4.1. Tricomi Equation $y\dfrac{\partial^2 w}{\partial x^2} + \dfrac{\partial^2 w}{\partial y^2} = 0$

The Tricomi equation is used to describe near-sonic flows of gas.

4.1-1. Particular solutions of the Tricomi equation:

$$w = Axy + Bx + Cy + D,$$
$$w = A(3x^2 - y^3) + B(x^3 - xy^3) + C(6yx^2 - y^4),$$

where A, B, C, and D are arbitrary constants.

4.1-2. Particular solutions of the Tricomi equation with even powers of x:

$$w = \sum_{k=0}^{n} \varphi_k(y) x^{2k},$$

where the functions $\varphi_k = \varphi_k(y)$ are defined by the recurrence relations

$$\varphi_n(y) = A_n y + B_n, \quad \varphi_{k-1}(y) = A_k y + B_k - 2k(2k-1)\int_0^y (y-t)t\varphi_k(t)\,dt,$$

where A_k and B_k are arbitrary constants ($k = n, \dots, 1$).

4.1-3. Particular solutions of the Tricomi equation with odd powers of x:

$$w = \sum_{k=0}^{n} \psi_k(y) x^{2k+1},$$

where the functions $\psi_k = \psi_k(y)$ are defined by the recurrence relations

$$\psi_n(y) = A_n y + B_n, \quad \psi_{k-1}(y) = A_k y + B_k - 2k(2k+1)\int_0^y (y-t)t\psi_k(t)\,dt,$$

where A_k and B_k are arbitrary constants ($k = n, \dots, 1$).

4.1-4. Separable particular solutions of the Tricomi equation:

$$w(x,y) = \bigl[A\sinh(3\lambda x) + B\cosh(3\lambda x)\bigr]\sqrt{y}\,\bigl[CJ_{1/3}(2\lambda y^{3/2}) + DY_{1/3}(2\lambda y^{3/2})\bigr],$$
$$w(x,y) = \bigl[A\sin(3\lambda x) + B\cos(3\lambda x)\bigr]\sqrt{y}\,\bigl[CI_{1/3}(2\lambda y^{3/2}) + DK_{1/3}(2\lambda y^{3/2})\bigr],$$

where A, B, C, D, and λ are arbitrary constants, $J_{1/3}(z)$ and $Y_{1/3}(z)$ are the Bessel functions, and $I_{1/3}(z)$ and $K_{1/3}(z)$ are the modified Bessel functions.

References

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