1. $\dfrac{\partial w}{\partial t} = \dfrac{\partial^2 w}{\partial x^2} + w\dfrac{\partial w}{\partial x}.$

Burgers equation. It is used for describing wave processes in acoustics and hydrodynamics.

1°. Solutions:

$$w(x,t) = \lambda + \frac{2}{x + \lambda t + A},$$
$$w(x,t) = \frac{4x + 2A}{x^2 + Ax + 2t + B},$$
$$w(x,t) = \frac{6(x^2 + 2t + A)}{x^3 + 6xt + 3Ax + B},$$
$$w(x,t) = \frac{2\lambda}{1 + A\exp(-\lambda^2 t - \lambda x)},$$
$$w(x,t) = -\lambda + A\frac{\exp\bigl[A(x - \lambda t)\bigr] - B}{\exp\bigl[A(x - \lambda t)\bigr] + B},$$

where A, B, and λ are arbitrary constants.

2°. Other solutions can be obtained using the following formula (Hopf–Cole transformation):

$$w(x,t) = \frac{2}{u}\frac{\partial u}{\partial x},$$

where $u = u(x,t)$ is a solution of the linear heat equation, $u_t = u_{xx}$.

3°. The Cauchy problem with the initial condition:

$$w = f(x) \quad \text{at} \quad t = 0, \qquad -\infty < x < \infty.$$

Solution:

$$w(x,t) = 2\frac{\partial}{\partial x}\ln F(x,t), \qquad F(x,t) = \frac{1}{\sqrt{4\pi t}}\int_{-\infty}^{\infty} \exp\left[-\frac{(x-\xi)^2}{4t} - \frac{1}{2}\int_0^{\xi} f(\xi')\,d\xi'\right] d\xi.$$

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