3. $i\dfrac{\partial w}{\partial t}+\dfrac{\partial^2 w}{\partial x^2}+f(|w|)w=0.$

Nonlinear Schrodinger equation (Schrödinger equation) of general form; $f(u)$ is a real function of a real variable.

1°. Suppose $w(x,t)$ is a solution of the Schrodinger equation in question. Then the function

$$w_1=e^{-i(\lambda x+\lambda^2 t+C_1)}w(x+2\lambda t+C_2,\,t+C_3),$$

where C_1, C_2, C_3, and λ are arbitrary real constants, is also a solution of the equation.

2°. Traveling-wave solution:

$$w(x,t)=C_1\exp\bigl[i\varphi(x,t)\bigr],\qquad \varphi(x,t)=C_2x-C_2^2t+f(|C_1|)t+C_3.$$

3°. Multiplicative separable solution:

$$w(x,t)=u(x)e^{i(C_1t+C_2)},$$

where the function $u=u(x)$ is defined implicitly by

$$\int\frac{du}{\sqrt{C_1u^2-2F(u)+C_3}}=C_4\pm x,\qquad F(u)=\int uf(|u|)\,du.$$

Here, $C_1,\,\dots,\,C_4$ are arbitrary real constants.

4°. Solution:

$$w(x,t)=U(\xi)e^{i(Ax+Bt+C)},\qquad \xi=x-2At,\tag{1}$$

where the function $U=U(\xi)$ is determined by the autonomous ordinary differential equation $U''_{\xi\xi}+f(|U|)U-(A^2+B)U=0$. Integrating yields the general solution in implicit form:

$$\int\frac{dU}{\sqrt{(A^2+B)U^2-2F(U)+C_1}}=C_2\pm\xi,\qquad F(U)=\int Uf(|U|)\,dU.\tag{2}$$

Relations (1) and (2) involve arbitrary real constants A, B, C, C_1, and C_2.

5°. Solution (A, B, and C are arbitrary constants):

$$w(x,t)=\psi(z)\exp\bigl[i(Axt-\tfrac{2}{3}A^2t^3+Bt+C)\bigr],\qquad z=x-At^2,$$

where the function $\psi=\psi(z)$ is determined by the ordinary differential equation $\psi''_{zz}+f(|\psi|)\psi-(Az+B)\psi=0$.

6°. Solutions:

$$w(x,t)=\pm\frac{1}{\sqrt{C_1t}}\exp\bigl[i\varphi(x,t)\bigr],\qquad \varphi(x,t)=\frac{(x+C_2)^2}{4t}+\int f\bigl(|C_1t|^{-1/2}\bigr)\,dt+C_3,$$

where C_1, C_2, and C_3 are arbitrary real constants.

7°. Solution:

$$w(x,t)=u(x)\exp\bigl[i\varphi(x,t)\bigr],\qquad \varphi(x,t)=C_1t+C_2\int\frac{dx}{u^2(x)}+C_3,$$

where C_1, C_2, and C_3 are arbitrary real constants, and the function $u=u(x)$ is determined by the autonomous ordinary differential equation $u''_{xx}-C_1u-C_2^2u^{-3}+f(|u|)u=0$.

8°. There is an exact solution of the form

$$w(x,t) = u(z)\exp\left[iAt + i\varphi(z)\right], \qquad z = kx + \lambda t,$$

where A, k, and λ are arbitrary real constants.

See also special cases of the nonlinear Schrodinger equation:

- Schrodinger equation with a cubic nonlinearity ,
- Schrodinger equation with a power-law nonlinearity .

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