1. $$\frac{\partial w}{\partial t} = a\frac{\partial}{\partial x}\left(w^m \frac{\partial w}{\partial x}\right).$$

Heat equation with a power-law nonlinearity. This equation occurs in nonlinear problems of heat and mass transfer and flows in porous media.

1°. Solutions:

$$w(x,t) = (\pm kx + k\lambda t + A)^{1/m}, \quad k = \lambda m/a,$$

$$w(x,t) = \left[\frac{m(x-A)^2}{2a(m+2)(B-t)}\right]^{\frac{1}{m}},$$

$$w(x,t) = \left[A|t+B|^{-\frac{m}{m+2}} - \frac{m}{2a(m+2)}\frac{(x+C)^2}{t+B}\right]^{\frac{1}{m}},$$

$$w(x,t) = \left[\frac{m(x+A)^2}{\varphi(t)} + B|x+A|^{\frac{m}{m+1}}|\varphi(t)|^{-\frac{m(2m+3)}{2(m+1)^2}}\right]^{\frac{1}{m}}, \quad \varphi(t) = C - 2a(m+2)t,$$

where A, B, C, and λ are arbitrary constants. The second solution for $B > 0$ corresponds to blow-up regimes (the solution increases without bound on a finite time interval).

2°. There are solutions of the following forms:

$$w(x,t) = (t+C)^{-1/m}F(x) \qquad \text{multiplicative separable solution;}$$

$$w(x,t) = t^{\lambda}G(\xi), \quad \xi = xt^{-\frac{m\lambda+1}{2}} \qquad \text{self-similar solution;}$$

$$w(x,t) = e^{-2\lambda t}H(\eta), \quad \eta = xe^{\lambda mt} \qquad \text{generalized self-similar solution;}$$

$$w(x,t) = (t+C)^{-1/m}U(\zeta), \quad \zeta = x + \lambda\ln(t+C),$$

where C, β, and λ are arbitrary constants.

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