Topic	**Tapering of windowed time series**
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Version	February 2012; **DOI: 10.2312/GFZ.NMSOP-2_IS_14.1**

As described in Chapter 14, a largely used method of seismic data analysis, which is often applied to study geophysical processes, consists in the spectral analysis of seismic signals after having carried out a Fourier transform of the time series.

However, for the analysis of noise data the recordings of some tens of minutes are usually subdivided into much shorter time windows (usually 30 s or 60 s). For earthquake recordings the spectral estimation of specific phases within a seismogram, particular those recorded at local and regional distances, can be difficult due to the difficulties in isolating a particular phase. On the contrary, mathematical principles require infinite long time series for performing a Fourier transform and, hence, such windowing will cause the Fourier transform to develop non-zero values predominantly at lower frequencies (commonly called spectral leakage, i.e. some frequencies tend to leak to other frequencies).

Therefore, it is standard practice to multiply the data windows by a taper before performing a Fourier transform. The taper consist in a function smoothly decaying to zero near the ends of each window, aimed at minimizing the effect of the discontinuity between the beginning and end of the time series. Although spectral leakage cannot be prevented it can be significantly reduced by changing the shape of the taper function in a way to minimize strong discontinuities close to the window edges.

Different types of tapers are shown in Figure 1. In seismic data analysis cosine tapers are often used, being both effective and easy to calculate, though bell-shaped and triangle functions are sometimes applied, too. In a mathematical form, the cosine taper can be written as

$$c(t) = \begin{cases} \frac{1}{2}\left(1 - \cos\frac{\pi}{a}t\right) & \text{for } 0 \leq t \leq a \\ 1 & \text{for } a \leq t \leq (1-a) \\ \frac{1}{2}\left(1 - \cos\frac{\pi}{a}(1-t)\right) & \text{for } (1-a) \leq t \leq 1 \end{cases}$$

with time t and taper ratio a. The cosine window represents an attempt to smoothly set the data to zero at the boundaries while not significantly reducing the level of the windowed transform.

Such tapering will lead to the following effects: It reduces the leakage of spectral power from a spectral peak to frequencies far away and it coarsens the spectral resolution by a factor $1 / (1 - a)$ for the above cosine tapers.

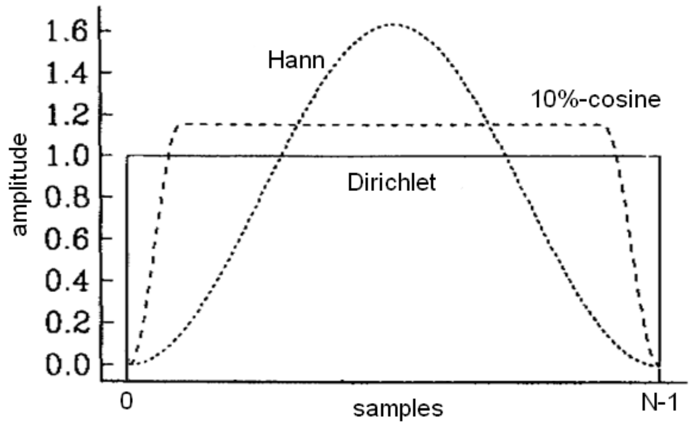

Figure 1 Comparison between the Dirichlet sharp-edged rectangle window function with two common taper functions.

The result of applying the cosine taper family of windows with a length of 30 s is presented in Figure 2. As the taper ratio *a* is decreased, or equivalently, as the shape of the window becomes increasingly rectangular, the bandwidth of the Fourier transform decreases. Therefore, a decreased taper ratio will increase the amount of leakage. As can be seen in Figure 2 (a), spectra from the untapered time series show a significant overestimation in spectral energy for low frequencies (leakage), resulting in a much lower H/V amplitude. Below 0.5 Hz the spectra for all three components show the same level of amplitude, limiting the effective bandwidth of the spectra when no tapering is applied.

Conversely, as the taper ratio is increased or the window shape becomes more like that of a 'cosine bell', the bandwidth increases, reducing the amount of leakage and giving rise to an increase in the resolution of closely-spaced low-frequency spectral components. The amplitudes of the Fourier spectra are significantly reduced, resulting in a clear H/V peak and a decrease of the spectral components towards lower frequencies.

Therefore, if the real spectrum contains a strong low-frequency component, cutting the time series into short time windows without tapering may strongly distort the observed spectrum. However, one should keep in mind that too much tapering may seriously impair the statistical efficiency of the estimates. In general, it is difficult to give precise recommendations which tapers to use in any specific situation (see, e.g., the discussion in Bingham et al., 1967). Yet, as these authors could show, it is recommendable to reduce the leakage more efficiently by increasing the length N of the time window and at the same time decreasing the taper ratio *a*, such that the length (N*a*) of the cosine half-bells is kept constant.

Usually, a value of *a* = 5% for window lengths of 30 s or 60 s is good enough for frequencies down to 0.2 Hz, whereas a further increase of the taper ratio *a* up to *a* = 20% does not imply further changes in the spectra (see Figure 2).

For further readings, we refer to Harris (1978), Oppenheim et al. (1999) and Park et al. (1987).

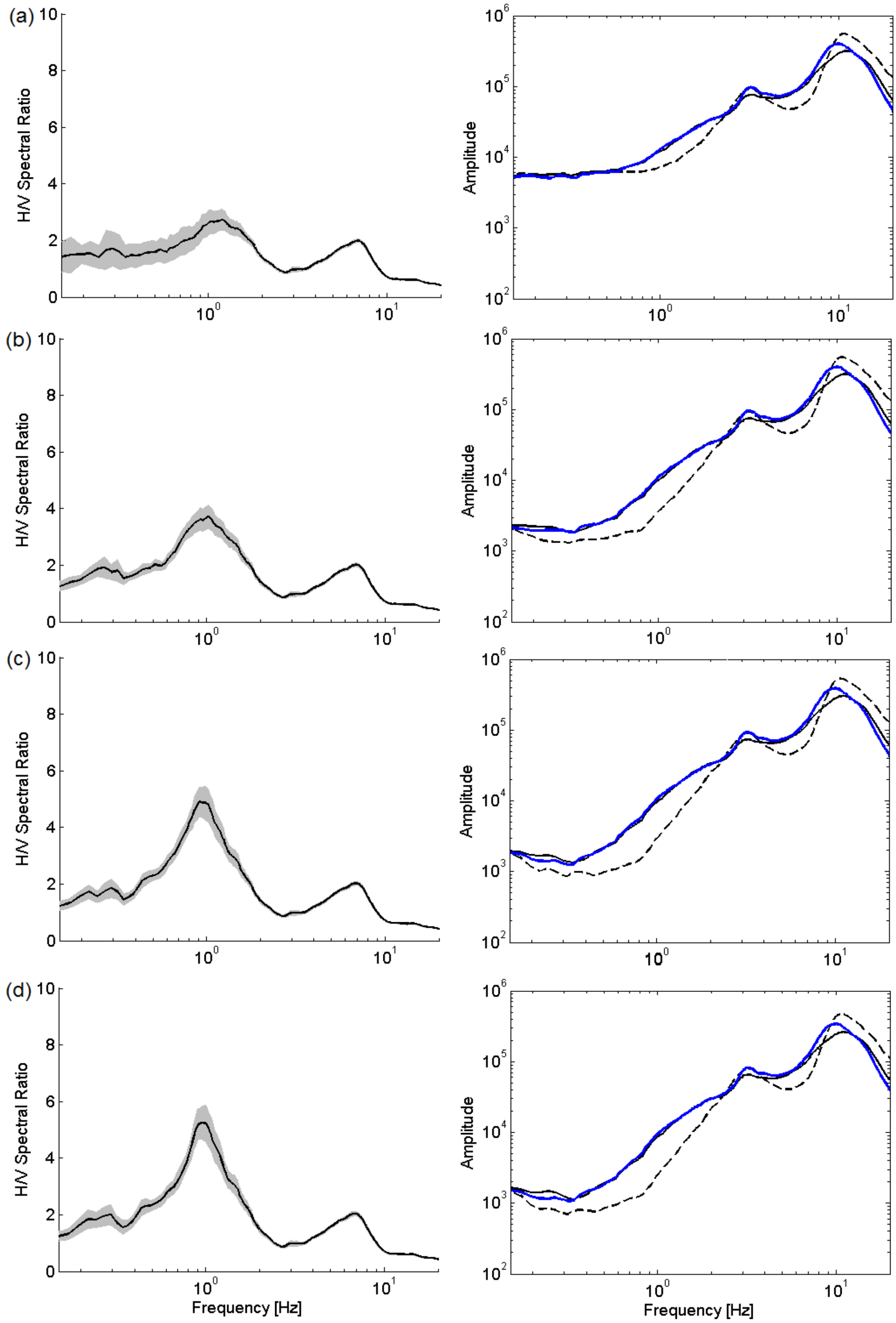

Figure 2 Effect of different cosine tapering for the same data set using a window length of 30 s. **Left column:** Thick lines show medium H/V spectral ratio plus/minus one standard deviation using (a) no tapering or cosine tapering with (b) a = 1%, (c) a = 5%, (d) a = 20%. **Right column:** The corresponding Fourier spectra for both horizontal (continuous black and blue lines) and vertical (dashed line) components.

References

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