THE VIRTUES AND VICES OF EQUILIBRIUM AND THE FUTURE OF FINANCIAL ECONOMICS

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The Virtues and Vices of Equilibrium and the Future of Financial Economics

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The use of equilibrium models in economics springs from the desire for parsimonious models of economic phenomena that take human reasoning into account. This approach has been the cornerstone of modern economic theory. We explain why this is so, extolling the virtues of equilibrium theory; then we present a critique and describe why this approach is inherently limited, and why economics needs to move in new directions if it is to continue to make progress. We stress that this shouldn't be a question of dogma, and should be resolved empirically. There are situations where equilibrium models provide useful predictions and there are situations where they can never provide useful predictions. There are also many situations where the jury is still out, i.e., where so far they fail to provide a good description of the world, but where proper extensions might change this. Our goal is to convince the skeptics that equilibrium models can be useful, but also to make traditional economists more aware of the limitations of equilibrium models. We sketch some alternative approaches and discuss why they should play an important role in future research in economics. © 2008 Wiley Periodicals, Inc. Complexity 14: 11–38, 2009

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1. INTRODUCTION

The concept of equilibrium has dominated economics and finance for at least 50 years. Motivated by the sound desire to find a parsimonious description of the world, equilibrium theory has had an enormous impact on the way that economists think, and indeed in many respects defines the way economists think. Nonetheless, its empirical validity and the extent of its scope still remains a matter of debate. Its proponents argue that it has been enormously successful, and that it at least qualitatively explains many aspects of real economic phenomena. Its detractors argue that when one probes to the bottom, most of its predictions are essentially unfalsifiable and therefore are not in fact testable scientific theories. We attempt to give some clarity to this debate. Our view is that while equilibrium theory is useful, there are inherent limitations to what it can ever achieve. Economists

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must expand the scope of equilibrium theory, explore entirely new approaches, and combine equilibrium methods with new approaches.

This article is the outcome of an eight-year conversation between an economist and a physicist. Both of us have been involved in developing trading strategies for hedge funds, giving us a deep appreciation of the difference between theories that are empirically useful and those that are merely aesthetically pleasing. Our hedge funds use completely different strategies. The strategy the economist developed uses equilibrium methods with behavioral modifications to trade mortgage-backed securities; the strategy the physicist developed uses time-series methods based on historical patterns to trade stocks, an approach that is in some sense the antithesis of the equilibrium method. Both strategies have been highly successful. We initially came at the concept of equilibrium from very different points of view, one very supportive and the other very skeptical. We have had many arguments over the last eight years. Surprisingly, we have more or less come to agree on the advantages and disadvantages of the equilibrium approach and the need to go beyond it. The view presented here is the result of this dialogue.

This article is organized as follows: In Section 2 we present a short qualitative review of what equilibrium theory is and what it is not, and in Section 3 we discuss the related idea of market efficiency and its importance in finance. In Section 4 we present what we think are the accomplishments and strengths of equilibrium theory, and in Section 5 we discuss its inherent limitations from a theoretical perspective. In Section 6 we review the empirical evidence for and against equilibrium models. Then in Section 7 we develop some further motivation for nonequilibrium models, illustrated by a few problems that we think are inherently out of equilibrium. We also compare equilibrium in physics and economics and use nonequilibrium models in physics to motivate corresponding models in economics. In Section 8 we review several alternatives approaches, including now established approaches such as behavioral and experimental economics. We also describe some less established approaches to dealing with bounded rationality, specialization and heterogeneous agents. We make a distinction between the structural properties of a model (those depend on the structure of institutions and interactions) vs. the strategic properties (those that depend on the strategies of agents) and discuss problems in which structure dominates (so equilibrium may not be very important). We also discuss how ideas from biology may be useful for understanding markets. Finally in Section 9 we present some conclusions.

2. WHAT IS AN EQUILIBRIUM THEORY?

In this section we explain the basic idea and the motivation for the neoclassical concept of economic equilibrium, and discuss some variations on the basic theme.

Market equilibrium models are designed to describe the market interactions of rational agents in an economy. The prototype is the general equilibrium theory of Arrow and Debreu [1, 2]. The economy consists of a set of goods, such as seed, corn, apples, and labor time, and a set of agents (households) who decide what to buy and sell, how and what to produce, and how much to consume of each good. Agents are characterized by their endowments, technologies, and utilities. The endowment of an agent is the set of all goods she inherits: for example, her ability to labor, the apples on the trees in her backyard and so on. The technology of an agent is her collection of recipes for transforming goods into others, like seed into corn. She might buy the seed and hire the labor to do the job, or use her own seed and labor. Some recipes are better than others, and not all agents have access to the same technologies. The utility of an agent describes how much happiness she gets out of consuming the goods. The agent does not get pleasure out of each good separately, but only in the context of all the goods she consumes taken together.

The goal is to model the decisions of agents incentivized solely by their own selfish goals, and to deduce the consequences for the production, consumption, and pricing of each good. The model of Arrow and Debrieu is based on four assumptions:

1. Perfect competition (price taking). Each agent is small enough to have a negligible effect on prices. Agents “take” the prices as given, in the sense that they are offered prices at which they can buy or sell as much as they want, but the agents cannot do anything to change the prices.
2. Agent optimization of utility. At the given prices each agent independently chooses what to buy, what and how to produce, and what to sell so as to maximize her utility. There is no free lunch—the cost of all purchases must be financed by sales. An agent can only buy corn, for example, by selling an equal value of her labor, or apples, or some other good from her endowment.
3. Market clearing. The aggregate demand, i.e. the total quantity that people wish to buy, must be equal to the aggregate supply, i.e. the total quantity that people wish to sell.
4. Rational expectations. Agents make rational decisions based on perfect information. They have perfect models of the world. They know the prices of every good before they decide how much to buy or sell of any good.

To actually use this theory one must of course make it concrete by specifying the set of goods in the economy, how production depends on labor and other inputs, and a functional form for the utilities. General equilibrium theory was revolutionary because it provided closure, giving a framework connecting the different components of the economy to

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each other and providing a minimum set of assumptions necessary to get a solution. The behavior of markets is an extraordinarily complicated and subtle phenomenon, but neoclassical economists believe that most of it can be explained on the basis of equilibrium.

The advent of general equilibrium theory marked a major transition in the discipline of economics. It gave hope for a quantitative explanation of the properties of the economy in one grand theory, causing a sea change in the way economics is done and in the kind of people that do it. Mainstream economics shifted from a largely qualitative pursuit, often called "political economy," to a highly mathematical field in which articles are often published in theorem-proof format.

We do not have the space to describe the many applications and consequences of equilibrium in any detail. The most important consequence for our purposes is efficiency, which we shall come to shortly. In the remainder of this section we discuss some logical questions one might raise about whether equilibrium exists and how and whether it is attainable. Then we give a few examples of how equilibrium models are being continuously extended to include more phenomena, and develop the concept of financial equilibrium, our central topic here.

2.1. Existence of Equilibrium and Fixed Points

In equilibrium everybody's plans are fulfilled. An agent who plans to buy three ears of corn by selling two apples at the given prices will find agents who want to sell him the three ears of corn and others who want to buy his two apples. A mother who goes to the store to buy her children bread will find as much as she wants to buy at the going prices on the shelves; yet at the end of the day the store will not be left with extra bread that goes to waste. One might well wonder how this could happen without any central coordinator. In short, why should there be an equilibrium? This is called the existence problem.

In 1954 Arrow, Debreu, and McKenzie simultaneously showed that there always is an equilibrium, no matter what the endowments and technologies and utilities, provided that each utility displays diminishing marginal utility of consumption and each technology displays diminishing marginal product. The margin is a very famous concept in economics that was discovered by Jevons, Menger, and Walras in the so-called marginal revolution of 1871. Apples display diminishing marginal utility if the more apples an agent consumes, the less additional happiness he achieves from one more apple. Diminishing marginal product means that the more a good like labor is used in production, the less extra output comes from one additional unit (hour) of input.

The method of proof used by all these authors was first to guess a vector of prices, one for each good, then to compute demand and supply at these prices, and then to increase the prices for the goods with excess demand and to decrease the prices of goods with excess supply. This defines a map from price vectors to price vectors. A price vector generates an equilibrium if and only if it is mapped into itself by this map (for then there is no excess demand or excess supply). By a clever use of Brouwer's fixed point theorem, it was shown that there must always be a fixed point and therefore an equilibrium.

2.2. Getting to Equilibrium

2.2.1. Tatonnement

Who sets the equilibrium prices? How does the economy get to equilibrium? Just because there is an equilibrium does not mean it is easy to find. One suggestion of the marginalist Leon Walras was that the equilibrium map could be iterated over and over. Starting from a guess, one could keep increasing and decreasing prices depending on whether there was too much demand or supply. He called this groping (tatonnement in French) toward equilibrium. Unfortunately, after many years of analysis, it became clear that in general this tatonnement can cycle without ever getting close to equilibium.

2.2.2. Omniscience

An alternative is that the agents know the characteristics of their competitors and know that they are rational and will not sell their goods at less than they could get elsewhere; if it is common knowledge that everybody is rational, and if the characteristics are common knowledge, then each agent will himself solve for equilibrium and the prices will simply emerge.

This explanation for the emergence of prices has an analogue in Nash equilibrium in game theory. A game is a contest in which each player makes a move and receives a payoff that depends on this move as well as the moves of the other players. The question is how does a player know what move to make without knowing what moves his opponents will make? And how can he guess what they will do without knowing what they think he will do? This circular conundrum was resolved by Nash in 1951 who basically showed that there is always at least one self-consistent fixed point in the space of strategies. To be more specific, a Nash equilibrium is an assignment of moves to each player such that if each player thought the others would follow their assignments, then he would want to follow his. In game theory the question also arises: who announces this assignment? If the players are all perfectly logical, and are aware of all their characteristics, they should deduce the assignment.

2.3. Financial Equilibrium Models

2.3.1. Time, Uncertainty, and Securities

One of the main limitations of general equilibrium theory (as described so far) is that it is cast in a nontemporal setting. There is only one time period, so agents don't need to worry about the future. They don't look ahead and they

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don't speculate. This simplifies matters enormously but comes at the heavy cost that the theory is powerless to deal with temporal change. Another limitation to the theory so far is that agents only trade goods. In the real world they trade stocks and bonds and other financial securities. Financial equilibrium extends general equilibrium to allow for time, uncertainty, and financial securities.

Uncertainty about the future is modeled in terms of states of nature that unfold as the nodes in a tree [2, 3]. Each state represents the condition of things like the weather or political events whose future behavior is unknown, but exogenous to the economy. States are given, and while they can affect the economy, the economy does not affect them. A change in the state of nature is often called a shock. The agents in the economy do not know the future states of nature, but they do know the tree from which the future states of nature are drawn and the probability for reaching each node. For example, suppose we assume that the dividends of an agricultural company are made once a year and take on only one of two possible values, "high" or "low," depending on the weather that year. Then we can organize these as a binary tree in which the n^{th} level with 2^n nodes corresponds to the n^{th} year in the future, and a path from the beginning to the end of the tree corresponds to a particular sequence of high and low dividends.[1] If the situation was more complicated so that we had to separate the weather in the western hemisphere from the eastern hemisphere, then each node would have four branches instead of two and there would be 4^n paths. The agents assign probabilities to each branch of the tree, giving a probability for reaching each node and thus a probability measure on the terminal states in the tree.

Financial models also extend the notion of equilibrium by generalizing the notion of what can be traded to include securities, which are promises to deliver certain goods at certain points in time. Such promises can be contingent on future states of nature. Examples are stocks (which promise future dividends that change depending on the profits of the firm), bonds (which promise the same amount each period over a fixed maturity) and so on. The existence of such securities is important because it allows agents to decrease their risk. It also allows them to speculate in ways that they might not otherwise be able to. The implication of such securities for social welfare has proved to be highly controversial.

In the general equilibrium model each agent is aware of her total consumption of every good, and makes her decisions in that context. That is why every decision is a marginal decision: an increase in her consumption of one good must be evaluated in light of the decrease in her consumption of some other good that might be required to balance her budget. In the financial equilibrium extension we are describing, each agent is assumed to be aware of what her consumption will turn out to be at every node, and to compute her utility on that basis. One popular way to make that calculation is to evaluate the utility of consumption at any given node exactly as agents evaluated their utility of consumption in the general equilibrium model (which is basically a special case of the financial model with one node). One could then multiply the utility of consumption at a given node by the probability of ever reaching the node. Finally, one could discount that number again (i.e., multiply it by a factor less than 1), depending on how far away from the origin of the tree the node is, to account for the impatience of consumers. Summing over all the nodes gives the expected discounted utility of consumption. An agent who contemplates increasing her consumption of some good in some node in the financial model must evaluate her utility in light of the decrease in the holding of some financial security that might then be required for her to balance her budget, including the consequent loss of consumption in future states where that security would have paid dividends.

2.3.2. Financial Equilibrium

Once the model is extended to allow for the tree of states of nature, and the trading of securities, we need to extend the notion of equilibrium. This is done by following principles (1–4) above. Prices must now be given at every node of the tree. When agents optimize they must choose an entire action plan contemplating what they will buy and sell at each node in the tree; part of their contemplated trades will involve securities, and the other part commodities.[2] Any purchase at any node must be financed by a sale of equal value at the same node. The sale could be of other goods, or it could be of securities. The market for securities, as well as for goods, must clear at every node in the tree. Finally, the rational expectations hypothesis becomes much more demanding: Agents are not only aware of the prices of all goods and securities today, but are also aware of how all the prices depend on each node in the future.

2.4. Rational Expectations and Utility Maximization

Maximizing a utility function already embeds a large dose of rationality. The utility function is defined over the entire consumption plan. The agents are also presumed to know the tree of states of nature, and the probabilities of every branch, and to correctly forecast the

[1] *Financial models can also be formulated with continuous states and/or in continuous time.*

[2] *As the path of future states unfolds the agents will carry out the trades they anticipated making on that path. Of course only one path will eventually materialize, and the agents will never get a chance to implement the plans they had made for unrealized branches.*

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prices that will emerge at each node if they get there. So when an agent stops to buy an apple today, she does so only after considering what other fruit she will likely have for dinner, how likely it is she will pass the same shop tomorrow, what else she could spend the money on the day after tomorrow, and so on.

Not surprisingly, this rational utility maximization has received a great deal of attention and has been attacked from many different directions. In 2002 Kahneman received a Nobel prize for showing that real people do not have simple utility functions such as those that are typically assumed in standard economic theory, but instead have preferences that depend on context and even on framing, i.e. on the way things are presented [4]. The field of behavioral economics, which Kahneman exemplifies, investigates psychologically motivated modifications of the rationality and utility assumptions. There are a range of levels at which one can do this. At the opposite extreme from rational expectations, one can simply assume that agents have fixed beliefs, which might or might not correspond to reality [5–7]. At an in-between level one can use a so-called noise trader model in which some agents have fixed beliefs while others are perfectly rational [8, 9] (This can actually make the task of computing the equilibrium even harder, since the rational agents have to have perfect models of the noise traders as well as of each other). At a higher level one can assume that agents are not given the probabilities of states of nature a priori, but need to learn them [10, 11].

2.5. Extensions of Equilibrium

The rational expectations equilibrium model that we have described so far is highly idealized, and in the next sections we will present a critique outlining its problems. The main agenda of current economic theory is to extend it by modifying or generalizing the assumptions. For example, in 1958 Samuelson extended the finite tree equilibrium model to an infinite number of time periods, calling it the overlapping generations model, in order to study intergenerational issues like social security. In 2001 Akerlof, Spence and Stiglitz received the Nobel prize for their work on asymmetric information, i.e. for studying situations such as buying a used car, or negotiations between labor and management, in which the information sets of agents differ [12–14]. Another significant extension is to allow agents who sell securities to default on delivering the dividends those securities promise [15, 16]. This raises interesting problems such as how the market determines the amount of collateral that must be put up for loans [17].

In view of all these extensions the meaning of the word "equilibrium" is not always clear. One definition would be any model that could be interpreted as satisfying hypotheses (1–4) as described in the introduction to this Section. But we also wish to allow for some boundedly rational agents. Following the practice of most economists, we will define an equilibrium model as one in which at least some agents maximize preferences and incorporate expectations in a self-consistent manner. So, for example, a noise trader model is an equilibrium model as long as some of the agents are rational, but models in which all the agents act according to fixed beliefs are not equilibrium models.

3. EFFICIENCY

In equilibrium everyone is acting in his own selfish interest, guided only by the market prices, without any direction from a central planner. One wonders if anything coherent can come from this decentralization of control. One might expect coordination failures and other problems to arise. If activities were organized by rational cooperation, surely people could be made better off. Yet in the general equilibrium model of Arrow and Debreu, equilibrium allocations are always allocatively efficient. Allocative efficiency means that the economy is as productive as possible, or Pareto efficient, i.e. that there is no change in production choices and trades that would make everyone better off. In equilibrium everyone acts in his own selfish interest, yet the decisions they make are in the common good in the sense that not even a benevolent and wise dictator who told everyone what to produce and what to trade could make all the agents better off.

In financial equilibrium the situation is much more complicated and interesting. An important assumption underpinning much of the theory of financial economics is that of complete markets. The markets are complete if at any node in the tree it is possible to offset any future risk, defined as a particular future state, by purchasing a security that pays if and only if that state occurs. When all such securities exist so that markets are complete, financial equilibrium is necessarily Pareto efficient.[3]

In contrast, when markets are not complete, financial equilibrium is almost never Pareto efficient. Worse still, the markets that do exist are not used properly. When markets are incomplete, a benevolent and wise dictator can almost always make everyone better off simply by taxing and subsidizing the existing security trades [21]. What form such government interventions should take is a matter for economic policy. Is it a good approximation to assume that real markets are complete? If they are not complete, does the government have enough information to improve the

[3] *This was proved by Arrow [18] and Debreu [19] under conditions (10-94), including the rationality hypothesis on agents. Brock, Hommes and Wagener have shown that when one deviates from rational expectations, for example by assuming agents use reinforcement learning, adding additional securities can destabilize prices [20]. This suggests that there are situations where market completeness can actually decrease utility.*

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situation? These questions have been a matter of considerable debate.

Since allocative efficiency typically fails for financial economies, economists turned to properties of security prices that must always hold. Informational efficiency is the property that security prices are in some sense unpredictable. In its strongest form, informational efficiency is the simple statement that prices are a martingale [22, 23], i.e. that the current price is the best prediction of future prices.[4] This is important because it suggests that all the information held in the economy is revealed by the prices. If weather forecasts cannot improve on today's price in predicting future orange prices, then one can say that today's orange price already incorporates those forecasts, even if many traders are unaware of the forecasts.[5] Arbitrage efficiency means that through buying and selling securities it is not possible for any trader to make profits in some state without taking any risk (i.e. without losing money in some other state). Again this is important because it suggests that there are no traders who can always beat the market by taking advantage of less informed traders. Thus the uninformed should not feel afraid that they are not getting a fair deal. Informational efficiency and arbitrage efficiency are intimately related; in some contexts they are equivalent.

A consequence of the martingale property is that the price of every (finitely lived) asset must be equal to its fundamental value (also called present value if there is no uncertainty). The fundamental value of an asset is the discounted expected dividend payments of the asset over its entire life, where the expectation is calculated according to the martingale probabilities, and the discount is the risk free rate (the interest rate on securities such as treasury bonds for which the chance of default can be taken to be zero). This is important for investors who might feel too ignorant to buy stocks, because it suggests that on average everybody will get what he pays for.

From the point of view of finance, arbitrage efficiency and informational efficiency are particularly useful because they can be described without any explicit assumptions about utility. Many results in finance can be derived directly from arbitrage efficiency. For example, the Black-Scholes model for pricing options is perhaps the most famous model in finance. The option price can be derived from only two assumptions, namely that the price of the underlying asset is a random walk and that neither the person issuing the option nor the person buying it can make risk-free profits. Because it does not rely on utility, several people have suggested that arbitrage efficiency forms a better basis for financial economics than equilibrium; see for example Ross [25]. As we will argue later, because of all the problems with defining utility, this is a highly desirable feature.

4. THE VIRTUES OF EQUILIBRIUM

4.1. Agent-Based Modeling

Equilibrium theory focuses on individual actions and individual choices. By contrast Marx emphasized class struggles, without asking whether each individual in a class would have the incentive to carry on the struggle. Similarly Keynesian macroeconomics often posited reduced form relationships, such as a positive correlation between unemployment and inflation (called the Phillips curve), without deriving them from individual actions. Equilibrium theory is an agent based approach which does not admit any variables except those that can be explained in terms of individual choices.

The advantages of the agent based approach can be seen in the macroeconomic correlation between inflation and output. At its face, such a correlation suggests that the monetary authority can stimulate increased output by engineering higher inflation through printing money. According to equilibrium theory, if inflation causes higher output, there must be an agent-based explanation. For example, it may be that workers see their wages going up and are tempted to work harder, not realizing that the prices of the goods they will want to buy are also going up so that there is no real incentive to work harder. But if that is the explanation, then it becomes obvious that policy makers will not be able to rely on the Phillips curve to stimulate output by printing more money. Agents will eventually catch on that when their wages are rising, so are general prices.[6] That skepticism was validated during the stagflation of the 1970s, when there was higher inflation and lower output. If printing money sometimes causes higher output, it must be through a different channel, connected say to interest rates and liquidity constraints. Finding what these are at the agent level deepens the analysis and brings it closer to reality.

4.2. The Need to Take Human Reasoning into Account

The fundamental difference between economics and physics is that atoms don't think. People are capable of reasoning and of making strategic plans that take each other's ability to reason into account. Ultimately any economic model has to address this problem. By going to a logical extreme, equilibrium models provide a way to incorporate reasoning without having to confront the

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[4] More precisely, if no security pays a dividend in period $t+1$, then there is a probability measure on the branches out of any node in period t such that the expected price in period $t+1$ of any security (relative to a fixed security), given its (relative) price in period t, is just the period t price.

[5] Roll found in some Florida counties that today's orange prices predicted tomorrow's orange prices better than today's weather reports, and also that today's orange prices predicted tomorrow's weather better than today's weather forecasts [24].

[6] This has come to be called the Lucas critique [26].

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messiness of real human behavior. Even though this approach is obviously unrealistic, the hope is that it may nonetheless be good enough for many purposes.

4.3. Parsimony

4.3.1. Rationality

The rationality hypothesis is a parsimonious description of the world that makes strong predictions. Rational expectations equilibrium models have the advantage that agent expectations are derived from a single, simple and self-consistent assumption. Without this assumption one needs to confront the hard task of determining how agents actually think, and how they think about what others think. This requires formulating a model of cognition or learning, and thereby introducing additional assumptions that are usually complicated and/or ad hoc. Without rationality one needs either to introduce a set of behavioral rules of thumb or to introduce a learning model. While perfect rationality defines a unique or nearly unique model of the world, there are an infinite number of boundedly rational models. To paraphrase Christopher Sims, once we depart from perfect rationality, there are so many possible models it is easy to become lost in the wilderness of bounded rationality [27].

Perfect rationality is an impossible standard for any individual to attain, but the hope of the economist in making a rational expectations model is that in aggregate, people behave "as if" they were rational. This maybe true in some situations, and it may not be true in others. In any case, rational expectations models can provide a useful benchmark for understanding whether or not people are actually rational, which can serve as a starting point for more complicated models that take bounded rationality into account.

4.3.2. Succinctness

Equilibrium theory provides a unifying framework from which many different conclusions can be drawn. Rather than having to invent a new method to attack each problem, it provides a standardized approach, and a standardized language in which to explain each conclusion.

A standardized model does not rule out new theories. New equilibrium theories are of two forms. First, one can specialize the class of utilities, technologies, and endowments to try to draw a sharp and interesting conclusion that does not hold in general. If either the premise or conclusion can be empirically validated, one has found a law of economics. Second, one can extend the definition of equilibrium (adding say asymmetric information or default). The second form facilitates the discovery of many theories of the first form. Agreement on a unifying framework, such as equilibrium and hypotheses (1–4), makes it easier to evaluate new economic theories, and to make progress on applications. But of course it stultifies radically new approaches.

4.4. The Normative Purpose of Economics

One of the principal differences between physical and social science is that the laws of the physical world are fixed, whereas the laws of society are malleable. As human beings we have the capacity to change the world we live in. Economics thus has a normative as well as a descriptive purpose. A descriptive model describes the world as it is, while a normative model describes the world as it might be under a change in social institutions. The equilibrium model is meant to allow us to describe both. For each arrangement of social institutions, such as those that prevail today, we compute the equilibrium. The representation of agents by utilities (as well as endowments) enables us to evaluate the benefits to each person of the resulting equilibrium, and thereby implicitly the utility of the underlying institutions. Equilibrium theory then recommends the institutions that lead to the highest utilities. By contrast, consider a purely phenomenological model that assumes behavioral rules for the agents without deriving them from utility. Such a model is either silent on policy questions, or requires a separate ad hoc notion of desireability in order to recommend one institution over another.

The rationality hypothesis allows us to define equilibrium succinctly. But as a byproduct it shapes the economist's evaluation of the institutions. For example, one of the basic questions in economics is whether free markets organize efficient sharing of risks, leading to sensible production decisions. The equilibrium model is often used to show that under certain conditions, free market incentives lead selfish individuals to make wise social decisions. We called this allocative efficiency or Pareto efficiency. But of course the rationality of all the individuals is an indispensable hypothesis in the proof. If agents are misinformed about future production possibilities, or act whimsically, the free market economy will obviously make bad decisions.

Even if rational expectations doesn't provide a good model of the present, it may sometimes provide a good model of the future. The introduction of an equilibrium model can change the future for the simple reason that once people better understand their optimal strategies they may alter their behavior. A good example of this is the Black-Scholes model; as people began using it to buy and sell mispriced options, the prices of options more closely matched its predictions. Another example is the capital asset pricing model. In that model the optimal strategy for every investor is to hold a giant mutual fund of all stocks. Nowadays every investor who has learned a little finance does indeed think first of holding exactly such an index (though few investors hold only that).

Unfortunately, many economists have used the normative purpose of economics as an excuse to construct economic theories that are so far removed from reality that they are useless. For an economic theory to have useful normative value it must be sufficiently close to reality to inspire confidence that its

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conclusions give useful advice. Unless a theory can give some approximation of the world as it is, it is hard to have confidence in its predictions of how the world might be.

4.5. Why Does Wall Street Like the Equilibrium Model?

Many Wall Street professionals try to beat the market by actively looking for arbitrages. Fundamental analysts, for example, believe that they can find stocks whose prices differ substantially from their fundamental values. Statistical arbitrageurs believe they can find information in previous stock movements that will help them predict the relative direction of future stock movements. The activities of these Wall Street professionals seems proof that they think the equilibrium model is flawed. If they believe the market is in equilibrium, why are they pursuing strategies that cannot succeed in equilibrium?

Ironically, in fixed income and derivative asset markets the arbitrageurs themselves use the apparatus of equilibrium models to find arbitrages. Many people make good profits by betting that when prices make large deviations from equilibrium these deviations will eventually die out and return to equilibrium.

There are many ways Wall Street practitioners use equilibrium models. We review some of them below.

4.5.1. Conditional Forecasting Is Good Discipline

Many pundits, and even some professional economists (and especially macroeconomists), make unconditional forecasts. They say growth next quarter will be slow, but by the fourth quarter it should pick up and unemployment should start to decline. They do not often bother to say, for example, that their predictions might change if there is a messy war in Iraq. In the equilibrium model agents do not know what state will prevail tomorrow, but they do know what prices will prevail conditional on tomorrow's state. They make conditional forecasts, based on a tree of possible future states. Wall Street traders are trained in business schools and economics departments across the country to appreciate equilibrium models. Nowadays so many Wall Street traders make conditional forecasts using concrete trees of future possibilities that this assumption has become more plausible as a descriptive model of the world.

One of the virtues of making conditional forecasts is that it stimulates traders to find strategies (called riskless arbitrages) that will work no matter what the future holds. If one is lucky enough to find a riskless arbitrage, success is independent of the probabilities of the future states of nature. In the real world riskless arbitrages are rare (as equilibrium theory says they should be). Most real arbitrageurs actually make their money with risky arbitrages, and in this case the probabilities for branches of the tree become important. The estimation errors that are inherent in estimating these probabilities compound the risk. Not only might the future bring a state in which money will be lost, but the trader might also underestimate the probability of this state. But at least she can see the scenarios that constitute the model explicitly, and can compute the sensitivity of her expected profits to variations in the probabilities of these scenarios.

4.5.2. The Power of the No-Arbitrage Hypothesis

Tree building is most useful when the future possibilities are easy to imagine, when they are not too numerous to compute, and when there is a reliable guide to their probabilities. The hypothesis of no-arbitrage drastically reduces the number of states, making the computation feasible, and often even determines the probabilities.

One reason that financial practitioners like the no arbitrage assumption is because it makes things much simpler. Arbitrage efficiency requires a consistency between prices that drastically reduces the size of the hypothetical tree. For example, a mortgage derivative trader might worry about future interest rates of all maturities, including the overnight (i.e one day) rate, the yearly rate, the ten year rate, and so on. If there are 20 such rates, and each rate can take on 100 values, then each node in the tree will require 100^{20} branches! Using the overnight rate alone requires a much smaller tree with only 100 branches from each node. The crucial point is that if the trader assumes there will never be any arbitrage among interest rate securities of different maturities in the future, then the smaller tree will be sufficient to recover all the information in the larger tree even for a trader who cares about all the rates. Given the smaller tree and the probabilities of each branch, the trader can deduce all the other future long rates, conditional on the future overnight rates, without adding any new nodes to her tree. (Today's two day discount, for example, can be deduced as the product of today's one day discount and tomorrow's one day discount, averaged over all possible values of the one day discount tomorrow). Furthermore, if the trader assumes there is no arbitrage between current long maturity interest rate securities and future short interest rate securities and current interest rate options, then she can deduce the probabilities of each branch in the smaller overnight interest rate tree.

The homage arbitrage traders pay to arbitrage efficiency is the most compelling evidence that in some respects it is true. Again we see the irony that the road to arbitrage comes from assuming no arbitrage.[7]

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[7] *The mortgage trader assumes no arbitrage in the class of interest rate securities alone. By contrast, she does assume there is an arbitrage involving interest rate securities and mortgage securities. A mortgage derivative trader might assume there is no arbitrage involving interest rate securities and mortgage securities, but that there is an arbitrage once mortgage derivatives are added to the mix.*

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4.5.3. Risk Reduction

Hedging refers to the process of reducing specific financial risks. To hedge the value of a particular security one creates a combination of other securities, called a portfolio, that mimics the security that one wishes to hedge. By selling this combination of securities one can cancel or partially cancel the risk of buying the original security. Statistical arbitrageurs, for example, try to make bets on the relative movements of stocks without making bets on the overall movement of the stock market. The market risk can be hedged by making sure that the total position is market neutral, i.e. that under a movement of the market as a whole the value of the position is unaffected. This is typically done by maintaining constraints on the overall position, and can also be done by selling a security, such as a futures contract on a market index.

To properly hedge it is necessary to have a set of scenarios for the future. Such scenarios can be represented by means of a tree, exactly as we described for equilibrium models. Thus, the same structure we defined in order to discuss equilibrium is central to hedging risks. Wall Street professionals are very concerned about risks, and for this reason like the financial framework for understanding equilibrium.

Hedging and arbitrage are two sides of the same coin. In both hedging and arbitrage, the trader looks for a portfolio of securities that will pay off exactly what the risky asset does. By buying the asset and selling the portfolio, the trader is completely hedged. If the trader discovers that the cost of the hedging portfolio is more than the security, then hedging becomes arbitrage: The arbitrageur can buy the security and sell the portfolio and lock in a riskless profit.

5. DIFFICULTIES AND LIMITATIONS OF EQUILIBRIUM

While equilibrium should be an important component of the economist's tool kit, its dangers and limitations need to be understood (and all too often are not).

5.1. Falsifiability?

Empirical laws in economics are much harder to find than in physics or the other natural sciences. Realistic experiments for the economy as a whole are nearly impossible to conduct (though for some small scale phenomena there is a thriving experimental research effort underway) and many causal variables are impossible to measure or even to observe. Economists usually do not dream of finding "constants of behavior," analogous to the constants of nature found in physics; when they do, they do not expect more than a couple of significant digits.[8] We shall argue later that this may be a mistake, and that it is possible to find finely calibrated relationships in economic data.

Perhaps because of the difficulty of empirical work, economic theory has emphasized understanding over predictability. For example, to economists, equilibrium theory itself is most important not for any empirical predictions, but for the paradoxical understanding it provides for markets: Without any centralized coordination, all individual plans can perfectly mesh; though everyone is perfectly selfish, their actions promote the common good; though everybody is spending lots of time haggling and negotiating over prices, their behavior can be understood as if everybody took all the prices as given and immutable.

Equilibrium theory has not been built with an eye exclusively focused on testability or on finding exact functional forms. Most equilibrium models are highly simplified at the sacrifice of features such as temporal dynamics or the complexities of institutional structure. When it comes time to test the model, it is usually necessary to make auxiliary assumptions that put in additional features that are outside of the theory. All these factors make equilibrium theories difficult to test, and mean that many of the predictions are not as sharp as they seem at first. In many situations, whether or not the predictions of equilibrium theory agree with reality remains controversial, even after decades of debate.

Not only are there few sharp economic predictions to test, but the hypotheses of economic equilibrium also seem questionable, or hard to observe. A long-standing dispute is whether utility is a reasonable foundation for economic theory. The concept of utility as it is normally used in economic theory is purely qualitative. The functional form of utility is generally chosen for convenience, without any empirical justification for choosing one form over another. No one takes the functional form and the parameters of utility functions literally. This creates a vagueness in economic theory that remains in its predictions.

Psychologists have generally concluded that utility is not a good way to describe preferences, and have proposed alternatives, such as prospect theory [4]. Some economists have taken this seriously, as evidenced by the recent Nobel prize awarded to Kahneman. However, most theory is still built around conventional utility functions, and so far the alternatives are not well-integrated into the mainstream. Attempts that have been made to develop economic models based on prospect theory still do not determine the parameters a priori (and in fact they have more free parameters). Thus so far it is still not clear whether more general notions of preferences can be used to make sharper and more quantitative economic predictions.[9]

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[8] *Okun's Law for example, which states that every 1% increase in unemployment is accompanied by a 3% decrease in GNP, has one significant digit.*

[9] *One approach that has gained some attention is hyperbolic discounting [28]. Hyperbolic agents care a lot more about today than tomorrow, yet they act today as if they will never care about the difference between 30 days from today and 31 days from today.*

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The other big problem that is intrinsic to equilibrium theories concerns expectations about the probabilities of future states. It is difficult to measure expectations. In practice we only observe a single path through the tree of future states. The particular path that is observed historically may be atypical, and may not be a good indication of what agents really believed when they made their decisions. Thus, even when we have a good historical record there is plenty of room to debate the conclusions.

All these problems occur in testing the assertion that markets are arbitrage efficient, as we discuss later. Real arbitrages are rarely risk free. Instead, they involve risk, and determining whether one arbitrage is better than another involves measuring risk. Future risks may not match historical risks. If a skillful trader produces excess returns (above and beyond the market as a whole) with a high level of statistical significance, a champion of market efficiency can always argue that this was possible only because the asset had "unobserved risk factors." What seems like a powerful scientific prediction from one point of view can look suspiciously like an unfalsifiable belief system from another.

5.2. Parsimony and Tractability Are Not the Same

While we have argued that equilibrium models are parsimonious this does not necessarily mean that they are easy to use. The mathematical machinery required to set up and solve an equilibrium model can be cumbersome. Finding fixed points is much more difficult than iterating dynamic maps. Thus incorporating equilibrium into a model comes at a large cost, and may force one to neglect other factors, such as the real structure of market institutions. As a result of this complexity most equilibrium models end up being qualitative toy models, formulated over one or two time periods, whose predictions are too qualitative to be testable.

5.3. Realism of the Rationality Hypothesis?

It seems completely obvious that people are not rational. The economic model of rationality not only requires them to be super-smart, it requires them to have God-like powers of omniscience. This should make anyone skeptical that such models can ever describe the real world. Rational expectations (i.e., knowing the probabilities of each branch in the tree and knowing what the prices will be at each node) is often justified by the argument that people behave "as if" they were rational. Skeptics do not find such arguments convincing, particularly without supporting empirical evidence. There are many reasons to be suspicious of equilibrium models, and many situations where the cognitive tasks involved are sufficiently complicated that the a priori expectation that an equilibrium theory should work is not high.

The conceptual problems with perfect rationality can be broken down into several categories:

- Omniscience. To take each others' expectations into account agents must have an accurate model of each other, including the cognitive abilities, utility, and information sets of all agents. All agents construct and solve the same tree. They must also agree on the probabilities of the branches. More realistically one must allow for errors in model building, so that not all agents have the same estimates or even the same tree of possibilities.
- Excessive cognitive demands. The cognitive demands the equilibrium model places on its agents can be preposterous in the sense that the calculations the agents need to make are extremely time consuming to perform. Even given the tree and the probabilities and the conditional prices, it may be difficult for an agent to compute her optimal plan. But how does the agent know the conditional prices unless she herself computes the entire equilibrium based on her knowledge of all the other agents' utilities and endowments? These computational problems can be intractable even if all agents are fully rational, and they can become even worse if some agents are rational and others aren't.
- Behavioral anomalies. There is pervasive evidence of irrationality in both psychological experiments and real world economic behavior. Even in simple situations where people should be able to deal with the difficulties of computing an equilibrium it seems many do not do so [29, 30].
- Modeling cognitive cost and heterogeneity. Real agents have highly diverse and context dependent notions of rationality. Because models are expensive to create real agents take shortcuts and ignore lots of information. They use specialized strategies. The resulting set of decisions may be far from the rationality supposed in equilibrium, even when taken in aggregate.

5.4. Limited Scope

There are many interesting problems in financial economics that the equilibrium framework was never intended to solve. Under conditions that are rapidly evolving, e.g. where there is insufficient time for agents to learn good models of the situation they are placed in, there is little reason to believe that the equilibrium framework describes the real world. Some of the problems include

- Evolution of knowledge. In a real market setting the framework from which we view the world is constantly and unpredictably evolving. The models used by traders to evaluate mortgage securities in the 1980s were vastly more primitive than the models used to evaluate the same securities today. People have a difficult time imagining the possible ways in which knowledge will evolve, much less assigning probabilities to all of its states.
- Lack of tatonnement. The fact that an equilibrium exists does not necessarily

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- imply that it is stable. For stability it is necessary to show that when a system starts out of equilibrium it will necessarily move toward equilibrium. This requires the construction of a model for price formation out of equilibrium, a process that is called tatonnement. Such models suggest that there are many situations where prices will fail to converge to equilibrium [31].
- Inability to model deviations from itself. Equilibrium can't model deviations from itself. It provides no way to pose questions such as "How inefficient is the economy?". One would like to be able to understand questions such as the timescale for violations of arbitrage efficiency to disappear, but such questions are inherently outside of the equilibrium framework. (See the discussion in Section 7).

While the financial implications of equilibrium theory are on one hand extraordinarily powerful, on the other hand, their very power makes them almost empty. Economic theory says that there is very little to know about markets: An asset's price is the best possible measure of its fundamental value, and the best predictor of future prices. If that is true, there is no need to answer the kinds of questions that economists are asked all the time, such as "Are stocks going up or down?," or "Is this a good time to invest in real estate?." Only by going outside the equilibrium model can one raise many pressing financial questions.

It is worth noting that the mere fact that people so persistently ask such questions puts equilibrium theory into doubt. There are three possibilities: Either the people that do this are irrational, but if they are, then how can we expect equilibrium theory to describe them? Or they are rational, but if so, then their questions reflect that their understanding is deeper than that of the equilibrium theorist. The only other explanation is that the people who ask such questions do not invest an economically significant amount of money. But this seems implausible—almost everyone asks such questions.

6. EMPIRICAL EVIDENCE FOR AND AGAINST

Equilibrium theory teaches us that expectations are important, and this is no less true in evaluating the success or failure of equilibrium theory itself. The booster can point to significant successes and the detractor can point to significant failures.

The booster can assert that equilibrium theory has made many practical suggestions that have been followed and have been borne out as correct. Communism did not produce as much output as the free markets of capitalism. Diversifying your investments really is a good idea, and many index funds have been created to accommodate this desire. A hedge fund that can show its returns are independent of much of the rest of the economy can attract investors even if it promises a lower rate of return than competitors whose returns are highly correlated with the market.[10] Arbitrages are indeed hard to find, and even harder to exploit. Stock option prices are explained to a high degree of accuracy by the Black-Scholes formula. Prepayment behavior for prime mortgages can be explained by maximizing the utility of individual households (with some allowances for inattention and other irrational behaviors).

The skeptic counters that, with a few exceptions, most of the predictions of equilibrium theory are qualitative and have not been strongly borne out empirically in an unambiguous manner. To quote Ijiri and Simon: "To be sure, economics has evolved a highly sophisticated body of mathematical laws, but for the most part these laws bear a rather distant relation to empirical phenomena, and usually imply only qualitative relations among observables .... Thus, we know a great deal about the direction of movement of one variable with the movement of another, a little about the magnitudes of such movement, and almost nothing about the functional forms of the underlying relations" [32]. Since they said this in 1977 the situation has improved, but only a little. Apart from options and prime mortgage pricing (but not subprime), there are very few examples of economic theories that cleanly fit the data and also unambiguously exclude nonequilibrium alternatives. While some of the predictions of equilibrium models are in qualitative agreement with the data, there are few that can be quantitatively verified in a really convincing manner.

One of the conclusions of equilibrum theory is that relative prices should follow a martingale. It is well documented that over the long-term risky securities like stocks have had higher returns than safe securities like short term government bonds, at least in the United States.[11] This would seem to contradict the martingale pricing theorem. But the martingale pricing theorem says only that asset prices follow a martingale with respect to some probability measure, not necessarily with respect to the objective probabilities. The skeptic naturally complains that this is giving the theory too much freedom, making it close to tautological. Economists have responded by making more assumptions on the underlying utilities in the equilibrium model to limit how the set of allowable martingale

[10] *Though diversification is often sighted as evidence for equilibrium theory, the skeptic will point out that this only depends on portfolio theory, which is a simple result from variational calculus concerning statistical estimation and has nothing to do with equilibrium theory.*

[11] *Brown, Goetzmann and Ross point out that averaged over all western countries, stocks have not outperformed bonds [33]. An economic collapse, say during the Russian revolution, usually crushes stock prices more than bond prices. So the American experience may not be representative. Of course this illustrates the difficulty of testing the prediction.*

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probabilities can deviate from objective probabilities, making the theory falsifiable. The most famous model of that type is the so-called capital asset pricing model (CAPM) developed by Markowitz, Tobin, Linter, and Sharpe [34–37]. In one version of CAPM all utilities are quadratic.[12] Under this assumption the martingale probabilities are not arbitrary, but must lie in a one dimensional set, making the theory more straightforwardly testable. CAPM does explain why riskier securities should have higher returns than safe securities, and also provides a rigorous definition and quantification of the risk of a security. Data from the 1930s through the 1960s seemed to confirm the CAPM theory of pricing. But little by little the empirical validity of CAPM has unraveled. Of course there is always the possibility that a different specialization will give rise to another model that matches the data more closely. Or perhaps an extension of the equilibrium model involving default and collateral will be able to match the data, without creating so many free parameters as to make the theory unfalsifiable.[13]

A similar story can be told about the conclusion that asset prices should match fundamental values. Attempts to independently compute fundamental values based on dividend series do not match prices very well. For example, for the U.S. stock market Campbell and Shiller [38] show deviations between prices and fundamental values of more than a factor of two over periods as long as decades. This conclusion can be disputed on the grounds that fundamental values are not easily measurable, and perhaps none of the measures they use are correct (though they tried several alternatives). But whether the theory is wrong or whether it is merely unfalsifiable, the failure to produce a good match to fundamental values represents a major flaw in either case.

Perhaps even worse, there are situations where equilibrium makes predictions that appear to be false. For example, people seem to trade much more than they should in equilibrium [39]. Global trading in financial markets is on the order of a hundred times as large as global production [40, 41]. If people are properly taking each others' expectations into account why should they need to trade so much? One of the problems is that the theoretical predictions of how much people should trade are not very sharp, due at least in part to the problems discussed in the previous section. Nonetheless, it seems that even under the most favorable set of assumptions people trade far more than can be rationalized by an equilibrium model [42].

As already discussed in the previous section, one of the key principles of rational expectations is that investors should correctly process information, and thus price movements should occur only in response to new information. Studies that attempt to correlate news arrival with large market moves do not support this. For example, Cutler et al. examined the largest 100 daily price movements in the S&P index during a 40 year period and showed that most of the largest movements occur on days where there is no discernable news, and conversely, days that are particularly newsworthy are not particularly likely to have large price movements [43]. Other studies have shown that price volatility when markets are closed, even on non-holidays, is much lower than when the market is open [44]. The evidence seems to suggest that a substantial fraction of price changes are driven by factors unrelated to information arrival. While news arrival and market movements are clearly correlated, the correlation is not nearly as strong as one would expect based on rational expectations. As asserted in the previous section, markets appear to make their own news. One of the problems in answering this question empirically is that it is difficult to measure information arrival—"news" contains a judgement about what is important or not important, and is difficult to objectively reduce to a quantitative measure.[14]

7. MOTIVATION FOR NONEQUILIBRIUM MODELS: A FEW EXAMPLES

We begin by describing some empirical regularities in financial data that seem salient yet which equilibrium theory does not seem able to explain. Of particular importance is price volatility. Not only do many people think that prices change more than they should under equilibrium, but also volatility displays an interesting temporal correlation structure that seems to be better explained by nonequilibrium models. This correlation is an example of a power law, a functional form that seems to underly many of the regularities in financial economics. In physics power laws can't be generated at equilibrium and are a signature of nonequilibrium behavior. Economic and physical equilibrium are quite different, however, as

[12] *This is an example of the first kind of progress equilibrium theory makes, in which specialized assumptions give rise to more precise conclusions, as we discussed in Section 4.3.2.*

[13] *If successful, this would be an example of the second kind of progress equilibrium theory makes, when the model is extended to incorporate new concepts, but retains methodological premises 1-4, as we discussed in Section 4.3.2.*

[14] *Engle and Rangel [45] demonstrate that there is a positive correlation between low frequency price volatility and macroeconomic fundamentals. Because their results involve both a longtitudinal and a cross-sectional regression involving both developed and undeveloped countries the interpretation of the correlations that they observe is not obvious. No one disputes that prices respond to information—the question is, "What fraction of price movements can be explained by information?." Timescale is clearly critical, i.e. one expects more correlation to information arrival at lower frequencies.*

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we try to make clear. Statistical testing for power laws is difficult and the existence of power laws and their relevance for economics have generated a great deal of debate, which we briefly review.

Next we observe that one of the most frequently cited pieces of evidence for equilibrium is that arbitrage opportunities tend to disappear. There is substantial evidence that real markets are efficient at some level of approximation. But we present anecdotal evidence that efficiencies disappear surprisingly slowly. We would like a theory of the transition to equilibrium, or of tatonnement as it has sometimes been called. We end this section with a brief discussion of one of the most remarkable episodes in financial markets of the last ten years, which illustrates both the value and the limitations of the equilibrium model.

7.1. Some Economically Important Phenomena May Not Depend on Equilibrium

There are many empirically observed properties of markets that have so far not been explained under the equilibrium framework. One famous example is called clustered volatility. This refers to the fact that there are substantial and strongly temporally correlated changes in the size of price movements at different points in time [46]. If prices move a lot on a Tuesday, they will probably move a lot the Wednesday after, and probably (but not as likely) move a lot on Thursday.[15] The hard core rational expectations booster would say that there is nothing to explain. If the standard equilibrium theory is correct, changes in price are caused by the receipt of new information, so clustered volatility is simply a reflection of nonuniform information arrival. The rate of information arrival has to do with meteorology, sociology, political science, etc. It is exogenous to financial economics and so it is someone else's job to explain it, if such a question is even interesting.

An alternative point of view is that, due to a lack of a perfect rationality, the market is not in perfect equilibrium and thus prices do not simply passively reflect new information. Under this view the market acts as a nonlinear dynamical system: agents process information via decision-making rules that respond to prices and other inputs, and prices are formed as a result of agent decisions. Since this information processing is imperfect, the resulting feedback loop amplifies noise. As a result a significant component of volatility is generated by the market itself. This point of view is supported by the fact that there are now many examples of nonequilibrium models that generate clustered volatility, and in some cases there is a good match to empirical data. For many of these models it is possible to include equilibrium as a special case; when this is done the overall volatility level drops and clustered volatility disappears (see Section 8.3). Another relevant point is that volatility is widely believed to exhibit long-memory, which we discuss shortly in the section on power laws.

Since the discussion of properties of markets that are not currently explained by equilibrium makes more sense in the context of alternative models, we will return to review this in more depth when we discuss nonequilibrium heterogeneous agent models in Section 8.3. As we discuss in the next section, the questions of when equilibrium theory is irrelevant or simply wrong can blur together.

7.2. Power Laws

As we already mentioned, clustered volatility and many other phenomena in economics are widely believed to display power laws. Loosely speaking, a power law is a relationship of the form $y = kx^{\alpha}$, where k and α are constants, that holds for asymptotic values of a variable x, such as $x \to \infty$ [52, 53]. The first observation of a power law (in any field) was made by Pareto, who claimed that this functional form fit the distribution of the incomes of the wealthiest people in many different countries [54]. Power laws are reported to occur in many aspects of financial economics, such as the distribution of large price returns[16] [47, 55–66], the size of transactions [67, 68], the autocorrelation of volatility [69–74] the prices where orders are placed relative to the best prices [75–77], the autocorrelation of signs of trading orders [78–81], the growth rate of companies [82–84], and the scaling of the price impact of trading with market capitalization [85]. The empirical evidence for power laws in economics and their relevance for economic theory remains controversial [86]. But there is a large literature claiming that power laws exist in economics and unless one is willing to cast aside this entire literature, it seems a serious problem that there is no explanation based on equilibrium.

This problem might be resolved in one of several ways: Either as in physics, (1) power laws represent nonequilibrium phenomena from an economic point of view and thus provide a motivation for nonequilibrium theory, or (2) power laws are consistent with economic equilibrium, but still need to be explained by some other means, or (3) equilibrium is a key concept in understanding power

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[15] *Mandelbrot and Clark suggest that it is possible to describe clustered volatitlity as if time gradually speeds up and slows down, according to some random process that is independent of the direction of prices [47–49]. It has also been suggested that the correct notion of trading time can be measured by either transaction volume or frequency of transactions [50]; more careful analysis reveals that this is not correct, and that fluctuations of volume are dominated by fluctuations in liquidity [51].*

[16] *Under time varying volatility the distribution of returns can be interpreted as a mixture of normal distributions with varying standard deviations. This generically fattens the tails, and for the right distribution of standard deviations can produce a power law.*

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laws but we just don't know how to do this yet.[17] Or perhaps the existence of power laws is just an illusion. But it seems that power laws fit the data to at least a reasonable degree of approximation. Furthermore, there are many nonequilibrium models that naturally generate power laws, and for such models it is easy to test that the phenomena they generate are indeed real power laws.

It may seem strange to single out a particular functional form for special attention. There are several reasons for viewing power laws as important. One of them comes from extreme value theory [89], which says that in a certain sense there are only four possible convergent forms for the tail of a probability distribution,[18] and power laws are one of them. The main motivation for looking at tail distributions is that they provide a clue to underlying mechanisms: different mechanisms or behaviors tend to generate different classes of tail distributions.

Statistically classifying a tail distribution is inherently difficult because one does not know a priori where the tail begins. In extreme value theory a power law is more precisely stated as a relation of the form $y = K(x)x^{\alpha}$, where $K(x)$ is a slowly varying function. A slowly varying function $K(x)$ satisfies $\lim_{x\to\infty} K(tx)/K(x) = 1$, for every positive constant t. There is a great variety of possible slowly varying functions; with a finite amount of data it is only possible to probe to a finite value of x and the slowly varying function may converge slowly. In practice one tends to look for a threshhold x beyond which the points $(\log x, \log y)$ almost lie on a straight line. (This can be made precise using maximum likelihood estimation, including the estimation of the threshold x [90]).

In the last 20 years or so the phenomenon of power laws has received a great deal of attention in physics. Power laws are special because they are self-similar, i.e. under a scale change of the dependent variable $x' = cx$, treating K as a constant, the function $f(x') = c^{\alpha} f(x)$ remains a power law with the same exponent α but a modified scale $K' = c^{\alpha}K$. One of the places where power laws are widespread is at phase transitions, such as the point where a liquid changes into a solid. Self-similarity proved to be a powerful clue to understanding the physics of phase transitions and led to the development of the renormalization group, which made it possible to compute the properties of phase transitions precisely. The main assumption of the renormalization group is that the physics is invariant across different scales, i.e. the reason the observed phenomena are power laws is because their underlying physics is self-similar. This is an important clue about mechanism.

In physics power laws are associated with nonequilibrium behavior and are viewed as a possible signature of nonequilibrium dynamics [52, 53, 91]. It was shown more than 100 years ago by Boltzmann and Gibbs that in physical equilibrium, energies obey an exponential distribution; thus a power law indicates nonequilibrium behavior. As discussed in Section 7.4, equilibrium in physics is quite different from equilibrium in economics, but it seems telling that the current set of financial models in which power laws appear are (economic) nonequilibrium models, as discussed in Section 8.3.

7.3. The Progression Toward Market Efficiency

The theory of market (arbitrage) efficiency is justified by the assertion that if there were profit-making opportunities in markets, they would be quickly found and exploited, and the resulting trading activity would change prices in ways that would remove them. But is this really true? Anecdotal empirical evidence suggests otherwise, as illustrated in Figure 1, where we plot the performance of two proprietary predictive trading signals developed by Prediction Company (where one of the authors was employed). The performance of the signals is measured in terms of their correlation to future price movements on a two week time scale over a twenty three year period.[19] In the case of signal 1 we see a gradual degradation in performance over the twenty three year period. The correlation to future prices is variable but the overall trend is clear. At the beginning of the period in 1975 it averages about 14% and by 1998 it has declined to roughly 4%. Nonetheless, even in 1998 the signal remains strong enough to be profitably tradeable.

Does this evidence support market efficiency? On one hand the answer is yes: The usefulness of the signal declines with time. On the other hand it is no: It takes at least 23 years to do this, and at the end of the period the opportunity for profits remains present, even if greatly diminished. The performance of signal 2 is even more surprising, and clearly does not support market efficiency. Due to a structural change in the market, signal 2 begins in 1983 and subsequently grows in predictive power over the next decade and a half. This is the opposite of what

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[17] See articles by Calvet and Fisher for examples where power laws seem to be consistent with equilibrium [87, 88].

[18] The four possible tail behaviors of probability distributions correspond to distributions with finite support; thin tailed distributions such as Gaussians, exponentials or log-normals; distributions where there is a critical cutoff above which moments don't exist (namely power laws); and distributions that lack any regular tail behavior at all. Almost all commonly used distributions are in one of the first three categories.

[19] The signals were developed based on data from 1991 to 1998. Afterward data from 1975 to 1990 was acquired and the model was tested on this data without alteration. Thus the earlier data is completely out of sample and the enhanced performance prior to 1991 cannot be due to overfitting.

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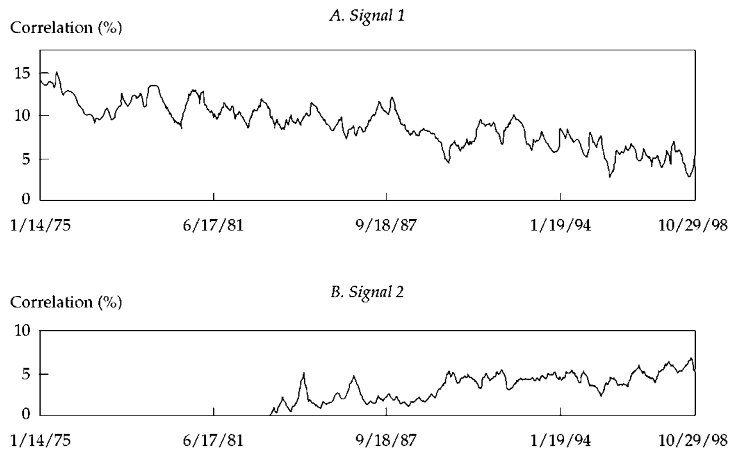
FIGURE 1



The inefficiency of the market with respect to two financial strategies over a 23-year period. The *y* axis plots the historical performance for a backtest of two trading signals developed by Prediction Company. Performance is measured in terms of the correlation of each signal to the movement of stock prices two weeks in the future.

standard dogma about efficient markets would suggest.

The need for a nonequilibrium theory is apparent. Equilibrium theory predicts that markets are perfectly efficient, and thus violations of efficiency cannot be addressed without going outside it. The time scale for the degradation of a profitable strategy is an inherently disequilibium phenomenon.

An estimate of the time scale for the progression toward efficiency can be made by taking into account what is needed to learn a good model and to acquire the capital to exploit it. Assume that there is a structural change in the market, suddenly creating a new inefficiency. For example, suppose that lots of mortgages are pooled together and securitized for the first time. On account of lack of historical data or irrational fear (say of adverse selection), the security might sell for a very low price. How long will this inefficiency take to disappear? Assume also that the only possible strategies for removing the inefficiency are econometric, i.e. they are based purely on statistical time series modeling. In this case one can approximate the rate at which the strategy can be discovered and exploited based on its Sharpe ratio S, which is defined as the ratio of the expected excess return of the strategy compared to its risk as measured by the standard deviation of its profits. Under the optimistic assumption that excess returns are normally distributed, the statistical significance with which an inefficiency with Sharpe ratio S can be detected is roughly $S\sqrt{t}$, where t is the time the inefficiency has been in existence. On an annual time scale a trading strategy with a Sharpe ratio of one is considered highly desirable.[20] Thus to detect the existence of an inefficiency at two standard deviations of significance requires four years. Accumulating a statistically significant track record based on live trading, which is normally a prerequisite to raising capital, takes another four years. We thus expect that the typical time scale before such strategies become profitable should be at least eight years.[21] One also has to address the question of how prices are altered and what is required to make the inefficiency disappear. As investors start to recognize the opportunity and exploit it they will make it shrink (say by bidding up the price of the mortgage backed security) and thus make it harder for others to see, extending the time until its superior profitability is completely extinguished [92]. In any case this argument at least indicates that we should not be surprised that inefficiencies disappear slowly.

As originally pointed out by Milton Friedman, the idea of a fully efficient market is inherently contradictory. To remove market inefficiencies we must have traders who are motivated to exploit them. But if the market is perfectly efficient there is no possibility to make excess profits. While efficiency might be true at first order, it cannot be true at second order; there must be on-going violations of efficiency that are sufficiently large to keep traders motivated. Developing a theory capable of quantitatively explaining this is a major challenge for economics, and a significant motivation for developing a nonequilibrium theory.

But what about signal 2 in Figure 1? In this case, the inefficiency actually grows for a period of more than a decade. We believe this is an example of a structural change. In fact the information available for constructing the signal was not even available prior to 1983, and only became

[20] *The historical Sharpe ratio of the S&P index is about 0.4, and the Sharpe ratio of many famous investors, such as Warren Buffet or George Soros, is significantly less than one.*

[21] *There is always the possibility of conducting more sophisticated research to speed up the learning time. Instead of observing the returns of the securitized MBS, one could find historical records of individual loan performance from before the securitization. But this data is hard to get, and the partial data that can be obtained might not be selected in the same way the securitized loans were.*

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available after a rule change in reporting. There are many examples of sudden structural changes in markets, for example the introduction of a new derivative such as a mortgage-backed security, a market for a new type of good such as an internet company, or the introduction of a new technology such as computers that enables new financial strategies such as program trading. When the ongoing introduction of financial innovations is combined with the slow time scale for discovering such signals, perhaps the persistent existence of inefficiencies is not so surprising.

7.4. Equilibrium and Nonequilibrium in Physics and Economics

To understand the motivations for nonequilibrium models, and to give some insight into how this might be useful in economics, we make a comparison with physics. The very name "nonequilibrium" indicates something that is defined in terms of its opposite, so before we can discuss nonequilibrium models in physics we have to make it clear what equilibrium means and how it differs from the same concept in economics.

As in economics, the word equilibrium in physics has several meanings. In mechanics the word "equilibrium" sometimes denotes the fairly trivial notion that forces are balanced, so there is no acceleration. The more common use of the term is in thermodynamics or statistical mechanics, where the word "thermal equilibrium" means that two systems in thermal contact reach the same temperature and cease to exchange energy by heat. A system in equilibrium will experience no change when isolated from its surroundings, and exists at the minimum of a thermodynamic potential. This is typically a good approximation only when a system has been left undisturbed for a sufficiently long period of time.

While the notion of equilibrium in physics is fundamentally different from that in economics, it does have some similarities. Both involve the simplifying assumption that the basic set up of the underlying situation is not changing in time. In economic equilibrium this is implicit in the sense that in order to imagine that rationality is a reasonable assumption all agents must have time to make good models of the world. In practice good models cannot be built without a learning period, in a setting that is reasonably constant. In physics, equilibrium explicitly requires time independence. Equilibrium in economics allows explicit time dependences, but requires that agents have a full understanding of them.

To illustrate how nonequilibrium models are used in physics, suppose you put a can of cold beer in a warm room. Heat initially flows from the room into the beer—the system is out of equilibrium. With time the beer gets warmer, and eventually to a good approximation the beer comes to room temperature and the system is at equilibrium.

Equilibrium thermodynamics tells us some useful things, such as the change in the pressure inside the can of beer between the time it is taken out of the refrigerator and when it comes to room temperature. There are, however, many things it doesn't tell us, such as how long will it take for the beer to get warm (The analogy to the problem of inefficiencies in the previous section should now be clear). From an empirical point of view, anyone who drinks beer knows that on a hot day it will become unpleasantly warm in less than an hour. A more quantitative, first principles prediction is possible using Fourier's law, which is a nonequilibrium principle stating that the rate of change in temperature is proportional to the temperature difference between the beer and the room. This implies that the temperature of the beer converges to that of the room exponentially. This can be derived from the heat equation (which is a standard diffusion equation and would look very familiar to financial economists). Furthermore, by making more detailed models it is possible to estimate the parameters of the diffusion process, so that we can roughly estimate when the beer will become too warm, even if we have no previous experience drinking beer in warm rooms.[22] Thus we see that in physics, equilibrium theory only explains part of what we need to know—to fully understand what is going on we need a more detailed nonequilibrium theory that can explain the dynamics of the transitions between equilibrium states. One important advantage of a nonequilibrium theory is that it makes it possible to know when the equilibrium theory is valid.

The fact that we have no similar disequilibrium theory in economics is clearly a serious problem. If there were such a theory, we would know when equilibrium applies and when it doesn't. There have been many attempts to create such a theory. Examples can be found in Franklin Fisher's book, *Disequilibrium foundations of equilibrium economics* [31]. This book reviews a research program involving many prominent economists that attempted to make models of the approach to equilibrium under the Walrasian framework. They focused on the problem of tatonnement, i.e. the process by which prices arrive at their equilibrium values. This body of theory was not very successful. In retrospect the difficulty was that the models required strong and detailed assumptions about the market mechanism and the conclusions that could be derived were not very general. This serves as a cautionary tale. The hope for a new attempt is that we have new tools (the computer), much better data, and perhaps a few new ideas.

Another often-used point of comparison between equilibrium in physics and economics concerns power laws.

[22] *This is not at all trivial. The rate of heat transfer depends on the heat conductivity of the can, on the convection properties of air, and several other factors. It is nonetheless possible to make estimates based on first principles.*

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As already mentioned, physical equilibrium does not yield power laws. This can be explained fairly trivially: Under the Jaynes formulation of statistical mechanics, deducing the correct probability distribution of states is just a matter of maximizing entropy subject to any imposed physical constraints, such as those that come from conservation laws. So, for example, for a closed system with a fixed number of particles, conservation of energy is just a constraint on the mean energy, and maximizing entropy subject to this constraint yields an exponential distribution of energies, which is not a power law. One can also work this out in more general settings, e.g. when the energy or the number of particles is not conserved, and a similar answer is obtained. It is not clear that this pertains in economics. For example, doing the same calculation under a constraint on the mean logarithm yields a power law; if there were economic systems where this constraint were natural (e.g., because of the use of logarithmic utility), power laws might emerge naturally. And as we have already stressed, in any case the notions of equilibrium in economics and physics are quite different.

7.5. The Sub-Prime Mortgage Crisis of 2007

As we are writing this in the late summer of 2007, a financial crisis is unfolding that provides a good illustration of the value of equilibrium models, at the same time that it illustrates the need for non-equilibrium models. The crisis began in sub-prime mortgages and appears to be spreading to seemiingly unrelated sectors. This will take some time to understand completely, but in cartoon form the crisis unfolded something like this:

- Low interest rates, a rising housing market, and confidence in the risk-reducing power of mortgage-backed securities led to a proliferation of sub-prime mortgages, often under questionable terms.
- A drop in housing prices made it clear that many sub-prime borrowers would eventually have to default.
- This stressed capital providers and caused them to tighten credit, for example by demanding more collateral for the same loans. One effect was that subprime borrowers themselves found it harder to refinance, which paradoxically created more defaults and higher losses.
- Another effect is that the prices of companies that depend on credit to do business were depressed.
- Institutions (including the largest banks in the United States) holding AAA rated securities called CDOs backed by bonds which in turn were backed by sub-prime mortgages lost something like $100 or $200 billion. Curiously these banks announced their losses piecemeal. One week a bank would announce $10 billion of losses, the next week they would add another $10 billion, then two weeks later another $10 billion.
- In mid August 2007, in the midst of the sub-prime mortgage crisis, there was a sudden meltdown of many statistical arbitrage hedge funds trading equities. Stocks held by these hedge funds made unusual but systematic relative movements—stocks owned by these hedge funds dropped in value, while those shorted by these funds rose in value.[23]
- One popular explanation is that the institutions losing money in sub-prime mortgages sold positions in other markets, including stocks held by stat arb hedge funds. This caused the stat arb hedge funds to take losses and to liquidate more stocks, causing more losses and more liquidations.
- The President announced that the crisis might be leading the country into a recession.

This crisis involves some phenomena that are inherently out of equilibrium. At the same time it shows the relevance of the equilibrium model.

The crisis was originally precipitated by levels of credit that are difficult to justify as rational. Homeowners got several hundred thousand dollar loans with no money down (i.e. the loan was equal to the appraisal value of the house), and representations about income were taken at face value (i.e. the income was not verified, but only stated by the homeowner himself). While housing prices were rising, this caused no trouble. If the homeowner did not pay, foreclosure could bring enough money to make the loan good, or more common, its threat could induce the homeowner to refinance into another loan. As housing prices started to decline, homeowners who in fact had no income could not pay, and foreclosures are not expected to yield nearly enough cash to repay the loans.

As of this writing, the crisis is based entirely on expectations, including expections about future home prices. The actual realized losses from foreclosures so far have only been around 1%. The $250 billion loss that the banks and hedge funds have been forced to take is based on the expectation that eventually, in several years, 50% of the homeowners (2.5 million families) will be thrown out of their houses with losses of 50 cents on the dollar for each foreclosed loan.[24] The equilibrium model is thus right to put so much emphasis on the importance of expectations. We have yet to see whether these expectations are rational.

One important contributing factor is that collateral levels have changed dramatically. Before the crisis, one could get a sub-prime loan with almost no money down. As the crisis developed, the required downpayment jumped to 25%. Since most sub-prime borrowers have

[23] *This is based on anecdotal conversations with hedge fund managers. See also [93].*

[24] *There is $1 trillion in outstanding sub-prime loans. Losses of 50% × 50% × $1trillion = $250 billion.*

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little free cash, this meant that effectively the sub-prime mortgage market dried up overnight. In the past, subprime borrowers who made 36 straight payments, demonstrating they deserved a better credit rating, would refinance into cheaper prime mortgages with nothing down. The refinancing rate was over 70% cumulatively for these people. This is important because the subprime interest rates typically reset to higher rates after 36 months. Now these same people are being asked to put 25% down to refinance into a new mortgage. Many of these homeowners will not be able to refinance, and they will eventually default, which will force further foreclosures, further depressing housing prices.

The collateral tightening is spreading throughout the mortgage market. Multibillion dollar hedge funds now have to put up twice as much cash to buy mortgage derivatives as they had three months before (that is, instead of borrowing 90% of the price using a mortgage derivative as collateral, they can only borrow 80%).

A striking feature of the crisis is that the situation evolved rapidly and appeared to be driven partly by emotion—the word "fear," which is not an equilibrium concept, appeared in almost every newspaper article covering these events.

The crisis underlined the role of specialized players: banks, mortgage backed derivatives experts, stock funds using valuation models, and quants building statistical arbitrage strategies. Key aspects are the heterogeneous nature of the strategies and their linkages to each other through institutional groupings and cross-market exposure.

Is the crisis a simple consequence of irrationality, completely orthogonal to the equilibrium model? It is not possible that lenders were so foolish that they did not imagine the possibility that housing prices might fall. But they apparently underestimated the amount of fraud in the loans. The brokers arranging the loans did not own them. The loans were sold and repackaged into bonds that were in turn sold to hedge funds and other investors. Since the brokers knew they would not bear the losses from defaulted loans, they evidently did not have enough incentive to check the truthfulness of the borrowers. And the investors apparently did not monitor the brokers carefully enough, perhaps lulled by the good behavior of the loans during the period of housing appreciation. There was a breakdown of rationality.

The next incredible blunder was that the big banks bought so many collateralized debt obligations (CDOs). To be sure, they only bought those securities in the CDOs that were rated AAA by the rating agencies. But when they do a deal the rating agencies are paid a commission for rating the securities and if they do not approve enough AAA securities the deal does not get done and they get no money. The rating agencies do not own the securities themselves, and so they do not have the best incentive to get their ratings right. It is amazing that the banks did not take this into account and naively bought the AAA securities thinking they were truly safe.

Though many aspects of this crisis involve out-of-equilibrium phenomena like irrational optimism, equilibrium theory can still be very insightful in understanding what happened. In fact, it partially predicted the crisis.

In the first place the brokers who winked at bad loans while collecting their commisions, and the rating agencies who gave high grades to bad bonds while collecting their commisions, were both responding to their incentives, just as equilibrium theory would predict, and as some economists had predicted.

But more importantly, the swift transition from lax collateral levels permitting too many risky loans to overly tight collateral levels strangling the sub-prime market is typical of the so-called leverage cycle in equilibrium theory. Once the equilibrium model is extended to allow for default on promises, and to allow for the posting of collateral to guarantee loans, equilibrium determines the collateral levels that will be used for promises (equivalently the leverage borrowers will choose). The kind of bad news that increases uncertainty also increases equilibrium collateral levels. In normal times these collateral levels are too lax, and in crisis times they are too tight (see refs. [17, 94]).

8. IF NOT EQUILIBRIUM, THEN WHAT?

Ken Arrow began a lecture he gave at the Santa Fe Institute a few years ago by saying that "economics is in chaos." In saying this he drew a contrast to the situation in say, 1970, when rational expectations-based equilibrium theories were producing many new results and it looked as though they might be able to solve many of the major problems in economics. At the time the path for a young theoretical economist was clear. Now, in contrast, this is up in the air. While the majority of economists still use rational expectations as the foundation for what they do, even the mainstream is investigating perturbations of this foundation, and some are seeking an entirely new foundation. In this chapter we will sketch some possible directions. Many of these approaches are overlapping and we try to stress the connections between them.

We are not suggesting that equilibrium theory should be thrown out: as emphasized earlier, there are many situations where it is extremely useful. What we do argue is that it should be one among many tools in the economist's toolbox. During the rise of the neoclassical paradigm over the last fifty years the emphasis on equilibrium theory has been so strong that most theoretical economists have been trained in little else.[25] As a result economics has suffered.

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[25] *The obvious exception is econometrics, which is an essential part of economics, and should remain so. But econometric models are not founded on a priori models of agent behavior, and are not theories in the sense that we are using the word here.*

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Disequilibrium models are not new in economics. Keynes, for example, made models that were essentially dynamical, without imposing equilibrium conditions. These models fell out of favor for the good reason that many of them failed. In the 1950s and 1960s Keynesians accepted the inflation-output trade-off we mentioned earlier, but government efforts to exploit that relationship collapsed completely during the stagflation of the 1970s (when there was both high inflation and low output). These failures were driving forces in the modern equilibrium approach to macroeconomics, led by Lucas. It is clear that for many problems in economics the ability to incorporate human reasoning is necessary. However, there may be other ways to accomplish this goal, and there may also be other important economic problems where human reasoning is not the central issue.

8.1. Behavioral and Experimental Finance

As already stressed in Section 5.3, one of the principal problems with rational expectations equilibrium is the lack of realism of the agent model. The importance of work in this direction was acknowledged by the mainstream with the Nobel prizes of Daniel Kahneman, who is one of the pioneers behavioral economics, and Vernon Smith, who is one of the pioneers of experimental economics. Because these important fields are already well-covered in many review articles [29, 30, 95, 96] we will not review them here, but rather only make a few comments.

We think that a proper characterization of human behavior is an essential part of the future of economics. This is not going to be easy; people are complicated and their behavior is malleable and context dependent. Experimental economics offers hope for categorizing the spectrum of human behavior and predicting how people will behave in given situations, but the results are difficult to reduce to quantitative mathematical form. So far behavioral finance has done a good job of documenting the many ways in which real investors are not rational, and has shown that this has important financial consequences. Although there has been some progress in understanding the implications of these facts for classical problems such as saving and asset pricing [28, 97], the jury is still out concerning whether this can be done in a fully quantitative manner.

One must also worry that behavioral finance might be the victim of its own success. If we discover new empirical rules for the irrational component of human behavior, might not these rules be violated as people become aware of them? For example, it has been widely shown that people are strongly prone to overconfidence. If this knowledge becomes widespread, will savvy investors learn to compensate? In financial markets there is a lot of money on the table and the motivation for overcoming irrational behavior is large. Of course, if this were to occur that would already be a major achievement for the field.

8.2. Structure vs. Strategy

In comparing theoretical economists to theoretical biologists, Paul Krugman has commented that the two are not so different as it might seem: both make extensive use of game theoretic models [98]. This observation is true, but it obscures a very important matter of degree. In biology game theory models are one of many approaches used by theoreticians. In economics, game theory models and their corollary form, equilibrium models, are almost the only approach. From a certain point of view this is natural—the need to take into account strategic interactions in economics is larger than it is in biology. The problem is that strategic interactions are not the only important factor in economic models. Other factors can also be important, such as the nature of economic institutions, and how the interactions of agents aggregate to generate economic phenomena at higher levels. We will call the aspects of a problem that do not depend on strategy its structure. In economics this occurs when equilibrium plays a role that is minor compared to other factors, such as interaction dynamics or budgetary constraints. We give several examples below. The substantial effort needed to capture the strategic aspects of a problem in an equilibrium model can often cause the structural aspects of a problem to be short-changed. In some cases the importance of structural factors may dominate over strategic considerations, and models that place too much emphasis on strategic interactions without proper emphasis on structural properties will simply fail to capture the essence of the problem. Of course in general one must take both into account.

8.2.1. Examples Where Structure Dominates Strategy

As an example of what we mean, consider traffic. Cars are driven by people, who anticipate the decisions of themselves and others. Yet an examination of the literature on traffic and traffic control suggests that game theory and equilibrium play only a minor role. Instead the dominant theories look more like physics [99, 100]. Traffic is modeled by analogy with fluid flow. The road is a structural constraint and cars are interacting particles. Appropriately modified versions of the same ideas that are used to understand the phases of matter in physics are used to understand traffic: A road with only a few cars is like a gas, a crowded road where traffic is still flowing is like a liquid, and a traffic jam is like a solid. In the liquid phase perturbations can propagate and induce traffic jams; the equations that are used to understand this are familiar to fluid dynamicists.[26] This is not because equilibrium

[26] *In the medium density range, when drivers have delayed reactions or over-reactions to random variations in the traffic flow, a breakdown in traffic flow is caused by a dynamic instability of the stationary and homogeneous solution.*

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is not relevant, but rather because driving is tightly constrained by roads, traffic control, driving habits, and response times. The strategic considerations are fairly simple and their importance is dominated by the mechanics of particles (automobiles) moving under structural constraints.

The same holds true for crowd dynamics, where game theoretic notions of equilibrium would also seem to be important. In moving through a crowd one tries to avoid collisions with one's neighbors, while trying to reach a given goal, such as a subway entrance. One's neighbors are thinking about the same thing. There are lots of strategic interactions and it would seem that game theory should be the key modeling principle. In fact this does not seem to be the most important consideration for modeling the movement of a crowd.

The problem of crowd dynamics becomes particularly important in extreme situations when the density of people is high. For example, on January 12, 2006 roughly three million Muslims participated in the Hajj to perform the stoning ritual. There was a panic and several hundred people died as a result of either being crushed or trampled to death. In an effort to understand why this occured the output of video cameras was processed in order to reduce the crowd dynamics to a set of quantitative measurements. Analysis by Helbing et al. [101] showed that high densities of people facilitate the formation of density waves, similar to those that occur in driven granular media.[27] The crowd dynamics amplify small perturbations and create pressure fluctuations that caused people to be crushed. The insights that were obtained by the study of the Hajj data led to several modifications in the methods of crowd control, and to a safe Hajj pilgrimage the following year.

The skeptical economist might respond that these are not economic problems, and just because physical models such as fluid dynamics apply to crowd behavior does not imply that they should be useful in economics. We included the examples above because they provide clear examples of situations that involve people, where despite the presence of strategic interactions, their ability to reason is not a dominant point in understanding their aggregate behavior. We now discuss a few problems in economics where similar considerations apply.

8.2.2. Distribution of Wealth and Firm Size

As originally observed by Pareto, the distribution of income displays robust regularities that are persistent across different countries and through time [54]. For low to medium income it has a functional form that has been variously described as exponential or log-normal [102, 103], but for very high incomes it is better approximated by a power law. Since the early efforts of Champernowne, Simon, and others, the most successful theories for explaining this have been random process models for the acquisition and transfer of wealth [102, 104–108]. If these theories are right, then the distribution of wealth, which is one of the most remarkable and persistent properties of the economy, has little to do with the principles of equilibrium theory, and indeed little to do with human cognition. Other problems with a similar flavor include the distribution of the sizes of firms, the size of cities, and the frequency with which words are used [32, 105, 109–113].

8.2.3. Zero Intelligence Models of Auctions

The continuous double auction, which is perhaps the most commonly used price formation mechanism in modern financial markets, provides a nice example of how in some cases it can be very useful to focus attention on structural rather than strategic properties. In a continuous double auction buyers and sellers are allowed to place or cancel trading orders whenever they like, at the prices of their choosing. If an order to buy crosses a preexisting order to sell, or vice versa, there is a transaction. The way in which orders are placed is obviously closely related to properties of prices, such as volatility and the spread (the price gap between the best standing sell order and the best standing buy order). There have been many attempts to model the continuous double auction using equilibrium theory. One famous example is the Glosten model [114], which predicts the expected equilibrium distribution of orders under a single period model. Tests of this model show that it does not match reality very well [115].

In contrast, an alternative approach to this problem assumes a zero intelligence model.[28] This approach was originally pioneered by Becker [116], who showed that some aspects of supply and demand curves could be understood without any reliance on strategic thinking. The notion that a zero intelligence model might be useful for understanding economic phenomena was further developed by Gode and Sunder [117], who popularized this phrase and argued that observations of efficiency in classroom experiments on auctions could be explained without relying on agent

[27] *During a New Year's Eve celebration at Trafalgar Square in London in 1980 one of the authors (jdf) personally experienced the propagation of density waves through a large crowd. The density waves propagate remarkably slowly. Their propagation speed can be crudely estimated by regarding each person as an inverted pendulum.*

[28] *"Zero intelligence" loosely refers to models in which the agents have minimal cognitive capacity. It should more accurately be properly called "ϵ intelligence", since often agents do have some cognitive ability, e.g. the capacity to draw from a known distribution or to enforce a budget constraint.*

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intelligence. Zero intelligence models of the continuous double auction make the assumption that people place orders more or less at random. We say "more or less" because there is more than one way to characterize their behavior, and one can argue that some of these involve at least some strategic thinking, and so are not strictly speaking zero intelligence. The development of zero intelligence models of the continuous double auction has a long history [75, 77, 118–130].

In some cases zero intelligence models make successful predictions. For example, by assuming that order flow follows simple Poisson processes, the model of Daniels et al. [127, 129] derives equations of state relating statistical properties of order flow such as the rates of trading order placement and cancellation to statistical properties of prices such as volatility and the bid-ask spread. Some of these predictions are borne out by empirical data [130]. The development of such zero intelligence models has guided more empirically grounded "low intelligence models," which refine the statistical processes for order placement and cancellation. For certain categories of stocks this results in accurate quantitative prediction of the distribution of volatility and also of the distribution of the spread [77].

These models can be criticized because they fail in an important respect: the resulting price series are not efficient. This is a failing, but it illustrates an important point: the fact that a good prediction of the distribution of volatility can be made from a model that does not satisfy efficiency suggests that the distribution of volatility may not depend on efficiency. Instead, the structural properties of the continuous double auction appear to be more important. In this case by "structural properties" we mean those that come directly from the structure of the auction, e.g., the rules under which transactions take place, the dynamics of removal and deposition of orders, and their interactions with price formation. These models suggest that for many purposes the structure of the institution used to make trades plays an important role.

The zero intelligence approach is similar to the equilibrium model in the sense that both approaches are parsimonious but sometimes highly unrealistic. By making the simple assumption that people do not think at all, one can focus on aspects of a problem that would be missed by an equilibrium analysis. Equilibrium and zero intelligence models provide ways to enter the wilderness of bounded rationality without becoming totally lost; the two approaches are complementary because they enter this wilderness from opposite sides.

Of course, many problems, including the double auction, require an understanding of both structure and strategy, and the art of good modeling is to find the right compromise in emphasizing these two facets of economics. A good recent example is the theory of Wyart et al. who show that a mixture of market efficiency and structural arguments can be used to explain the relationship between the spread, price impact and volatility in transaction time [131].

8.3. Bounded Rationality, Specialization, and Heterogeneous Agents

As originally pointed out by Adam Smith, the cognitive limitations of real agents cause them to specialize.[29] In finance, agents use specialized trading strategies. A few common examples are fundamental valuation, technical trading (interpreting patterns in historical price movements), many forms of derivative pricing, statistical arbitrage, market making, index arbitrage, and term structure models. The champions of heterogeneous models believe that to understand asset pricing as it manifests itself in the real world, it is necessary to understand the behaviors of at least some of these types of agents and their inter-relationships and interactions.

During the last couple of decades a new school of boundedly rational heterogeneous agent (BRHA) models has emerged in finance. The primary motivation of this work has been to explain phenomena such as bubbles and crashes, clustered volatility, excess volatility, excess trading volume and the heavy tails of price returns. As discussed in Sections 6 and 7.1, these phenomena have so far not been explained by equilibrium theory. In contrast there are now many BRHA models that produce these phenomena. The typical BRHA models that have been constructed so far study a single asset and assume that exogenous information enters through a stochastic process corresponding to the dividends paid by that asset. The most common technique is to simulate the resulting market behavior on a computer, though some studies also derive analytic results. This style of work often goes under the name of agent-based modeling [134–136].

Roughly speaking BRHA models can be divided into those that identify classes of agents a priori and those that use learning to generate heterogeneous agents de novo. Examples of models that assume heterogeneous agents a priori are found in references [5, 6, 92, 137–149]. The most commonly studied categories of agents are value investors, who price stocks according to their fundamentals, and trend followers, who buy stocks that have recently increased in price and sell stocks that have recently

[29] *Equilibrium models often begin with asymmetric information a priori. See for example the classic paper of Grossman and Stiglitz [132], which can be viewed as an example of a rational treatment of heterogeneous agents emerging from idiosyncratic information. Noise trader models provide another example [8]. For a modern treatment of agents who are a priori the same but choose to specialize and learn different things on account of information processing constraints see [133].*

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decreased in price. The learning models assume heterogeneous learning algorithms (or starting points) a priori and then let the strategies evolve as learning takes place, thereby generating the heterogeneous strategies de novo. Examples of this approach are given in references [134, 150–156]. This approach lets strategies co-evolve in response to the conditions created by each other, as well as in response to exogenously changing fundamentals. The typical result produces a highly heterogeneous population of trading strategies.

Simulations of such systems show that they naturally generate the apparently nonequilibrium phenomena mentioned earlier, including clustered volatility, heavy tails, bubbles and crashes, and excess trading. In most of these models the primary cause of clustered volatility is changes in the populations of different kinds of trading strategies. The population can shift due to exogenous conditions, but most of the shifting appears to be largely random, due to context-dependent fluctuations in the profitability of one group vs. another. Changes in the exogenously provided fundamentals are amplified with feedback. In some cases it is possible to show that in an analogous equilibrium model none of these phenomena would arise [150, 154, 155]. Furthermore, in some cases it is possible to calibrate these BRHA models against real data and get a good fit for properties such as the distribution of returns and the autocorrelation function of volatility [152, 153, 156–158]. Thus the more developed models in this class make testable predictions. Note that such studies do not claim that information arrival does not have an effect, but only that a substantial component of volatility, and in particular its interesting statistical properties, are internally generated.

Most of these BRHA models produce uncorrelated prices and arbitrage efficiency between the simulated strategies. In the more sophisticated models, capital is reallocated so that more profitable agents gain capital, giving them a greater effect on price formation. After sufficient simulation time these models reach a state in which the remaining strategies are all equally profitable (and are in this sense arbitrage efficient). When this occurs the autocorrelations in prices also disappear. This is not complete efficiency in the sense that it is possible (at least in principle) to introduce new strategies that are not within the generated set, that can exploit nonlinear price patterns and temporarily make profits. (For some examples see [92].) After enough time for capital to reallocate the advantage of those strategies also disappears. This shows that that the prediction of efficiency by equilibrium models is not as striking as it might seem—the simulation models discussed above generate results that are difficult to distinguish from true efficiency. Thus, these models provide a means, albeit crude, to take reasoning into account, and satisfy many of the desiderata for modeling agent behavior discussed in Section 4.

A great deal remains to be done in this area. Many of the the models mentioned above illustrate the principle (by generating clustered volatility), but they do not make quantitative predictions about the real economy. Most of them are not very parsimonious. There are still major open questions about the necessary and sufficient conditions required for generating the phenomena of interest. For instance, it is not absolutely clear what features of the models that generate the phenomena are incompatible with equilibrium. It is even possible that some equilibrium model might also generate the same phenomena, but equilibrium models are so hard to compute that examples have not yet been discovered. Further validation and testing and simplification are needed before we can have a high degree of confidence that the detailed explanations of these BRHA models are correct. Still more work is required to apply the lessons of BRHA models to asset pricing and market design. Nonetheless, these models apper to confirm the physics lesson that certain kinds of empirical regularities (power laws) are the consequences of nonequilibrium behavior of the market.

8.4. Finance Through the Lens of Biology

The influence of biology on economics dates back at least to Alfred Marshall [92, 159–165]. Both biology and economics involve specialized agents evolving through time under strong selection pressure, and it is not surprising that they share many features, such as extensive use of game theory. In this section we consider other aspects of biology that are not captured by game theory and ask whether financial economics might not benefit by taking some lessons from biology.

Note that the biological point of view is particularly complementary with that of BRHA modeling. Biology is after all a study of specialized organisms and their interactions with each other, and thus provides a lens for viewing many aspects of financial markets.

8.4.1. Taxonomy of Financial Strategies

Taxonomy is the study of classification. By sorting complex phenomena such as organisms into categories we compress an enormous amount of information into a comprehensible scheme. Taxonomy has historically been one of the principal activities of biologists. Perhaps the most famous taxonomist is Carl Linnaeus, who in the middle of the eighteenth century introduced the hierarchical classification of organisms into categories at varying levels of differentiation, proposed a list of characteristics for classification, and created the standardized nomenclature that is still largely used today. The taxonomic classification of organisms has benefited from the participation of hundreds of thousands or perhaps millions of individuals for more than three hundred years and continues to be a field of active research and sometimes heated debates in biology.

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Although taxonomy might seem to be a very limited end in and of itself, it has proved to be an important prerequisite for the development and testing of the theory of evolution in biology.

An economist, Adam Smith, was one of the first to point out the importance of specialization, and in some branches of economics, such as industrial economics, this is a field of active research. Not so in finance. To our knowledge there are only two articles that gather empirical data on the nature of real trading strategies in financial markets (and both of these papers only address the frequency of usage of two preassigned categories of strategies, technical trading, and value investing) [166, 167]. The diversity and specialized functions performed by the trading desks of major financial institutions suggest that there is an enormous amount waiting to be done if we are to understand the taxonomy of real trading strategies. The BRHA models mentioned above suggest that such diversity is important and plays a key role in price formation. If this endeavor is to enter the realm of hard science it needs a grounding in empirical data. We believe that market taxonomy should become a major field of study by financial economists. Electronic data storage presents the possibility of gathering data with information about the identity of agents, which can potentially form the basis for such taxonomies. Preliminary results in this direction have already yielded interesting results [168, 169].

8.4.2. Market Ecology

For biological systems ecology is the study of organisms and their relationship to their environment, including each other. Simulations of BRHA models make it clear that understanding the relationships between financial strategies is also important in financial economics. Market ecology is an attempt to develop a theory that gives a deeper and more universal understanding of this [92].

Under the analogy to biology a financial strategy can be viewed as a species, and the capital invested in it can be viewed as its population. If strategies are successful they accumulate more capital, which causes them to have more influence in setting prices. The price provides the principal channel through which different strategies interact—the movements of prices influence the buying and selling activity of strategies, which in turn affects prices. On a longer timescale the movements of prices also affects profitability, which alters the capital invested in strategies and is the main driver of the selection process underlying market evolution.

In most BRHA models, price setting is done using a standard method, such as market clearing, but even when this is the case, it can be very useful to view the interactions of strategies in terms of their market impact. The market impact is the price movement corresponding to the initiation of a trade of a given size, and is equivalent (up to a constant of proportionality) to the demand elasticity of price originally introduced by Marshall [Farmer, et al. unpublished research]. Market impact is an increasing function of trading volume. As the capital invested in a strategy grows, market impact lowers realized returns and ultimately limits the trading capital that any given strategy can support. With a proper understanding of market impact it is possible to derive differential equations for the dynamics of capital flows between strategies, analogous to the generalized Lotka-Volterra models of population biology [92].

Market impact also provides a way to understand the strength of the interactions between strategies and to classify them in ecological terms. In particular, it is possible to make a pairwise classification of strategies according to their influence on each other's profits by understanding how a variation in the capital associated with strategy j influences the profits of strategy i. Letting ρ_i be the return of strategy i and C_j be the capital invested in strategy j, one can define a gain matrix $G_{ij} = \partial\rho_i/\partial C_j$. If $G_{ij} < 0$ and $G_{ji} < 0$ strategies i and j have a competitive relationship; if $G_{ij} > 0$ and $G_{ji} < 0$ then they have a predator-prey relationship, in which i preys on j, and if $G_{ij} > 0$ and $G_{ji} > 0$ they have a mutualistic (or symbiotic) relationship. The ecological view asserts that the set of such relationships in the market is critical to its function, and that the proper diversity of financial strategies may be critical for market stability. This view also provides an evolutionary framework in which to understand the path toward efficiency.

Until recently this ecological view of markets was essentially hypothetical and it was not clear that the resulting models were testable. Recently, however, the acquisition of data sets in which there is information about brokerages or individual investors has opened up the possibility of calibrating such models and testing their predictions against real data [168, 169]. It will be interesting to determine whether one can link properties of financial ecologies to properties of price formation. For example, do alterations in financial ecologies influence price volatility?

8.4.3. Evolutionary Finance

The three essential elements of evolutionary theory are descent, variation, and selection. Financial ecologies are formed by these precisely these forces:

- Descent. Financial strategies are passed down through time as traders change and start new firms.
- Variation. Old strategies are modified and improved to create new strategies.
- Selection. Successful strategies proliferate and unsuccessful strategies disappear. Success is substantially based on profitability, but is also influenced by other factors such as marketing and psychological appeal.

The view that profit selection is important is not new, having been championed as a force for creating efficient

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markets by central figures in financial economics such as Milton Friedman and Eugene Fama [22, 170]. The theory of market efficiency amounts to the assertion that it is possible to short-circuit the evolutionary process and go directly to its endpoints, in which evolution has run its course and profits of well-developed strategies are reduced to low levels. The suggestion that the timescales for this process are not short, as presented in Section 7.3, motivate more detailed studies of the transient properties of the evolutionary process and its dynamics. This view implicitly underlies all the BRHA models we have discussed so far, and is also now being given a more mathematical underpinning [171, 172].

What is still largely lacking is an empirical foundation on which to build an evolutionary theory. This ultimately depends on developing a taxonomy of real financial strategies together with a database of studies of financial ecologies as they change through time.

8.5. The Complex Systems Viewpoint

Complex systems refers to the idea that systems composed of simple components interacting via simple rules can give rise to complex emergent behaviors, in which in a certain sense the whole is greater than the sum of its parts. With his introduction of the concept of the "invisible hand" Adam Smith was one of the earliest to articulate this point of view. The general equilibrium theory of Arrow and Debreu can be regarded as an attempt to cut through the complexity of individual interactions and reduce the invisible hand to a tractable mathematical form. It appears, though, that for many purposes this approach is just too simple—to understand real financial economies we need to do the hard work of understanding who the agents are, what factors cause them to change, and how they interact with each other. To do this we need better taxonomic studies of real markets, better simulation models, and better theory. To the external observer, current research in financial economics has so far largely failed to capture the richness of real markets, which provide some of the best examples of complex systems. More robust contact between financial economics and the emerging field of complex systems could help remedy this situation.

9. CONCLUSION

We have written this article with a dual purpose. On one hand, we worry that physicists often misunderstand the equilibrium framework in economics, and fail to appreciate the very good reasons for its emergence. On the other hand, the majority of economists have become so conditioned to explain everything in terms of equilibrium that they do not appreciate that there are many circumstances in which this is unlikely to be appropriate. We hope that physicists will begin to incorporate equilibrium into their models when appropriate, and that economists will become more aware of analogies from other fields and begin to explore the possibilities of alternatives to the standard equilibrium framework.

Our own belief is that one must choose modeling methods based on the context of the problem. In situations where the cognitive task to be solved is relatively simple, where there is good information available for model formation, and where the estimation problems are tractable, rational expectations equilibrium is likely to provide a good explanation. Some examples where this is true include option pricing, hedging, or the pricing of mortgage-backed securities. In other cases where the cognitive task is extraordinarily complex, such as the pricing of a new firm, or where estimation problems are severe, such as portfolio formation, human models may diverge significantly from rational models, and the equilibrium framework may be a poor approximation. For good science one must choose the right tool for the job, and in this case the good scientist must use an assortment of different tools. Close-mindedness in either direction is not likely to be productive.

As we have stressed, equilibrium theory is an elegant attempt to find a parsimonious model of human behavior in economic settings. It can be criticized, though, as a quick and dirty method, a heroic attempt to simplify a complex problem. Now that we have begun to understand its limitations, we must begin the hard work of laying new foundations that can potentially go beyond it.

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