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# Five crisp and fuzzy models for supply chain of an automotive manufacturing system

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**Abstract.** Supply Chain Management (SCM) is a new approach to production planning. It integrates the components of supply chain in a holistic manner. Modeling this large-scale system, which contains all effective enterprises in production such as raw material suppliers, part manufacturers, assembly plants, distribution organizations, and the like, is challenging for managers, engineers and researchers. This paper concentrates on supply chain system modeling with fuzzy linear programming, and fuzzy expert system for an automobile plant. First, a linear programming model is developed in such a way that while the input data is fuzzy, the constraints are crisp. In the second linear model, the coefficients of the model are crisp while the constraints are fuzzy. In the third model, we aggregate the first and the second models into one fuzzy linear programming where all constraints and coefficients are fuzzy. In each case, we compare the results with those of classical SC models. Finally, a rule based fuzzy expert system for SC is developed and the results are compared with those of the classical and fuzzy LP models. The results of the fuzzy expert system show its superiority over the former crisp and fuzzy linear programming models.

**Keywords:** fuzzy theory, supply chain management (SCM), fuzzy linear programming, expert systems

## 1 Introduction

The main focus of supply chain management (SCM) is organizing internal and external resources of an economic enterprise comprehensively. The main components of a SC system are: (a) Materials Flow, (b) Information Flow, and (c) Buyer-Seller Relations (see Fig. 1). Besides these three components of a SC system, there are some other building-blocks for the system such as raw material suppliers, manufacturers of parts, assemblers, Original Equipment Manufacturers (OEMs), distributors, retailers, customers, etc. (Fazel Zarandi and Saghiri, 2003)[30].

The first component of a SC system is Material flow. Previous analyses of this component have mainly been studied in fields of inventory, cost, price, and quality considerations (Bloemhof-Ruwaard, 1995[4]; Chandra and Fisher, 1994[5]; Lui ,1999[14]). Petrovic et. al (1999)[20] examine uncertainties in SC by focusing on "decentralized control of each inventory" and "partial coordination of the inventory control". Petrovic et. al (1998)[19] tried to identify the stock level and order quantities for inventories in a SC, with a consideration of two sources of uncertainty in a SC system: "customer demand" and "external supply of raw materials". Gerchak (2000)[7] investigates cost reduction in manufacturing processes and supply chain, by considering the uncertainties in projects. He Qi-Ming and Jewkes (2000)[8] also investigate inventory costs in a SC by presenting two algorithms for computing average total cost per product. Samroengraja (1999)[22] studies overall supply chain costs by proposing alternative ordering policies as a strategy in coping with the order volatility

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in a SC. Cost analysis, storage and warehousing, and quality management systems are some other interesting areas in SC (Mullins, 1999[17]; Elinger, 2000[1]).

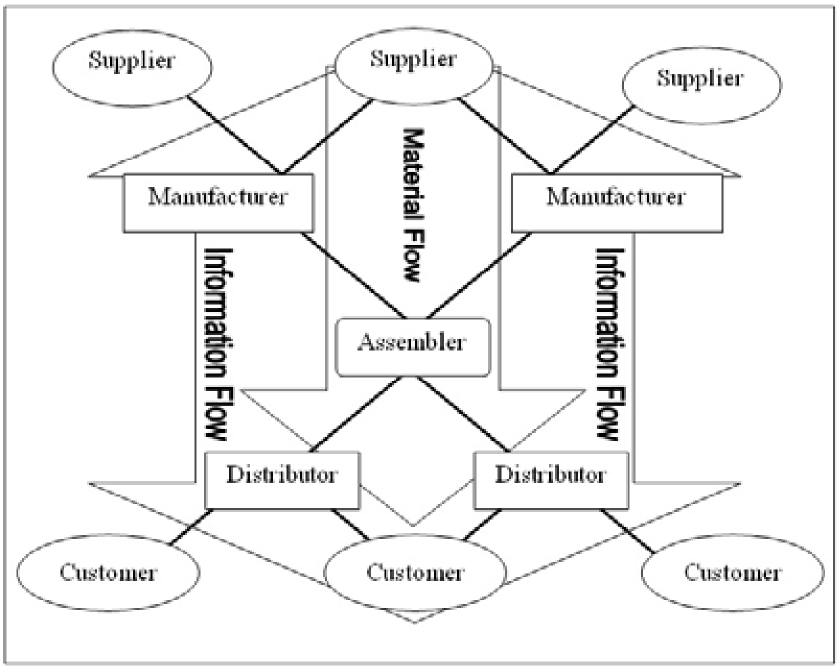


**Fig. 1.** Material Flow, Information Flow, and Buyer-Seller Relations in a supply chain system (Fazel Zarandi et al., 2002)[32]

The second component of a SC is information flow. Companies' Information Technology (IT) strategies in their SC, role of the IT in cost reduction of SCM, and IT-SC inter-relationships are some of the main aspects of information flow in SC (Lancioni, 2000[12]; Lancioni and Smith, 2000[13])

The third component of a SC system is Buyer-Seller Relations. Monczka et al. (1998)[16] investigate success factors in strategic supplier alliances. Stuart (1993)[24] surveys influencing factors in supplier partnerships as well as potential and strategic benefits of such a policy. Ohmae (1989)[18] has defined a global logic of strategic partnerships and alliances aspects: "World competition", "need to market expansion and finding or developing new market" and some other related topics in Business Planning Process. Heide and John (1990)[9] pursue alliances in industrial purchasing that lead supply chains to more unification and harmonized operation.

As a matter of fact, in a way to achieving maximum efficiency in performance of its components, SC could be modeled as a large and complex system. However, modeling real world SC systems are very difficult. The main problems with classical SC models are high complexity, lack of flexibility and uncertainty embedded in them.

The hypothesis in this paper is that fuzzy system modeling and approximate reasoning are suitable tools to represent three main aspects of real systems: complexity, flexibility, and reliability. Our approach for the development of SC models in this paper is to investigate a gradual transition from a totally crisp LP model to a completely fuzzy one and next to a linguistic fuzzy expert system model to see if the behavior of the system can better be represented.

The rest of the paper is organized as follows: first, uncertainties in SC systems will be investigated and a brief review of fuzzy logic and fuzzy set theory will be presented. Then, a case study of a SC system will be demonstrated for an automobile industry. Models for such a system are developed through classical and fuzzy operation research (OR) models in three steps: (i) a linear programming (LP) model with fuzzy coefficients, (ii) an LP model with fuzzy bounds and objective function, and (iii) an LP model with fuzzy coefficients, fuzzy bounds and objective function. Finally, the same case study will be modeled and analyzed via fuzzy expert system (FES) approach. The results of all four approaches are compared and their advantages and disadvantages are articulated.

## 2 Fuzzy concepts for SCM

Real world production management, planning, and control problems are usually imprecise, complex, and critically depend on human activities. However managers are to interact in an intelligent way with this environment. Thus, they have to reach out for new kind of reasoning based on such situation. (Turksen and Fazel Zarandi, 1999)[27]. A SC system usually contains several sub-systems with unlimited relations and interfaces. Each subsystem and its interfaces with others in the context of Material Flow, Information Flow, and Suppler-Buyer relations naturally contain a lot of uncertainties. It is a challenge to model a SC with an integrated approach and to capture relations between different elements of such a chain (Chen and Tzeng 2000)[6]. Petrovic et al. (1999)[20] demonstrate the uncertainties in SC systems as follows:

"··· A real SC operates in an uncertain environment. Different sources and types of uncertainty exist along the SC. There are possible events, uncertainties in judgment, some lack of evidence, as well as a lack of certainty in available evidence. They appear in customer demand, production processes and supply sources. Each facility in the SC must deal with uncertain demand imposed by succeeding facilities and uncertain delivery of the preceding facilities in the SC ··· " (Petrovic et al., 1999)[20].

On the other hand, influencing several factors on the performance of a SC system as well as its dependence on human activities, make it naturally complex. In a SC system design and modeling, we have to deal with a large-scale socio-technical system where classical quantitative approaches do not provide satisfactory answers–see Fig. 2 (Sugeno and Yasukawa, 1993[26]; Turksen, 1992[28]).

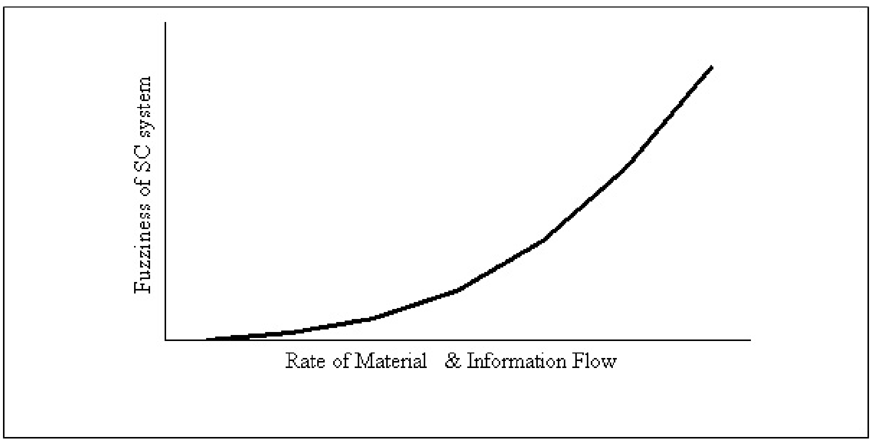


**Fig. 2.** Relation between SC system fuzziness and flow of material and information in it

Concerning above characteristics of SC system, applying fuzzy concepts and theory seems appropriate in SCM. In mathematical foundations of the fuzzy theory, there exists enough flexibility subject to certain axioms. In fuzzy systems, it is not difficult to merge hard and soft constraints. Moreover, the main source of difficulty for constructing reasonable production planning and scheduling stems from conflicting nature of the criteria and goals. Fuzzy approach provides tools to satisfy constraints to certain degree and to take into account the relative importance in a format easily understood by human experts (Turksen, 1992[28]; Zimmerman, 1996[33]).

Our hypothesis is that SC is a complex system with imprecise parameters and conditions and it can be analyzed and modeled by using fuzzy set theory, more appropriately. In sum, the fuzzy systems approach demonstrates many advantages in real world applications like SC systems that could be summarized as follows (Turksen and Fazel Zarandi, 1999)[27].

(1) Fuzzy systems are conceptually easy to understand.

(2) Fuzzy systems are flexible, and with any given system, it is easy to mange it or layer more functionality on top of it without staring again from scratch.

(3) Fuzzy systems can model most nonlinear functions of arbitrary complexity.

(4) Fuzzy systems are tolerant of imprecise data.

(5) Fuzzy systems can be built on top of the experience of experts.
(6) Fuzzy systems can be blended with conventional control techniques.
(7) Fuzzy systems are based on natural language.
(8) Fuzzy systems provide better communication between the experts and the managers.

## 3 SC modeling for an automotive industry

In this section, we develop several models for SC systems of an automotive industry. In these SC system models, there exist 6 components, as shown in Fig. 3. The tasks of each enterprise, as shown in Fig. 4, are: customer, marketing and sales, manufacturing and assembly (production plant), purchasing (procurement enterprise), and parts and raw materials suppliers, where all of them are interconnected.

• The main focus of the research is on the role of marketing and procurement in selling and purchasing of goods in the SC, with a concentration on:

• The role of procurement enterprise in purchasing most proper raw materials, which directly affect the operational costs of the production plant.

• The role of procurement enterprise in negotiations with suppliers on price, delivery, quality, etc.

• The role of marketing enterprise in selling the product with the maximum possible price.

• The role of procurement and marketing enterprises in finalizing a deal in minimum time. The stronger actions of these two enterprise will lead to sails contract for production plant in a shorter time and accordingly the investment costs of the production plant will be reduced.

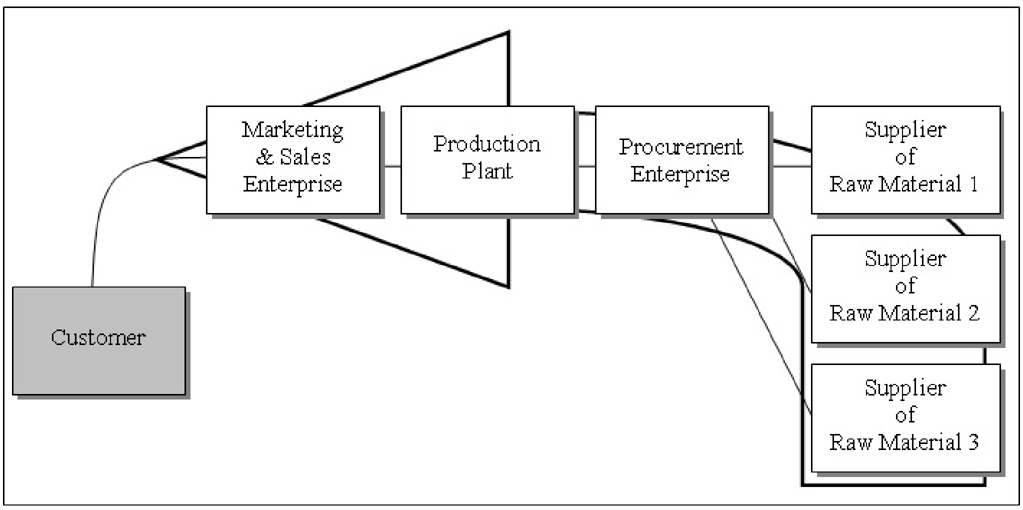


**Fig. 3.** A schematic of the SC example

The goal of modeling the SC is maximization of profit in dollars. The variables and parameters of the system are as follows:

$P$: The price of final product;
$M$: Costs to be paid by production plant in marketing sector;
$S$: Costs to be paid by production plant in procurement sector;
$O$: Operational costs of the production plant;
$T$: Lead time of the arrangement until a contract is finalized with a customer;
$S_1$: Price of first raw material;
$S_2$: Price of second raw material;
$S_3$: Price of third raw material;
$\pi_{\min}$: The lowest price which is acceptable by production plant to sell its product;
$\pi_{\max}$: The maximum rate in which the product could be sold; Market situation dictate this rate.

$\rho_{1\max}$: Maximum price of the first raw material;
$\rho_{1\min}$: Minimum price of the first raw material;
$\rho_{2\max}$: Maximum price of the second raw material;
$\rho_{2\min}$: Minimum price of the second raw material;
$\rho_{3\max}$: Maximum price of the third raw material;
$\rho_{3\min}$: Minimum price of the third raw material;
$\zeta_{med}$: Average operational costs of the factory;
$\zeta_{\min}$: Minimum operational costs of the factory;
$\alpha$: Annual interest rate;
$I$: Investment of the production factory, on making moulds, prototyping, and providing other conditions;
$\theta_1$: The rate, on which, each dollar paid in procurement enterprise will lead to a decrease in $\rho_{1\max}$ in that rate;
$\theta_2$: The rate, on which, each dollar paid in procurement enterprise will lead to a decrease in $\rho_{2\max}$ in that rate;
$\theta_3$: The rate, on which, each dollar paid in procurement enterprise will lead to a decrease in $\rho_{3\max}$ in that rate;
$\beta$: The rate, on which, each dollar paid in marketing enterprise will lead to an increase in $\pi_{\min}$ in that rate;
$\chi$: The rate, on which, each dollar paid in procurement enterprise will lead to a decrease in $\zeta_{med}$ in that rate;
$\tau_{\max}$: Maximum acceptable time for production plant for finalizing the deal;
$\delta$: The rate, on which, each dollar paid in marketing enterprise will lead to a decrease in investment costs $(\alpha \mathrm{I} \tau_{\max})$ in that rate;
$\varepsilon$: The rate, on which, each dollar paid in a procurement enterprise will lead to a decrease in investment costs $(\alpha \mathrm{I} \tau_{\max})$;
$\phi$: Available budget for costing in marketing and procurement enterprises.

This model is to be interpreted in different situations: (i) while there are exact data available for decision making and there is not any vagueness in the model, (ii) while parameters of the model are not exactly determined, (iii) while constraints are fuzzy, (iv) while goals faces some uncertainties, (v) while these uncertainties happen concurrently. These conditions are investigated in this paper in different models developed as follows:

### 3.1 Crisp (LP) model

Considering the variables and the parameters of the system, the SC problem using classical LP approach can be modeled as follows:

$$\max Z = P - [(S_1 + S_2 + S_3) + S + M + O + \alpha IT] \tag{1}$$

Subjet to:

$$S_1 \geq \rho_{1\max} - \theta_1 S \tag{2}$$
$$S_2 \geq \rho_{2\max} - \theta_2 S \tag{3}$$
$$S_3 \geq \rho_{3\max} - \theta_3 S \tag{4}$$
$$S_1 \geq \rho_{1\min} \tag{5}$$
$$S_2 \geq \rho_{2\min} \tag{6}$$
$$S_3 \geq \rho_{3\min} \tag{7}$$
$$P \leq \pi_{\min} + \beta M \tag{8}$$
$$P \leq \pi_{\max} \tag{9}$$

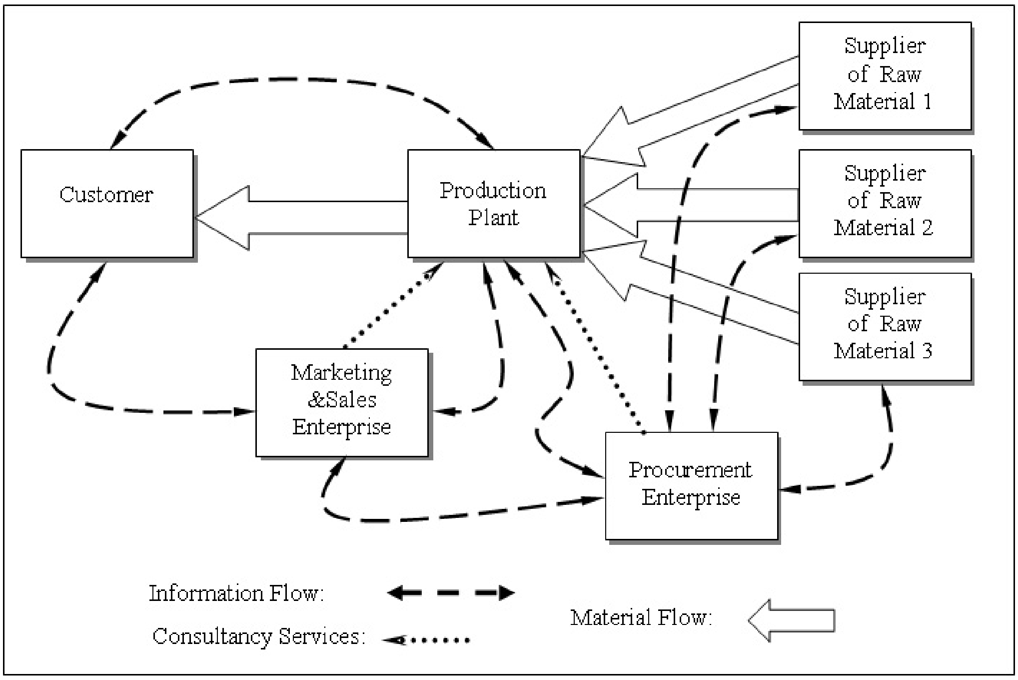


**Fig. 4.** Relationship diagram between different sectors of the SC

$$O \geq \zeta_{med} - \chi S \quad (10)$$
$$O \geq \zeta_{\min} \quad (11)$$
$$\alpha IT \geq \alpha I \tau_{\max} - \delta M - \varepsilon S \quad (12)$$
$$M + S \leq \phi \quad (13)$$
$$T \leq \tau_{\max} \quad (14)$$

This model is a basic for all further models in this research. Its elements are as follows: Objective function is maximizing of profit (income minus expenses). Constraints (2) $\sim$ (9) are related to procurement enterprise, since this department affects purchasing raw materials from suppliers and sales of products through marketing and sales enterprise to customers. The proposed price of raw materials by each supplier is in interval $[\rho_{i\min}, \rho_{i\max}]$, where $\rho_{i\max}$ is first offer and $\rho_{i\min}$ is lowest possible price. In such a situation, production plant prefers to spend \$S in procurement department to negotiate with suppliers to decrease $\rho_{i\max}$. However, this investment in procurement department has different effect on price reduction, which is shown by parameters $\theta_i$. Hence, constraints (2) $\sim$ (7) try to illustrate $S$ and $S_i$ relations. The same situation exists for marketing department, which is shown by constrains (8) and (9). Constrains (10) and (11) show the effect of procurement department on operational costs, and (12) demonstrates their effect on speed up of supply and sales operation. Finally, constrains (13) is the budget bound and (14) illustrates the time restriction.

The parameters and the coefficients of the model have been determined after a long negotiation and discussion with the experts of the production plant and tuning them with some mathematical and statistical models as follows:

$$\pi_{\min}: 11, \quad \pi_{\max}: 16, \quad \rho_{1\max}: 6.5, \quad \rho_{1\min}: 5, \quad \rho_{2\max}: 1.7, \quad \rho_{2\min}: 1.2, \quad \rho_{3\max}: 1,$$
$$\rho_{3\min}: 0.5, \quad \zeta_{med}: 1.2, \quad \zeta_{\min}: 0.8, \quad \alpha: 0.2, \quad I: 0.625, \quad \theta_1: 1.1, \quad \theta_2: 1.3, \quad \theta_3: 1.4,$$
$$\beta: 1.5, \quad \chi: 1.2, \quad \delta: 0.05, \quad \tau_{\max}: 1, \quad \varepsilon: 0.05, \quad \phi: 1.5$$

Inserting the above parameters to the above LP model, a SC model is determined as follows:

$$\max Z = P - [(S_1 + S_2 + S_3) + S + M + O + 0.125T] \tag{15}$$

Subject to :

$$S_1 \geq 6.5 - 1.1S \tag{16}$$
$$S_2 \geq 1.7 - 1.3S \tag{17}$$
$$S_3 \geq 1 - 1.4S \tag{18}$$
$$S_1 \geq 5 \tag{19}$$
$$S_2 \geq 1.2 \tag{20}$$
$$S_3 \geq 0.5 \tag{21}$$
$$P \leq 11 + 1.5M \tag{22}$$
$$P \leq 16 \tag{23}$$
$$O \geq 1.2 - 1.2S \tag{24}$$
$$O \geq 0.8 \tag{25}$$
$$0.125T \geq 0.125 - 0.05M - 0.02S \tag{26}$$
$$M + S \leq 1.5 \tag{27}$$
$$T \leq 1 \tag{28}$$
$$P, M, S, O, T, S_1, S_2, S_3 \geq 0 \tag{29}$$

We used CPLEX software to solve this crisp linear program. The results of the model are shown in Tab. 1.

**Table 1.** Solution of the crisp LP model

| Var. | Solution ($) |
|---|---|
| $Z$ | 2.5364 |
| $P$ | 12.67 |
| $S_1$ | 6.077 |
| $S_2$ | 1.2 |
| $S_3$ | 0.5 |
| $M$ | 1.115 |
| $S$ | 0.385 |
| $O$ | 0.8 |
| $T$ | 0.492 |

### 3.2 LP model with fuzzy coefficients

Real world situations are not often crisp. In this section, the former crisp LP model is modified to a fuzzy LP using fuzzy mathematics. For this purpose, we use both linguistic and nonlinguistic variables for gathering data from managers and decision makers of industry mentioned above. Thus, changing the linguistic variables into the language of the model, fuzzy numbers and fuzzy logic leads model designers to solve a realistic model in comparison with the crisp LP model (Zimmermann, 1996)[33]. In the following subsections, first the crisp model will be transferred to the model with fuzzy coefficients. Then, the situation in which bounds and goals are fuzzy is considered. Finally, the above models will be merged into an aggregated fuzzy model with fuzzy objectives, bounds and coefficient.

The linguistic input data of the model are transferred into fuzzy numbers. The procedure of this transformation is as follows: First, a new interview has been carried out with associated experts. They were requested to explain their ideas in more detail. At this stage, because there were additional explanations from the experts, the interviewer determined a wider range for each former crisp variable value. Upper and lower bounds of this range were extracted by the acquisition of all hidden points in the decision makers mind.

For example, finding the meaning of “almost 1.5” is a critical task in designing a perfect model when a manager states it as a marginal profit in marketing sector. In this discussion, the interviewer should clarify which price rate is expected to be invested in marketing sector. The reply could be: “between \$1.3-\$1.8, but more possible between \$1.4 –\$1.7.” With such information, forming an approximate membership function that represents a decision maker’s decisions can be easy. A trapezoidal fuzzy number can be determined. Alternate to this direct interview method to determine linguistic variables, there are several fuzzification approaches, which could be used such a Singleton Fuzzifier, Gaussian Fuzzifier and triangular Fuzzifier. However, this study is based on direct interviews which seems provide to reasonably reliable data.

According to above process, the coefficients of the model are generated via Trapezoidal fuzzy numbers as follows (Fig. 5):

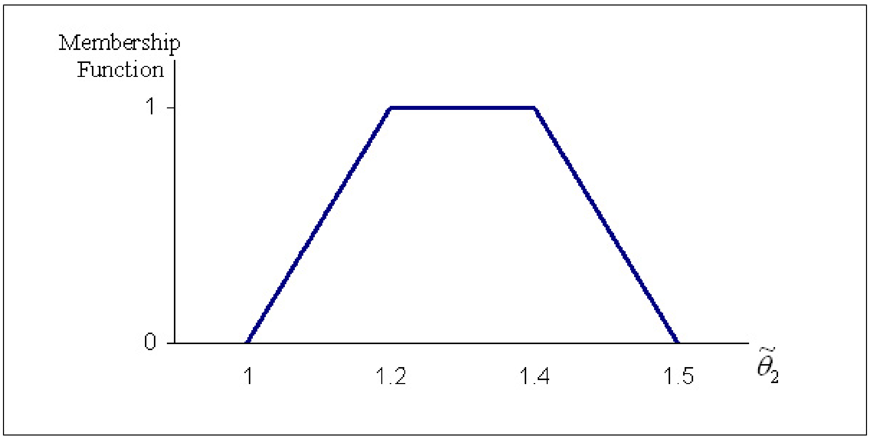


**Fig. 5.** The trapezoidal fuzzy number for $\tilde{\theta}_2$: (1, 1.2, 1.4, 1.5)

$$\tilde{\theta}_1 : (0.9, 1, 1.2, 1.4) \tag{30}$$

$$\tilde{\theta}_2 : (1, 1.2, 1.4, 1.5) \tag{31}$$

$$\tilde{\theta}_3 : (1.1, 1.3, 1.5, 1.65) \tag{32}$$

$$\tilde{\beta} : (1.3, 1.4, 1.65, 1.8) \tag{33}$$

$$\tilde{\chi} : (1, 1.1, 1.24, 1.3) \tag{34}$$

$$\tilde{\delta} : (0.03, 0.04, 0.06, 0.08) \tag{35}$$

$$\tilde{\varepsilon} : (0, 0.01, 0.03, 0.035) \tag{36}$$

After determining the parameters of the model, they were inserted into the model. Since the LP model could work with crisp numbers, defuzzification process should be performed. Among different defuzzification methods General Mean Values (GMV) is more prevalent because of its satisfactory results (McCahon and Lee, 1992)[15]. According to the procedure of the GMV, each fuzzy number, along with its range and membership functions is calculated as follows:

$$m(\tilde{A}) = \frac{\int_S x\mu_{\tilde{A}}(x)dx}{\int_S \mu_{\tilde{A}}(x)dx} \tag{37}$$

Thus, the above fuzzy numbers are defuzzified via Borland C++ package and are as follows:

$$\tilde{\theta}_1 : (0.9, 1, 1.2, 1.4) = 1.13 \tag{38}$$

$$\tilde{\theta}_2 : (1, 1.2, 1.4, 1.5) = 1.27 \tag{39}$$

$$\tilde{\theta}_3 : (1.1, 1.3, 1.5, 1.65) = 1.38 \tag{40}$$

$$\tilde{\beta} : (1.3, 1.4, 1.65, 1.8) = 1.54 \tag{41}$$

$$\tilde{\chi} : (1, 1.1, 1.24, 1.3) = 1.16 \tag{42}$$

$$\tilde{\delta} : (0.03, 0.04, 0.06, 0.08) = 0.053 \tag{43}$$

$$\tilde{\varepsilon} : (0, 0.01, 0.03, 0.035) = 0.018 \tag{44}$$

Next, the fuzzy LP model with fuzzy coefficient is formed as follows:

$$\max Z := \tilde{P} - [(\tilde{S}_1 + \tilde{S}_2 + \tilde{S}_3) + \tilde{S} + \tilde{M} + \tilde{O} + 0.125\tilde{T} \tag{45}$$

Subject to :

$$\tilde{S}_1 \geq 6.5 - 1.13\tilde{S} \tag{46}$$

$$\tilde{S}_2 \geq 1.7 - 1.27\tilde{S} \tag{47}$$

$$\tilde{S}_3 \geq 1 - 1.38\tilde{S} \tag{48}$$

$$\tilde{S}_1 \geq 5 \tag{49}$$

$$\tilde{S}_2 \geq 1.2 \tag{50}$$

$$\tilde{S}_3 \geq 0.5 \tag{51}$$

$$\tilde{P} \leq 11 + 1.54\tilde{M} \tag{52}$$

$$\tilde{P} \leq 16 \tag{53}$$

$$\tilde{O} \geq 1.2 - 1.16\tilde{S} \tag{54}$$

$$\tilde{O} \geq 0.8 \tag{55}$$

$$0.125\tilde{T} \geq 0.125 - 0.053\tilde{M} - 0.018\tilde{S} \tag{56}$$

$$\tilde{M} + \tilde{S} \leq 1.5 \tag{57}$$

$$\tilde{T} \leq 1 \tag{58}$$

Tab. 2 shows the solution of this model, where, it is rather improved in comparison to the crisp one. That is, while the value of Z in crisp model is equal to 2.54, in fuzzy LP it is 2.64 (Fig. 6).

**Table 2.** Solution of the LP model with defuzzified coefficients

| Var. | Solution ($) |
|---|---|
| $Z$ | 2.64 |
| $P$ | 12.704 |
| $S_1$ | 6.055 |
| $S_2$ | 1.2 |
| $S_3$ | 0.5 |
| $M$ | 1.106 |
| $S$ | 0.394 |
| $O$ | 0.8 |
| $T$ | 0.47 |

### 3.3 LP model with fuzzy bounds and objective function

In this section, we assume that the bounds and objective function of the model are fuzzy to examine the behavior of the model.

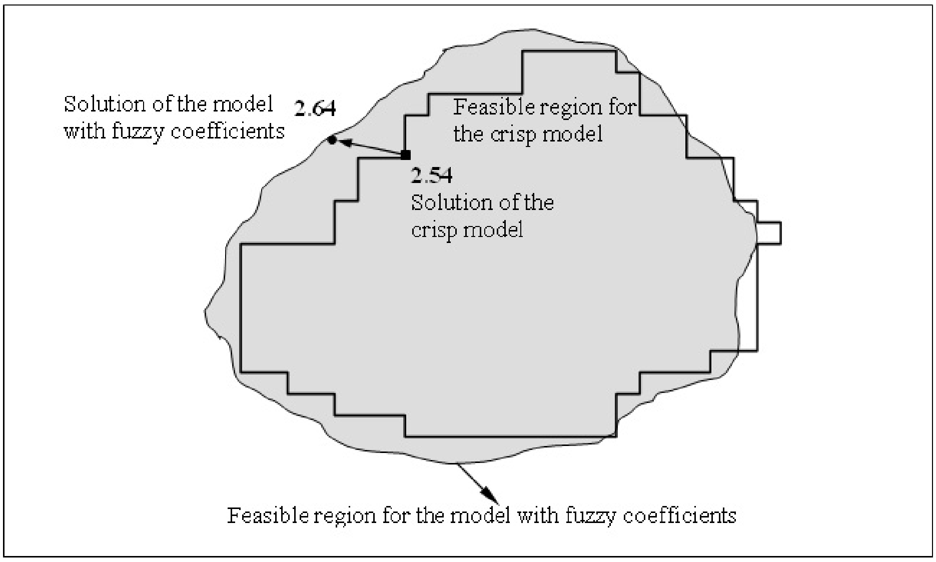


**Fig. 6.** Schematic comparison between crisp LP model and LP model with defuzzified coefficients

LP models are generally stated in the following form:

$$\max f(x) = z = c^T x \tag{59}$$

Subject to :

$$Ax \le b \tag{60}$$

$$x \ge 0 \tag{61}$$

$$c, x \in R^n, b \in R^m, A \in R^{m\times n} \tag{62}$$

Having fuzzy objective function $\tilde{G}$ and fuzzy bounds $\tilde{C}$ in variable space $X$, the fuzzy decision $\tilde{D}$ can be represented by (Bellman and Zadeh, 1970)[3]:

$$\tilde{D} = \tilde{G} \cap \tilde{C} \tag{63}$$

The "and" composition of goals and bounds should be interpreted as "logical and", where, the "logical and" corresponds to the set theoretic intersection (Zimmermann, 1996)[33]. The membership function of $\tilde{D}$ using t-norm (Turksen and Fazel Zarandi, 1999[27]; Schweizer and Sklar, 1983[23]) is also as follows:

$$\mu_{\tilde{D}} = t(\mu_{\tilde{G}}, \mu_{\tilde{C}}) \tag{64}$$

It should be noted that the Zadeh's standard t-norm is defined as $\mu_{\tilde{D}} = \min(\mu_{\tilde{G}}, \mu_{\tilde{C}})$. Hence, fuzzy structure of the model resulted from intersection of fuzzy objective functions and fuzzy constraints can be represented by (Zimmermann, 1996)[33]:

$$\text{Find} \quad x \tag{65}$$

Subject to :

$$c^T x \tilde{\ge} z \tag{66}$$

$$Ax \tilde{\le} b \tag{67}$$

$$x \ge 0 \tag{68}$$

Substituting $B = \begin{bmatrix} -c^T \\ A \end{bmatrix}$ and $d = \begin{bmatrix} z \\ b \end{bmatrix}$, the model is changed to

$$\text{Find} \quad x \tag{69}$$

Subject to :

$$B^T x \tilde{\geq} d \tag{70}$$

$$x \geq 0 \tag{71}$$

Each rows of such a model could be represented by a fuzzy set with membership function$\mu_i(x)$. As indicated before, i.e., equation (63) and (64), the membership function of the "decision" represented by the intersection of fuzzy sets as follows (Schweizer and Sklar, 1983)[23]:

$$\mu_{\tilde{D}}(x) = \min_i\{\mu_i(x)\} \tag{72}$$

Interpretation of $\mu_i(x)$ is not difficult. That is, when $\mu_i(x)$ is equal to 1 a constraint is completely satisfied, and when it is 0 the bound is totally not satisfied. Thus, $\mu_i(x)$ can be represented as follows:

$$\mu_i(x) = \begin{cases} 1 & if\ B_i x \leq d_i \\ \in [0,1] & if\ d_i \leq B_i x \leq d_i + p_i \\ 0 & if\ B_i x \geq d_i + p_i \end{cases} \tag{73}$$

where, $p_i$ shows the degree to which the $i$th constraint could alter. When the membership functions are linear, then,

$$\mu_i(x) = \begin{cases} 1 & if\ B_i x \leq d_i \\ 1 - \frac{B_i x - d_i}{p_i} & if\ d_i \leq B_i x \leq d_i + p_i \\ 0 & if\ B_i x \geq d_i + p_i \end{cases} \tag{74}$$

Now assuming $\lambda$ to be $\min_i\{\mu_{i(x)}\}$, the above model can be modified as follows (Zimmermann, 1996):

$$\max \quad \lambda \tag{75}$$

$$\text{Such that} : \lambda p_i + B_i x \leq d_i + p_i \qquad I = 1, \cdots, m+1 \tag{76}$$

$$x \geq 0 \tag{77}$$

Thus, our fuzzy SC model where the objective functions and constraints are fuzzy can be illustrated as follows:

$$\max \quad \lambda \tag{78}$$

Such that :

$$-\boldsymbol{0.3}\lambda + \tilde{S}_1 \geq 6.5 - 1.1\tilde{S} - \boldsymbol{0.3} \tag{79}$$

$$-\boldsymbol{0.1}\lambda + \tilde{S}_2 \geq 1.7 - 1.3\tilde{S} - \boldsymbol{0.1} \tag{80}$$

$$-\boldsymbol{0.1}\lambda + \tilde{S}_3 \geq 1 - 1.4\tilde{S} - \boldsymbol{0.1} \tag{81}$$

$$-\boldsymbol{0.2}\lambda + \tilde{S}_1 \geq 5 - \boldsymbol{0.2} \tag{82}$$

$$-\boldsymbol{0.1}\lambda + \tilde{S}_2 \geq 1.2 - \boldsymbol{0.1} \tag{83}$$

$$-\boldsymbol{0.1}\lambda + \tilde{S}_3 \geq 0.5 - \boldsymbol{0.1} \tag{84}$$

$$\boldsymbol{0.5}\lambda + P \leq 11 + 1.5\tilde{M} + \boldsymbol{0.5} \tag{85}$$

$$\boldsymbol{0.75}\lambda + \tilde{P} \leq 16 + \boldsymbol{0.75} \tag{86}$$

$$-\boldsymbol{0.2}\lambda + \tilde{O} \geq 1.2 - 1.2\tilde{S} - \boldsymbol{0.2} \tag{87}$$

$$-\boldsymbol{0.2}\lambda + \tilde{O} \geq 0.8 - \boldsymbol{0.2} \tag{88}$$

$$-\boldsymbol{0.005}\lambda + 0.125\tilde{T} \geq 0.125 - 0.05\tilde{M} - 0.02\tilde{S} - \boldsymbol{0.005} \tag{89}$$

$$\boldsymbol{0.3}\lambda + \tilde{M} + \tilde{S} \leq 1.5 + \boldsymbol{0.3} \tag{90}$$

$$\boldsymbol{0.3}\lambda + \tilde{T} \leq 1 + \boldsymbol{0.3} \tag{91}$$

$$-\boldsymbol{0.2}\lambda + \tilde{P} - \tilde{S}_1 - \tilde{S}_2 - \tilde{S}_3 - \tilde{M} - \tilde{S} - \tilde{O} - 0.125\tilde{T} \geq Z - \boldsymbol{0.2} \tag{92}$$

It should be noted that italic numbers in each row shows the tolerance of $p_i$. Moreover, (92) is the objective function of the crisp model, and $Z$ (Solution) is considered to be 3.10 as a lower bound for it.

Therefore, the solution of the problem using CPLEX is shown in Tab. 3.

As shown in Fig. 7, this solution shows an improvement obtained from former models.

**Table 3.** Solution of the LP model with fuzzy constraints and objective function

| Var. | Solution ($) |
|---|---|
| $Z$ | 3.2 |
| $P$ | 13.067 |
| $S_1$ | 5.95 |
| $S_2$ | 1.16 |
| $S_3$ | 0.458 |
| $M$ | 1.242 |
| $S$ | 3.846 |
| $O$ | 0.715 |
| $T$ | 0.4245 |
| $\lambda$ | 0.576 |
| $Z$ (Solution) | 3.10 |

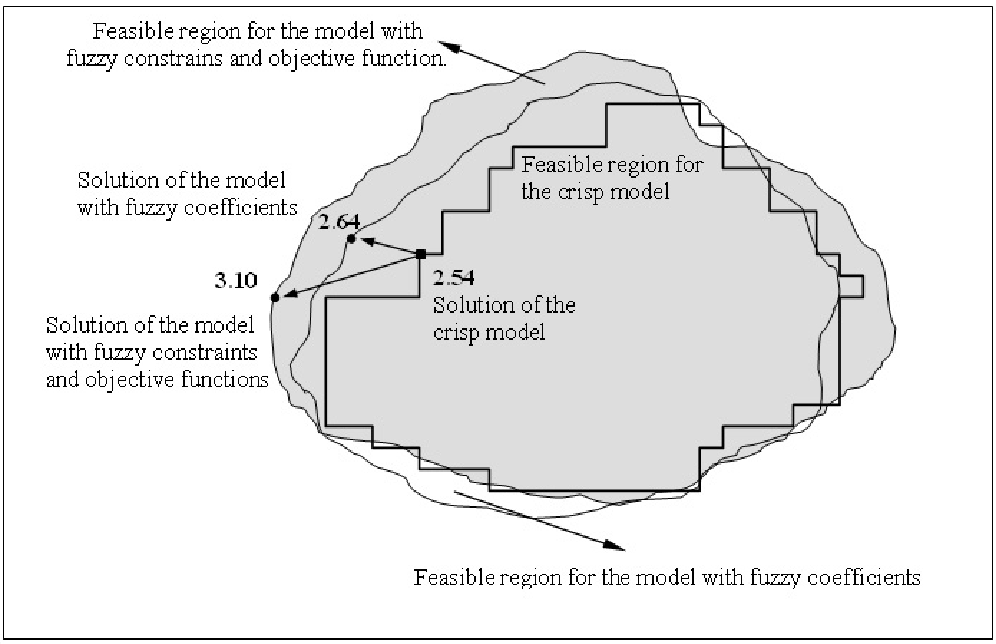


**Fig. 7.** Schematic comparison among crisp LP model, LP model with fuzzy coefficient, and LP with fuzzy constraints and objective function

### 3.4 LP model with fuzzy coefficients, bounds and objective function

In this section, all of the parameters and constraints of the LP model are assumed to be fuzzy. The procedure for the development this model is as follows:

**Step 1.** Construct the crisp model (as shown in section 3.1).

**Step 2.** Fuzzify of the coefficients of the crisp model (as shown in section 3.2).

**Step 3.** Transfer the last model to a decision making by intersecting objective function with bounds (as shown in section 3.3).

**Step 4.** Solving the model.

Thus, following the explained step-by-step procedures in section 3.1, 3.2 and 3.3, the final LP model with fuzzy coefficient and constraints can be easily developed as follows:

$$\max \quad \lambda \tag{93}$$

Such that :

$$-\boldsymbol{0.3}\lambda + \tilde{S}_1 \tilde{\geq} 6.5 - 1.1\tilde{S} - \boldsymbol{0.3} \tag{94}$$

$$-\boldsymbol{0.1}\lambda + \tilde{S}_2 \tilde{\geq} 1.7 - 1.3\tilde{S} - \boldsymbol{0.1} \tag{95}$$

$$-\boldsymbol{0.1}\lambda + \tilde{S}_3 \tilde{\geq} 1 - 1.4\tilde{S} - \boldsymbol{0.1} \tag{96}$$

$$-\boldsymbol{0.2}\lambda + \tilde{S}_1 \tilde{\geq} 5 - \boldsymbol{0.2} \tag{97}$$

$$-\boldsymbol{0.1}\lambda + \tilde{S}_2 \tilde{\geq} 1.2 - \boldsymbol{0.1} \tag{98}$$

$$-\boldsymbol{0.1}\lambda + \tilde{S}_3 \tilde{\geq} 0.5 - \boldsymbol{0.1} \tag{99}$$

$$\boldsymbol{0.5}\lambda + P \tilde{\leq} 11 + 1.5\tilde{M} + \boldsymbol{0.5} \tag{100}$$

$$\boldsymbol{0.75}\lambda + \tilde{P} \tilde{\leq} 16 + \boldsymbol{0.75} \tag{101}$$

$$-\boldsymbol{0.2}\lambda + \tilde{O} \tilde{\geq} 1.2 - 1.2\tilde{S} - \boldsymbol{0.2} \tag{102}$$

$$-\boldsymbol{0.2}\lambda + \tilde{O} \tilde{\geq} 0.8 - \boldsymbol{0.2} \tag{103}$$

$$-\boldsymbol{0.005}\lambda + 0.125\tilde{T} \tilde{\geq} 0.125 - 0.05\tilde{M} - 0.02\tilde{S} - \boldsymbol{0.005} \tag{104}$$

$$\boldsymbol{0.3}\lambda + \tilde{M} + \tilde{S} \tilde{\leq} 1.5 + \boldsymbol{0.3} \tag{105}$$

$$\boldsymbol{0.3}\lambda + \tilde{T} \tilde{\leq} 1 + \boldsymbol{0.3} \tag{106}$$

$$-\boldsymbol{0.2}\lambda + \tilde{P} - \tilde{S}_1 - \tilde{S}_2 - \tilde{S}_3 - \tilde{M} - \tilde{S} - \tilde{O} - 0.125\tilde{T} \tilde{\geq} Z - \boldsymbol{0.2} \tag{107}$$

As shown in Tab. 4, the $Z$ (Solution) in this case is equal to 3.154, which has been rather improved in comparison to previous models. The comparison of the solution of different LP models is shown in Fig. 8.

**Table 4.** Solution of the LP model with fuzzy coefficients, constraints, and objective function

| Var. | Solution ($) |
|---|---|
| $Z$ | 3.2 (as an input) |
| $P$ | 13.075 |
| $S_1$ | 5.94 |
| $S_2$ | 1.16 |
| $S_3$ | 0.46 |
| $M$ | 1.2 |
| $S$ | 0.39 |
| $O$ | 0.72 |
| $T$ | 0.41 |
| $\lambda$ | 0.614 |
| $Z$ (Solution) | 3.154 |

This improvement in development of the fuzzy models has several advantages, such as clear benefit in simplifying data gathering and works with linguistic variables. This creates flexibility in model building. In this way, more realistic representation of system behavior is also obtained by the introduction of tolerances.

### 3.5 Fuzzy expert system model for SC

In this section, we first develop a new fuzzy expert system based on rule bases for SC. Then, we compare the result with those of former crisp and fuzzy LPs developed for SC.

The main problems with mathematical models are their complexity and inability to use natural languages (Turksen, 1992)[28]. Fuzzy expert systems are valuable tools to come up with these problems of Operation

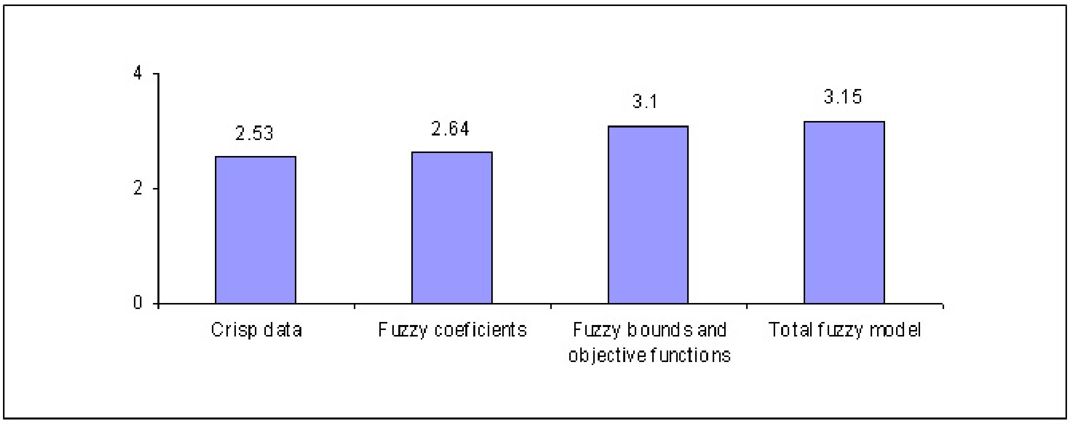


**Fig. 8.** Comparison of the solution in different steps of LP models development

Research (OR) models. For more information of fuzzy expert systems please see the works of Kusiak and Chen (1988)[11], McCahon and Lee (1992)[15], Kerr and Ebsary (1988)[10], Rayson (1985)[21], and Turksen and Fazel Zarandi(1999)[27], Fazel Zarandi and Saghiri (2006)[31].

In this section, a rule base system is designed for modeling the SC. Here the generation of the rules contains several steps. Briefly, this process was based on the direct interviews with associated managers, decision makers, and experts of the under study automotive plant. In this way, 24 of them have been selected, and the procedure was explained for them step-by-step. Then their ideas were gathered in straight personal interviews. Afterwards, thirty five preliminary rules were established, and offered to the experts to receive their ideas. Next, through a long discussions and negotiations with them, the rule based system was modified and nineteen rules with multi input single output were generated.

In establishing the rule base system, first, we used the same inputs as LP models. Then, we mapped the inputs into fuzzy sets. The fuzzy membership functions assigned to each input vector are determined by one or more linguistic variables that are antecedents of fuzzy rules. The output of the system is the "unit benefit", B. Accordingly, the rules generated by consensus of experts are as follows.

**1- IF** $P$ **isr** *ALMOST HIGH* **and** $M$ **isr** *HIGH* **THEN** $B$ **isr** *ALMOST HIGH*

***also***

**2- IF** $S$ **isr** *HIGH* **and** $O$ **isr** *LOW* **THEN** $B$ **isr** *HIGH*

***also***

**3- IF** $T$ **isr** *SHORT* **and** $M$ **isr** *HIGH* **THEN** $B$ **isr** *HIGH*

***also***

**4- IF** $S$ **isr** *HIGH* **and** $T$ **isr** *SHORT* **THEN** $B$ **isr** *HIGH*

***also***

**5- IF** $S$ **isr** *HIGH* **and** $S1$ **isr** *LOW* **and** $S2$ **isr** *LOW* **and** $S3$ **isr** *LOW* **THEN** $B$ **isr** *HIGH*

***also***

**6- IF** $P$ **isr** *HIGH* **and** $S1$ **isr** *LOW* **and** $S2$ **isr** *LOW* **and** $S3$ **isr** *LOW* **THEN** $B$ **isr** *HIGH*

***also***

**7- IF** $P$ **isr** *MED* **and** $O$ **isr** *MED* **and** $S1$ **isr** *HIGH* **and** $S2$ **isr** *HIGH* **and** $S3$ **isr** *HIGH* **THEN** $B$ **isr** *AVE*

***also***

**8- IF** $P$ **isr** *MED* **and** $O$ **isr** *MED* **and** $T$ **isr** *LONG* **THEN** $B$ **isr** *AVE*

***also***

**9- IF** $S$ **isr** *MED* **and** $O$ **isr** *MED* **and** $S1$ **isr** *LOW* **and** $S2$ **isr** *LOW* **and** $S3$ **isr** *LOW* **THEN** $B$ **isr** *AVE*

***also***

**10- IF** $P$ **isr** *MED* **and** $M$ **isr** *MED* **THEN** $B$ **isr** *AVE*

***also***

**11- IF** $P$ **isr** *LOW* **and** $M$ **isr** *LOW* **THEN** $B$ **isr** *LOW*

***also***

**12- IF** $S$ **isr** *LOW* **and** $O$ **isr** *HIGH* **and** $S1$ **isr** *HIGH* **and** $S2$ **isr** *HIGH* **and** $S3$ **isr** *HIGH* **THEN** $B$ **isr** *LOW*

***also***

**13- IF** $P$ **isr** *LOW* **and** $O$ **isr** *MED* **and** $T$ **isr** *LONG* **THEN** $B$ **isr** *LOW*

***also***

**14- IF** $P$ **isr** *LOW* **and** $S1$ **isr** *HIGH* **and** $S2$ **isr** *HIGH* **and** $S3$ **isr** *HIGH* **THEN** $B$ **isr** *LOW*

***also***

**15- IF** $P$ **isr** *LOW* **and** $O$ **isr** *HIGH* **THEN** $B$ **isr** *LOW*

***also***

**16- IF** $S$ **isr** *LOW* **and** $T$ **isr** *LONG* **THEN** $B$ **isr** *ALMOST LOW*

***also***

**17- IF** $S1$ **isr** *HIGH* **and** $S2$ **isr** *HIGH* **and** $S3$ **isr** *HIGH* **THEN** $B$ **isr** *LOW*

***also***

**18- IF** P **isr** *ALMOST HIGH* **and** $M$**isr** *HIGH* **and** $T$ **isr** *SHORT* **THEN**$B$ **isr** *ALMOST HIGH*

***also***

**19- IF** $S$ **isr** *HIGH* **and** $O$ **isr** *ALMOST LOW* **and** $T$ **isr** *SHORT* **and** $S1$ **isr** *LOW* **and** $S2$ **isr** *LOW* **and** $S3$ **isr** *LOW* **THEN** $B$ **isr** *ALMOST HIGH*

These rules are summarized in Tab. 5.

**Table 5.** Summarized rule base system for a SC

| Inputs: | $P$ | $M$ | $S$ | $O$ | $T$ | $S_1$ | $S_2$ | $S_3$ | B |
|---|---|---|---|---|---|---|---|---|---|
| Rule 1 | ALMOST HIGH | HIGH | | | | | | | ALMOST HIGH |
| Rule 2 | | | HIGH | LOW | | | | | HIGH |
| Rule 3 | | HIGH | | | SHORT | | | | HIGH |
| Rule 4 | | | HIGH | | SHORT | | | | HIGH |
| Rule 5 | | | HIGH | | | LOW | LOW | LOW | HIGH |
| Rule 6 | HIGH | | | | | LOW | LOW | LOW | HIGH |
| Rule 7 | MED | | | MED | | HIGH | HIGH | HIGH | AVE |
| Rule 8 | MED | | | MED | LONG | | | | AVE |
| Rule 9 | | | MED | MED | | LOW | LOW | LOW | AVE |
| Rule 10 | MED | MED | | | | | | | AVE |
| Rule 11 | LOW | LOW | | | | | | | LOW |
| Rule 12 | | | LOW | HIGH | | HIGH | HIGH | HIGH | LOW |
| Rule 13 | LOW | | | MED | LONG | | | | LOW |
| Rule 14 | LOW | | | | | HIGH | HIGH | HIGH | LOW |
| Rule 15 | LOW | | | HIGH | | | | | LOW |
| Rule 16 | | | LOW | | LONG | | | | ALMOST LOW |
| Rule 17 | | | | | | HIGH | HIGH | HIGH | LOW |
| Rule 18 | ALMOST HIGH | HIGH | | | SHORT | | | | ALMOST HIGH |
| Rule 19 | | | HIGH | ALMOST LOW | SHORT | LOW | LOW | LOW | ALMOST HIGH |

We have used Fuzzy Toolbox of MATLAB6 to demonstrate the input-output membership functions (Fig. 9). The assigned figures are based on mean value of each entry, and the defuzzification method is centroid approach (see McCahon and Lee, 1992[15]; Yager and Filev, 1994[29]).

For comparing the results of the fuzzy rule base system with fuzzy LP models, the fuzzy rule base system is executed with the same inputs. In this respect, the inputs for the final solution of the last fuzzy LP model, as the best model amongst LP models, which contains fuzzy bounds, coefficient and fuzzy objective function, are inputed to the fuzzy rule base system. That is, we Choose these inputs to compare the behavior of the best solution of the LP models we have investigated. Aggregation of the rules through OR, i.e., Max, operator while rules have been fired by inputs leads to the results shown in Fig. 10 (Baldwin, 1981[2]; Sugeno and Yasukawa, 1993[24]; Sugeno and Kang, 1988[25]).

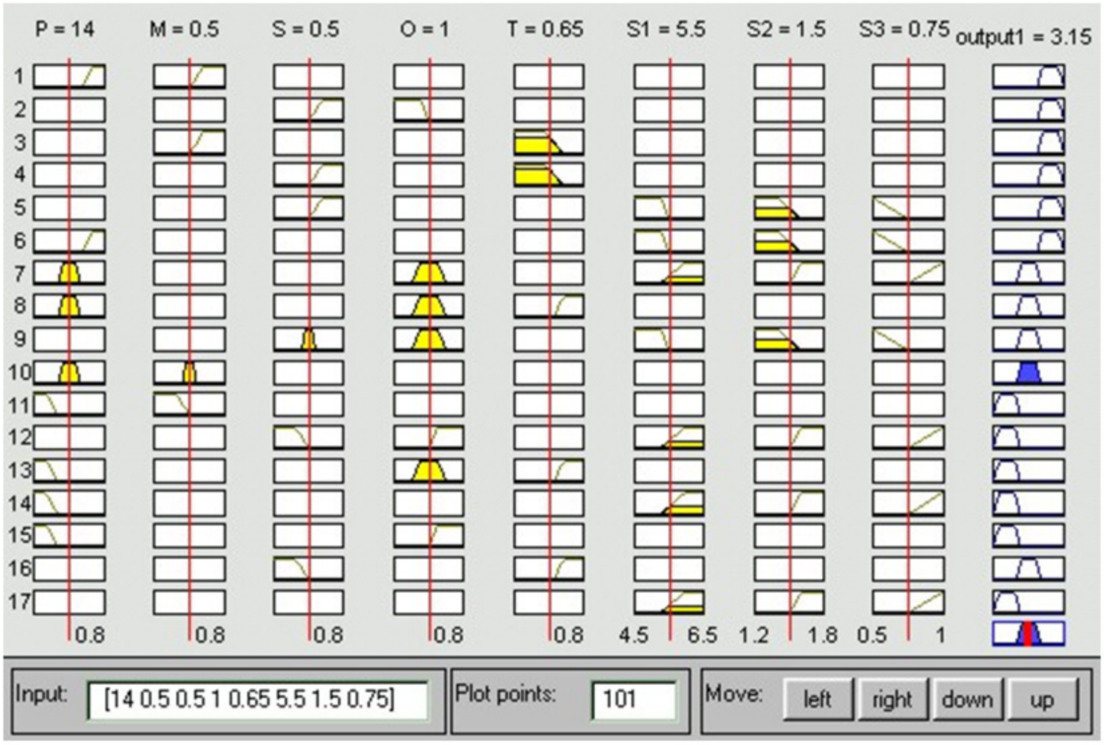


**Fig. 9.** Graphical view of the rule base system


**Fig. 10.** Result of the rule base system with the inputs which were solution of the LP model

As shown in Tab. 6, while the result of fuzzy LP is 3.154, the result of fuzzy expert system is 4.78, which is result of considering real situatoin. Tab. 6 represents comparison between all results, produced by different methods. Furthermore, Fig. 11, illustrates a schematic comparison between these models.

**Table 6.** The comparison between LP models and fuzzy rule base model

| Model Type | Solution |
|---|---|
| LP crisp model | 2.54 |
| LP model with fuzzy coefficients | 2.64 |
| LP model with fuzzy constraints and objective function | 3.10 |
| LP model with fuzzy coefficient, constraints, and objective functions | 3.154 |
| Fuzzy rule base model, with inputs from last LP model | 4.78 |

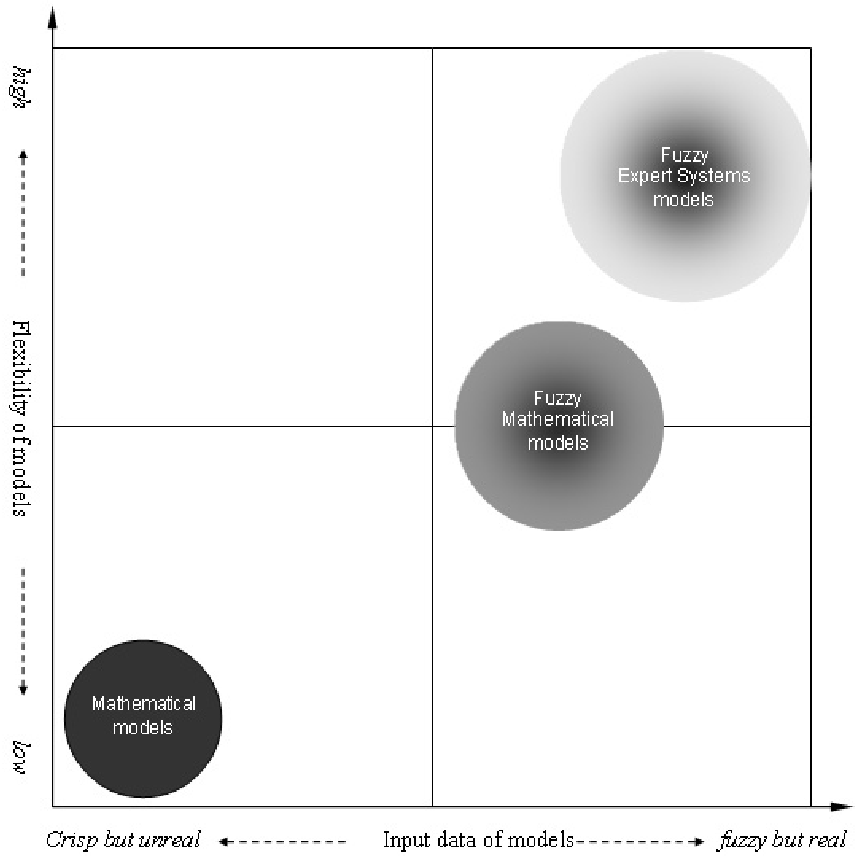


**Fig. 11.** Conceptual comparison between developed models

## 4 Conclusions and future works

The main purpose of this research was to develop the most appropriated SC model for an automotive industry. For this purpose, we developed several models, i.e., five models, step-by-step. We started with very crisp model and gradually we moved to a very fuzzy one. The knowledge acquisition was gradually improved and fine tuned as we moved from one model to the next and trained the experts in providing more specific knowledge. First, a SC model was developed for this plant with classical LP. Then we gradually fuzzified objective function, constraints and coefficients of the system, and developed a fuzzy LP model to analyze and predict the behavior of the system. This approach shows different aspects of modeling SC in real world setting. In each stage, we compared the results to demonstrate the advantages of the application of fuzzy methodology in SC. However, these advantages are not just due to the results, but also because of more flexibility of the model in considering the real-world situation. Finally, we developed a fuzzy expert system (FES) based on rule bases that described the behavior of SC as we have extracted from and negotiated with the experts. The results of the FES showed the superiority of approximate reasoning in qualitative SC models.

With reference to very few works on fuzzy modeling of SC system, this research was an attempt to show how we can model a SC from different points of view. However, there exists potentiality more future work. For example, in fuzzificatoin of LP models, there is a need to find an algorithmic approach for an improved representation of such models. Also in this paper we used several heuristic approaches for rule generation. For this purpose, the application of other approaches such as Neural Networks and/or fuzzy clustering algorithms may produce better results.

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