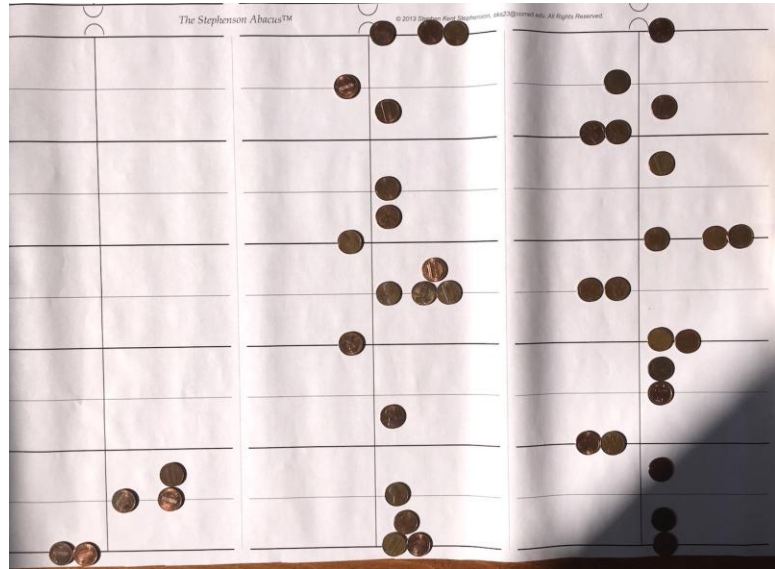


There are ancient Babylonian texts that seem to indicate that an abacist would use two tablets to work on large numbers; e.g., in the geometric progression of doubling. This paper recreates a procedure to do doubling using two tablets of the [3rd abacus design](#).

At the right are the results when the calculation was terminated at the max for two tablets.

A count of the doublings was kept at the lower left and reads 68 ((50 + 10) + (10 - 2)).

The resulting number can be read from the top of the middle tablet to its bottom, then continuing at the top of the right tablet and down to its bottom. The number [reads](#) as 295,147,905,179,352,825,856; over 295 quintillion, accurate in every digit.



The calculation starts by placing one token on the bottom line of the right tablet on the positive side with a count of zero doublings. The first doubling is done by adding one more token next to the first and adding one to the count. The second doubling is done by sliding one token up to the space above and the other to the left of the bar resulting in $5-1=4$; one is added to the count.

After each doubling the number is made readable by, where necessary, adding or deleting zero pairs, replacing 3 with $5-2$, and replacing $10-5$ with 5.

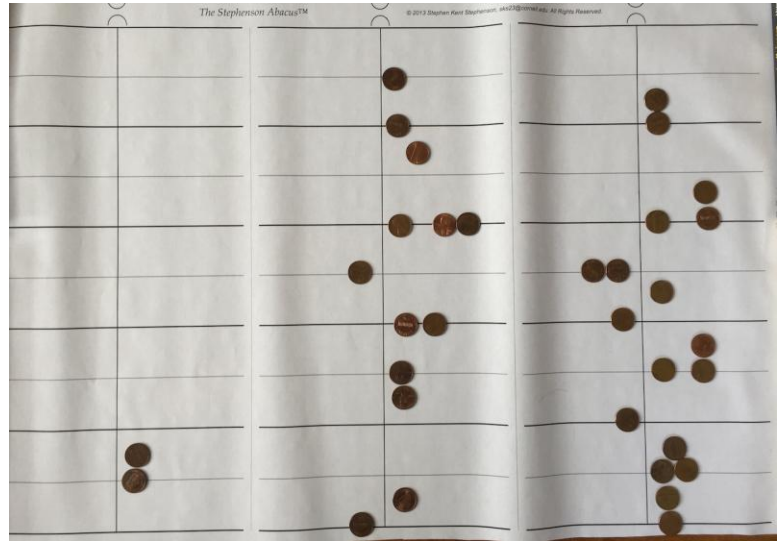
Before the next doubling the amount of tokens is reduced where possible by removing zero pairs and changing 3 to $5-2$ and $10-5$ to 5. That should result in no more than two tokens on each line and no more than one token in each space between lines.

Doubling starts with the most significant digit. A token in a space is moved up to the next line ($5*2=10$). If a line has one token on it another token is added ($1*2=2$), but if the line has two tokens on it one is moved up to the space above and the other to the opposite side of the vertical bar ($2*2=5-1$). Repeat for each decreasing significant digit. When the least significant digit has been doubled add one to the counter at the bottom of the left table.

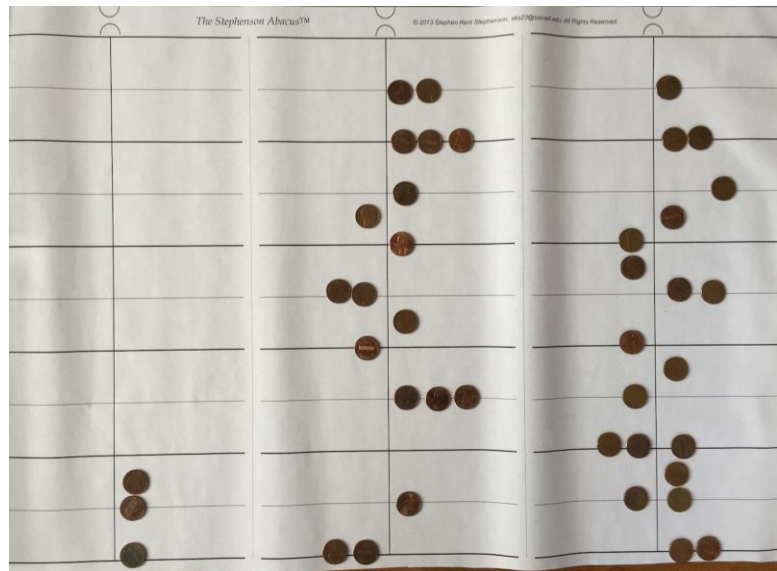
Repeating the above three paragraphs eventually fills up the right tablet. The first plan of using two tablets was to move the tokens on the top half of the right tablet to the bottom half of the middle tablet. Then both tablets would be doubled in each step and any resulting tokens in the top half of the right tablet would be accumulated into the bottom half of the middle tablet and removed from the right tablet. But the most that was ever accumulated was contained in one digit.

So the next plan accumulated the top digit of the right tablet into the bottom digit of the middle tablet after each doubling; thereby giving an effective number size of $10+(10-1)=19$ digits. The next three images show the doubling of a number that requires the accumulation of the top digit of the right table into the bottom digit of the middle table.

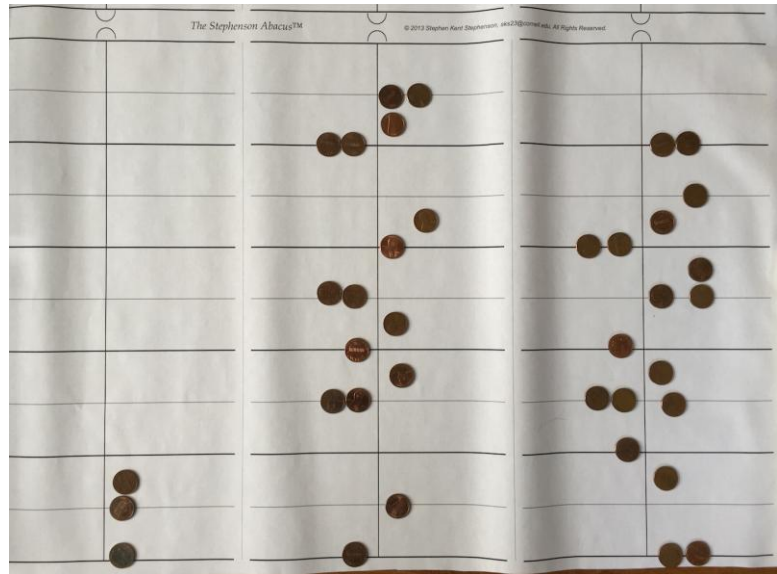
This is $2^{60} =$
1,152,921,504,606,846,976.



This is 2^{60} doubled.

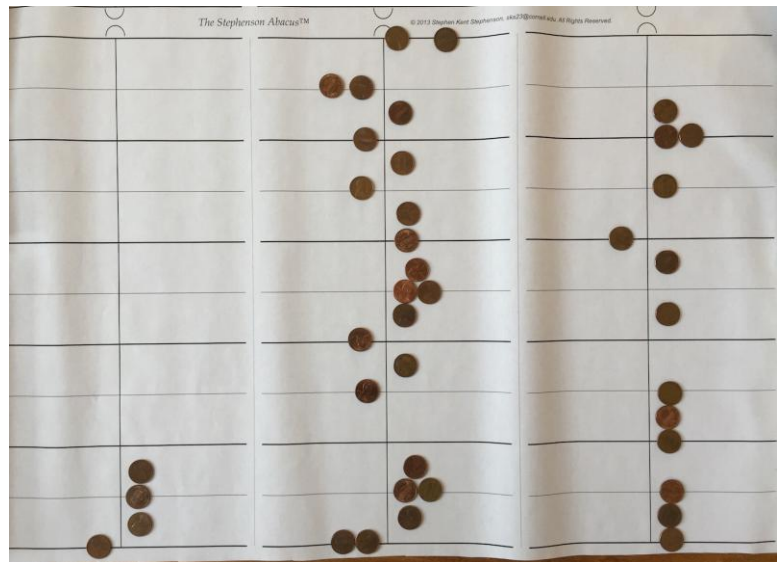


This is 2^{60} doubled, made readable, and top digit of right table has been moved and accumulated into bottom digit of middle table. So the result is now $2^{61} = 2,305,843,009,213,693,952$.

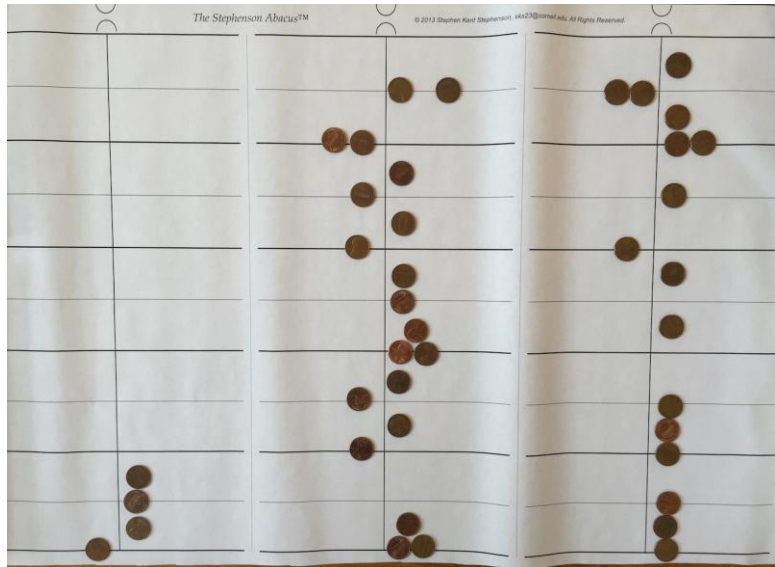


The max for this method is $2^{64} = 18,446,744,073,709,551,616$.

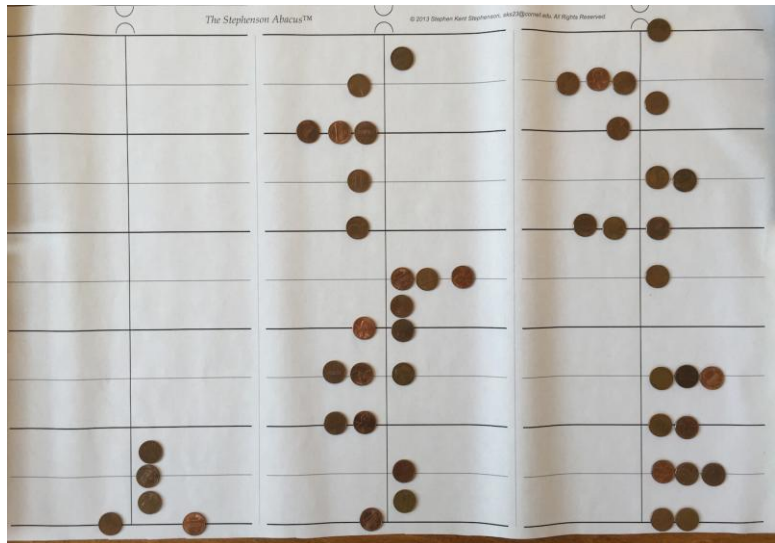
But look at that useless and empty digit at the top of the right table. Since the multiplier is 2 there will be no carry other than 1, so only the top line is needed as the accumulator. I.e., at the end of a doubling, any token on the top line of the right tablet that was not part of a 10–1 or 10–2 pair would be removed and accumulated into the bottom line of the middle tablet.



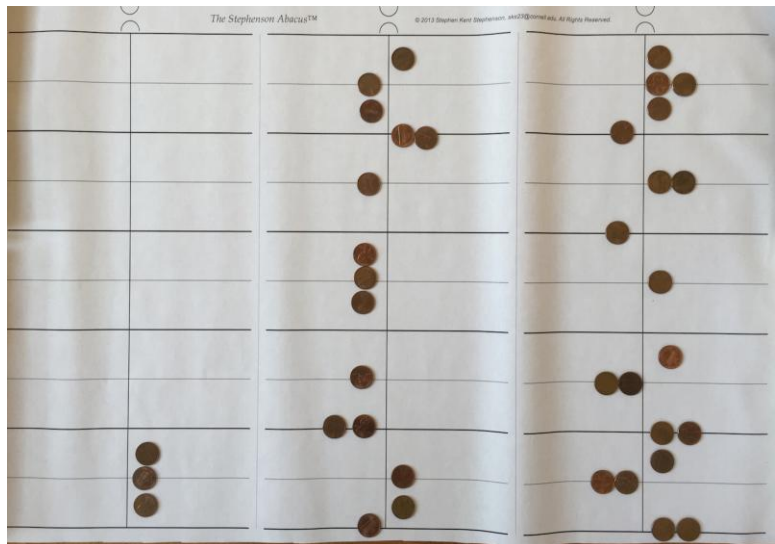
Here the bottom digit of the middle table has been moved into the top (empty) digit of the right table and all the other digits in the middle table have been shifted down one digit. Now the calculation can continue.



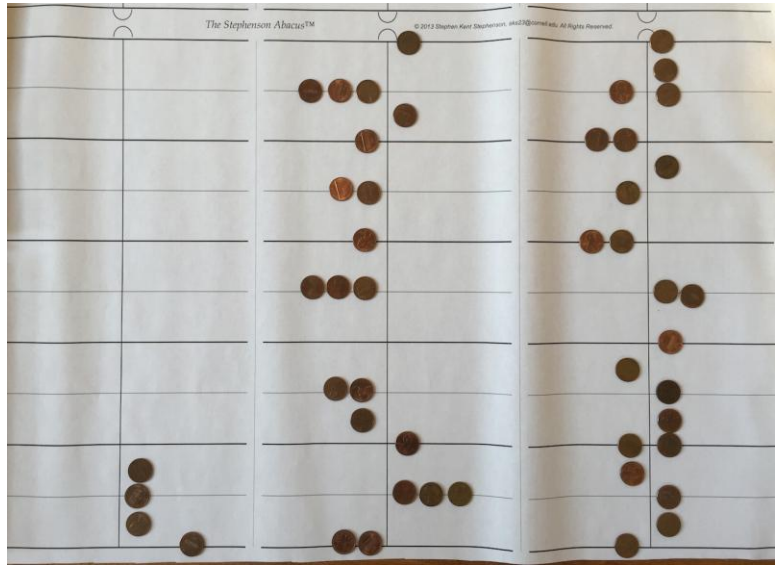
Double.



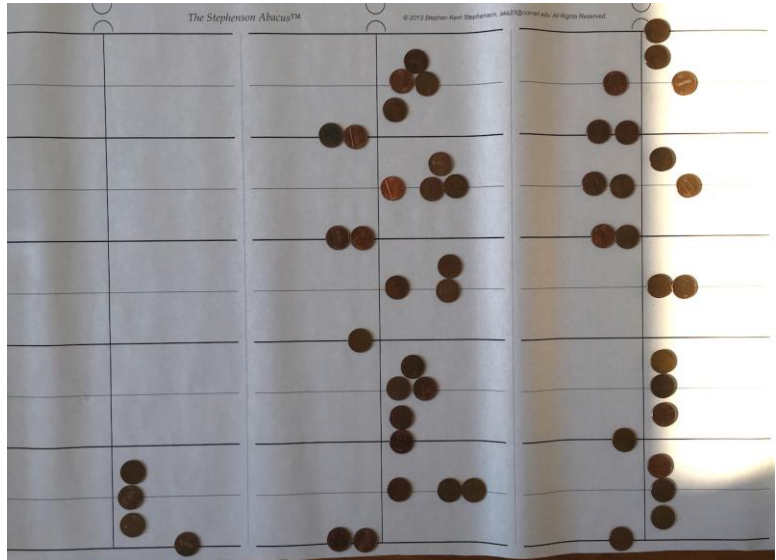
Reduce.



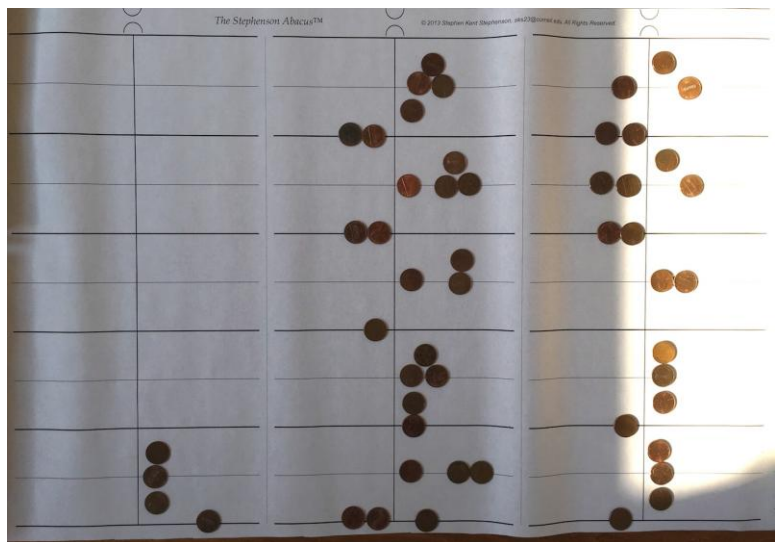
Duplicate (skipping make readable this time).



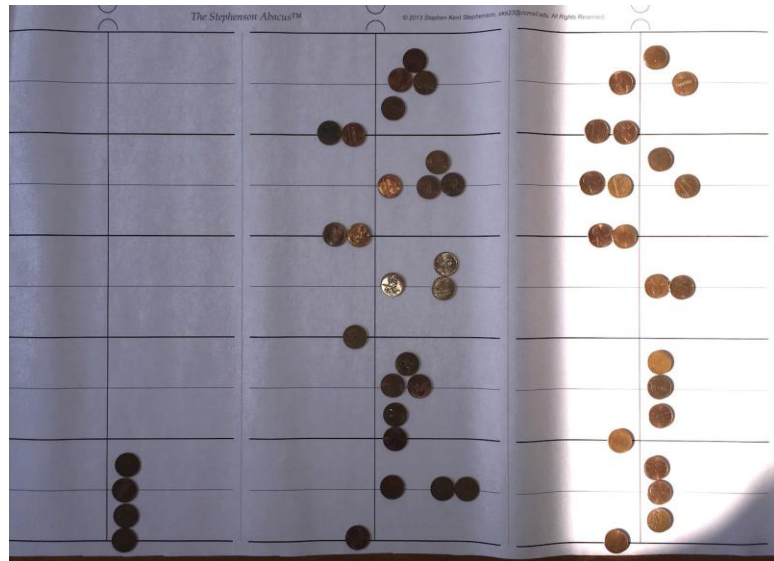
Make readable (short of moving accumulation token).



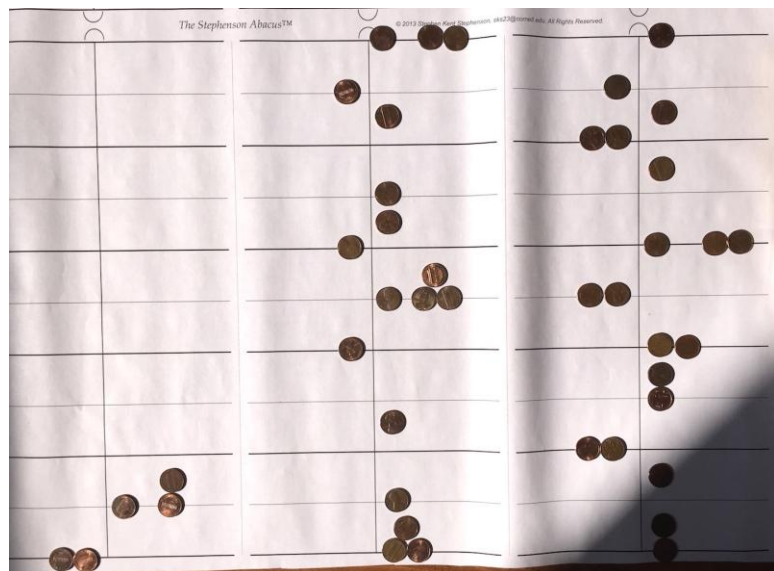
Move accumulation token.



Finish make readable $\Rightarrow 2^{66} =$
73,786,976,294,838,206,464.



Continue to final result $\Rightarrow 2^{68} =$
295,147,905,179,352,825,856;
over 295 quintillion and accurate in
every digit.



What about geometric sequence's with multipliers greater than 2?

The table below gives a possible iteration procedure for each multiplier from 3 to 32. The procedures use the terms bar# and edge# which are names of the two numbers that can be represented simultaneously on a line abacus. Their definitions are:

bar# := number formed by tokens next to the vertical bar of the table(s); and
edge# := number formed by tokens on left and right edges of the table(s).

Multiplier	Expanded	Calculation plan for each iteration
3	$2 + 1$	a. Copy bar# to edges. b. Double bar#. c. Add edge# into bar#.
4	2^2	a. Double bar#. b. Double bar#.
5	$2^2 + 1$	a. Copy bar# to edges. b. Double bar#. c. Double bar#. d. Add edge# into bar#.
6	$(2 + 1) * 2$	a. Copy bar# to edges. b. Double bar#. c. Add edge# into bar#. d. Double bar#.
7	$2^3 - 1$	a. Copy bar# to opposite edges. b. Double bar#. c. Double bar#. d. Double bar#. e. Add edge# into bar#.
8	2^3	a. Double bar#. b. Double bar#. c. Double bar#.
9	$10 - 1$	a. Copy bar# to opposite edges. b. Shift bar# up 1 digit. c. Add edge# into bar#.
10	10	a. Shift bar# up 1 digit.
11	$10 + 1$	a. Copy bar# to edges. b. Shift bar# up 1 digit. c. Add edge# into bar#.
12	$10 + 2$	a. Copy bar# to edges one digit up . b. Double bar#. c. Add edge# into bar#.
13	$1 + 2 + 10$	a. Copy bar# to edges. b. Double bar#. c. Add copy of edge# into bar#. d. Shift edge# up one digit. e. Add edge# into bar#.
14	$10 + 2^2$	a. Copy bar# to edges one digit up . b. Double bar#. c. Double bar#. d. Add edge# into bar#.

Multiplier	Expanded	Calculation plan for each iteration
15	$2^4 - 1$	a. Copy bar# to opposite edges. b. Double bar#. c. Double bar#. d. Double bar#. e. Double bar#. f. Add edge# into bar#.
16	2^4	a. Double bar#. b. Double bar#. c. Double bar#. d. Double bar#.
17	$2^4 + 1$	a. Copy bar# to edges. b. Double bar#. c. Double bar#. d. Double bar#. e. Double bar#. f. Add edge# into bar#.
18	$(10 - 1) * 2$	a. Copy bar# to opposite edges. b. Shift bar# up 1 digit. c. Add edge# into bar#. d. Double bar#.
19	$10 * 2 - 1$	a. Copy bar# to opposite edges. b. Shift bar# up one digit. c. Double bar#. d. Add edge# into bar#.
20	$10 * 2$	a. Shift bar# up one digit. b. Double bar#.
21	$10 * 2 + 1$	a. Copy bar# to edges. b. Shift bar# up one digit. c. Double bar#. d. Add edge# into bar#.
22	$(10 + 1) * 2$	a. Copy bar# to edges one digit up . b. Add edge# into bar#. c. Double bar#.
23	$(1 + 10) * 2 + 1$	a. Copy bar# to edges. b. Shift bar# up one digit. c. Add copy of edge# into bar#. d. Double bar# e. Add edge# into bar#.
24	$(2 + 1) * 2^3$	a. Copy bar# to edges. b. Double bar#. c. Add edge# into bar#. d. Double bar#. e. Double bar#. f. Double bar#.

Multiplier	Expanded	Calculation plan for each iteration
25	$(2 + 1) * 2^3 + 1$	a. Copy bar# to edges. b. Double bar#. c. Add copy of edge# into bar#. d. Double bar#. e. Double bar#. f. Double bar#. g. Add edge# into bar#.
26	$13 * 2 = ((1 + 2) + 10) * 2$	a. Copy bar# to edges. b. Double bar#. c. Add copy of edge# into bar#. d. Shift edge# up one digit. e. Add edge# into bar#. f. Double bar#.
27	$9 * 3 = (10 - 1) * (1 + 2)$	a. Copy bar# to opposite edges. b. Shift bar# up 1 digit. c. Add edge# into bar#. d. Copy bar# to edges. e. Double bar#. f. Add edge# into bar#.
28	$14 * 2 = (10 + 2^2) * 2$	a. Copy bar# to edges one digit up . b. Double bar#. c. Double bar#. d. Add edge# into bar#. e. Double bar#.
29	$30 - 1 = (3 * 10) - 1 = ((1 + 2) * 10) - 1$	a. Copy bar# to edges. b. Double bar#. c. Add copy of edge# into bar#. d. Shift bar# one digit up. e. Negate edge# f. Add edge# into bar#.
30	$3 * 10 = (1 + 2) * 10$	a. Copy bar# to edges. b. Double bar#. c. Add edge# into bar#. d. Shift bar# one digit up.
31	$30 + 1 = (3 * 10) + 1 = ((1 + 2) * 10) + 1$	a. Copy bar# to edges. b. Double bar#. c. Add copy of edge# into bar#. d. Shift bar# one digit up. e. Add edge# into bar#.
32	2^5	a. Double bar#. b. Double bar#. c. Double bar#. d. Double bar#. e. Double bar#.