remarks on unification

richard waldinger

herbrand award

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[joint work with zohar manna]

jacques herbrand

- studied with emmy noether, von neumann, artin.

- logic was not highly regarded.

- proved consistency of mathematics.

- reconciled with godel's finding.

- died in mountain climbing accident at age 23!

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herbrand and unification

- robinson credited herbrand with unification.

- herbrand treatment analogous to solving sets of simultaneous equations.

- did not have notion of substitution as object.

- no idea of composition of substitution.

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synthesis of unification

- find most general unifier if one exists.

- otherwise, indicate non-unifiable.

  - expression or tuples of expressions.

boyer moore generalization

- "inventor's paradox" [Polya]
- to prove a theorem, strengthen it to get the benefit of a stronger induction hypothesis.

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strengthening of unification theorem

- termination proof does not go through.

- must specify that algorithm yields "idempotent" unifier.

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idempotent unifiers

- $\theta \circ \theta = \theta$

- no variable on left and right side.

- $\{x \leftarrow f(y)\}$ is idempotent

- $\{x \leftarrow f(z), y \leftarrow z, \ z \leftarrow x\}$ is not.

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recursive calls in unification: function application

- **unify(f(s_1,...,s_n), f(t_1,...,t_n)) =**

  unify($\langle s_1,...,s_n \rangle$, $\langle t_1,...,t_n \rangle$)

- reduces size of inputs

- does not increase set of distinct variables.

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recursive calls in unification: lists

- **unify($\langle s_1,...,s_n \rangle$ $\langle t_1,...,t_n \rangle$) =**

  let θ be unify(s_1, t_1) in

  unify($\langle s_2\theta, ..., s_n\theta \rangle$, $\langle t_2\theta,..., t_n\theta \rangle$)

- arguments may get bigger
- if θ is idempotent, set of distinct variables is reduced.

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termination of unification

- one recursive call reduces size of inputs but may leave variable set the same.

- another recursive call may increase size but reduces set of variables in the arguments.

- lexicographic ordering:

  - <variable set, size of expression>

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proposal: discovery during synthesis

- discover generalization.
- discover well-founded relation.

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synthesis of unification still unfinished

- larry paulson

- alessandro armando et al

- daniele nardi

- lars-henrik eriksson

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herbrand's notion still useful

- alberto martelli and ugo montanari used systems of equations.

- notion used in e.g. constraint logic programming.

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