid of friction, as already described (Art. 172). A pair of equal weights A, B are attached by a silken thread, which passes over the pulley; when one of the weights is set in motion, its weight is completely counterbalanced by the other: we may consider it not to be

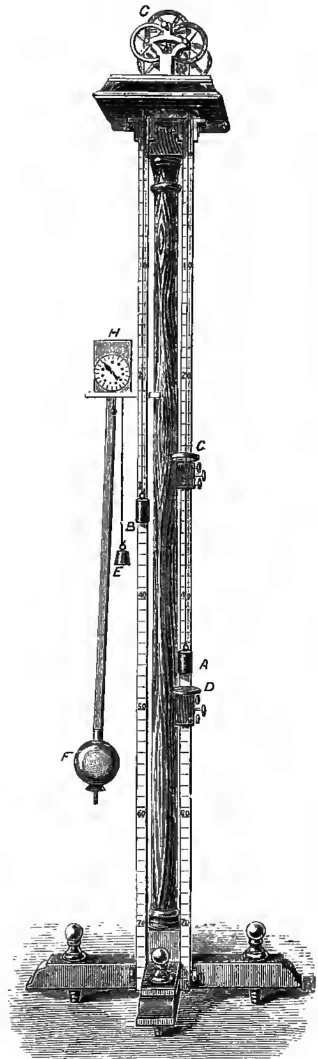

FIG. 66.

acted upon by any forces, and you see that it moves uniformly, as far as the length of the thread will permit.

488. If we try to conceive a body free in space, and not acted upon by any force, it is more natural to suppose that such a body, when once started, should go on moving uniformly for ever, than that its velocity should be altered according to what must be some arbitrary law. The true proof of the first law of motion is, however, that all consequences properly deduced from it, in combination with other principles, are found to be verified. Astronomy presents us with the best examples. The calculation of the time of an eclipse is based upon laws which in themselves assume the first law of motion; hence, when we invariably find that an eclipse occurs precisely at the moment at which it has been predicted, we have a splendid proof of the sublime truth which the first law of motion expresses.

THE EXPERIMENT OF GALILEO FROM THE TOWER OF PISA.

489. The contrast between heavy and light bodies is so marked that it is difficult at first to admit that a heavy body and a light body will fall from the same height in the same time. That they do so Galileo proved by dropping a heavy ball and a light ball together from the top of the Leaning Tower at Pisa. They were found to reach the ground simultaneously. We shall repeat this experiment on a smaller scale, and then we shall ascertain what the phenomenon teaches.

490. The apparatus used is that of Fig. 67. It consists of a stout framework supporting a pulley H at a height of about 20 feet above the ground. This pulley

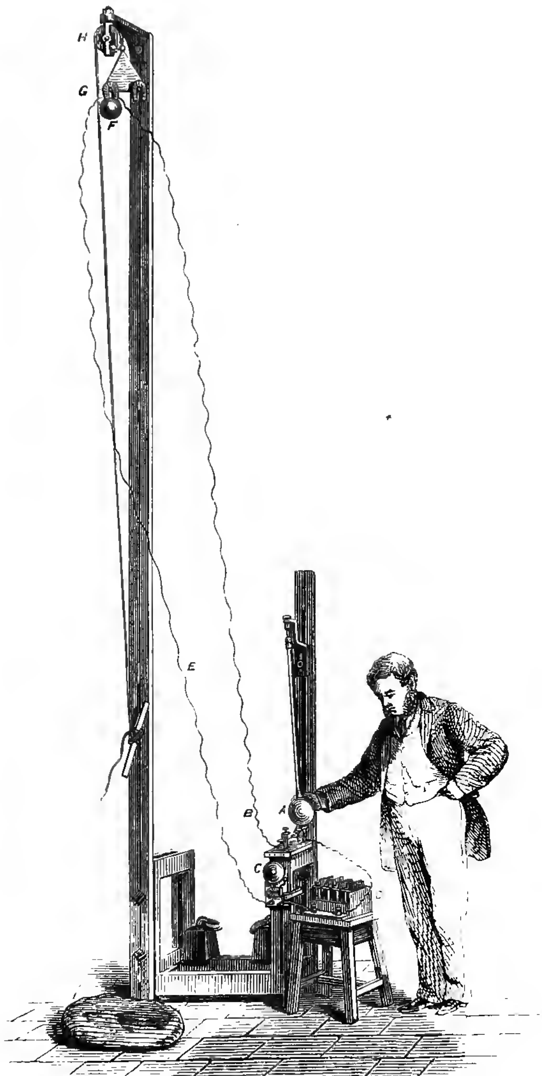

FIG. 67.

carries a rope; one end of the rope is attached to a triangular piece of wood, to which two electro-magnets G are fastened. The electro-magnet is a piece of iron in the form of a horse-shoe, around which is coiled a long wire. The horse-shoe becomes a magnet immediately an electric current passes through the wire; it remains a magnet as long as the current passes, and returns to its original condition the moment the current ceases. Hence, if I have the means of controlling the current, I have complete control of the magnet; you see this ball of iron remains attached to the magnet as long as the current passes, but drops the instant I break the current. The same electric circuit includes both the magnets; each of them will hold up an iron ball F when the current passes, but the moment the current is broken both balls will be released. Electricity travels along a wire with prodigious velocity. It would pass over many thousands of miles in a second; hence the time that it takes to pass through the wires we are employing is quite inappreciable. A piece of thin paper interposed between the balls and the magnets will ensure the balls being dropped simultaneously; when this precaution is not taken one or both of the balls may hesitate a little before commencing to descend. A long pair of wires E, B must be attached to the magnets, the other ends of the wires communicating with the battery D; the triangle and its load is hoisted up by means of the rope and pulley, and the magnets thus carry the balls up to a height of 20 feet; the balls we are using weigh about 0·25 lb. and 1 lb.

491. We are now ready to perform the experiment. I break the circuit; the two balls are disengaged simultaneously; they fall side by side the whole way, and reach the ground together, where it is well to place a

cushion to receive them. Thus you see the heavy ball and the light ball fall in the same amount of time from the same height.

492. But these balls are both of iron ; let us compare together balls made of different substances, iron and wood for example. A flat-headed nail is driven into a wooden ball of about 2″·5 in diameter, and by means of the iron head of the nail I can attach this ball to the magnet ; this wooden ball is on one magnet, while an iron ball is on the other. I repeat the experiment in the same manner, and you see these also fall together ; finally, when an iron ball and a cork ball are dropped, the latter is within two or three inches of its weighty companion when the cushion is reached : this small difference is due to the unequal effect of the resistance of the air on the two different balls. There can be no doubt that in a vacuum all bodies of whatever size or material would fall precisely in the same time.

493. How is the fact that all bodies fall in the same time to be explained ? Let us first consider two iron balls. Take two equal particles of iron : it is evident that these fall in the same time ; they would do so if they were very close together, even if they were touching, but then they might as well be in one piece ; and thus we should find that a body consisting of two particles takes the same time to fall as one particle (omitting of course the resistance of the air). Thus it appears most reasonable that two balls of iron, even though unequal in size, should fall in the same time.

494. The case of the wooden ball and the iron ball will require a little thought in order to realize thoroughly how much Galileo's experiment really proves. We must first explain the meaning of the word *mass* in mechanics.

495. It is not correct to define *mass* by the introduction of the idea of weight, because the *mass* of a body is something independent of the existence of the earth, whereas weight is produced by the attraction of the earth. It is true that weight is a convenient means of measuring *mass*, but this is only a consequence of the property of gravity which the experiment proves, namely, that the attraction of gravity for a body is proportional to its *mass*.

496. Let us select as the unit of mass the mass of a piece of platinum which weighs 1 lb.; it is then evident that the mass of any other piece of platinum will be expressed by the ratio which it bears to the standard piece: but how are we to determine the mass of some other substance, such as iron? A piece of iron has the same mass as a piece of platinum, if the same force acting on either of the bodies for the same time produce the same velocity. This is the proper test of the equality of masses. The mass of any other piece of iron will be represented by the number of times it contains a piece equal to that which we have just compared with the platinum; similarly of course for other substances.

497. The magnitude of a force acting for a given time is measured by the product of the mass set in motion and the velocity which it has acquired. This is a truth established, like the first law of motion, by indirect evidence.

498. Let us now apply these principles to explain the experiment which showed us that a ball of wood and a ball of iron fall in the same time. Forces act upon the two bodies for the same time, but the magnitude of the forces must be proportional to the mass of each body multiplied into its velocity, and, since the bodies fall simultaneously, their velocities are equal. The forces

acting upon the bodies are therefore proportional to their masses; but the force acting on each body is the attraction of the earth, therefore, the attraction of gravitation upon different bodies is proportional to their masses.

499. This may be illustrated by contrasting the attraction of gravitation with that of a magnet. A magnet attracts iron powerfully and wood not at all; but the earth recognizing no such difference, draws all bodies towards it with forces proportional to their masses.

THE SPACE DESCRIBED BY A FALLING BODY IS PROPORTIONAL TO THE SQUARE OF THE TIME.

500. It is necessary for us to inquire into the law by which we can ascertain the distance a body will fall in a given time; it is not possible to experiment directly upon this subject, as in two seconds a body will fall 64 feet and acquire a prodigious velocity; we can, however, resort to Atwood's machine (Fig. 66) as a means of diminishing the motion. For this purpose we require a pendulum with a clock whose pendulum beats seconds.

501. On one of the equal weights A I place a slight brass rod, whose weight gives a preponderance to A, which would consequently descend. I hold the loaded weight in my hand, and release it simultaneously with the tick of the pendulum. I observe that it descends 5″ before the next tick. Returning the weight to the place from whence it started, I release it again, and I find that at the second tick of the pendulum it has travelled 20″. Similarly we find that in three seconds it descends 45″. It greatly facilitates these experiments to use a little stage which is capable of being slipped up and down the scale, and which can be clamped to the scale in

any position. By actually placing the stage at the distance of 5″, 20,″ or 45″ below the point from which the weight starts, the coincidence of the tick of the pendulum with the tap of the weight on its arrival at the stage is very marked.

502. These three distances are in the proportion of 1, 4, 9; that is, as the squares of the numbers of seconds 1, 2, 3. Hence we may infer that in falling bodies the space described is proportional to the square of the time.

503. The motion of the bodies in Atwood's machine is very different from the motion of a body falling freely, but the nature of the law in the two cases is the same. In a body falling freely, the space described is proportional to the square of the time. Atwood's machine cannot, without some difficulty, tell us the actual space through which a body falls in one second. If we can find this distance by other means, we shall easily be able to find the space through which a body will fall in any number of seconds.

A BODY FALLS 16′ IN THE FIRST SECOND.

504. The apparatus by which this important truth may be demonstrated is shown in Fig. 67. A part of it has been already employed in repeating the experiment of Galileo, but two other parts must now be used which will be briefly explained.

505. At A is shown a pendulum which vibrates once every second; it need not be connected with any clockwork to sustain the motion, as when once set vibrating it will continue to swing some hundreds of times. When this pendulum is at the middle of its swing, the bob just touches a slender spring, and presses it slightly

downwards. The electric current which circulates about the magnets G already described passes through this spring when in its natural position; but when the spring is pressed down by the pendulum, the current is interrupted. The consequence is that, as the pendulum swings backwards and forwards, the current is broken once every second. There is also in the circuit a little electric alarum bell C, which is so arranged that, when the current passes, the hammer is drawn from the bell; but, when the current is interrupted, a spring forces the hammer against the bell and strikes it. When the circuit is closed, the hammer is again drawn back. The pendulum and the bell are in the same circuit, and thus every vibration of the pendulum produces a stroke of the bell. We may regard the strokes from the bell as the ticks of the pendulum rendered audible to the whole room.

506. You will now understand the mode of experimenting. I draw the pendulum aside so that the current passes uninterruptedly. An iron ball is attached to one of the electro-magnets, and it is then gently hoisted up until the height of the ball from the ground is about 16′. A cushion is placed under the ball in order to receive it when it falls. You are to keep your eyes upon the cushion while you listen for the bell. All being ready, the pendulum, which has been held at a slight inclination, is released. The moment the pendulum reaches the middle of its swing it touches the spring, rings the bell, breaks the current which circulated around the magnet, and as there is now nothing to keep up the ball, the ball falls to the cushion; but just as it arrives at the cushion, the pendulum has a second time broken the circuit, and you observe the falling of the ball upon the cushion to be identical with

the second stroke of the bell. As these strokes are repeated at intervals of a second, it follows that the ball has fallen 16′ in one second. If the magnet be raised a few feet higher, the ball may be seen to reach the cushion after the bell is heard. If the magnet be lowered a few feet, the ball reaches the cushion before the bell is heard.

507. We have previously shown that the space is proportional to the square of the time. We now see that when the time is one second, the space is 16′. Hence if the time were two seconds, the space would be $4 \times 16 = 64$ feet; and in general the space in feet is equal to 16 multiplied by the square of the time in seconds.

508. By the help of this rule we are sometimes enabled to ascertain the height of a perpendicular cliff, or the depth of a well. For this purpose it is convenient to use a stop-watch, which will enable us to measure a short interval of time accurately to one-fifth of a second. One person drops a stone into a well; a second observer, who has the watch, starts it the moment the stone moves. He then listens carefully till he hears the sound of the stone striking the water at the bottom of the well, and then he stops the watch. The interval recorded shows the time of descent; the square of the number of seconds (taking account of fractional parts) multiplied by 16 gives the depth of the well.

THE ACTION OF GRAVITY IS INDEPENDENT OF THE MOTION OF THE BODY.

509. We have already learned that the effect of gravity in moving a body does not depend upon the nature of the body. We have now to learn that its effect is un-

influenced by any motion which the body may possess. Gravity pulls a body down 16′ per second, if the body start from rest. But suppose a stone be thrown upwards with a velocity of 20 feet, where will it be at the end of a second? Did gravity not act upon the stone, it would be at a height of 20 feet. The principle we have stated tells us that gravity will draw this stone towards the earth through a distance of 16′, just as it would have done if the stone had started from rest. Since the stone ascends 20′ in consequence of its own velocity, and is pulled back 16′ by gravity, it will, at the end of a second, be found at the height of 4′. If, instead of being shot up vertically, the body had been projected in any other direction, the result would have been the same; gravity would have brought the body at the end of one second 16′ nearer the earth than it would have been had gravity not acted. For example, if a body had been shot vertically downwards with a velocity of 20′, it would in one second have moved through a space of 36′.

510. We shall prove one case of this remarkable property by experiment. The principle of doing so is as follows:—Suppose we take two bodies, A and B. If these be held at the same height from the ground and released together, of course they reach the ground at the same instant; but if A, instead of being merely dropped, be projected with a horizontal velocity at the same moment that B is released, it is still found that A and B reach the ground together.

511. You may very simply try this on a level floor. In your left hand hold a marble, and drop it at the same instant that your right hand throws another marble horizontally. It will be seen that the two marbles reach the ground together.

512. A more accurate mode of making the experiment is shown by the apparatus of Fig. 68.

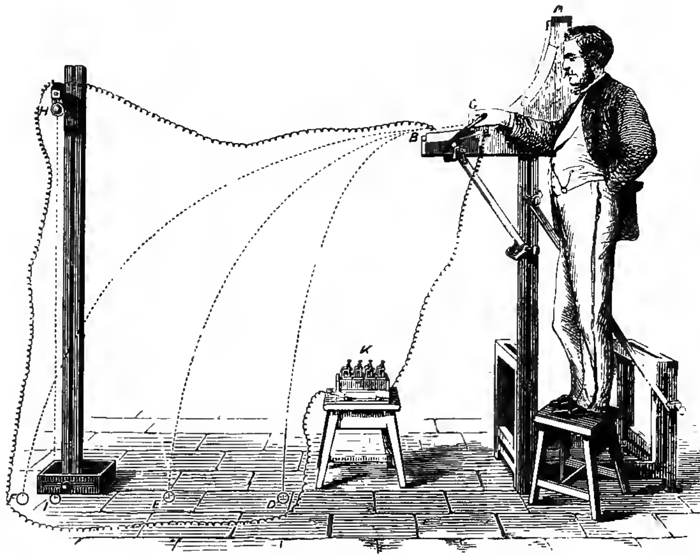

FIG. 68.

In this we have an arrangement by which we ensure that one ball shall be released just as the other is projected. At AB is shown a piece of wood about 2″ thick; the circular portion (2′ radius) on which the ball rests is grooved, so that the ball only touches the two edges and not the bottom of the groove. Each edge of the groove is covered with tinfoil C, but the pieces of tinfoil on the two sides must not communicate. One edge is connected with one pole of the battery K, and the other edge with the other pole, but the current is unable to pass until a communication by a conductor is opened between the two edges. The ball G supplies the bridge; it is covered with tinfoil, and therefore, as long as it is upon the groove, the circuit is complete; the groove is so placed that the tangent to it at the lowest point B is horizontal, and therefore, when the ball rolls down the curve, it is projected from the bottom in a horizontal direction. A spring is shown in the figure; by drawing back the ball G, when embraced by the spring, I can communicate to the ball any amount of velocity within reasonable limits. At H we have an electro-magnet, the wire around which forms part of the circuit we have been considering. This magnet is so placed that a ball suspended from it is precisely at the same height above the floor, as the tinned ball is at the moment when it leaves the groove.

513. You now understand the mode of proceeding. Let the tinned ball be called G, and H be the ball attached to the electro-magnet; as long as G is on the curve, H is held up, but the moment G leaves the curve, H is let fall. We find invariably that whatever be the velocity with which G is projected, it reaches the ground at the same instant as H arrives there. Various dotted lines in the figure show the different paths which G may traverse; but whether it fall at D, at E, or at I, the time is invariably the same as that taken by H. Of course, if G were not projected horizontally, we should not have arrived at this result: all we assert is, that whatever be the motion of a body, it will be at the end of a second, sixteen feet nearer the earth (if possible) than it would have been if gravity had not acted. If the body be projected horizontally, its descent is due to gravity alone, and is neither accelerated nor retarded by the horizontal velocity. What this experiment proves is, that the mere fact of a body having velocity does not affect the action of gravity upon the body.

514. Though we have only shown that a horizontal velocity does not affect the action of gravity, yet neither

does a velocity in any direction. This is verified, like the first law of motion, by the complete accordance of the consequences deduced from it with observed facts.

515. We may summarize these results by saying that no matter what be the material of which a body is composed, whether it be large or small, moving or at rest, if gravity act upon the body for t seconds, it will be $16t^2$ feet nearer the earth at the end of that time, than it would have been had gravity not acted.

516. A proposition which is of some importance may be introduced here. Let us suppose a certain velocity and a certain force. Let the velocity be such that a point starting from A, Fig. 69, would in one second move

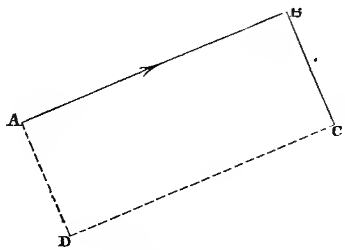

FIG. 69.

uniformly to B. Let the force be such that if it acted on a particle originally at rest at A, it would in one second draw the particle to D; if then the force and the velocity act together, where will the particle be at the end of the second? Complete the parallelogram A B C D, and the particle will be found at C. By what we have seen the force will discharge its duty whether the body have any velocity or not. The force will make the particle move to a distance A D, in a direction parallel to A D from whatever position the particle would have assumed, had the force not acted; but had the force not acted, the particle would have been found at B: hence, when the force does act, the particle must be found at C, since B C is equal and parallel to A D.

HOW THE FORCE OF GRAVITY IS MEASURED.

517. From the formula

$$\text{Space} = 16t^2,$$

we learn that a body falls through 64′ in 2 seconds; therefore, since it falls 16′ in the first second, it must fall 48′ in the second second. Let us examine this. After falling for one second, the body acquires a certain velocity, and with that velocity it commences the next second. Now, according to what we have just seen, gravity will act during the next second, quite independently of whatever velocity the body may have previously had. Hence in the second second gravity pulls the body down 16′, but the body moves altogether through 48′; therefore it must move through 32′ in consequence of the velocity which has been impressed upon it by gravity in the first second. We learn by this that when gravity acts for a second, it produces a velocity such that, if the body be conceived to move uniformly with the velocity acquired, the body would in one second pass through 32′.

518. In three seconds the body falls 144′, therefore in the third second it must have fallen

$$144' - 64' = 80';$$

but of this 80′ only 16′ could be due to the action of gravity impressed during that second: the rest,

$$80' - 16' = 64',$$

is due to the velocity with which the body commenced the third second.

519. We see therefore that after the lapse of two seconds gravity has communicated to the body a velocity of 64′ per second; we should similarly find, that at the end of the third second, the body has a velocity of 96′, and in general at the end of t seconds a velocity of $32t$. This proves the remarkable truth that the velocity developed by gravity is proportional to the time.

520. This law points out to us that the proper way of measuring gravity is by the velocity produced in a falling body at the end of one second. Hence we are accustomed to say that g (as gravity is generally designated) is 32. We shall afterwards show in the lecture on the pendulum (XVIII.) how the value of g can be obtained accurately. From the two equations, $v = 32t$ and $s = 16t^2$, it is easy to infer another very well known formula, namely, $v^2 = 64s$.

THE PATH OF A PROJECTILE IS A PARABOLA.

521. We have already seen, in the experiments of Fig. 68, that a body projected horizontally describes a curved path on its way to the ground. We are now going to examine into the nature of this path. The movement being rapid, it is difficult to follow the path sufficiently to ascertain its nature; we must therefore adopt special means for definitely observing the form. This can be done by the apparatus represented in Fig. 70.

B C is a quadrant of wood 2″ thick; it contains a groove, along which the ball B will run when released. A series of cardboard hoops are properly placed on a black board, and the ball, when it leaves the quadrant, will pass through all these hoops without touching any, and finally fall into a basket placed to receive it. The quadrant must be secured firmly, and the ball must always start from precisely the same place. This may be done by bringing the ball home against a little ledge at the top of the quadrant. The hoops are easily adjusted: if the ball run down the quadrant two or three times, we can see how to place the first hoop in its right position, and secure it by drawing pins; then by a few more trials

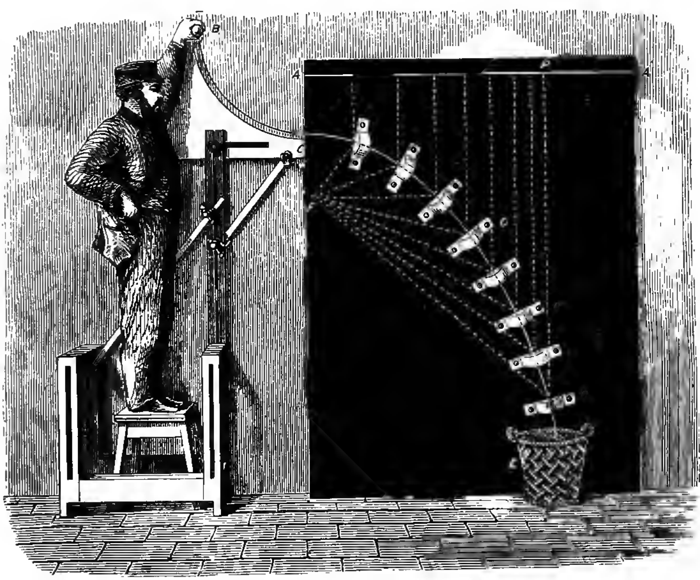

FIG. 70.

the next hoop is to be adjusted, and so on for the whole eight.

522. The curved line from the bottom of the quadrant, which passes through the centre of the hoops, is the path in which the ball moves; this curve is a parabola, of which F is the focus and the line A A the directrix.

It is a property of the parabola that the distance of any point on the curve from the focus is equal to its perpendicular distance from the directrix. This is shown in the figure. For example, the dotted line F D, drawn from F to the lowest hoop D, is equal in length to the perpendicular D P let fall from D on the directrix A A.

523. The direction in which the ball is projected is in this case horizontal, but, whatever be the direction of projection, the path is a parabola. This can be proved directly from the theorem of Art. 516.

LECTURE XVI.

THE FORCE OF INERTIA.

Inertia is a Force.—The Hammer.—The Storing of Energy.—The Fly-wheel.—The Punching Machine.

INERTIA IS A FORCE.

524. A BODY unacted upon by force will continue for ever at rest, or for ever moving uniformly in a straight line. This is asserted by the first law of motion (Art. 486). When a force tries to change either the velocity or the direction of the motion, the body resists. The force with which a body resists interference is called the force of inertia.

525. Let us see how we can make the existence of this force manifest. I have here an india-rubber spring; if I pull it at both ends it is stretched, but pulling one end will not stretch it: hence, whenever we find a spring to be stretched, there must be a force pulling it at each end. Here is a heavy weight, 25 lbs., attached to a wire which hangs from the ceiling. I fasten one end of the spring to the weight and pull it; the weight is moved, but to move it the spring was stretched. Hence there must have been a force exerted by the weight to stretch the spring. This is the force of inertia, which the weight manifested when a force endeavoured to

disturb it from its position of rest. The ball is now swinging to and fro. If, holding one end of the spring, I endeavour to stop the ball, you see the spring is stretched again: this is due to the force of inertia, with which the weight, now in motion, seeks to avoid being stopped.

526. If I place a weight upon the table, the spring attached to it will be stretched before the weight can be moved; but in this case, the friction between the table and the weight has to be overcome in addition to the inertia, and therefore I have preferred the swinging ball where there is no friction.

527. We can also show inertia to be a force, according to the strict definition of force. The inertia of one body can produce or destroy motion in another. Two equal balls of putty, or some other substance possessing no elasticity, when thrown one against the other with equal velocities, destroy one another's motion by the collision, and come to rest; the motion of each ball is stopped by the inertia of the other. Here we have the force of inertia, manifested by the destruction of motion. Had one of the balls been at rest, it would be put in motion when struck by the other ball: the striking ball loses some of its velocity; this it cannot do without exerting force on the body which has arrested it, and this force it is which causes the arresting body to be put in motion. Inertia can stretch a spring; it can put a body in motion, or it can stop motion; and therefore it is in every respect a force.

528. Notwithstanding what I have said, perhaps some of you feel a difficulty in recognizing this force. You may say it is the blow which has sent on the ball; so it is, but the blow only has its efficacy in consequence of this property of inertia which all matter possesses.

Another point which presents some difficulty, is the uncertain amount of the force. The force of inertia, developed when a body is stopped, depends upon the manner in which it is stopped. If suddenly, the inertia is enormous; if gradually, the inertia is very small. This will, it is hoped, be made clear presently.

529. Inertia is a property inherent in matter. Friction can be avoided or diminished, inertia cannot. Could inertia be evaded, the din of battle must cease, for missiles would be powerless, and blows would lose their efficacy; railway collisions would be harmless and explosions without danger; but these advantages would be dearly gained: for were inertia suspended, the moon would fall upon the earth and the earth tumble into the sun.

THE HAMMER.

530. The hammer and other tools which give a blow, depend for their action upon inertia. The mere weight of the head of a hammer produces no effect, if only laid upon the nail; it requires to be brought down with a smart blow. What is the reason of this? We have here inertia acting as a mechanical power, overcoming the great resistance which the wood opposes to the entrance of the nail. The nail would probably require a direct pressure of some hundreds of pounds, were it not for assistance we receive from inertia.

531. We can study this property by the apparatus shown in Fig. 71. This consists of a tripod, at the top of which, about 9′ from the ground, is a stout pulley C; the rope is about 15′ long, and to each end of it a 14 lb. weight is attached. These weights are shown at A and B.

I raise A up to the pulley, leaving B upon the ground; I then let go the rope, and down falls A: it first pulls the slack rope through, and then, when A is about 3′ from the ground, the rope becomes tight, B gets a violent

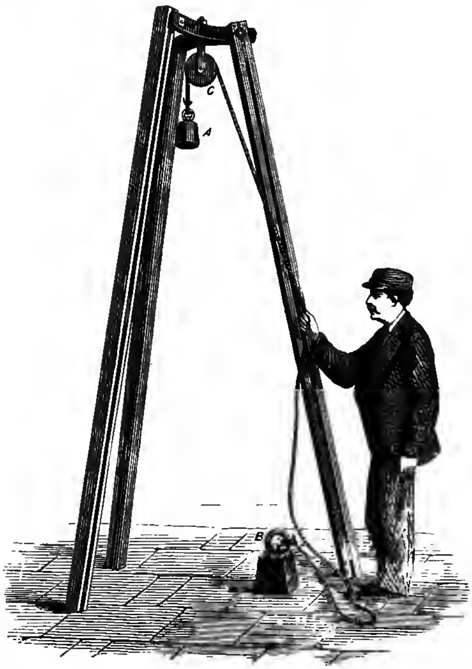

FIG. 71.

chuck and is lifted into the air. What has raised B? It cannot be the mere weight of A, because that being equal to B, could only just balance B, and is insufficient to raise it. You may say it was the 'chuck' which raised it; so it was, only give the 'chuck' the proper name which belongs to it in mechanics. It must have been a force which raised B; that force could not have been the mere weight of A, yet it was produced by A when its motion was arrested. A was not stopped completely; it only lost some of its velocity, but it could not lose any velocity without opposing resistance: this resistance must take the form of a pull on the rope by which A was held back, and the force of inertia thus produced and transmitted by the rope was added to the weight of A in pulling up B. You see, therefore, that there were two distinct forces concerned in the process.

532. Let us remove the 14 lb. weight from B, and attach there a weight of 28 lbs., A remaining the same as before (14 lbs.). I raise A to the pulley; I allow it to fall. You observe that B, though double the weight of A, is again chucked up after the rope has become tight. We can only explain this by the supposition that the force of inertia is sufficient, when added to the weight of A, to raise up 28 lbs. Hence the inertia must be greater than 14 lbs.; for, were it only equal to 14 lbs., B would not be raised up, though it would be balanced.

533. Finally, let us remove the 28 lbs. from B, put on 56 lbs., and repeat the experiment again; you see that even the 56 lbs. is raised up several inches. Here A, when aided by the force of inertia, has actually overcome a weight four times its amount. We have then, by the help of inertia, a mechanical power, for a small force has overcome a greater.

534. After B is raised by the chuck to a certain height it descends again, if heavier than A, and raises A. The height to which B is raised is of course the same as the height through which A descends while it is exerting the force of inertia. You noticed that the height through which 28 lbs. was raised, was considerably greater than that through which the 56 lbs. was raised. Hence we may draw the inference, that when A was deprived of its motion while passing through a short

space, it resisted with a greater force of inertia than when it was gradually deprived of that motion through a longer space. This is a most important point. Supposing I put a hundredweight at B, I have little doubt, if the rope were strong enough to bear the strain, that the inertia of A would raise B a little, but only a little: hence A would be deprived of its motion in a very short space, but the force of inertia exerted would be very great.

535. But it is clear that matters would not be much altered if A were to be stopped by some force, exerted from below rather than above; in fact, we may conceive the rope omitted, and suppose A to be a hammer-head falling upon a nail in a piece of wood. The blow would drive the nail slightly deeper, and the entire velocity of A would have to be destroyed while moving through a small distance: consequently the inertia of A would exert a large force. This explains the effect of a blow.

536. In the case that we have supposed, the weight merely falls upon the nail: this is actually the principle of the hammer used in pile-driving machines. A pile is a large piece of timber, pointed and shod with iron at one end: this end is driven down into the ground. Piles are required for various purposes in engineering operations. They are often intended to support heavy loads, such as buildings; they are therefore driven until the resistance with which the ground opposes their further entrance affords a guarantee that they shall be able to bear what is required.

537. The machine for driving piles consists essentially of a heavy mass of iron, which is raised to a height, and allowed to fall upon the pile. The resistance to be overcome depends upon the depth and nature of the soil: a

pile may be driven two or three inches with each blow, but the less the distance the pile enters each time, the greater is the actual force with which the inertia of the weight forces it downwards. In the ordinary hammer, the power of the arm imparts velocity to the hammer-head, in addition to that which is due to the fall; the effect produced is merely the same as if the hammer had fallen from a greater height.

538. Another point may be mentioned here. A nail will only enter a piece of wood when the nail and the wood are pressed together with sufficient force. The nail is urged by the hammer; but what is the force acting upon the wood? If this be lying on the ground, or against a wall, the reaction of the ground or the wall is ample; but in many cases the principal force on which the wood must rely, is its own inertia, by which it resists motion. If the wood be thin and unsupported at the back, the inertia is not sufficient to supply force enough, and the nail, consequently, does not enter. The usual remedy is obvious. Hold a heavy mass of iron close at the back of the wood: if the wood and iron together have sufficient inertia, the nail will enter

THE STORING OF ENERGY.

539. Our conceptions of inertia will be very much facilitated by some considerations founded on the principles of energy. In the experiment of Fig. 71 let A be 14 lbs., and B, on the ground, be 56 lbs. Since the rope is 15′ long, A is 3′ from the ground, and therefore 6′ from the pulley. I raise A to the pulley, and, in doing so, expend $6 \times 14 = 84$ units of energy. Energy is never lost, and therefore I shall expect to recover this

amount. I allow A to fall; when it has fallen 6′, it is then precisely in the same condition as it was before being raised, except that it has a considerable velocity of descent. In fact, the 84 units of energy have been expended in giving A a velocity. The strain raises B, and it ascends to a height X; to raise B, 56 × X units of work have been consumed. At the instant when B is at the height X, A must be at a distance of 6 + X feet from the pulley; hence the quantity of work performed by A is 14 × (6 + X). But the work done by A must be equal to that done upon B, and therefore

$$14\ (6 + x) = 56\ x,$$

whence X = 2. If there were no loss by friction, B would be raised 2′; but owing to friction, and doubtless also to the rigidity of the rope, B is not raised so much. The distance, as you see, is not even one foot. We may regard the work done in raising A as energy stored up, until A is allowed to fall, when the work is reproduced in a modified form.

540. Let us apply this principle to the pile-driving engine to which we have already referred; we shall then be able to see the actual magnitude of the force of inertia developed in producing the blow. Suppose the "monkey," that is the heavy mass of iron, weigh 560 lbs. (a quarter of a ton). A couple of men raise this by means of a small windlass to a height of 15′. It takes them perhaps a few minutes to do this; their energy is then stored up: they have expended 560 × 15 = 8,400 units of work. When the monkey reaches the top of the pile in its fall, it transfers to the pile the whole 8,400 units of work, and this is expended in forcing the pile into the ground. Suppose the pile to enter one inch, the reaction of the pile upon the monkey must be so great, that the number

of units of work performed in one inch is 8,400. Hence this reaction must be 8,400 × 12 = 100,800 lbs. If the reaction did not reach this amount, the monkey could not be brought to rest in the space of one inch. The reaction of the pile upon the monkey, and therefore the action of the monkey upon the pile, is about 45 tons. This is the actual pressure which has been exerted upon the pile.

541. If the stratum into which the pile is penetrating be more resisting than that which we have supposed,—for example, if the pile require a force of 100 tons to drive it in,—the same monkey with the same fall would still be sufficient, but the pile would not be driven so far with each blow. The pressure required is 224,000 lbs.: this exerted over a space of 0″·45 would be 8,400 units of work; hence the pile would be driven 0″·45. The more the resistance, the less the penetration produced by each blow. A pile which is permanently intended to bear a very heavy load, must be driven until it enters but little with each blow.

542. We may compare the pile-driver with the mechanical powers in one respect, and contrast it in another. In each, we have machines which receive energy and restore it modified into a greater power exerted over a smaller distance; but while the mechanical powers restore the energy at one end of the machine, simultaneously with their reception of it at the other, the pile-driver is a reservoir which receives the energy and does not restore it until all has been received.

543. We have, then, a class of mechanical powers, of which a hammer may be taken as the type, which depend upon the storage of energy; the force of the arm is stored in the hammer throughout its whole descent, to

be instantly transferred to the nail in the blow. Inertia is the property of matter which qualifies it for storing energy. Energy is developed by the explosion of gunpowder in a cannon. This energy is applied in overcoming the inertia of the ball: the ball strikes the target, and its inertia causes it to give a terrific blow. Here we see energy stored in a rapidly moving body, a case to which we shall presently return.

544. But energy can be stored in many ways; gunpowder is itself energy in a compact and storable form. The efforts which we make in forcing air into an air-cane are not lost; our energy is there stored for us, to be reproduced in the discharge of a number of bullets. During the few seconds occupied in winding a watch, the watch is given a small charge of energy which it economizes over the next twenty-four hours. In using a bow my energy is stored up from the moment I begin to pull the string until I release the arrow.

545. Many machines of extensive use depend upon this principle. In the clock or watch the demand for energy to sustain the motion is constant, while the supply is only occasional; in other cases the supply is constant, while the demand is only occasional. I may mention a good illustration of this. Suppose it be required *occasionally* to hoist heavy weights up to a great height. If an engine sufficiently powerful to raise the weights be employed, the engine will be idle except when the weights are being raised; and if the engine were to have much idle time, the waste of fuel in keeping up the fire during the intervals would make the arrangement very uneconomical. It would be a far better plan to have a smaller engine: and even though this were not powerful enough to raise

one of the weights directly, yet we might be able, by keeping the engine continually working and storing up its energy, to produce enough energy in the twenty-four hours to raise all the weights which it would be necessary to lift in the same time.

546. Let us suppose we want to raise slates from the bottom of a quarry to the surface. A large pulley is mounted at the top of the quarry, and over this a rope is passed: to each end of the rope a bucket is attached, so that when one bucket is at the bottom the other is at the top, and their sizes and that of the pulley are so arranged that the buckets can pass with safety. A reservoir is established at the top of the quarry on a level with the pulley, and an engine is set to work constantly pumping up water from the bottom of the quarry into the reservoir. Each of the buckets has a large tank attached to it, which can be quickly filled or emptied. The lower bucket is loaded with slates, and when ready for work, the man at the top fills the tank of the upper bucket with water: this bucket becomes so heavy that it descends and raises the slates. When the heavier bucket reaches the bottom, the water from its tank is let out into the lower reservoir, from which the engine pumps, and the slates are removed from the bucket which has been raised. The two buckets are then ready for the same operation again. If the slates be raised at intervals of ten minutes, the energy of the engine will be sufficient if, in ten minutes' work, it can pump up enough water to fill one tank; therefore, by the aid of the water, we are able to accumulate for one effort the whole power of the engine for ten minutes. The same water may of course be used over and over again.

THE FLY-WHEEL.

547. One of the best means of storing up energy is by setting a heavy body in rapid motion. This has already been referred to in the case of the cannon-ball. In order to render this method practically available for the purposes of machinery, the heavy body we use is a fly-wheel, and the rapid motion imparted to it is that of rotation about its axis. A very large amount of energy can by this means be stored in a convenient and accessible form.

548. We shall illustrate the principle by the apparatus of Fig. 72. This represents a fly-wheel of iron B: its dia-

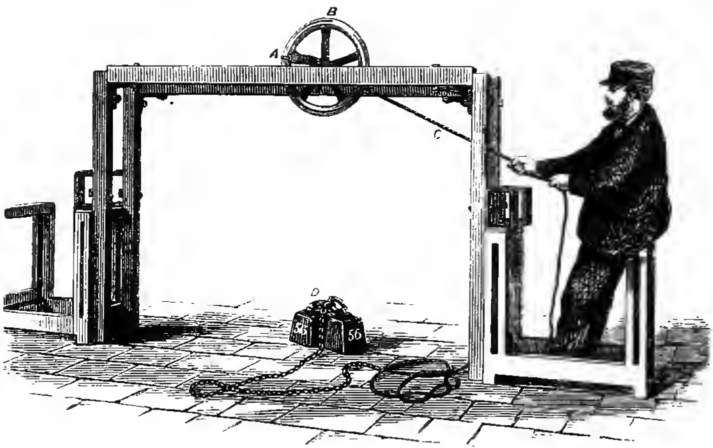

FIG. 72.

meter is 18″, and its weight 26 lbs.; the fly is carried upon a shaft (A) of wrought iron $\frac{3}{4}''$ in diameter. We shall store up a quantity of energy in this wheel, by setting it in rapid motion, and then we shall see how it will return to us the energy we have imparted.

549. A rope is coiled around the shaft; by pulling this rope the wheel is made to turn round: thus the rope is the medium by which my energy shall be imparted to the wheel. I need not catch hold of the rope directly, but I can attach it to the hook of the spring balance (Fig. 9); by taking the ring of the balance in my hand, I see by the index the amount of the force I am exerting. I find that when I walk backwards as quickly as is convenient, pulling the rope all the time, the scale shows a strain of about 50 lbs. What is it that produces the strain on the scale? There must be a force of 50 lbs. pulling at each end. My hand imparts one of these forces; the other is imparted by the inertia with which the wheel resists. To set the wheel rapidly in motion, I pull about 20′ of rope from the axle, so that I have imparted to the wheel somewhere about $50 \times 20 = 1{,}000$ units of energy. The rope is fastened to the shaft, so that, after the rope has been all unwound, the wheel begins to wind it in again. By measuring the time in which the wheel made a certain number of coils of the rope around the shaft, I am able to see that the wheel is rotating at about the rate of 600 revolutions per minute.

550. Let us see how the stored-up energy can be withdrawn. A piece of pine $24'' \times 1'' \times 1''$ requires a force of about 300 lbs. applied to its centre to produce fracture when both ends are supported. I arrange such a piece of pine near the wheel. As the shaft is winding in the rope, a tremendous chuck would be given to anything which tried to stop the rope. If I tied the end of the rope to the piece of pine, the chuck would break the rope; therefore I have fastened one end of a 10′ length of chain to the rope, and the other has been tied round the middle of the pine-rod. The wheel first winds in the rope, then

the chain takes a few turns before it tightens, when crack goes the bar of pine. The wheel had no choice; it must either stop or break the bar: but nature forbids it to stop without exerting its great force of inertia, and that force was sufficient to break the bar. Here I never exerted a force greater than 50 lbs. in setting the wheel in motion. The wheel stored up and modified my energy into a force of 300 lbs., which, however, had only to be exerted over a very small distance.

551. But we may show the experiment in another way, which is that represented in the figure (72). We see the chain is there attached to two 56 lb. weights. The mode of proceeding is that already described. The rope is first wound round the shaft, then by pulling the rope the wheel is made to revolve; the wheel then begins to wind in the rope again, and when the chain tightens the two 56 lbs. are raised up to a height of 3 or 4 feet. Here, again, the force has been stored and modified. But though the fly-wheel will keep energy stored up, it does so at some cost: the energy is continually being wasted on friction and the resistance of the air; in fact, the energy would altogether disappear in a little time, and the wheel would come to rest; it is therefore desirable to make the wheel yield up what it has received as soon as convenient.

552. We can easily see the part which a fly-wheel fulfils in a steam-engine. The action of the steam upon the piston varies according to the different parts of the stroke; the fly-wheel obviates the inconvenience which would arise from this irregularity. Its great inertia makes it but little affected by the exuberant action of the piston when its power is a maximum, while the same inertia sustains the motion when the piston is

giving no assistance. The fly-wheel is a vast reservoir into which the engine pours its energy, sudden floods alternating with droughts; but these succeed each other so rapidly, and the area of the reservoir is so vast, that its level remains uniform, and therefore the supplies sent out to the consumers are regular and unvaried. The consumers of the energy stored in the fly-wheel of an engine are the machines in the mill; they are supplied by shafts which traverse the building, conveying, by their rotation, the energy originally condensed within the coal from which combustion has set it free.

THE PUNCHING MACHINE.

553. When energy has been stored in a fly-wheel, it can be withdrawn either as a small force acting over a great distance, or a large force over a small distance. In the latter case the fly-wheel acts as a mechanical power, and it is in this form that it is used in the very important machine to be next described. A model of the punching machine is shown in Fig. 73.

The punching machine is usually worked by a steam-engine, but a handle will move the model. The handle turns a shaft on which the fly-wheel F is mounted. On the shaft is a small pinion D of 40 teeth: this works into a large wheel E of 200 teeth, so that, when the fly and the pinion have turned round 5 times, E will have turned round once. C is a circular piece of wood called a cam, which has a hole bored through it, between the centre and circumference; by means of this hole, the cam is mounted on the same axle as E, to which it is rigidly fastened, so that the two must revolve together. A is a lever of the first order, whose fulcrum is at A: the power-end of this lever rests upon the cam C; the other end B contains the punch. As the wheel E revolves it carries with it the cam: this raises the lever and forces the punch down a hole in a die into which it fits exactly. The plate of metal to be punched is placed under the punch before it is depressed by the cam, and the pressure drives the punch through, cutting out a cylindrical piece of metal from the hole: this model will, as you see, punch ordinary tin-plate.

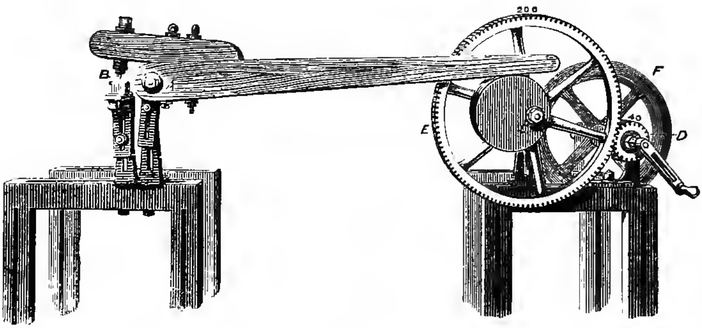

FIG. 73.

554. Let us examine the mode of action. The fly-wheel being made to rotate rapidly, the punch is depressed once for every 5 revolutions of the fly; the resistance which the metal opposes to being punched is very great, but the leverage at which the lever acts is about 12. When the punch comes down on the surface of the metal, one of three things must happen: either the motion must stop suddenly, or the machine must be strained and injured, or the metal must be punched. But the motion cannot be stopped suddenly, because, before this could happen, an infinite force of inertia would be developed by the fly-wheel, which must make something yield. If therefore we make the machine sufficiently massive to

prevent yielding, the metal must be punched. Punching machines are enormously strong, as it is necessary to make the punching of the metal easier than breaking the machine.

555. We shall be able to calculate, from what we have already seen in Art. 249, what is the magnitude of the force required for punching. We there saw that about 22·5 tons of pressure was necessary to shear a bar of iron one square inch in section. Punching does not differ much from shearing, for in each case a certain area of iron has to be cut; the area in punching is measured by the surface of the cylinder of iron which is cut out.

556. Suppose a plate be 0″·8 thick, and it be required to punch out a hole 0″·5 in diameter; the area of iron that has to be cut across is $\frac{22}{7} \times \frac{1}{2} \times \frac{4}{5} = 1·26$ square inches: hence, since 22·5 tons per square inch are required for shearing, this hole will require $22·5 \times 1·26 = 28·4$ tons. A pressure of about 28 tons must therefore be exerted upon the punch: this will require from the cam a pressure of a little over 2 tons upon its end of the lever. Though the iron must be cut out to a depth of 0″·8, yet it is obvious that almost immediately after the punch has penetrated the surface of the iron, the cylinder must be entirely cut and begin to emerge from the other side of the plate. We shall probably be correct in supposing that the punching is completed when the punch has entered 0″·1, and that it is only during this space that the great pressure of 28 tons has to be exerted; only a small pressure is afterwards necessary to overcome the friction which opposes the motion of the cylinder of iron. Hence, though so great a pressure has been required, yet the number of units of energy is not very large; it is

$$\tfrac{1}{120} \times 2,240 \times 28 = 523.$$

Therefore the number of units of energy actually required is less than that which would be expended in raising 1 cwt. up 5′.

557. The fly-wheel is here an accumulator of energy. The time that is actually occupied in the punching is extremely small, and the sudden expenditure of 523 units is gradually restored by the engine: a small engine is therefore sufficient to work one of these machines; they proceed exactly on the same principle as the water accumulator already mentioned. If the fly-wheel contain 50,000 units of energy, the sudden call for 523 units will not perceptibly affect its velocity. There is therefore an advantage in having a very heavy fly sustained at a high speed for the working of a punching machine.

LECTURE XVII.

CENTRIFUGAL FORCE.

The Nature of Centrifugal Force.—The Action of Centrifugal Force upon Liquids.—The Applications of Centrifugal Force.—The Permanent Axes.

THE NATURE OF CENTRIFUGAL FORCE.

558. A BODY in motion will resist any force which tends to make it deviate from a straight line. This resistance is a force of inertia. It is just as much due to inertia as the resistance with which a body endeavours to preserve its condition of rest or motion, which we have already considered. The force which resists deviation is usually called centrifugal force.

559. We noticed as one of the principal difficulties in recognizing the force of inertia, that its amount depends on the manner in which the velocity was changed; so we find that the amount of centrifugal force depends on the manner in which the direction is changed.

560. I shall show you, by direct experiment, the existence of centrifugal force. You have already learned that, whenever a spring has been stretched, force has been exerted. A spring can be stretched by centrifugal force. The apparatus we use is shown in Fig. 74.

The essential part of the machine consists of two balls A, B, each 2″ in diameter: these are thin hollow spheres of silvered brass. The balls are supported on arms P A, Q B, which are attached to a piece of wood, P Q, capable of turning round an axle at C. The arm A P is rigidly fixed to P Q at P; the other arm, B Q, is capable of turning round a pin at Q. An india-rubber door-spring is shown at F; one end of this is secured to P Q, the other end to the moveable arm, Q B. If the arm Q B be turned so as to move B away from C, the spring F must be stretched.

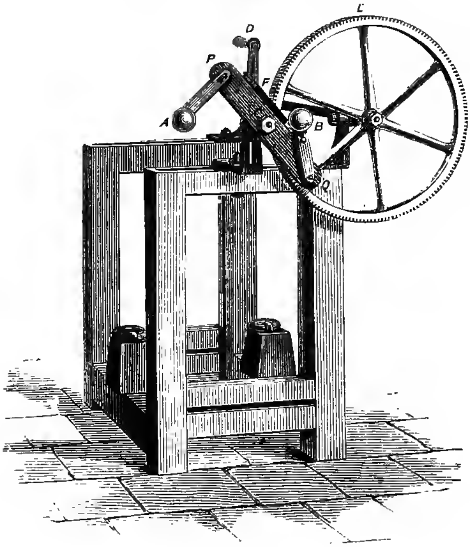

FIG. 74.

A pinion is mounted on the same socket with C; this is behind P Q, and therefore not seen in the figure: this pinion is made to revolve rapidly by the large wheel E, when E is turned by the handle D.

561. The room being darkened, a beam from the lime-light is allowed to fall on the apparatus: the reflection of the light is seen in the two silvered balls as two bright points. When D is turned, the balls move round

rapidly, and you see the points of light reflected from them describe circles. The ball B when at rest is 4″ from C, while A is 8″ from C; hence the circle described by B is smaller than that described by A. The appearance presented is that of two concentric luminous circles. As the speed increases, the inner circle enlarges till the circles blend into one. By increasing the speed still more, you see the circle whose diameter is enlarging actually exceeding the fixed circle, and its size continues to increase until the highest velocity which it is safe to employ has been communicated to the machine.

562. What is the explanation of this? The arm A is fixed and the distance AC cannot alter, hence A describes the fixed circle. B, on the other hand, is not fixed; it can recede from C, but only if there be a force impelling it to do so sufficient to stretch the spring F. There must, therefore, be a force urging B away from C, when B spins round, and this force must become greater when the velocity is increased. This is evident because the more the spring is stretched, the greater must be the force employed in stretching it.

563. This experiment, then, proves that there is a force which tends to drive a body moving in a circle away from the centre of that circle: this is what we call centrifugal force.

It also teaches us that centrifugal force increases when the velocity increases.

564. We can see the magnitude of this force by the same apparatus. The ball B weighs 0·1 lb. I find that I must pull it with a force of 3 lbs. in order to draw it to a distance of 8″ from C; that is, to the same distance as A is from C. Hence, when the diameters of the circles in which the balls move are equal, the cen-

trifugal force repelling B from the centre must be 3 lbs.; that is, it must be nearly thirty times as strong as gravity.

565. What is the cause of this remarkable force? Let us conceive a weight attached to a string to be swung round in a circle, a portion of whose arc is shown in Fig. 75.

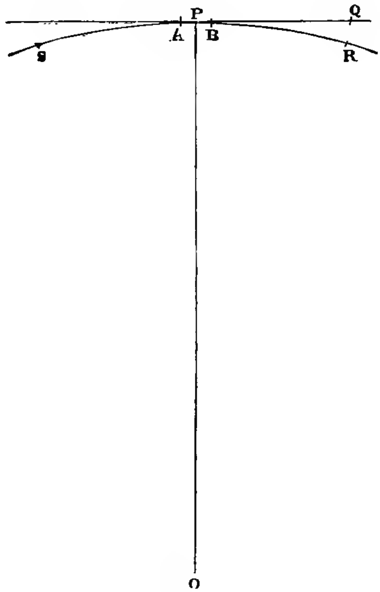

FIG. 75.

Suppose the weight be at S and moving towards P, and let a tangent to the circle be drawn at P. Take two points on the circle, A and B, very near P; the small arc AB does not differ perceptibly from the part AB on the tangent line: hence, when the particle arrives at A, it is a matter of indifference whether it travels in the arc AB, or along the line AB. Let us suppose it to move along the line. By the first law of motion, a particle moving in the line AB would continue to do so; hence, if the particle be allowed, it will move on to Q: but the particle is not allowed to move to Q; it is found at R. Hence it must have been withdrawn by some force.

566. This force is supplied by the string to which the weight is attached. The constant change from the natural motion of the weight is constantly opposed by the inertia of the weight, and this opposition is called centrifugal force. Should the string be released, the body flies off in the direction of the tangent P Q, to the circle at the point which the body occupied at the instant of release.

567. The centrifugal force increases in proportion to the square of the velocity. If I double the speed with which the weight is whirled round in the circle, I quadruple the strain with which centrifugal force tends to break the string. If the speed be trebled, the force is increased ninefold, and so on. If the velocities with which two bodies are moving in two circles be equal, the centrifugal force in the smaller circle is greater than that of the larger circle, in the proportion of the radius of the larger circle to that of the smaller.

THE ACTION OF CENTRIFUGAL FORCE UPON LIQUIDS.

568. I have here a small bucket nearly filled with water: to the handle a piece of string is attached. If I whirl the bucket round in a vertical plane sufficiently fast, you see no water escapes, although the bucket is turned upside down once in every revolution. This is because the centrifugal force which tends to repel the water from the centre is greater than the force of gravity, and consequently the water does not fall out.

569. The action of centrifugal force upon liquids is

also shown by the experiment which is represented in Fig. 76.

A glass beaker about half full of water is mounted so that it can be spun round rapidly. The motion is given by means of a large wheel turned by a handle, as shown in the figure. When the rotation commences, the water is seen to rise up against the glass sides and form a hollow in the centre.

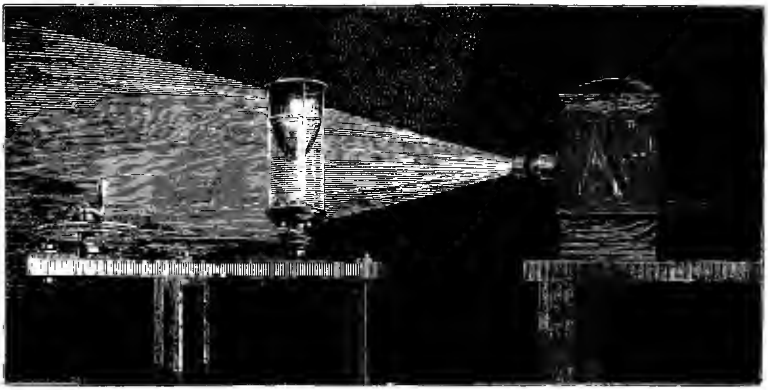

FIG. 76.

570. In order to demonstrate this clearly, I turn upon the vessel a beam from the lime-light. I have previously dissolved a little quinine in the water. The light of the lamp is transmitted through a piece of dense blue glass. When the light thus coloured falls on the water, the quinine imparts a bluish luminosity to the whole mass. This remarkable property of quinine, which is known as fluorescence, enables you to see distinctly the hollow in the water.

571. You observe that as the speed becomes greater the hollow increases, and that if I turn the wheel rapidly the water is driven out of the glass. The curved

surface which the water assumes is that which would be produced by the revolution of a parabola about its axis.

572. The explanation is simple. Directly the glass begins to revolve, the friction of its sides upon the water makes the water rotate; but when this happens, the particles of water fly from the centre by centrifugal force, and thus the liquid becomes elevated against the sides of the glass.

573. But you may ask why all the particles of the water acted upon by centrifugal force should not go to the circumference, and thus line the inside of the glass with a hollow cylinder of water? The answer is easy; such an arrangement could not exist in a liquid. The lower parts of the cylinder must bear the pressure of the water above, and therefore have more tendency to flatten out than the upper portions. This tendency could not be overcome by the centrifugal force, as that is equal on all parts at the same distance from the axis of the cylinder.

574. A very beautiful experiment, which we shall now show, was devised by M. Plateau, for the purpose of studying a liquid removed from the action of gravity.

The apparatus employed is represented in Fig. 77. A glass vessel 9″ cube is filled with a mixture of alcohol and water. The relative quantities are so proportioned that the fluid is of the same specific gravity as sweet oil. This is possible, because sweet oil is heavier than alcohol and lighter than water. In practice, however, it is found difficult to realize this exactly; the best plan is to make two alcoholic mixtures so that oil will just float on one of them, and just sink in the other. The lower half of the glass is to be filled with the former mixture, and the upper half with the latter. If, then, sweet oil be carefully introduced, it will form into a beautiful sphere in the middle of the vessel, as shown in the figure. The oil is then a liquid freed from the action of terrestrial gravity, and forms a sphere in consequence of the mutual action of its particles.

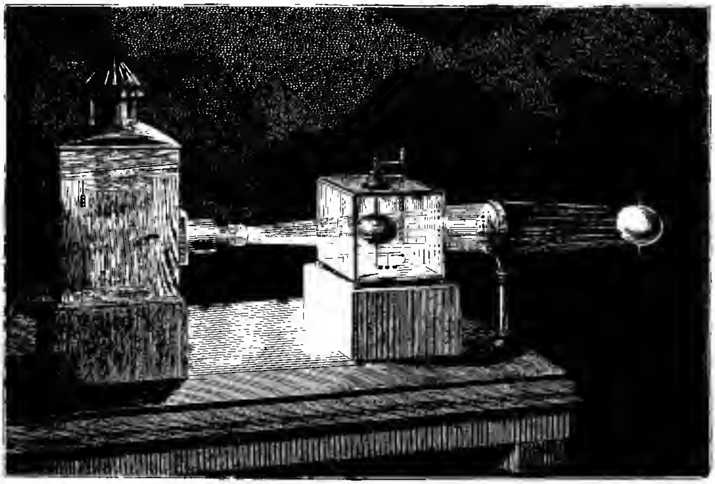

FIG. 77.

A vertical spindle passes through the vessel. On this there is a small disk at the middle of its length, about which the sphere of oil arranges itself symmetrically. To the end of the spindle a handle is attached. When the handle is turned round slowly, the friction of the disk and spindle communicates a motion of rotation to the sphere of oil. We have then a liquid spheroidal mass endowed with a movement of rotation; and we can study the effect of centrifugal force upon the form. We first see the sphere flatten down at its poles, and bulge at the equator. In order to show the phenomenon to those who may not be near to the vessel, the sphere can be projected on the screen by the help of the lime-light lamp and a lens. We first see on the screen the yellow circle, and then, as the movement begins, this gradually changes into an ellipse. But a very remarkable modification of the appearance is shown when the handle is turned somewhat rapidly. The ellipsoid gradually flattens down until, when a certain velocity is attained, the surface actually becomes indented at the poles, and then flies from the axis altogether. Consequently the liquid assumes the form of a beautiful ring, and the appearance on the screen is shown in Fig. 78.

FIG. 78.

575. The explanation of the phenomenon of the ring depends on more than centrifugal force; as the sphere of oil spins round in the liquid, its surface is retarded by friction; so that when the velocity reaches a certain amount, the centrifugal force drives the internal portions of the sphere, which are in the immediate neighbourhood of the spindle, out into the outer portions, whose centrifugal force, owing to the retardation, is considerably diminished.

576. The earth was, we believe, originally in a fluid condition. It had then, as it has now, a rotation around an axis; the centrifugal force arising from this rotation caused the earth to be slightly protuberant at the equator, just as we have seen the sphere of oil bulging out under the action of centrifugal force.

577. The centrifugal force on the earth has another effect besides that of making the equator protuberant. Bodies have their weight slightly diminished by the

effect of this force, which acts in opposition to gravity. This effect is greatest at the equator, where it amounts to $\frac{1}{289}$th of the weight; it gradually diminishes as the latitude increases, and is nothing at the poles (Art. 387).

THE APPLICATIONS OF CENTRIFUGAL FORCE.

578. Centrifugal force has some applications in the mechanical arts; we shall mention two of them. The first is to the governor-balls of a steam-engine; the second is to the process of sugar-refining.

An engine which turns a number of machines in a factory should work uniformly. Irregularities of motion may be productive of loss and various inconveniences. An engine would work irregularly either from variation in the production of steam, or from the demands upon the power being lessened or increased. Even if the first of these sources of irregularity could be avoided by care, it is clear that the second could not. Some machines in the mill are occasionally stopped, others occasionally set in motion, and the engine generally tends to go faster the less it has to do. It is therefore necessary to provide means by which the speed shall be restrained within narrow limits, and it is obviously desirable that the contrivance used for this purpose should be self-acting. We must, therefore, have some arrangement which shall admit more steam to the cylinder when the engine is moving too slowly, and less steam when it is moving too quickly. The valve which is to regulate this must, then, be worked by some force which depends upon the velocity of the engine; this at once points to centrifugal force as the proper force to be employed, since it depends upon velocity. Such was the train of reasoning which led to the happy invention of the governor-balls: these are shown in Fig. 79.

A B is a vertical spindle which is turned by the engine. P P is a piece firmly attached to the spindle and turning with it. P W, P W are arms terminating in weights W W; these are balls of iron, generally very massive: the arms are free to turn round pins at P P. At Q Q links are placed, attached to another piece R R, which is able to slide up and down the shaft. When A B rotates, W and W are carried round, and therefore fly outwards by centrifugal force; to do this they must evidently pull the piece R R up the shaft. We can easily imagine an arrangement by which R R shall be made to shut or open the steam-valve according as it ascends or descends. The problem is then solved, for if the engine begin to go too rapidly, the balls

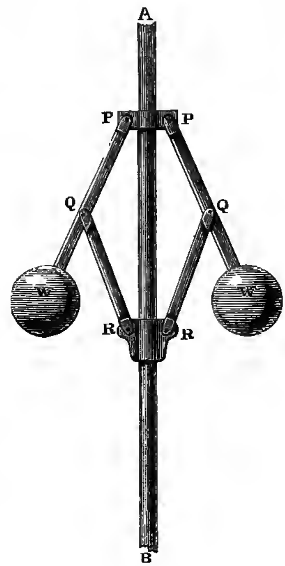

FIG. 79.

fly out further by the increased centrifugal force: this movement raises the piece R R, which diminishes the supply of steam, and consequently checks the speed. On the other hand, when the engine works too slowly, the balls fall in towards the spindle, the piece R R descends, the valve is opened, and a greater supply of steam is admitted. This beautiful contrivance is indispensable in engines which are employed in manufactories. There are other governors occasionally employed which depend also on centrifugal force; some of these are more sensitive than the governor-balls; but they are elaborate machines, and are only employed under exceptional circumstances.

579. The application of centrifugal force to sugar-refining is a very beautiful modern invention. To explain it I must briefly describe the process of refining.

The raw sugar is dissolved in water, and the solution is purified by filtration through flannel and animal charcoal. The syrup is then boiled. In order to preserve the colour of the sugar, and to prevent loss, this boiling is conducted *in vacuo*, as by this means the temperature required is much less than would be necessary with the ordinary atmospheric pressure.

The evaporation having been completed, crystals of sugar form throughout the mass of syrup. To separate these crystals from the liquor which surrounds them, the aid of centrifugal force is called in. A mass of the mixture is placed into a large iron tub, the sides of which are perforated with small holes. The tub is then made to rotate with prodigious velocity; its contents instantly fly off to the circumference, the liquid portions find an exit through the perforations in the sides, but the crystals are left behind. A little clear syrup is then sprinkled over the sugar while still rotating: this washes from the crystals the last traces of the coloured liquid, and passes out through the holes; when the motion ceases, the inside of the tub contains a layer of perfectly pure white sugar, several inches thick, ready for the market.

580. Centrifugal force is peculiarly fitted for this purpose; each particle of liquid is itself acted on by the force, and strives to get out in consequence. The action on the sugar is very different from what it would have been

had the mass been subjected to pressure by a screw-press or otherwise; the particles immediately acted on in that case have to transmit the pressure to those within; and the consequence would be that, while the crystals of sugar on the outside would be crushed and destroyed, the water would only be very imperfectly driven from the interior: water could lurk in the interstices of the sugar, which remain notwithstanding the pressure.

581. But with the centrifugal force the water must go, not because it is pushed by the crystals, but because of its own inertia; and it is found that the water can be perfectly expelled with a velocity less than that which would be necessary to produce centrifugal force enough to make the crystals injure each other.

THE PERMANENT AXES.

582. There are some curious properties of centrifugal force which remain to be considered. These we shall investigate by means of the apparatus of Fig. 80. This consists of a pair of wheels B C, by which a considerable velocity can be given to a horizontal shaft. This shaft is connected by a pair of bevelled wheels D with a vertical spindle F. The machine is worked by a handle A, and the object to be experimented upon is suspended from the spindle.

583. I first take a disk of wood 18″ in diameter; a hole is bored in the margin of this disk; through this hole a rope is fastened, by means of which the disk is suspended from the spindle. The disk hangs of course in a vertical plane.

584. I now begin to turn the handle round gently, and you see the disk begins to rotate about the vertical

diameter; but, as the speed increases, the motion becomes a little unsteady; and finally, when I turn the handle very rapidly, the disk springs up into a horizontal plane, and you see it like the surface of a small table: the rope by which the disk is suspended swings round and round in a cone, so rapidly that it is hardly seen.

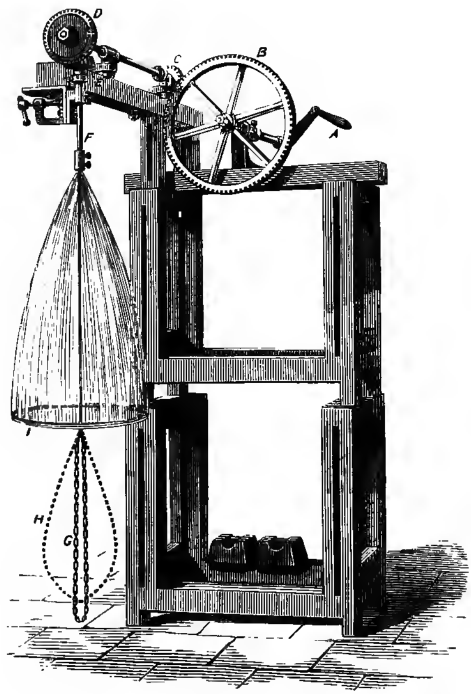

Fig. 80.

585. We may repeat the experiment in a different manner. I take a piece of iron chain about 2′ long, G; I pass the rope through the two last links of its extremities, and suspend the rope from the spindle. When I commence to turn the handle, you see the chain gradually opens out into a loop H; and as the speed increases,

the loop becomes an almost perfectly circular ring. Still increasing the speed, I find the ring becomes unsteady, till finally it rises into a horizontal plane. The ring of chain in the horizontal plane is shown at I. When the motion is further increased, the ring swings about violently, and so I cease turning the handle.

586. The principles of centrifugal force will explain these remarkable results; we shall only describe that of the chain, as the same explanation will suffice also for the disk of wood. We shall begin with the chain hang-

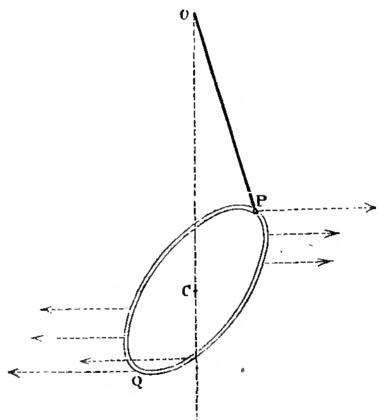

FIG. 81.

ing vertically from the spindle: the moment rotation commences, the chain begins to spin about a vertical axis; the effect of the centrifugal force is to make the parts of the chain fly outwards from this axis; this is the cause of the looped form H which the chain assumes. As the speed is increased more and more the loop gradually enlarges into a circle, because the centrifugal force increases with the velocity. But we have also to inquire into the cause of the remarkable change of position which the ring undergoes; instead of continuing to rotate about a vertical diameter, it comes into a horizontal plane. This will be easily understood with the help of Fig. 81. Let O P represent the rope attached to the ring, and O C be the vertical axis. Suppose the ring to be spinning about the axis O C, when O C was a diameter; if then, from any cause, the ring be slightly displaced, we can show that centrifugal force will tend to drive the ring further from the vertical plane, and force it into the horizontal plane. Let the ring be in the position represented in the figure; then, since it revolves about the vertical line O C, the centrifugal force upon P and Q is urging these parts of the ring outwards in the direction of the arrows, thus evidently tending to bring the ring into the horizontal plane.

587. In Art. 104, we have explained what is meant by stable and unstable equilibrium; we have here found a precisely analogous phenomenon in motion. The rotation of the ring about its diameter is unstable, for the minutest deviation of the ring from this position is fatal; centrifugal force immediately acts to augment the deviation more and more, until finally the ring is brought into the horizontal plane. Once in the horizontal plane, the motion there is stable, for if the ring be displaced the tendency of centrifugal force is to restore it to the horizontal. Centrifugal force is therefore the cause of the chain opening out into the ring, and also of the ring assuming and retaining the horizontal position.

588. The ring, when in a horizontal plane, rotates permanently about the vertical axis through its centre; this axis is called permanent, to distinguish it from all other directions, as being the only axis about which the motion is stable.

589. We may show another experiment with the chain: instead of passing the rope through the links at its ends, I pass the rope through the centre of the chain, and allow the ends of the chain to hang downwards. I now turn the handle; instantly the parts of the chain fly outwards in a curved form; by increasing the velocity, the parts of the chain at length come to be almost in a straight line. This phenomenon is easily explained by centrifugal force.

LECTURE XVIII.

THE SIMPLE PENDULUM.

Introduction.—The Circular Pendulum.—Law connecting the Time of Vibration with the Length.—The Force of Gravity determined by the Pendulum.—The Cycloid.

INTRODUCTION.

590. If a weight be attached to a piece of string, the other end of which hangs from a fixed point, we have what is called a simple pendulum. The pendulum is of the utmost importance in science, as well as for its practical applications as a time-keeper. In this lecture and the next we shall treat of its general properties; and the last will be devoted to the practical applications. We shall commence with the simple pendulum, as already defined, and prove, by experiment, the remarkable property which was discovered by Galileo. The simple pendulum is often called the circular pendulum.

THE CIRCULAR PENDULUM.

591. We first experiment with a pendulum on a large scale. Our lecture theatre is 32 feet high, and there is a wire suspended from the ceiling 27′ long; to the end of this a ball of cast iron weighing 25 lbs.

is attached. This wire when at rest hangs vertically in the direction O C (Fig. 82).

I draw the ball from its position of rest to A; when released, it slowly descends to C, where it was before; it then moves on the other side to B, and back again to my hand at A. The ball—or to speak more precisely, the centre of the ball—moves in a circle, whose centre is the point O in the ceiling from which the wire is suspended.

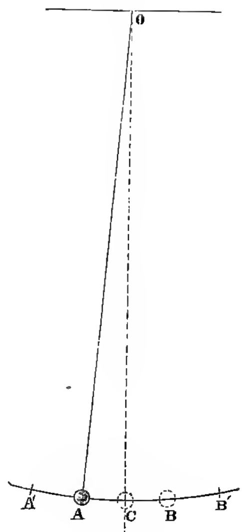

FIG. 82.

592. What causes the motion of the pendulum when the weight is released? It is the force of gravity; by moving the ball to A I raise it a little, and therefore, when I release the ball, gravity acts; to return to C again is the only manner in which the mode of suspension will allow the ball to fall. But when the ball reaches its position of rest C, what forces it onwards to B?—for gravity must be acting against the ball during the journey from C to B. The first law of motion explains this. In travelling from A to C the ball acquires a certain amount of velocity, which becomes greatest at C; hence at C the ball has a tendency to go on, and it is only when the ball has arrived at B that gravity has conquered the force of inertia, and begins to make the ball descend.

593. You see, the ball continues moving to and fro—oscillating, as it is called—for a long time. The fact is, that it would oscillate for ever, were it not for the resist-

ance of the air, and for some loss of energy at the point of suspension.

594. By the time of an oscillation is meant the time of going from A to B, but not back again. The time of our long pendulum is nearly three seconds.

595. It is with reference to the time that Galileo made his great discovery. He found that whether the pendulum were swinging through the arc A B, or whether it had been brought to a more distant point A′, and so was describing the arc A′ B′, the time of oscillation remained the same. The arc through which the pendulum oscillates is called its amplitude, so that we may euunciate this truth more concisely by saying that the time of oscillation is independent of the amplitude. The means by which Galileo proved this, would hardly be adopted in modern days. He allowed a pendulum to perform a certain number of vibrations, say 100, through the arc A B, and he counted his pulse during the time; he then counted the number of pulsations while the pendulum vibrated 100 times in the arc A′ B′, and he found the number of pulsations in the two cases to be equal. Assuming, what is probably true, that Galileo's pulse remained uniform throughout the experiment, this result showed that the pendulum took the same time to perform 100 vibrations, whether it swung through the arc A B, or through the arc A′ B′. This discovery it was which first suggested the employment of the pendulum as a means of keeping time.

596. We shall adopt a different method to show that the time does not depend upon the amplitude. I have here an arrangement which is represented in Fig. 83. It consists of two pendulums A D and B C, each 12′ long, and suspended from two points A B, about 1′ apart, in

the same horizontal line. Each of these pendulums carry a weight of the same size: they are in fact identical.

597. I take one of the balls in each hand. If I withdraw each of them from its position of rest through

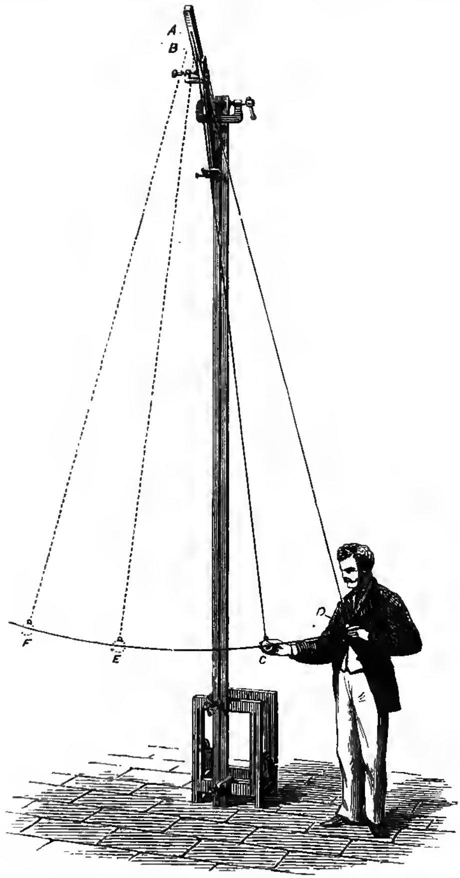

Fig. 83.

equal distances and then release them, both balls return to my hands at the same instant. This might have been expected from the identity of the circumstances.

598. I next withdraw the weight C in my right hand to a distance of 1′, and the weight D in my left hand to a distance of 2′, and release them simultaneously. What happens? I keep my hands steadily in the same position, and I find that the two weights return to them at the same instant. Hence, though one of the weights moved through an amplitude of 2′ (C E) while the other moved through an amplitude of 4′ (D F), the times occupied by each in making two oscillations are identical. If I draw the right-hand ball away 3′, while I draw the left hand only 1′ from their respective positions of rest, I still observe the same result.

599. In two oscillations we can see no effect on the time produced by the amplitude, and we are correct in saying that, when the amplitude is only a small fraction of the length of the pendulum, it has no effect. But if the amplitude of one pendulum were very large, we should find that its time of oscillation is slightly greater than that of the other, though to detect the difference would require a delicate test. One consequence of what is here remarked will be noticed in Art. 654.

600. We next inquire whether the weight which is attached to the pendulum has any effect upon the time of vibration. Using the 12′ pendulums of Fig. 83, I place a weight of 12 lbs. on one hook and one of 6 lbs. on the other. I withdraw one in each hand; I release them; they return to my hand at the same moment. Whether I withdraw the weights through long arcs or short arcs, equal or unequal, they invariably return together, and both therefore have the same time of vibration. With other weights of iron the same result is always obtained; hence we learn that, besides being independent of the amplitude, the time of vibration is also independent of the weight.

601. Finally, let us see if the material of the weight have any effect. I place a ball of wood on one hook and a ball of iron on the other; I swing them as before: the vibrations are still isochronous, that is, performed in equal times. A ball of lead is found to swing in the same time as a ball of brass, and both in the same time as a ball of iron or of wood.

602. In this we may be reminded of the experiments on gravity (Art. 492), where we showed that all bodies fall to the ground in equal times, whatever be their size or material; in both cases the fact proved is the same, that gravity acts upon all bodies proportionally to their masses, though the bodies be composed of very different substances. It was by means of experiments upon the pendulum that Newton proved that the weights of different bodies are in the proportion of their masses.

LAW CONNECTING THE TIME OF VIBRATION WITH THE LENGTH.

603. We have seen that the time of vibration of a pendulum depends neither upon its amplitude, material, nor weight; we have now to learn on what the time does depend. It depends upon the length of the pendulum. The shorter a pendulum the less is its time of vibration. We shall proceed to find by experiment the relation between the time and the length of the cord by which the weight is suspended.

604. I have here (Fig. 84) two pendulums AD, BC, one of which is 12′ long and the other 3′; they are mounted side by side, and the weights are at the same distance from the floor. I take one of the weights in each hand, and withdraw them to the same distance from the position of rest. I release the balls simultaneously; C moves off

rapidly, arrives at the end C′ while D has only reached D′, and returns to my hand just as D has completed one oscillation. I do not seize C; it goes off again, and returns again exactly at the same moment as D reaches my hand. Thus you see that C has performed four oscillations while D has made two. This proves to us that when one of

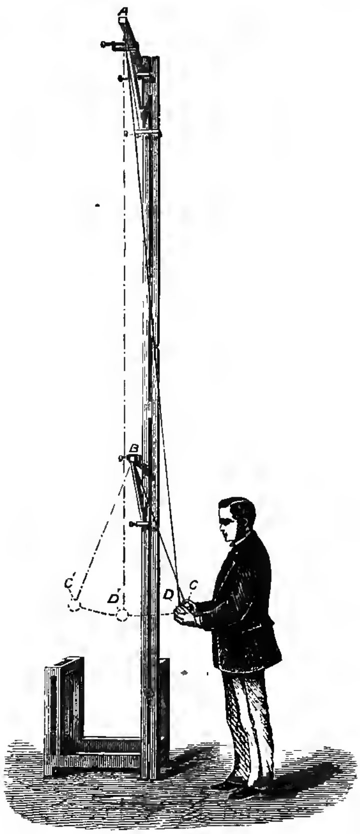

FIG. 84.

two pendulums is a quarter the length of the other, the time of vibration in the short pendulum is exactly half the time of vibration in the long pendulum.

605. We shall repeat the experiment with the pendulum 27′ long, which is suspended from the ceiling, and compare it with a pendulum 3′ long, which is suspended near it. I withdraw the weights and release them as before; and you see that the weight of the small pendulum returns twice to my hand while the long pendulum has not yet returned; but that, keeping my hands steadily in the same place throughout the experiment, the long pendulum returns exactly at the same instant as the short pendulum returns for the third time. Hence we learn that a pendulum 27′ long takes three times as much time for its vibration as a 3′ pendulum.

606. The lengths of the three pendulums on which we have experimented (27′, 12′, 3′), are in the proportions of the numbers 9, 4, 1; and the times of the oscillations are proportional to 3, 2, 1: hence we learn that the time of vibration of a pendulum is proportional to the square root of the length of the pendulum.

607. But the time of vibration must also depend upon gravity; for it is only owing to gravity that the pendulum makes vibrations; and it is evident that, if gravity were increased, the time of vibration would be diminished: hence the expression for the time of vibration must be proportional to the square root of the length, and must also be diminished when gravity is increased.

It is found by calculation, and the result is confirmed by experiment, that the time of vibration is represented by the expression,

$$3{\cdot}1416\sqrt{\frac{\text{Length}}{\text{Force of gravity.}}}$$

608. The force of gravity in London (Art. 517) is 32·1908, so that the time of vibration of a pendulum in

London is $0 \cdot 5537 \sqrt{\text{length}}$: the length of a pendulum which vibrates in one second, at London, is $3' \cdot 2616$.

MODE OF FINDING GRAVITY BY THE PENDULUM.

609. The pendulum affords the proper means of determining the force of gravity at any place on the earth. We have seen that the time of vibration can be expressed in terms of the length and the force of gravity; so conversely, when the length and the time of vibration are known, the force of gravity can be determined; the expression for gravity is—

$$\text{Length} \times \left(\frac{3 \cdot 1416}{\text{Time}}\right)^2.$$

610. It is, of course, quite impossible to observe the time of one vibration with any degree of accuracy; but supposing we observe a large number of vibrations, say 100, and find the time taken to perform them, we shall then find the time of one oscillation by dividing the entire time by 100. The amplitude of the oscillations may diminish, but they are still performed in the same time; and hence, if we are sure that we have not made a mistake of more than one second in the whole time, there cannot be an error of more than 0·01 second, in the time of one oscillation. By taking a still larger number of oscillations, the time may be determined with the utmost precision, so that this part of the inquiry presents no difficulty.

611. But the length of the pendulum has also to be ascertained, and this does present some difficulties. The ideal pendulum whose length is required, is supposed to be composed of a very fine, perfectly flexible cord, at the end of which a particle without appreciable size is

attached; but this is very different from the pendulum which we must employ. We are not sure of the exact position of the point of suspension, and, although we use a perfect sphere for the weight of the pendulum, the distance between its centre and the point of suspension is not precisely the length of the simple pendulum that would vibrate isochronously. Owing to these circumstances, the measurement of the pendulum is embarrassed by considerable difficulties, which have only been overcome by the most lavish expenditure of mechanical skill.

612. We shall perform, in a very simple way, an experiment for the purpose of determining the force of gravity. I have here a silken thread which is fastened by being clamped between two pieces of wood. A cast-iron ball 2″·54 in diameter is suspended from this piece of silk. The distance from the point of suspension of the silk to the ball is 24″·07, as well as it can be measured.

The length of the ideal pendulum which would vibrate isochronously with this pendulum is 25″·37, being about 0″·03 greater than the distance from the point of suspension to the centre of the sphere.

613. The length having been ascertained, the next point to be determined is the time of vibration. For this purpose I use a stop-watch, which can be started or stopped instantaneously by touching a little stud: this watch will indicate time accurately to one-fifth of a second. It is necessary that the pendulum should swing in a small arc, as otherwise the oscillations are not strictly isochronous. It is quite sufficient amplitude to allow the ball to move to and fro through a few tenths of an inch.

614. In order to observe the vibrations easily, I have

mounted a little telescope, through which I can view the top of the ball. In the eye-piece of the telescope a vertical wire is fastened, and I count each vibration just as the silk of the ball passes the vertical wire. Taking my seat with the stop-watch in my hand, I write down the position of the hands of the stop-watch; and then look through the telescope. I see the silk thread slowly moving to and fro, crossing the vertical wire at every vibration; just as it passes the wire on one occasion, I touch the stud and start the watch. I allow the pendulum to make 300 vibrations, and suddenly, as the silk arrives at the vertical wire for the 300th time, I stop the watch; on reference I find that 241·6 seconds have elapsed since the time the watch was started. To avoid error, I repeat this experiment, with precisely the same result: 241·6 seconds are again required for the completion of 300 vibrations.

615. It is desirable to commence counting the vibrations when the pendulum is at the middle of its stroke, rather than when it arrives at its highest point. In the former case the pendulum is moving with the greatest rapidity, and therefore the identity of the thread with the vertical wire in the telescope can be noticed with the most perfect definiteness.

616. The time of one vibration is therefore found, by dividing 241·6 by 300, to be 0·805 second. This is certainly correct to less than a thousandth part of a second. We have, then, a pendulum whose length is $25''\cdot37 = 2'\cdot114$, vibrating in 0·805 second; and from this we find that gravity is $2'\cdot114 \times \left(\frac{3\cdot1416}{0\cdot805}\right)^2 = 32\cdot196$. This result agrees with what has been determined by very careful measurement.

Another method of finding gravity from the oscillations of a pendulum will be described in the next lecture (Art. 638).

THE CYCLOID.

617. If the amplitude of the vibration of a circular pendulum bear a large proportion to the radius, the time of oscillation is slightly greater than if the amplitude be very small. In this case the weight moves in the arc of a circle.

618. But there is a curve in which a weight may be made to move where the time of vibration is precisely the same, whatever be the amplitude. This curve is called a cycloid. This is the curve which is described

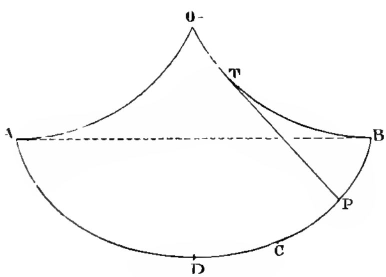

FIG. 85.

by a nail in the circumference of a wheel, when the wheel rolls along the ground. Thus, if a circle (Fig. 85) rolled along the line AB, a point on its circumference describes the cycloid ADCPB. This curve does not differ very much at its lower part from a circle whose centre is a certain point O above the curve.

619. Suppose we had a piece of wire carefully shaped to the curve ADCPB, and that a ring could slide along this wire without friction, it would be found that, whether

the ring be allowed to drop from C, P or B, it would fall to D precisely in the same time; the ring would of course rise upon the wire to an equal height on the other side of D, and would continue to vibrate for ever. In vibrations upon the cycloid, the amplitude is absolutely without effect upon the time.

620. Owing, however, to the fact that a frictionless wire is impossible, we cannot adopt this method; but we can avail ourselves of a remarkable property of a cycloid. OA (Fig. 85) is a curve consisting of a half cycloid; in fact, OA is just the same as BD, moved into a different position, so also is OB. If a string of length OD be suspended from the point O, and have a weight attached to it, the weight will describe the cycloid, provided that the string wrap itself along the arcs OA and OB; thus, when the weight has moved from D to P, the string is wrapped along the curve through the space OT, the part TP only being free. This arrangement will always force the point P to move in the cycloidal arc.

621. We are now in a condition to ascertain experimentally, whether the time of oscillation in the cycloid be independent of the amplitude. We use for this purpose the apparatus shown in Fig. 86. DCE is the arc of the cycloid; two strings are attached at O, and equal weights A, B are suspended from them; C is the middle point of the arc. The time A will take to fall through the arc AC is of course half the time of its oscillation. If, therefore, I can show that A and B both take the same time to fall down to C, I shall have proved that the vibrations are isochronous.

622. Holding, as shown in the figure, A in one hand and B in the other, I release them simultaneously, and you see the result,—they both meet at C: even if I

bring A up to E, and bring B down close to C, the result is the same. The motion of A is so rapid that it arrives at C just at the same instant as B. When I bring the two balls on the same side of C, and release them simultaneously, A overtakes B just at the moment when it is passing C. Hence, under all circumstances, the times of descent are equal.

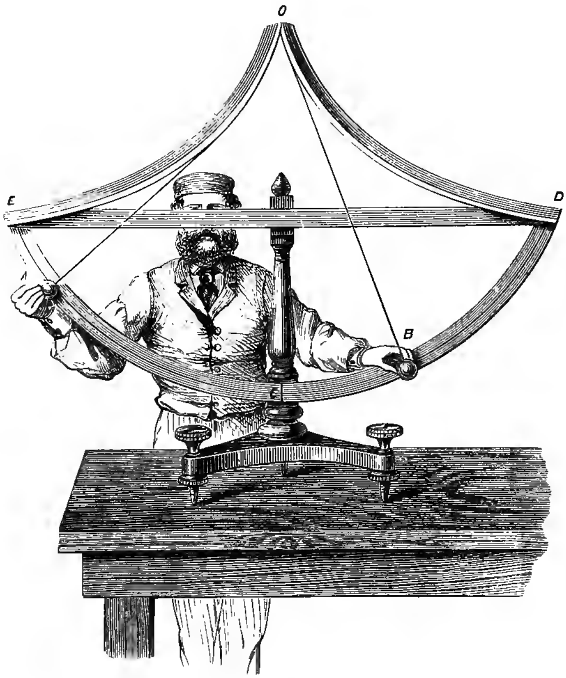

FIG. 86.

623. It will be noticed that the ball B, in the position shown in the figure, is almost as free as if it were merely suspended from O, for it is only when the ball is some distance from the lowest point that the side arcs produce

any appreciable effect upon the thread O B. The ball swings from B to C nearly as in a circle whose centre is O. Hence, in the circular pendulum, the vibrations when small are isochronous, for in that case the cycloid and the circle become indistinguishable.

LECTURE XIX.

THE COMPOUND PENDULUM AND THE COMPOSITION OF VIBRATIONS.

The Compound Pendulum.—The Centre of Oscillation.—The Centre of Percussion.—The Conical Pendulum.—The Composition of Vibrations.

THE COMPOUND PENDULUM.

624. PENDULOUS motion is met with in many other forms besides that of the simple pendulum, which consists of a weight and a cord. In fact, any body which rotates about an axis may oscillate like a pendulum. A body thus vibrating is called a compound pendulum. Every pendulum is more or less a compound pendulum, for the ideal form, which consists of an indefinitely small weight attached to a perfectly flexible and imponderable string, is an abstraction which can only be approximately imitated in nature.

625. The first pendulum of this class which we shall notice is the common clock pendulum (Fig. 87). This consists of a wooden or steel rod A E, to which a brass or leaden bob B is attached. This pendulum is suspended by means of a steel spring C A, which being very flexible, allows the pendulum to vibrate with considerable freedom. The use of the screw at E will be explained in

Art. 665. A pendulum like this vibrates isochronously, when the amplitude is small, but it is not easy to see precisely what the length of the simple pendulum is which would swing in the same time. In the first place, we are uncertain as to what is virtually the point of suspension, for the spring, though flexible, will not yield at the point C to the same extent as a string; thus the effective point of suspension is really a little lower than C. The other extremity is still more uncertain, for the weight, so far from being a single point, is not exclusively in the neighbourhood of the bob, for the rod of the pendulum has a weight that is appreciable. This form of pendulum cannot therefore be used where it is necessary to determine the length with accuracy.

FIG. 87.

626. When the length of a pendulum is to be measured, we must adopt other means of supporting it than that of suspension from a spring, in order to have a definite point from which to measure. To illustrate the mode that is to be adopted, I take here an iron bar 6′ long and 1″ square, which weighs 19 lbs. I wish to support this at one end so that it can vibrate freely, and at the same time have a definite point of suspension. I have here two small prisms of steel E (Fig. 88) fastened to a brass frame; these prisms are called knife-edges, though they are far more blunt than any knife—in fact, the edges meet at about an angle of 45°: this frame and the knife-edges can be placed on the end of the bar, and can be fixed there by tightening two nuts. The object of having the knife-edges on a

sliding frame is that they may be applicable to different parts of the bar with facility. In some instruments used in experiments requiring extreme delicacy, the knife-edges which are attached to the pendulum are supported upon plates of agate; the edges are adjusted on the same horizontal line, and the pendulum really vibrates about this line, as about an axis. For our purpose it will be sufficient to support the knife-edges upon small pieces of steel. A B, Fig. 88, represents one side of the top of the iron bar; E is the knife-edge projecting from it, with its edge perpendicular to the bar; there is of course a similar edge on the other side. C D is a steel plate whose upper surface is polished; this piece of steel is firmly secured to the framework. There is of course a similar piece on the other side, supporting the other knife-edge. The bar, thus carried by its knife-edges, will, when once started, vibrate backwards and forwards for an hour, as there is very little friction between the edges and the pieces which support them.

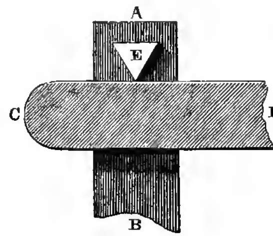

FIG. 88.

627. The general appearance of the apparatus, when mounted, is shown in Fig. 89. A B is the bar: at A the knife-edges and the framework are shown, and also the pieces of steel which support the knife-edges. The whole is carried by a horizontal beam bolted to two uprights; a glance at the figure will explain the arrangements made to secure the steadiness of the apparatus; the knife-edges shown at B will be referred to presently (Art. 636).

628. This bar, as you see, vibrates to and fro; and we shall determine the length of a simple pendulum which

would vibrate in the same period of time. The length might be deduced by finding the time of vibration, and then calculating from Art. 609. This would be the most accurate mode of proceeding, but I have preferred to adopt a simple method which does not require calculation.

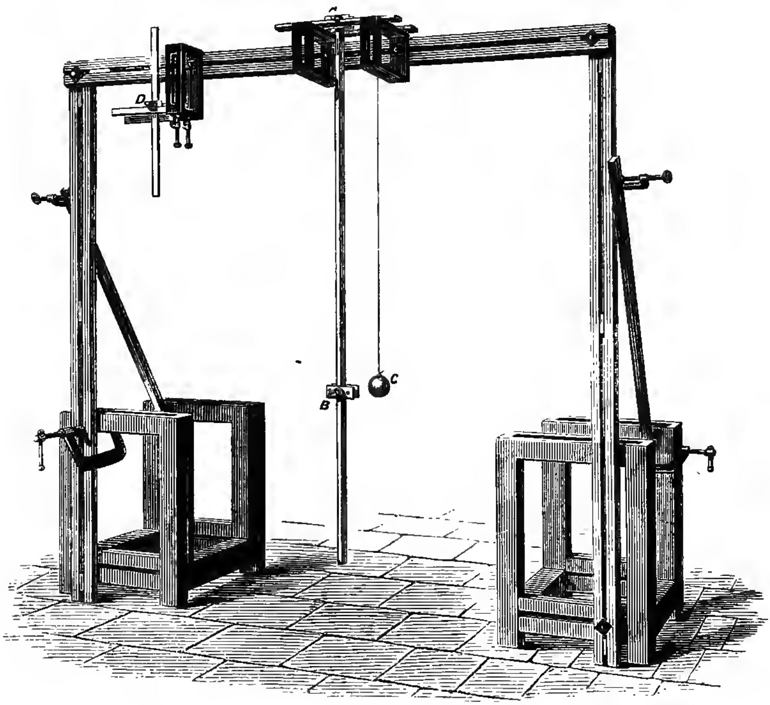

FIG. 89.

A simple pendulum, consisting of a fine cord and a small iron sphere C, is mounted behind the knife-edge, Fig. 89. The point from which the cord is suspended lies exactly in the line of the two knife-edges, and there is an adjustment for lengthening or shortening the cord at pleasure.

629. I first let out 6′ of cord, so that the simple pen-

dulum has the same length as the bar. Taking the ball in one hand and the bar in the other, I draw them aside, and you see, when I release them, that the bar performs two vibrations and returns to my hand before the ball. Hence the length of the isochronous simple pendulum is certainly less than the length of the bar; for we see that a pendulum of that length is too slow.

630. I now shorten the cord until it is only half the length of the bar; and, repeating the experiment, I see that the ball returns to my hand before the bar, and therefore the simple pendulum is too short. Hence we learn that the isochronous pendulum is greater than half the length of the bar, and less than the whole length.

631. Let us try a simple pendulum two-thirds of the length of the bar. I repeat the experiment, and find that the ball and the bar return to my hand precisely at the same instant. Therefore two-thirds of the length of the bar is the length of the isochronous simple pendulum.

632. In every uniform bar the time of vibration about one end is the same as that of a simple pendulum, whose length is two-thirds of the bar; the rod we have used is not strictly uniform, because of the knife-edges; but their weight (1·5 lb. each) may be neglected when compared with 19 lbs., the weight of the bar.

633. For this rule to be verified, it is essentially necessary that the knife-edges be placed at one end of the bar; to illustrate this we may examine the oscillations of the small rod, shown at D (Fig. 89). This rod is also of iron 24″ × 0″·5 × 0″·5, and it is suspended from a point near the centre by a pair of knife-edges; if the knife-edges could be placed so that the centre of gravity of the whole lay in the line of the edges, it is evident that the bar would rest indifferently, however it were

placed, and would not oscillate. If then the edges be very near the centre of gravity, we can easily understand that the oscillations may be very slow, and this is actually the case in the bar D. By the aid of the stop-watch, I find that one hundred vibrations are performed in 248 seconds, and that therefore each vibration occupies 2·48 seconds. The length of the simple pendulum which has 2·48 seconds for its period of oscillation, is about 20′. Had the knife-edge been at one end, the length of the simple pendulum would have been

$$24'' \times \tfrac{2}{3} = 16''.$$

THE CENTRE OF OSCILLATION.

634. We have already explained that the isochronous pendulum is that simple pendulum whose period of oscillation equals that of a compound pendulum. Thus, for example, in the 6′ bar already described (Art. 626), this length is 4′. If I measure off from the knife-edges a distance of 4′, and mark this point upon the bar, the point is called the *centre of oscillation.* The centre of oscillation in any compound pendulum is at a distance from the knife-edge, equal to the length of the corresponding simple pendulum. A bar 72″ long will vibrate in a shorter time when the knife-edge is 15″·2 from one end than when it has any other position. The length of the corresponding simple pendulum is 41″·6.

635. In the bar D the centre of oscillation would be at a distance of 20′ below the knife-edges; and in general the position will vary with the position of the knife-edges.

636. In the 6′ bar B is the centre of oscillation. I take another pair of knife-edges and place them on the bar, so

that the line of the edges passes through B. I now lift the bar carefully and turn it upside down, so that the edges B rest upon the steel plates. In this position one-third of the bar is above the axis of suspension, and the remaining two-thirds below it. A is of course now at the bottom of the bar, and is on a level with the ball, C: the pendulum is made to oscillate about the knife-edges B, and the time of its vibration may be approximately determined by direct comparison with C, as already explained. I find that, when I allow C and the bar to swing together, they both vibrate precisely in the same time. You will remember, that when the ball was suspended by a string of 4′, its vibrations were isochronous with those of the bar when suspended from the edges A. Now, without having altered C, but making the bar vibrate about B, I find that the time of oscillation of the bar is still equal to that of C. Therefore, the period of oscillation about A is equal to that about B. Hence, when the bar is vibrating about B, its centre of oscillation must be 4′ from B, that is, it must be at A: so that when the bar is suspended from A, B is the centre of oscillation; while, when the bar is suspended from B, A is the centre of oscillation. This is a most remarkable truth. It may be more concisely expressed by saying that the centre of oscillation and the centre of suspension are reciprocal.

637. Though the proof that we have given of this curious law applies only to a uniform bar, yet the law is itself true in general, whatever be the nature of the compound pendulum.

638. We alluded in the last lecture (Art. 611) to the difficulty of measuring with accuracy the precise length of a pendulum; an ingenious philosopher, Captain Kater, saw in the reciprocity of the centres of oscillation and

suspension, a method by which this difficulty could be evaded. We shall explain the principle. Let one pair of knife-edges be at A. Let the other pair of knife-edges, B, be placed as near as possible to the centre of oscillation. We can test whether B has been placed correctly: for the time taken by the pendulum to perform 100 vibrations about A should be equal to the time taken to perform 100 vibrations about B. If the times are not quite equal, B must be moved slightly until they are found to be exactly equal. Now the length of the isochronous simple pendulum is precisely equal to the distance between the knife-edges A, B; but the distance, from one edge to the other edge, presents none of the difficulties in its exact measurement which we had before to contend with: it can be found with precision. Hence, knowing the length of the pendulum and its time of oscillation, gravity can be found in the manner already explained.

639. I have adjusted the two edges of the 6′ bar as nearly as possible at the centres of oscillation and suspension, and we shall proceed to test the correctness of the positions. Mounting the bar first by the knife-edges at A, I set it vibrating. I take the stop-watch already referred to (Art. 613), and record the positions of its hands. I then place my finger on the stud, and, just at the moment when the bar is at the middle of one of its vibrations, I start the watch. I count a hundred vibrations; and when the pendulum is again at the middle of its stroke, I stop the watch, and find it records an interval of 110·4 seconds. Thus the time of vibration is 1·104 seconds. Reversing the bar, so that it vibrates about its centre of oscillation B, I now find that 110·0 is the time occupied by one hundred vibrations counted in the same manner as before; hence 1·100 seconds is the time of one

vibration about B: thus, the periods of the vibrations are very nearly equal, as they differ only by $\frac{1}{250}$th part of a second.

640. It would be difficult to render the times of oscillation exactly equal by altering the position of B. In Kater's pendulum the two knife-edges are first placed so that the periods are as nearly equal as possible. The final adjustments are given by moving a small sliding-piece on the bar until it is found that the times of vibration about the two edges are identical. We shall not, however, use this refinement in a lecture experiment; I shall adopt the mean value of 1·102 seconds. The distance of the knife-edges is about 3′·992; hence gravity may be found from the expression (Art. 609)

$$3'{\cdot}992 \times \left(\frac{3{\cdot}1416}{1{\cdot}102}\right)^2$$

The value thus deduced is 32′·4, which is too large by about two or three inches.

641. With proper care Kater's pendulum can be made to give a very accurate result. It is to be adjusted so that there shall be no perceptible difference in the number of vibrations in twenty-four hours, whichever edge be the axis of suspension; the distance between the edges is then to be measured with the last degree of precision by comparison with a proper standard.

THE CENTRE OF PERCUSSION.

642. The centre of oscillation in a body moving about a fixed axis is identical with another remarkable point, called the centre of percussion. We proceed to examine some of the properties of a body thus suspended with reference to percussion.

For the purposes of this experiment the method of suspension by knife-edges is too delicate to be adopted; the knife-edges would be injured by the blows which must be given.

643. We shall first use a rod suspended from a pin about which the rod can rotate. A B, Fig. 90, is a pine rod 48″ × 1″ × 1″, free to turn around B. Suppose this rod hang at rest. I take a stick in my hand, and, giving the rod a blow, I make it vibrate; the rod will immediately act upon the pin at B; but the immediate effect upon B will be very different according to the position at which the blow is given. If I strike the upper part of the rod at D, the action of A B upon the pin is a pressure to the left. If I strike the lower part at A, the pressure is to the right. But if I strike the point C, which is distant from B by two-thirds of the length of the rod, there is no pressure upon the pin. In fact, for a blow below C, the pressure is to the right; for one above C, it is to the left; for one at C it is nothing.

FIG. 90.

644. We can easily verify this by holding one extremity of a rod between the finger and thumb of the left hand, and striking it in different places with a rod held in the right hand; the pressure of the rod, when struck, will be felt by the fingers, and the circumstances already stated can be verified.

645. But a more complete way of investigating the subject is shown in Fig. 91. F B is a rod of wood, which is suspended from a beam by the string F G. A piece of paper is fastened to the rod at F by means of a small slip of wood which is clamped firmly to the

rod; the other ends of this piece of paper are similarly clamped at P and Q.

646. When the rod receives a blow on the right-hand side of A, we find that the piece of paper is broken across at E, because the end F has been driven by the blow towards Q, and consequently caused the fracture of the paper at a place, E, where it had been specially narrowed. I remove the pieces of paper, and replace them by a new piece precisely similar. I now strike the rod at B,—a smart tap is all that is necessary,—and the piece of paper breaks at D. Finally replacing the pieces of paper by a third piece, I find that when I give the rod a tap (not a violent blow) at C, neither D nor E are broken.

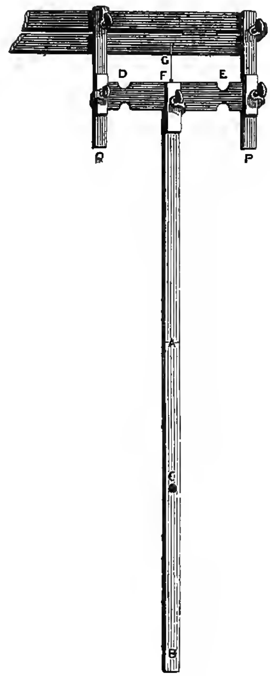

FIG. 91.

647. This point C, where the rod can receive a blow without producing a strain upon the extremity, is called the centre of percussion. We see, from its being two-thirds of the length of the rod distant from F, that it is identical with the centre of oscillation of the rod, if vibrating about knife-edges at F. It is true in general, whatever be the shape of the body, that the centre of oscillation is identical with the centre of percussion.

648. The principle embodied in what has been said of the centre of percussion has many applications. Every cricketer knows well that there is one part of his bat

from which the ball flies without giving his hands any unpleasant shock. The explanation is simple. The bat may be regarded as a body suspended from his hands; and if the blow be given with the centre of percussion of the bat, there is no shock experienced. In a hammer the centre of percussion is in the head, consequently a nail can receive a violent blow from the head, without injury to the hand which holds the handle of the hammer.

THE CONICAL PENDULUM.

649. I have here a tripod (Fig. 92) which supports a heavy ball of cast iron by a string 6′ long. If I withdraw the ball from its position of rest, and merely release it, the ball vibrates to and fro, the string continues always in the same plane, and the motion is that produced by the circular pendulum. If at the same instant that I release the ball, I impart to it a slight push in a direction not passing through the position of rest, the ball describes a curved path, returning to the point from which it started. This motion is that of the conical pendulum, because the string supporting the ball describes a cone.

650. In order to examine the nature of the motion, we can make the ball depict its own path. At the opposite point of the ball to that from which it is suspended, a hole is bored, and in this I have fitted a camel's-hair paint-brush filled with ink. I bring a sheet of paper on a drawing-board under the vibrating ball; and you see the brush traces an ellipse upon the paper, which I quickly withdraw.

651. By starting the ball in different ways, I can make

it describe very different ellipses: here is one that is extremely long and narrow, and here another almost circular. Pushing the ball with the proper velocity perpendicularly to the line joining its position to the position of rest, I can make the string describe a right

FIG. 92.

cone, and the ball a horizontal circle, but it requires some care and several trials in order to succeed in this. When the ellipse becomes very narrow, the motion passes by insensible gradations into that of the common pendulum, and the brush traces a straight line.

652. When the ball is moving in a circle, its velocity is uniform; when moving in an ellipse, its velocity is greatest at the extremities of the least axes of this ellipse, and least at the extremities of the greatest axes; but, when the ball is vibrating to and fro, as in the ordinary circular pendulum, the velocity is greatest at the middle of each vibration, and vanishes of course each time the pendulum reaches the extremity of its swing. It is very remarkable, that under all circumstances the brush traces an ellipse upon the paper; for the circle and the straight line are only extreme cases, the one being a very round ellipse and the other a very flat one. The brush will never trace any other form of curve.

653. How are we to explain the form of the path? To do so fully would require more calculation than would be admissible here, but we can give a general account of the phenomenon.

Let us suppose that the ellipse A C B D, Fig. 93, is the path described by a particle when suspended by a string from a point vertically above O, the centre of the ellipse. To produce this motion I withdraw the particle from its position of rest at O to A. If merely released, the particle would swing over to B, and back again to A; but I do not simply release it, I give it a velocity impelling it in the direction A T. Through O draw C D parallel to A T. If I had taken the particle at O, and, without withdrawing it from its position of rest, had started it off in the direction O D, the particle would continue for ever to vibrate backwards and forwards from C to D. Hence, when I release the particle at A, and give it a velocity in the direction A T, the particle commences to move under the action of two distinct vibrations, one parallel to A B, the other parallel to C D. What is the effect of these two vibrations impressed simultaneously upon the same particle? They are performed in the same time, since all vibrations are isochronous. We must conceive one motion starting from A towards O at the same moment that the other commences to start from O towards D. After the lapse of a short time, the body has moved through A Y in its oscillation towards O, and in the same time through O Z in its oscillation towards D; it is therefore found at X. Now, when the

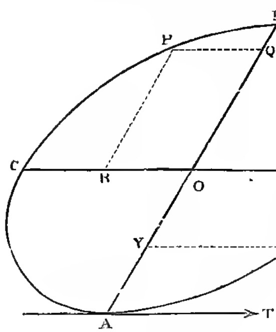

FIG. 93.

particle has moved through a distance equal and parallel to A O, it must be found at the point D, because the motion from O to D takes the same time as from A to O. Similarly the particle must pass through B, because in the time occupied in going from A to B, the particle has had time to go from O to D, and back again. The particle is found at P, because, after the vibration returning from B has arrived at Q, the movement from D to O has travelled on to R. In this way the particle may be traced completely round its path by the composition of the two motions. It can be proved that the path is an ellipse, and not any other curve, by reasoning founded upon the fact that the times of vibration are equal.

654. Close examination reveals a very interesting circumstance connected with this experiment. It may be observed that the ellipse described by the body is not quite fixed in position, but that it gradually moves round in its plane. Thus, in Fig. 92, the ellipse which is being traced out by the brush will gradually change its position to the dotted line shown on the board. The ellipse moves round in the same direction as that in which the ball is moving. This phenomenon is more marked with an ellipse whose dimensions are considerable in proportion to the length of the string. In fact, if the ellipse be very small, the change of position is imperceptible. The cause of this change is to be found in the fact already mentioned (Art. 599), that though the vibrations of a pendulum are very nearly isochronous, yet they are not absolutely so; the vibration in a long arc taking a minute portion of time longer than a vibration through a short arc.

This difference only becomes appreciable when the larger arc is of considerable magnitude with reference to the length of the pendulum.

655. How this produces the effect on the ellipse may be explained by Fig. 94. The particle is describing the ellipse A D C B in the direction shown by the arrows. This motion may be conceived to be compounded of vibrations A C and B D, if we imagine the particle to have been started from A with the right velocity in the right direction. Now, at the point A, the motion is for the instant perpendicular to O A; in fact, the motion is due for that moment exclusively to the vibration B D, and there is no movement parallel to O A. We may then define the extremity of the major axis of the ellipse to be the position of the particle, when the motion parallel

to that axis vanishes. Of course this applies equally to the other extremity of the axis C, and similarly at the points B or D there is no motion of the particle parallel to B D.

656. Let us follow the particle, starting from A until it returns there again. The movement is compounded of two vibrations, one from A to C and back again, the other along B D; from O to D, then from D to B, then from B to O, taking exactly double the time of one vibration from D to B. Now, if the time of vibration along A C were exactly equal to that along B D, these two vibrations would bring the particle back to A again; precisely under the same circumstances. But they do not take place in the same time; the motion along A C takes a shade longer, so that, when the motion parallel to A C has ceased, the motion along D B has gone past O to a point Q, very near O. Let A P = O Q, and when the motion parallel to A C has vanished, the particle will be found at P; hence P must be the extremity of the major axis of the ellipse. In the next revolution, the extremity of the axis will advance a little more, and thus the ellipse moves round gradually.

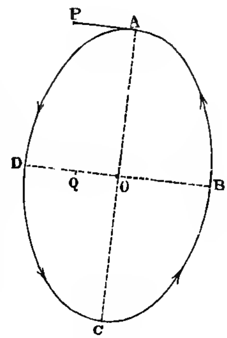

FIG. 94.

THE COMPOSITION OF VIBRATIONS.

657. We have learned to regard one motion in the conical pendulum, as compounded of two vibrations. The importance of the composition of vibrations justifies us in considering this subject experimentally in another

way. The apparatus which we shall employ is represented in Fig. 95.

A is a heavy iron ball weighing 25 lbs., suspended from the tripod by a cord whose length can be modified at pleasure: this ball itself forms the support of another

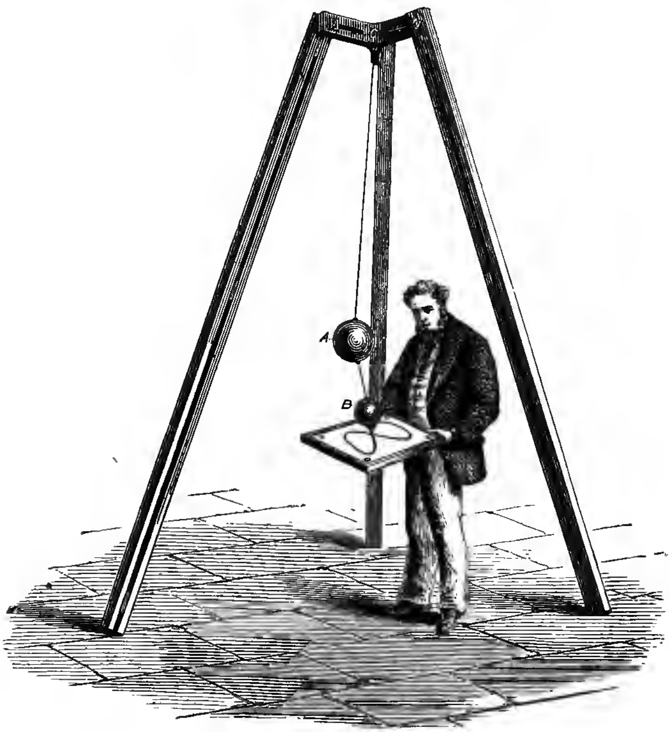

Fig. 95.

pendulum, B. The second pendulum is very light, being merely a globe of glass filled with sand. Through a hole at the bottom of the glass the sand runs out upon a drawing-board placed underneath to receive it.

Thus the little stream of sand writes its own history upon the drawing-board, and the curves traced out by the sand indicate the path in which the bob of the second pendulum has moved.

658. If the lengths of the two pendulums be equal, and their vibrations be in different planes, the curve described is an ellipse ; passing at one extreme into a circle, and at the other into a straight line. This is what we might have expected, for the two vibrations are each performed in the same time, and therefore the case is analogous to that of the conical pendulum of Art. 649.

659. But the curve is of a very different character when the cords are unequal. Let us study in particular the case in which the second pendulum is only one-fourth the length of the cord supporting the iron ball : this is actually the case represented in Fig. 95. The form of the path described by the sand is given in Fig. 96. The arrow-heads placed upon the curve show the manner in which it is formed. Let us suppose that the formation of the sand commences at A ; the curve goes on to B, to O, to C, to D, and back to A : this shows us that the bob of the lower pendulum must have performed two vibrations up and down, and one right and left. The motion is compounded of two vibrations at right angles to each other, and the time of one vibration is half that of the other.

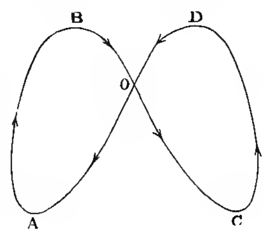

FIG. 96.

The time of vibration is proportional to the square roots of the length ; and, since the lower pendulum is one-fourth the length of the upper, its time of vibration is one-half. In this experiment, therefore, we have a confirmation of the law of Art. 606.

LECTURE XX.

THE MECHANICAL PRINCIPLES OF A CLOCK.

Introduction.—The Compensating Pendulum.—The Escapement.—The Train of Wheels.—The Hands.—The Striking Parts.

INTRODUCTION.

660. We come now to the most important practical application of the pendulum. The vibrations being always isochronous, it follows that, if we count the number of vibrations which the pendulum makes in a certain time, we shall be able to ascertain the amount of that time, provided we know the period of vibration of the pendulum. Let us suppose a pendulum 39·139 inches long; such a pendulum will in London vibrate exactly once a second, and is therefore called a seconds pendulum. If I set one of these pendulums vibrating, and devise means by which the number of its vibrations shall be recorded, I have a means of measuring time. This is in fact the principle of the common clock: the pendulum vibrates once a second, and the number of vibrations made from one epoch to another epoch is shown by the hands of the clock. For example, when the clock tells me that 15 minutes have elapsed, what it really shows is that the pendulum has made $60 \times 15 = 900$ vibrations, each of which has occupied one second.

661. One duty of the clock is therefore to count and record the number of vibrations of the pendulum; but the wheels and works have another part to discharge, and that is to sustain the motion of the pendulum. The friction of the air and the resistance experienced at the point of suspension are forces tending to bring the pendulum to rest; to counteract the effect of these forces, the pendulum must be continually supplied with fresh energy. This supply is communicated to the pendulum by the works of the clock, which will be more fully detailed presently.

662. When the clock is wound up, a store of energy is given to the machine, and this is doled out to the pendulum in a very small impulse, which it receives at every vibration. The clock-weight is of such a magnitude that it shall just be able to counterbalance the retarding forces when the pendulum has a proper amplitude of vibration. In all machines there is a certain amount of energy lost in setting the parts in motion, and in overcoming friction and other resistances; in clocks this represents the whole amount of the force, as there is no external work to be performed.

THE COMPENSATING PENDULUM.

663. A pendulum whose length is 39·139 inches vibrates exactly once a second in London. It is essential for the correct performance of a clock that the pendulum should vibrate at a constant rate; even the smallest irregularity will produce an appreciable effect upon the clock. Thus, suppose the pendulum vibrates in 1·001 seconds instead of in one second, the clock loses one-thousandth of a second at each beat; and, since there are 86,400 seconds in a day, it follows that the pendulum

will make only 86,400 − 86·3 vibrations in a day, and that therefore the clock will lose 86·3 seconds, or nearly a minute and a half daily.

664. For correct performance it is therefore essential that the time of vibration be rigidly constant. Now the time of vibration depends upon the length, and therefore it is necessary that the length of the pendulum be absolutely constant. If the length of the pendulum be altered by one-tenth of an inch, the clock will lose or gain nearly two minutes daily, according to whether the pendulum be lengthened or shortened. In general we may say that, if the pendulum be altered in length by K thousandths of an inch, the number of seconds gained or lost per day is 1·103 × K.

665. This explains the well-known practice of raising or lowering the bob of the pendulum when the clock is going too slow or too fast. Suppose the thread of the screw used in doing this have twenty threads to the inch; then one complete revolution of the screw will raise the bob through 50 thousandths of an inch, and therefore the effect on the rate will be 1·103 × 50 = 55 nearly. Thus, the rate of the clock will be altered by about 55 seconds daily. A screw by which this can be accomplished is shown in Fig. 87. Whatever be the screw, its effect can be calculated by the simple rule expressed as follows. Divide 1103 by the number of threads to the inch; the quotient is the number of seconds that the clock can be made to gain or lose daily by one revolution of the screw on the bob of the pendulum.

666. Let us suppose that the length of the pendulum has been properly adjusted so that the clock keeps accurate time. It is necessary that the pendulum should not alter in length. But there is an ever-present cause con-

stantly tending to change the length of the pendulum. That cause is heat. We shall first prove by actual experiment that bodies expand under the action of heat; then we shall consider the irregularities introduced into the motion of the pendulum by change of temperature; and, finally, we shall point out means by which these irregularities may be effectually counteracted.

667. I have here a brass bar a yard long; it is at present at the temperature of the room. If I heat the bar over a lamp, it becomes longer; but upon cooling, it returns to its original dimensions. These alterations of length are very small, indeed too small to be perceived except by careful measurement; but we shall be able to show you in a simple way that this bar does

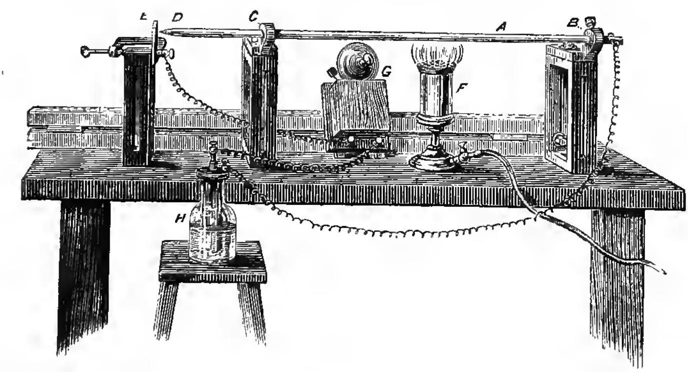

FIG. 97.

actually elongate when warmed. I place the bar A D in the supports shown in Fig. 97. It is firmly secured at B by means of a binding screw, and passes quite freely through C; if the bar elongate when it is heated by the lamp, the point D must approach nearer to E. At H is an electric battery, and at G an alarm clock rung by electricity. One wire of the battery connects H and G, another connects G with E, and a third connects H with the end of the brass rod. Now, until the electric current becomes completed, the alarm is dumb, and the current is not complete until the point touches E: when this is the case, the current rushes from the battery along the bar, then from D to E, from that through the alarm, and so back to the battery. I move the bar so that the point is not touching E, though extremely close to it. If I press E towards the point, you hear the alarum, showing that the circuit is complete; removing my finger, the alarm again becomes silent, because E springs back, and the current is interrupted.

668. I place the lamp under the bar: the bar begins to heat and to elongate; and, as it is firmly held at B, the point gradually approaches E; it has now touched E; the circuit is complete, and the alarm rings. If I withdraw the lamp, the bar cools. I can accelerate the process by touching the bar with a damp sponge; the bar contracts, breaks the circuit, and the bell stops: heating the bar again with the lamp, the bell again rings, to be again stopped by an application of the sponge. Now, though you have not been able to see the process, your ears have informed you that heat must have elongated the bar, and that cold has contracted it.

669. What we have proved with respect to a bar of brass, is true for a bar of any material; and thus, whatever be the substance of which a pendulum is made, the rod must be longer in hot weather than in cold weather: hence a clock will generally have a tendency to go faster in winter than in summer.

670. The amount of change thus produced is, it is true, small. For a pendulum with a steel rod, the difference of temperature between summer and winter

will cause a difference in the rate of five seconds daily, or about half a minute in one week. The amount of error thus introduced is of no great consequence in clocks, which are only intended for ordinary use; but in astronomical clocks, where seconds or even portions of a second are of the utmost importance, inaccuracies of this magnitude would be quite inadmissible.

671. There are, it is true, some substances—for example, slips of white deal—in which the rate of expansion is less than that of steel; consequently, the irregularities introduced by employing a pendulum whose rod is a slip of deal, would be less than that of the steel pendulum we have mentioned; but no substance is known which would not undergo greater variations than are admissible in the pendulum of an astronomical clock.

We must, therefore, devise some means by which the effect of temperature on the length of a pendulum can be avoided. Various means have been proposed for this purpose; we shall describe that which is generally adopted.

672. The mercurial pendulum (Fig. 98) is doubtless familiar to many; it is frequently used in clocks of good quality. The rod by which the pendulum is suspended is made of steel; and the bob consists of a glass jar of mercury. The distance of the centre of gravity of the mercury from the point of suspension may practically be considered as the length of the pendulum. The rate of expansion of mercury is about sixteen times that of steel: hence, if we had the bob formed of a column of mercury which was one-eighth part of the length of the steel rod, the compensation would be complete. For, suppose the temperature of the pendulum to be raised, the steel rod would be lengthened, and therefore the vase of mercury would be lowered; on the other hand, the column of

mercury would expand by an amount double that of the steel rod: thus the centre of the column of mercury would be raised by an amount exactly equal to that by which the steel was elongated; hence the centre of the mercury is raised by its own expansion as much as it is lowered by the expansion of the steel, and therefore it remains unaltered. By this contrivance the time of oscillation of the pendulum is rendered independent of the temperature. The bob of the mercurial pendulum is shown in Fig. 98. The screw is for the purpose of raising or lowering the entire vessel of mercury in order to make the rate correct in the first instance. It is of course essential that the vessel should contain the proper quantity of mercury.

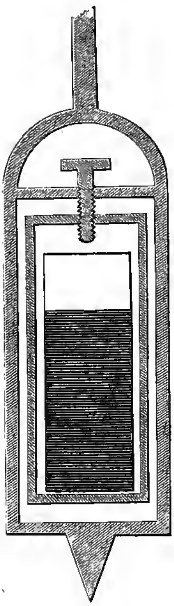

FIG. 98.

THE ESCAPEMENT.

673. Great labour, both of practical skill and theoretical investigation, has been lavished upon the very important part of a clock which is called the escapement. A good escapement is essential to the correct performance of the clock. The pendulum must have its motion sustained by receiving an impulse at every vibration: at the same time it is desirable that the vibration of the pendulum should be hampered as little as possible by mechanical connection. The isochronism of the pendulum, on which its utility as a time-keeper depends, is only a property of a pendulum which is swinging quite freely; hence we must

endeavour to approximate the clock pendulum as nearly as possible to a pendulum swinging quite freely. To effect this, and at the same time to maintain the arc of vibration constant, is the property of a good escapement.

674. A common form of escapement is shown in Fig. 99. The arrangement is somewhat different from that actually

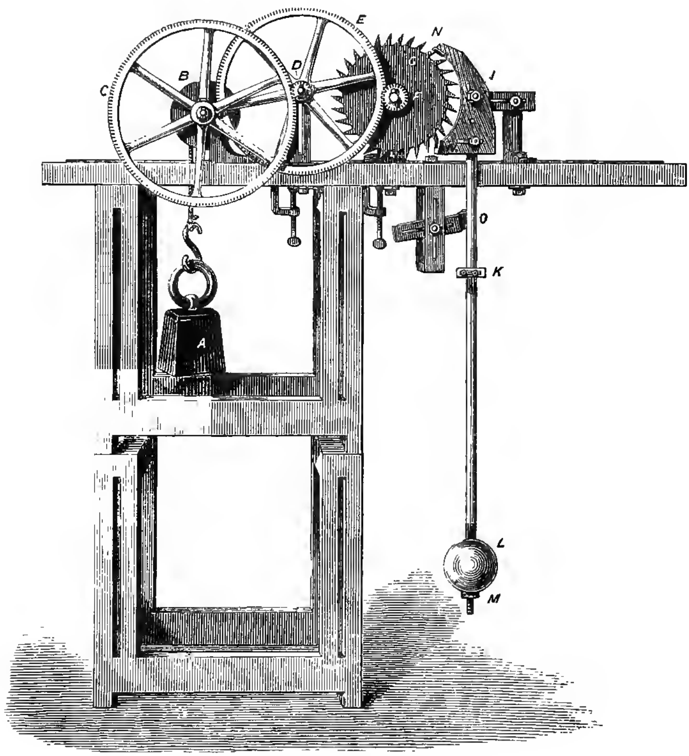

FIG. 99.

found in a clock; but I have constructed the machine in this way in order to show clearly the action of the different parts. G is called the escapement-wheel: it is surrounded by thirty teeth, and turns round once when the pendulum has performed sixty vibrations,—that is, once a minute. I represents the escapement; it turns about an axis and carries the fork K: this fork projects behind, and between the prongs the rod of the pendulum passes. The pendulum is itself suspended from a point O. At N, H are polished surfaces called the pallets: these fulfil a very important part.

675. The escapement-wheel is constantly urged to turn round by the action of the weight and train of wheels, of which we shall speak presently; but the action of the pallets regulates the rate at which the wheel can revolve. When a tooth of the wheel falls upon the pallet N, the latter is gently pressed away: this pressure is transmitted by the fork to the pendulum; as N moves away from the wheel, the other pallet H approaches the wheel; and by the time N has receded so far that the tooth slips from it, H has advanced sufficiently far to catch the tooth which immediately drops upon H. In fact, the moment the tooth is free from N, the wheel begins to turn in consequence of the weight A; but the wheel is quickly stopped by a tooth falling on H: the noise of this collision is the well-known tick of the clock. Now what happens? The pendulum is still swinging to the left when the tooth falls on H. The action of the tooth then tends to press H outwards, but the inertia of the pendulum in forcing H inwards is at first sufficient to overcome the outward pressure arising from the wheel; the consequence is that, after the tooth has dropped, the escapement-wheel moves back a little, or "recoils," as it is called. If you look at any ordinary hall clock, which has a second-hand, you will notice that after each second is completed the hand recoils before starting for the next second. The reason of this is, that the

second-hand is turned directly by the escapement-wheel, and that the inertia of the pendulum causes the escapement-wheel to recoil. But the constant pressure of the tooth soon overcomes the inertia of the pendulum, and H is gradually pushed out until the tooth is able to "escape;" the moment it does so the wheel begins to turn round, but is quickly brought up by another tooth falling on N, which has moved sufficiently inwards.

The process we have just described then recurs over again. Each tooth escapes at each pallet, and the escapements take place once a second; hence the escapement-wheel with thirty teeth will turn round once in a minute.

676. Now, how far does this escapement leave the pendulum free? When the tooth is pushing N, the pendulum is being urged to the left; the instant this tooth escapes, another tooth falls on H, and the pendulum, ere it has accomplished its swing to the left, has a force exerted upon it to bring it to the right. When this force and gravity combined have stopped the pendulum, and caused it to move to the right, the tooth soon escapes at H, and another tooth falls on N, then retarding the pendulum. Hence, except during the very minute portion of time that the wheel turns after one escapement, and before the next tick, the pendulum is never free; it is urged forwards when its velocity is great, but before it comes to the end of its vibration it is urged backwards: this escapement does not therefore possess the characteristics which we pointed out (Art. 673) as necessary for a really good escapement. For the ordinary purposes of time-keeping, however, the arrangement works sufficiently well, as the force which acts upon the pendulum is in reality extremely small. But for the refined uses of the astronomical clock, to

which we have already alluded, the performance of a recoil-escapement is inadequate.

The obvious defect in the recoil is the circumstance that the pendulum is retarded during a portion of its vibration; the impulse forward is of course necessary, but the retarding force is useless and injurious.

677. The "dead-beat" escapement was devised by the celebrated clockmaker Graham, in order to avoid this difficulty. If you observe the second-hand of a clock, controlled by this escapement, you will understand why it is called the dead beat: there is no recoil; the second-hand moves steadily over each second, and remains there fixed until it starts for the next second.

The wheel and escapement by which this effect is produced is shown in Fig. 100. A and B are the pallets, by the action of the teeth on which the motion is given to the crutch, which turns about the centre O; from the axis through this centre the fork descends, so that as the crutch is made to vibrate to and fro by the wheel, the fork is also made to vibrate, and thus sustain the motion of the pendulum. But the essential feature in which the dead-beat escapement differs from the recoil escapement is this: when the tooth escapes from the pallet A, the wheel turns; but the tooth which in the recoil escapement would have fallen on the other pallet, now falls on a surface D, and not on the pallet B. D is part of a circle whose centre is at O, the centre of motion; consequently, the tooth can neither affect the crutch, nor be affected by it, when the tooth lies on the surface D.

678. There is thus no recoil, and the pendulum is allowed to reach the extremity of its swing to the right unretarded; but when the pendulum is returning, the crutch moves until the tooth D passes from the circular

arc D on to the pallet B: instantly the tooth slides down the pallet, giving the crutch an impulse, and escaping when the point has traversed B. The next tooth that comes into action falls upon the circular surface C, whose centre is also at O: this tooth likewise remains at rest until the pendulum has finished its swing, and has com-

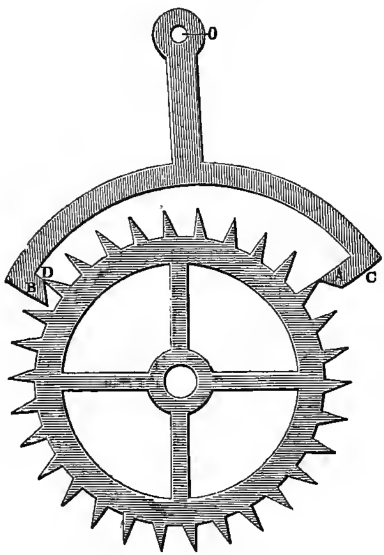

FIG. 100.

menced its return; then the tooth slides down A, and the process recommences as before.

679. The operations are so timed that the pendulum receives its impulse (which takes place when a tooth slides down a pallet) precisely when the pendulum is at the middle of the stroke; the pendulum is then unacted upon till it reaches a similar position in the next vibration. This impulse at the middle of the stroke does not affect the time of vibration, so that the pendulum works very freely.

680. There is still a certain minute resisting force acting to retard the pendulum. This arises from the pressure of the teeth upon the circular surfaces, for there is a certain amount of friction, however carefully the surfaces may be polished. This friction is not found practically to be a source of any appreciable irregularity.

In a clock furnished with a dead-beat escapement and a mercurial pendulum, we have a superb time-keeper.

THE TRAIN OF WHEELS.

681. We have next to consider the manner in which the supply of energy is communicated to the escapement-wheel, and also the mode in which the vibrations of the pendulum are counted. A train of wheels for this purpose is shown in Fig. 99. The same remark may be made about this train that we have already made about the escapement,—namely, that it is more designed to explain the principle clearly than to show the actual construction of a clock.

682. The weight A which animates the whole machine is attached to a rope, which is wound around a barrel B; the process of winding up the clock consists in raising this weight. On the same axle as the barrel B is a large toothed-wheel C; this wheel contains 200 teeth. The wheel C works into a pinion D, containing 20 teeth; consequently, when the wheel C has turned round once, the pinion D has turned round ten times. The large wheel E is on the same axle with the pinion D, and turns with D; the wheel E contains 180 teeth, and works into the pinion F, containing 30 teeth: consequently when E has gone round once, F will have turned round six times;

and therefore, when the wheel C and the barrel B have made one revolution, the pinion F will have gone round sixty times; but the wheel G is on the same shaft as the pinion F, and therefore, for every sixty revolutions of the escapement-wheel, the wheel C will have gone round once. We have already shown that the escapement-wheel goes round once a minute, and hence the wheel C must go round once in an hour. If therefore a hand be placed on the same axle with C, in front of a clock dial, the hand will go completely round once an hour; that is, it will be the minute-hand of the clock.

683. The train of wheels serves also to transmit the power of the descending weight and supply energy to the pendulum. In the clock model you see before you, the weight sustaining the motion is 56 lbs. The diameter of the escapement-wheel is about double that of the barrel, and the wheel turns round sixty times as fast as the barrel; therefore for every inch the weight descends, the circumference of the escapement-wheel must move through 120 inches. The force of 56 lbs. is therefore, at all events, reduced to the one hundred-and-twentieth part of its amount at the circumference of the escapement-wheel. This follows from the principles already explained in Arts. 191 and 192. In reality the force is even less than this, as the friction in such a train of wheels is considerable; therefore the actual force with which each tooth acts upon the pallet is only a few ounces.

684. In a good clock an extremely minute force need only be supplied to the pendulum, so that, notwithstanding 86,400 vibrations have to be performed daily, one winding of the clock in a week will supply sufficient energy to sustain the motion.

THE HANDS.

685. How is it that the hour-hand and the minute-hand are made to revolve with different velocities about the same dial? We shall be able to explain this by the help of Fig. 101.

G is a handle by which I can turn round the shaft which carries the wheel F, and the hand B. The wheel F contains 20 teeth; this wheel works into another wheel E, containing 80 teeth; the shaft which is turned by E carries another wheel D, containing 25 teeth; and D works into a wheel C, containing 75 teeth. C is capable of turn-

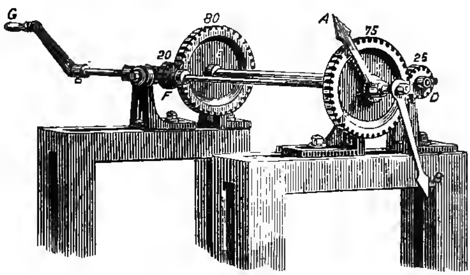

FIG. 101.

ing freely round the shaft, so that the motion of the shaft does not affect it, except through the intervention of the wheels E, F, and D. To C another hand A is attached, which therefore turns round simultaneously with C. Let us compare the motion of the two hands A and B. We suppose that the handle G is turned twelve times; then, of course, the hand B, since it is on the shaft, will turn twelve times. The wheel F also turns twelve times, but E has four times the number of teeth that A has, and therefore, when F has gone round four times, E will only have gone round once: hence, when F has revolved twelve times, E will have gone round three times. D turns with E, and therefore the twelve revolutions of the handle will have turned D round three times; but since C has 75 teeth and D 25 teeth, C will have only made one revolution, while D has made three revolutions; hence the hand A will have made only one revolution, while the hand B has made twelve revolutions.

We have already seen (Art. 682) how, by a train of wheels, one wheel can be made to revolve once in an hour. If that wheel be upon the shaft instead of the handle G, the hand B will be the minute-hand of the clock, and the hand A the hour-hand.

686. The action in this contrivance is worthy of attention. The choice of wheels which would answer is limited. For since the shafts are parallel, the distance from the centre of the wheel F to the centre of the wheel E, must be equal to the distance from the centre of the wheel C to the centre of the wheel D. But it is evident that the distance from the centre of F to the centre of E is equal to the sum of the radii of the wheels F and E. Hence the sum of the radii of the wheels F and E, must be equal to the sum of the radii of C and D; and since the number of teeth in the wheels are proportional to their radii, it follows that the sum of the teeth in E and F must be equal to the sum of the teeth in C and D. In the present case each of these sums is equal to one hundred.

687. Other arrangements of wheels might have been devised, which would give the required motion; for example, if F were 20, as before, and E 240, and if C and D were each equal to 130, the sum of the teeth in each pair would be 260. E would only turn round once for every twelve revolutions of F, and C and D would turn with the

same velocity as E; hence the motion of the hand A would be one-twelfth that of B. This plan requires larger wheels than the train already proposed.

THE STRIKING PARTS.

688. We have examined the essential features of the going parts of the clock; to complete our sketch of this instrument we shall describe the beautiful mechanism by which the striking is arranged. The model which I shall show you (Fig. 102) is, as usual, rather intended to illustrate the principles of the striking gear than to be an exact counterpart of the arrangement found in clocks. Some of the details are not reproduced in the model; but enough is shown to explain the principle, and to enable the model to work.

689. The duty which the striking part of a clock has to accomplish is this. When the hour-hand reaches certain points on the dial, the striking is to commence; and a certain number of strokes must be delivered. The apparatus has then both to initiate the striking and control the number of strokes; the latter is by far the more difficult duty. Two contrivances are in common use; we shall describe that which is used in the best clocks.

690. An essential feature of the striking gear in the repeating clock is the snail, which is shown at B. This piece revolves once in twelve hours, and is, therefore, attached to an axle which performs its revolution in exactly the same time as the hour-hand of the clock. In the model, the striking gear is shown detached from the going parts, but it is easy to imagine that the snail can receive this motion. The margin of the snail is

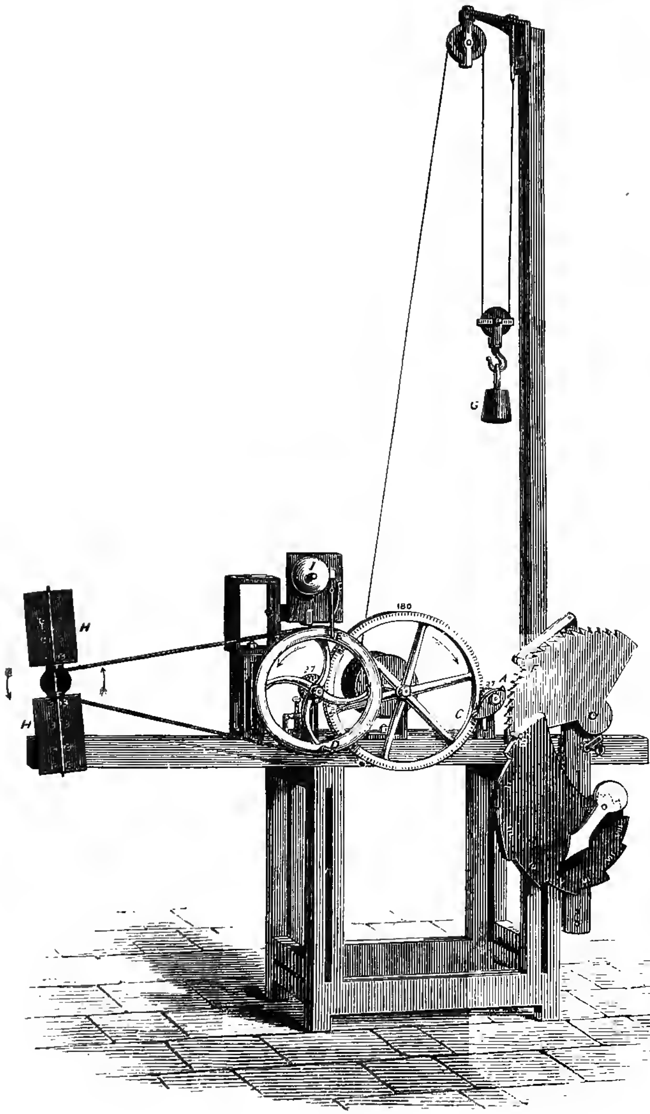

FIG. 102.

marked with twelve steps, numbered from one to twelve. The portions of the margin between each pair of steps is a part of the circumference of a circle, of which the axis of the snail is the centre. The correct figuring of the snail is of the utmost importance to the correct performance of the clock. Above the snail is a portion of a toothed wheel, F, called the rack; this contains about fourteen or fifteen teeth. When this wheel is free, it falls down until a pin comes in contact with the snail at B.

691. The distance through which the rack falls depends upon the position of the snail; if the pin come in contact with the part marked I., as it does in the figure, the rack will descend but a small distance, while, if the pin fall on the part marked VII., the rack will have a longer fall: hence as the snail changes its position with the successive hours, so the distance through which the rack falls changes also. The snail is so contrived that at each hour the rack falls on a lower step than it does in the preceding hour; for example, during the hour of three o'clock, the rack would, if allowed to fall, always drop upon the part of the snail marked III., but, when four o'clock has arrived, the rack would fall on the part marked IV.; it is to ensure this happening correctly that such attention must be paid to the form of the snail.

692. A is a small piece called the gathering pallet: it is so placed with reference to the rack that, at each revolution of A, the pallet raises the rack one tooth. Thus, after the rack has fallen, the gathering pallet gradually raises it.

693. On the same axle as the gathering pallet, and turning with it, is another piece C. The object of this piece C is to arrest the motion when the rack has been

raised sufficiently. On the rack is a projecting pin; the piece C passes free of this pin until the rack has been lifted to a certain height, when C is caught by the pin, and the motion is arrested. The magnitude of the teeth in the rack is so arranged with reference to the snail, that the number of lifts which the pallet must make in raising the rack is equal to the number marked upon the step of the snail upon which the rack had fallen; hence the snail has the effect of controlling the number of revolutions which the gathering pallet can make. The rack is retained by a detent F, after being raised each tooth.

694. The gathering pallet is turned by a small pinion of 27 teeth, and the pinion is worked by the wheel C, of 180 teeth. This wheel carries a barrel, to which a movement of rotation is given by a weight, the arrangement of which is evident: a second pinion of 27 teeth on the same axle with D is also turned by the large wheel C. Since these pinions are equal, they revolve with precisely equal velocities. The second pinion carries a large wheel D: over D the bell I is placed; its hammer E is so arranged that a pin attached to D strikes the bell once in every revolution of D. The action will now be easily understood. When the hour-hand reaches the hour, a simple arrangement raises the detent F; the rack then drops; the moment the rack drops, the gathering pallet commences to revolve and raises up the rack; as each tooth is raised a stroke is given to the bell, and thus the bell strikes until the piece C is brought to rest against the pin.

695. The object of the fan H is to control the rapidity of the motion: when its blades are placed more or less obliquely, the velocity is lessened or increased.

APPENDIX.

We shall now describe how the formulæ in the tables have been ascertained. The formulæ can be deduced by two different methods,—one that of graphical construction, the other that of least squares. The first method is the more simple and requires but little calculation; though neatness and care are necessary in constructing the diagrams. The second method will be described for the benefit of those who possess the requisite mathematical knowledge. The formulæ, in the form in which they have been recorded, have been deduced from the method of least squares, as the results are to a slight, though insignificant, extent more accurate than those of the method of graphical construction. This remark will explain why the terms in some of the formulæ are carried to a greater number of places of decimals than could be obtained by graphical construction.

We shall confine the numerical examples to Tables III. and IV., and show how the formulæ of these tables have been deduced by the two different methods.

Tables V., XIV., XVI., XXI., are to be found in the same manner as Tables III.; and Tables VI., IX., X., XI., XV., XVII., XVIII., XIX., XX., XXI., XXII., in the same manner as Table IV.

I.

THE METHOD OF GRAPHICAL CONSTRUCTION.

Table III.

A horizontal line APS, shown on a diminished scale in Fig. 103, is to be neatly drawn upon a piece of cardboard about $14'' \times 6''$. A scale which reads to the hundredth of an inch is to be used in the construction of the figure. A pocket lens will be found convenient in reading the small divisions. By means of a pair of compasses and the scale, points are to be marked upon the line APS, at distances 1″·4, 2″·8, 4″·2, 5″·6, 7″·0, 8″·4, 9″·8, 11″·2 from the origin A. These distances correspond to the magnitudes of the loads placed upon the slide on the scale of 0″·1 to 1 lb. Perpendiculars to APS are to be erected at the points marked, and distances F_1, F_2, F_3, &c., set off upon these perpendiculars. These distances are to be equal on the adopted scale, to the frictions for the corresponding loads. For example, we see from Table III., Experiment 3, that when the load upon the slide is 42 lbs., the friction is 12·2 lbs.; hence the point F_3 is found by measuring a distance 4″·2 from A, and erecting a perpendicular 1″·22. Thus, for each of the loads a point is deter-

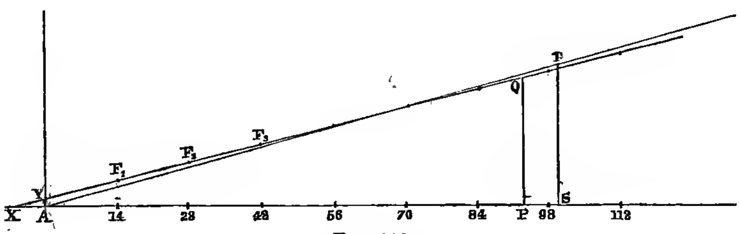

Fig. 103.

mined. The positions of these points should be indicated by making each of them the centre of a small circle 0″·1 diameter. These circles, besides neatly defining the points, will be useful in a subsequent part of the process.

It will be found that the points F_1, F_2, &c. are very nearly in a straight line. We assume that, if the apparatus and observations were perfect, the points would lie exactly in a straight line. The object of the construction is to determine the straight line, which on the whole is most close to all the points. If it be true that the friction is proportioned to the pressure, this line should pass through the origin A, for then the perpendicular which represents the friction is proportional to the line cut off from A, which represents the load. It will be found that a line AT can be drawn through the origin A, so that all the points are in the immediate vicinity of this line, if not actually upon it. A string of fine black silk about 15″ long, stretched by a bow of wire or whalebone, is a convenient straight-edge for finding the required line. The circles described about the points F_1, F_2, &c. will facilitate the placing of the silk line as nearly as possible through all the points. It will not be found possible to draw a line through A, which shall intersect all the circles; the best line passes below but very near to the circles round F_1, F_2, F_3, F_4, touches the circle about F_5, intersects the circles about F_6 and F_7, and passes above the circle round F_8. The line should be so placed that its depth below the point which is most above it, is equal to the height at which it passes above the point which is most below it..

From A measure AS, a length of 10″, and erect the perpendicular ST. We find by measurement that ST is 2″·7. If, then, we suppose that the friction for any load is really represented by the distance cut off by the line AT upon the perpendicular, it follows that

$$F : R :: 2''\cdot7 : 10''.$$
$$\text{or } F = 0\cdot27\ R.$$

This is the formula from which Table III. has been constructed.

Table IV.

By a careful application of the silk bow-string, XYQ can be drawn, which, itself in close proximity to A, passes more nearly through F_1, F_2, &c. than is possible for any line which passes exactly through A. XYQ will be found not only to intersect all the small circles, but to cut off a considerable arc from each. Measure off XP a distance of 10″, and erect the perpendicular PQ; then, if R be the load, and F the corresponding friction, we must have from similar triangles—

$$\frac{F - \dfrac{A\,Y}{0''\cdot1} \times 1 \text{ lb.}}{R} = \frac{P\,Q}{P\,X}.$$

By measurement it is found that $A\,Y = 0''\cdot14$, and $P\,Q = 2''\cdot53$.

We have, therefore,

$$F = 1{\cdot}4 + 0{\cdot}253\ R.$$

This is practically the same formula as

$$F = 1{\cdot}44 + 0{\cdot}252\ R,$$

from which the table has been constructed. In fact, the column of calculated values of the friction might have been computed from the formula we have deduced, without appreciably differing from what is found in the table.

II.

THE METHOD OF LEAST SQUARES.

TABLE III.

Let K be the coefficient of friction. It is impossible to find any value for K which will satisfy the equation,

$$F - KR = 0,$$

for all the observed pairs of values of F and R. We have then to find the value for K, which, upon the whole, best represents the experiments. $F - KR$ is to be as near zero as possible for each pair of values of F and R.

It is known to mathematicians that the best value of K is that which makes

$$(F_1 - KR_1)^2 + (F_2 - KR_2)^2 + \&c. + (F_m - KR_m)^2$$

a minimum.

In fact, it is easy to see that, if this quantity be small, each of the essentially positive elements,

$$(F - KR)^2,$$

of which it is composed, must be small also, and that therefore

$$F - KR$$

must always be nearly zero.

Differentiating the sum of squares and equating the differential coefficient to zero, we have according to the usual notation,

$$\Sigma R_1 (F_1 - K R_1) = 0;$$

$$\text{whence } K = \frac{\Sigma R_1 F_1}{\Sigma R_1^2}.$$

The calculation of K becomes simplified when (as is generally the case in the tables) the loads R_1, R_2, &c., R_m are of the form,

$$N,\ 2N,\ 3N,\ \&c.\ mN.$$

In this case,

$$\Sigma R_1 F_1 = N (F_1 + 2F_2 + 3F_3 + \&c. + m F_m).$$

$$\Sigma R_1^2 = N^2 (1^2 + 2^2 + \&c. + m^2)$$

$$= N^2 \frac{m (m+1)(2m+1)}{6}$$

$$\therefore K = 6 \frac{(F + 2F_2 + \&c. + m F_m)}{m (m+1)(2m+1) N}.$$

In the case of Table III.

$$m = 8,\ N = 14,$$

$$F_1 + 2F_2 + 3F_3 + m F_m = 770{\cdot}9;$$

$$\text{whence } K = 0{\cdot}27.$$

Thus the formula $F = 0{\cdot}27 R$ is deduced both by the method of least squares, and by the method of graphical construction.

Table IV.

The formula for this table is to be deduced from the following considerations.

No values exist for x and y, so that the equation

$$F = x + y R$$

shall be satisfied for all pairs of values of F and R, but the best values for x and y will make the quantity

$$(F_1 - x - y R_1)^2 + (F_2 - x - y R_2)^2 + \&c. + (F_m - x - y R_m)^2$$

a minimum.

Differentiating with respect to x and y, and equating the differential coefficients to zero, we have

$$\Sigma (F_1 - x - y R_1) = 0,$$

$$\Sigma R_1 (F_1 - x - y R_1) = 0.$$

This gives two equations for the determination of x and y. Suppose, as is usually the case, the loads be of the form,

$$N,\ 2\,N,\ 3\,N,\ 4\,N,\ \&c.\ m\,N,$$

and making

$$A = F_1 + F_2 + F_3 + \&c. + F_m$$
$$B = F_1 + 2\,F_2 + 3\,F_3 + \&c. + m\,F_m,$$

we have the equations

$$A - m\,x - \frac{m\,(m+1)}{2}\,N\,y = 0,$$

$$B - \frac{m\,(m+1)}{2}\,x - \frac{m\,(m+1)\,(2\,m+1)}{6}\,N\,y = 0.$$

Solving these, we find

$$x = \frac{2 + 4\,m}{m^2 - m}\,A - \frac{6}{m^2 - m}\,B.$$

$$y = \frac{12}{m^3 - m}\,\frac{B}{N} - \frac{6}{m^2 - m}\,\frac{A}{N}.$$

In the present case,

$m = 8,\qquad N = 14,\qquad A = 138{\cdot}4,\qquad B = 770{\cdot}9;$

whence $x = 1{\cdot}44$

$y = 0{\cdot}252,$

and we have the formula,

$$F = 1{\cdot}44 + 0{\cdot}252\,R.$$

INDEX.

(Reference is made to the numbers of the paragraphs, not to the pages.)

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THE END.

R. CLAY, SONS, AND TAYLOR, PRINTERS, BREAD STREET HILL.