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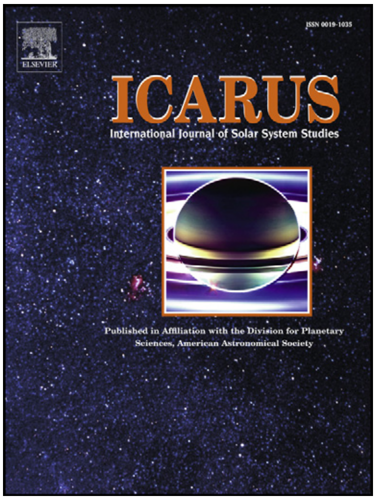

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Dynamical evolution of Earth's quasi-satellites: 2004 GU_9 and 2006 FV_{35}

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Abstract

We study the dynamical evolution of asteroids (164207) 2004 GU_9 and 2006 FV_{35}, which are currently Earth quasi-satellites (QS). Our analysis is based on numerical computation of their orbits, and we also applied the theory of co-orbital motion developed in Wajer (2009) to describe and analyze the objects' dynamics. 2004 GU_9 stays as an Earth QS for about a thousand years. In the present epoch it is in the middle of its stay in this regime. After leaving the QS orbit near 2600 this asteroid will move inside the Earth's co-orbital region on a regular horseshoe (HS) orbit for a few thousand years. Later, either HS-QS or HS-P transitions are possible, where P means "passing". Although 2004 GU_9 moves primarily under the influence of the Sun and Earth, Venus plays a significant role in destabilizing the object's orbit. Our analysis showed that the guiding center of 2006 FV_{35} moves deep inside the averaged potential well, and since the asteroid's argument of perihelion precesses at a rate of approximately $\dot{\omega} \approx -0.002°$/year, it prevents the QS state begin left for a long period of time; consequently the asteroid has occupied this state for about 10^4 years and will stay in this orbit for about 800 more years. Near 2800 the asteroid's close approach with Venus will cause it to exit the QS state, but probably it will still be moving inside the Earth's co-orbital region and will experience transitions between HS, TP (tadpole) and P types of motion.

Key Words: Asteroids, Dynamics

1 Introduction

Asteroids that are in the 1:1 mean motion resonance can be classified according to librational properties of the principal resonant angle, $\sigma = \lambda - \lambda_p$, where λ and λ_p are the mean longitude of the asteroid and the planet respectively. In the case of tadpole orbits the principal resonant angle librates around $\pm 60^\circ$, but for eccentric TP orbits these libration centers are displaced with respect to the equilateral locations at $\pm 60^\circ$ (Namouni and Murray, 2000). Horseshoe orbits are associated with librations of σ around 180° and the principal resonant angle of retrograde satellite orbits librates around 0°. These orbits, recently known as quasi-satellite orbits (Lidov and Vashkov'yak, 1994; Mikkola and Innanen, 1997), were predicted by Jackson in 1913 (Jackson, 1913) and correspond to the Henon "f-family" in the restricted three-body problem (Henon, 1969). Quasi-satellites move outside of the planet's Hill sphere at the mean distance from the associated planet of the order of $\mathcal{O}(e)$, where e is the eccentricity of the object. For sufficiently large values of eccentricity and/or high enough inclination transitions between QS and HS (or TP) orbits are possible, and there can exist compound orbits which are unions of the HS (or TP) and QS orbits (Namouni, 1999; Namouni *et al.*, 1999; Christou, 2000; Brasser *et al.*, 2004a)

So far quasi-satellites have been found for Venus, Earth and Jupiter. Venus currently has one temporary quasi-satellite object 2002 VE$_{68}$ (Mikkola *et al.*, 2004) and also one compound HS-QS orbiter (Brasser *et al.*, 2004a). The asteroid 2003 YN$_{107}$ was a QS of the Earth in the years 1996–2006 (Connors *et al.*, 2004). Also, as was shown by Connors *et al.* (2002), another Earth companion asteroid, 2002 AA$_{29}$, which moves on an HS orbit, in the future will be a QS of the Earth for several decades. Moreover, several objects which move (or will be moving in the future) on compound HS-QS and TP-QS orbits were recognized inside the Earth's co-orbital region[1] (see eg. Wiegert *et al.* (1998); Namouni *et al.* (1999); Christou (2000); Brasser *et al.* (2004a); Wajer (2008b)). Kinoshita and Nakai (2007) found that Jupiter has four quasi-satellites at present: two asteroids, 2001 QQ_{199} and 2004 AE_9, as well as two comets, P/2002 AR_2 LINEAR and P/2003 WC_7 LINEAR-CATALINA. Although a quasi-satellite has not been found for Saturn, Uranus and Neptune, Wiegert *et al.* (2000) investigated numerically the stability of test particles which move on quasi-satellite orbits around these giant planets. They concluded that quasi-satellites can exist around Saturn for times of $< 10^5$ years. Uranus and Neptune can possess primordial clouds of quasi-satellites (for times up to 10^9 years), although at low inclinations relative to their accompanying planet and over a restricted range of heliocentric eccentricities.

There are two confirmed objects which at present are quasi-satellites of the Earth: (164207) 2004 GU_9 (hereafter 2004 GU_9) and 2006 FV_{35} (Mikkola *et al.*, 2006; Stacey and Connors, 2009). In this paper we analyze the dynamical evolution of these asteroids. They are temporarily in the QS state. The first object has been in this regime for about 500 years. The time when the second asteroid transited into the QS state is unclear; this object has been a QS probably for over 10^4 years. We use a numerical method to discuss their orbital characteristics as well as the analytical method described in Wajer (2009) to

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[1] The co-orbital region is defined as $|a - a_p| \leq \epsilon$, where ϵ is the radius of the Hill sphere, and a and a_p are the object's and planet's orbital semimajor axes respectively. Objects which move in the co-orbital region are termed co-orbital objects (Namouni, 1999).

better understand the dynamics of these quasi-satellites.

Throughout this paper we use the following notations and conventions. As in the theoretical analysis described in the previous paper (Wajer, 2009) we assume that the orbit of a planet is circular with $a_p = 1$ AU and its mass, m_p, is equal to that of the Earth, r and r_p are the vector positions of the small body relative to the Sun and the planet, k is the Gaussian gravitational constant and the mass of the Sun equals 1. We use the set $(a, e, i, \omega, \Omega, M)$ as the osculating elements of the semi-major axis, eccentricity, inclination, argument of perihelion, longitude of ascending node, and mean anomaly of the asteroid orbit. Following the notation used before, unsubscribed quantities refer to the asteroid and the quantities with subscript p refer to the planet.

We say that an orbit of the asteroid is predictable within a time interval if the following properties are satisfied in this interval:

(1) The asteroid's nominal orbit as well as orbits of all considered virtual asteroids (VAs) [2] move in the same type of co-orbital motion;

(2) Difference between the Keplerian orbital elements a, e and i of an arbitrary VA and the nominal orbit of the object are very small compared to the orbital element of the nominal orbit, e.g. in the case of semimajor axis we must have $|a - a_0| \ll a_0$, where a and a_0 are the semimajor axis of the VA and the nominal orbit respectively. In case of the angular parameters ω, Ω and M the following inequality, e.g. for ω, $\min(|\omega - \omega_0|, 360^\circ - |\omega - \omega_0|) \ll \omega_0$ must hold. [3]

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[2] A swarm of fictitious asteroids with slightly different orbits all compatible with the observations.

[3] In case of ω, Ω and M, the values 0° and 360° are equivalent. In order to fix this ambiguity we should take the smallest value of $|\omega - \omega_0|$ and $360^\circ - |\omega - \omega_0|$. For

otherwise, we say that the orbit is unpredictable.

2 Observational material and method of numerical integration

The positional observations as well as physical information of 2004 GU_9 and 2006 FV_{35} were taken from the NeoDys pages[4] . The asteroids' orbit computations were done using the recurrent power series (RPS) method (Hadjifotinou and Gousidou-Koutita, 1998) for ten thousand years forward and backward. All eight planets, the Moon and Pluto were included in our integrations. When we studied the motion of 2004 GU_9 we used 125 positional observations covering almost a 8-year observational interval, and in the case of 2006 FV_{35} – 59 observations that cover a 14-year observational interval were taken into account.

For the purpose of this work, the orbits of the analyzed asteroids were cloned. A cluster of 100 VAs was randomly generated using Sitarski's orbital program package (Sitarski, 1998) which allows to create an arbitrary number of initial orbital element sets, fitting the observations within statistical uncertainties. The derived sample of VAs follows a Gaussian distribution in the 6-dimensional space of orbital elements. The osculating elements of the nominal asteroid orbit as well as the $1-\sigma$ uncertainties as fitted to the observations using Sitarski's package and used to generate the clone orbits are given in Table 1. [Table 1]

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inclination we have by definition $0 \leq i \leq 180°$ and both the values of $i = 0$ and $i = 180°$ represent different types of orbits (prograde and retrograde). It follows that in this case the definition $|i - i_0| \ll i_0$ works.

[4] http://unicorn.eis.uva.es/neodys/

3 Analysis of the results

3.1 Theoretical background

Previously, we developed, in the framework of the restricted three-body problem (CRTBP), an analytical method that allows one to identify and analyze the type of co-orbital motion for arbitrary values of eccentricity and inclination of the asteroid's orbit (Wajer, 2009). Below we briefly describe and summarize the results that have been employed in this paper.

Orbits of objects in 1 : 1 mean motion resonance can be decomposed into a slow guiding center motion described by the variables $\Delta a = a - a_p$ and σ, with a superimposed short period three-dimensional epicyclic motion viewed in the frame co-rotating with the Earth. By averaging the disturbing function:

$$\mathcal{R} = k^2 m_p \Big(\frac{1}{|\mathbf{r}_p - \mathbf{r}|} - \frac{\mathbf{r}_p \cdot \mathbf{r}}{r_p^3} \Big), \tag{1}$$

with respect to the fast angle λ_p it is possible to obtain the first integral that entirely determines the shape of the asteroid's guiding center trajectory (Brasser *et al.*, 2004b):

$$\bar{\mathcal{R}}(\sigma) + k^2 (\sqrt{(1+m_p)a} + \frac{1}{2a}) = const, \tag{2}$$

which, in the co-orbital region, can be written in the approximate form (Wajer, 2009):

$$(\Delta a)^2 = \frac{8 m_p}{3} (C - R(\sigma)), \tag{3}$$

where C is a constant, and $R(\sigma) = \bar{\mathcal{R}}(\sigma)/(k^2 m_p)$. Averaging with respect to the fast angle is justified due to regularity of e, i and ω of the asteroid. The regularity breaks down if the object remains close to the planet or in the vicinity of the separatrix (Namouni, 1999; Namouni *et al.*, 1999).

Examples of guiding center trajectories described by Eq. 3 in a $(\sigma, \Delta a)$ plane are shown in Fig. 1. For simplicity we assumed that the eccentricity, the inclination and the argument of the perihelion are constant in time, since the periods of their variations are significantly longer than the period of the principal resonant angle. The types of orbits for the 1:1 mean motion resonance are defined by the value of C and the libration centers, i.e. the specific values of σ around which the librations exist. These are determined by the minima of $R(\sigma)$. For example, orbits with values of C smaller than both R_- and R_+ (as well as $\sigma_- < \sigma < \sigma_+$) are QS, where the symbols R_+ and R_+ represents the maximum values of $R(\sigma)$ at σ_+ and σ_- respectively (see Fig. 1). In a similar manner, one can obtain the conditions for existence of other types of co-orbital motion. Finally, if we take into account secular changes of e, i and ω we will be able to explain the transitions between different types of orbits (see Wajer 2009 and references therein). [Fig. 1]

From the dynamical point of view variation of the asteroid's argument of perihelion is crucial because the values of extrema of the function $R(\sigma)$, which define possible types of co-orbital motion as well as transition conditions, strongly depend on ω (Namouni 1999, Christou 2000). In the case of QS motion Mikkola *et al.* (2006) found that permanently stable QS orbits can exist only for small enough inclination, i.e. $i < |e - e_p|$, otherwise transitions to other types of co-orbital motion can take place.

We analyzed the dynamical behavior of 2004 GU$_9$ and 2006 FV$_{35}$ by comparing the value of C with the two extrema R_+ and R_- of the function $R(\sigma)$. In our analysis the values and location of the extrema of $R(\sigma)$ as well as the value of C were obtained from the set of values of the orbital osculating elements of the asteroids calculated for every ten days.

3.2 Asteroid 2004 GU$_9$

3.2.1 Orbital behavior and stability

Asteroid 2004 GU$_9$ was discovered on April 13, 2004, at which time the apparent magnitude was 17.9. The asteroid, which is currently an Apollo type object, has a diameter of approximately 170 m–380 m and absolute visual magnitude of ∼21. Table 1 gives the nominal orbital elements of this object.

The orbital characteristics of 2004 GU$_9$ are illustrated in Fig. 2 by presentation of its orbit in two reference frames: non-rotating (Fig. 2a) and co-rotating with the Earth (Fig. 2b). The difference between the shape of the asteroid's and the Earth's orbit, as a result of the value of its eccentricity ($e = 0.137$) and inclination ($i = 13.6°$), is clearly seen. This asteroid has been a QS of the Earth for about 600 years and is near the halfway point of its time in this state (Mikkola *et al.*, 2006). It moves deep inside the Earth's co-orbital region ($|\Delta a| \leq 0.15\epsilon$) and its guiding center librates around $\sigma = 0°$ with amplitude 8°–10° and a period of about 70 years. Our numerical calculations show that during the QS regime its distance from the Earth remains beyond 0.11 AU.

[Fig. 2]

After leaving the present QS orbit in about 2500 the asteroid will move inside the Earth's co-orbital region ($|\Delta a| \leq 0.44\epsilon$) on a regular HS orbit with a period of about 350 years. Moreover, we determined that this HS phase will probably last up to 7600. After that time 90% of random VAs including the nominal orbit of the asteroid will transit to the QS state. The remaining 10% of VAs will still move in the HS orbit. We found that in the past, in contrast to Mikkola *et al.* (2006), 2004 GU$_9$ also experienced an HS-QS transition near 1100. According to our analysis, during the years 1100–1476, the object executed one compound HS-QS loop as shown in Fig. 3. The Mikkola's paper can not draw conditions of the initial values of the asteroids as well as of the method of numerical integration of orbits thus it is difficult to find the sources of the difference. [Fig. 3]

Our calculations show that the orbit of 2004 GU$_9$ is predictable within the time interval from 900 to 7600, while outside this time dynamical evolution starts to be unpredictable. We found that during the time span of our integrations [-12000; 8000] the nominal orbit and all considered VA orbits stay inside the Earth's co-orbital region and experience several HS-QS and HS-P transitions. Although the asteroid's eccentricity is not large enough to cross Venus' orbit, Venus seems to play a significant role in destabilizing the object's motion. When we excluded Venus from our numerical integrations the orbit of the asteroid is predictable within the assumed time of integration.

3.2.2 Current quasi-satellite phase

The dynamics of temporary capture of 2004 GU$_9$ into the QS state is quite similar to that of asteroid 2002 AA$_{29}$ (see Wajer 2009). The main difference is that both HS and QS phases last about ∼10 times longer than in the case of 2002 AA$_{29}$.

Fig. 4a shows the time evolution of R_+ and R_-, as well as C for this asteroid.

As we can see from the figure, up to about the year 1500, $C \approx R_-$. This implies that the asteroid's orbit was unstable, i.e. is sensitive to small perturbations [5]. We found that about 1250, when 2004 GU$_9$ moves in an HS orbit, the values of $R_\pm$ decrease until $\sim$ 1500. At that time $C > R_- = 2.69$, as shown in Fig. 4a; therefore there appears the possibility to transfer from the HS to the QS state. During the HS-QS transition $\omega \simeq 322°$ and the argument of perihelion starts to decrease at a rate of $\dot{\omega} \approx -0.9°/\text{year}$. Hence, the values of both R_+ and R_- increase so that $C < R_-$, R_+, as you can see in Fig. 4a, this causes the object to be trapped into the QS state. The values of R_+ and R_- tend to infinity about the year 2039 and 2189 respectively. Afterward, both R_+ and R_- decrease and near 2585 the asteroid can leave the QS state because at that time $C > R_+$. Then 2004 GU$_9$ starts to move in an HS state. [Fig. 4]

We found that if the asteroid moves in an HS orbit, $\dot{\omega} > 0$, and if it is a QS, $\dot{\omega} < 0$, in accordance with theory (Namouni, 1999). This behavior of ω causes the cycle of HS-QS transitions to repeat itself.

3.3 Asteroid 2006 FV$_{35}$

Asteroid 2006 FV$_{35}$ was discovered on March 29, 2006. Its absolute magnitude is 21.60 and its diameter is about 140 m–320 m. This object is currently an Apollo type object with high eccentricity ($e = 0.377$) and small inclination ($i = 7.1°$). The large value of e causes it to cross the orbits of both Earth and Venus (Fig. 5a). However, as one can see in Figs. 5b and 5c, which show, in a co-rotating frame, one QS loop of this object during the years 2000–2200 and one epicyclic loop respectively, the asteroid's average distance in a single loop [Fig. 5]

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[5] Compare with Sect. 3.3 in Wajer (2009).

is quite far, approximately 0.64 AU from Earth. In the QS regime the asteroid librates with amplitude of principal resonant angle $\sigma = 25^\circ$ and a period of libration of about 200 years.

The asteroid's argument of perihelion precesses at a rate of approximately $\dot{\omega} \approx -0.002^\circ$/year. It follows that the values of $R_\pm$ are near constant in time, as shown in Fig. 6. In this figure, where we also plotted the time evolution of C and R_0, one can obtain the relation $R_0 < C \approx \frac{R_0+R_\pm}{2} < R_\pm$; thus, the averaged potential well barrier prevents the asteroid leaving the QS phase for a long period of time. It is worth noting that in the case of 2006 FV$_{35}$ the condition of permanent stability holds, i.e. $i = 0.24$ rad $< |e - e_p| = 0.36$ (see Sect. 3.1). However, because of the large value of its eccentricity the perturbations from planets can influence the stability of the asteroid's orbit, and in consequence the asteroid may leave the QS orbit. Near 2800 the asteroid experiences a close approach with Venus (the asteroid passes within 0.05 AU of the planet). It implies that the value of C rapidly grows so that $C > R_\pm$ and consequently the object can leave the current QS state. [Fig. 6]

Our statistical analysis shows that 2006 FV$_{35}$ stays in the QS state for at least 10000 years – 87% of VAs exhibit this behavior. After 2800, when this object experiences close approaches with Venus, the asteroid's orbit will still remain inside the Earth's co-orbital region and will transit between TP, HS and P types of orbits. Further results of the statistical analysis of 2006 FV$_{35}$ can be summarized as follows:

(1) Within the assumed time of integration, i.e. [-8000; 12000], the orbital eccentricity of all VA orbits has nearly the same value, 0.38 – 0.39.
(2) In the past evolution the inclination stays in the interval $6.7^\circ - 7.1^\circ$ for all

100 VAs. In the future, the value of i of all VAs increase to $8.3° - 12.6°$.

(3) During the simulation period the longitude of the ascending node slowly decreases at a rate of $-0.013°$/year to $-0.009°$/year.

4 Summary and discussion

We have analyzed the orbital behavior of the Earth's current quasi-satellites 2004 GU$_9$ and 2006 FV$_{35}$ numerically and applied the theory of co-orbital motion in order to better understand the dynamics as well as to obtain qualitative information about the stability of these objects. 2004 GU$_9$ stays as an Earth QS for about a thousand years. In the present epoch it is in the middle of its stay in this regime. After leaving the QS orbit near 2600 this asteroid will move inside the Earth's co-orbital region on a regular HS orbit up to 7600 at least. Later, either HS-QS or HS-P transitions are possible. We have determined that the averaged potential well barrier prevents 2006 FV$_{35}$ leaving the QS phase for a long period; consequently this asteroid has occupied its present QS orbit for about 10^4 years and will stay in this state for about 800 more years. Our calculations have shown that near 2800 the asteroid's close approach to Venus will cause it to leave the present QS state. However, probably it will still be moving inside the Earth's co-orbital region and will experience transitions between HS, TP and P types of motion.

In this work we have shown that the main cause of the transitions between different different regimes in the motion of the co-orbital asteroids 2004 GU$_9$ and 2006 FV$_{35}$ is secular change in the argument of perihelion, as predicted and demonstrated numerically by Namouni (1999) (see also Christou, 2000 and Brasser *et al.*, 2004a).

Namouni (1999) found, using Hill's approximation of CRTBP, that for planar quasi-satellite objects the pericenter precession frequency is $0 > \dot{\omega} \propto e^{-3}$, i.e. $\dot{\omega}$ decreases as e increases. Such behavior is observed in the case of known Earth quasi-satellites, as shown in Table 2. In this table are listed current, past and future Earth quasi-satellites: 2002 AA$_{29}$, 2003 YN$_{107}$, 2004 GU$_9$ and 2006 FV$_{35}$, time of QS episode (t_{QS}) as well as the average distance from the Earth ($\overline{r_{QS}}$), variation of eccentricity and rate of precession of ω when the object is a QS [6]. As we can see, more eccentric objects stay as QS for a longer time and their distance from the planet is approximately of the order $\sim 2e$ (in AU). If the eccentricity $e \sim 10^{-2}$, the object is a QS for a few dozen years, while $e \sim 0.1$ – it is likely to remain in this regime for a time longer than 10^2–10^3 years. For sufficiently small inclination (or large eccentricity in order to satisfy Mikkola's criterion of permanent stability presented in Sect. 3.1) long-lived QS can exist. However, we must bear in mind that for highly eccentric objects, such as 2006 FV$_{35}$, the evolution of these orbits seems to be mostly caused by close encounters with the terrestrial planets. On the other hand, objects with small values of eccentricity, such as 2003 YN$_{107}$ (Connors *et al.*, 2004), can experience close approaches with the Earth, generating instability in their orbits.

[Table 2]

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[6] In the case of the asteroids 2002 AA$_{29}$ and 2003 YN$_{107}$ data collected in the table concern the years 2580–2620 and 1996–2006 respectively, when the objects are in a QS state, and are taken from Wajer (2008a).

discussions as well as for providing me with a number of VAs. I also thank the referees, Martin Connors and an anonymous referee for their valuable discussions and helpful reviews.

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Table 1

Osculating orbital elements of current quasi-satellites of Earth with their 1-σ uncertainties. Epoch: March 25, 2010 (JD 2455280.5), Equinox: J2000.0.

Object	a [AU]	e	$i[^\circ]$	$\Omega[^\circ]$	$\omega[^\circ]$	$M[^\circ]$	arc. [yr]
2004 GU9	1.000916164	0.1363817	13.64919	38.79359	280.85323	228.41396	7.88
1-σ	$5 \cdot 10^{-9}$	$5 \cdot 10^{-7}$	$4 \cdot 10^{-5}$	$3 \cdot 10^{-5}$	$4 \cdot 10^{-5}$	$4 \cdot 10^{-5}$	
2006 FV35	1.00104573	0.377568	7.1021	179.5703	170.8739	209.7604	14.03
1-σ	$5 \cdot 10^{-8}$	$8 \cdot 10^{-6}$	$2 \cdot 10^{-4}$	$1 \cdot 10^{-4}$	$2 \cdot 10^{-4}$	$5 \cdot 10^{-4}$	

Table 2

Comparison of QS states of past, present and future transient QS Earth companions.

Object	t_{QS} [years]	$\dot{\omega}$ [°/year]	e	$\overline{r_{QS}}$ [AU]
2002 AA_{29}	~ 50	-2.2	$0.035 - 0.061$	0.14
2003 YN_{107}	~ 10	-9.7	$0.014 - 0.045$	0.07
2004 GU_9	$\sim 10^3$	-0.9	$0.134 - 0.160$	0.30
2006 FV_{35}	$> 10^4$	~ -0.002	$0.377 - 0.396$	0.64

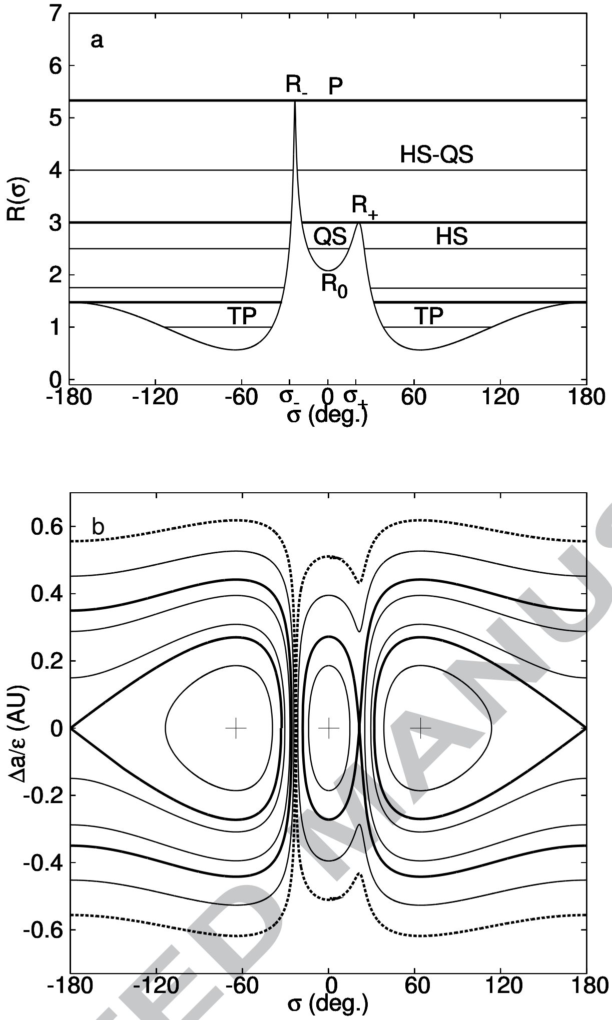

Fig. 1. (a) $R(\sigma)$ as a function of σ for $e = 0.2$, $i = 10°$, and $\omega = 75°$. TP, HS, QS and P denote respectively tadpole, horseshoe, quasi-satellite and passing orbits. Compound horseshoe and quasi-satellite orbits are denoted by HS-QS. The horizontal thin and thick lines correspond to the energy level (value of C). (b) Types of co-orbital orbits defined by $R(\sigma)$ for differential values of C. The separatrices are denoted by thick solid and thick dashed lines. The latter separate two different kinds of motion - librations and circulations.

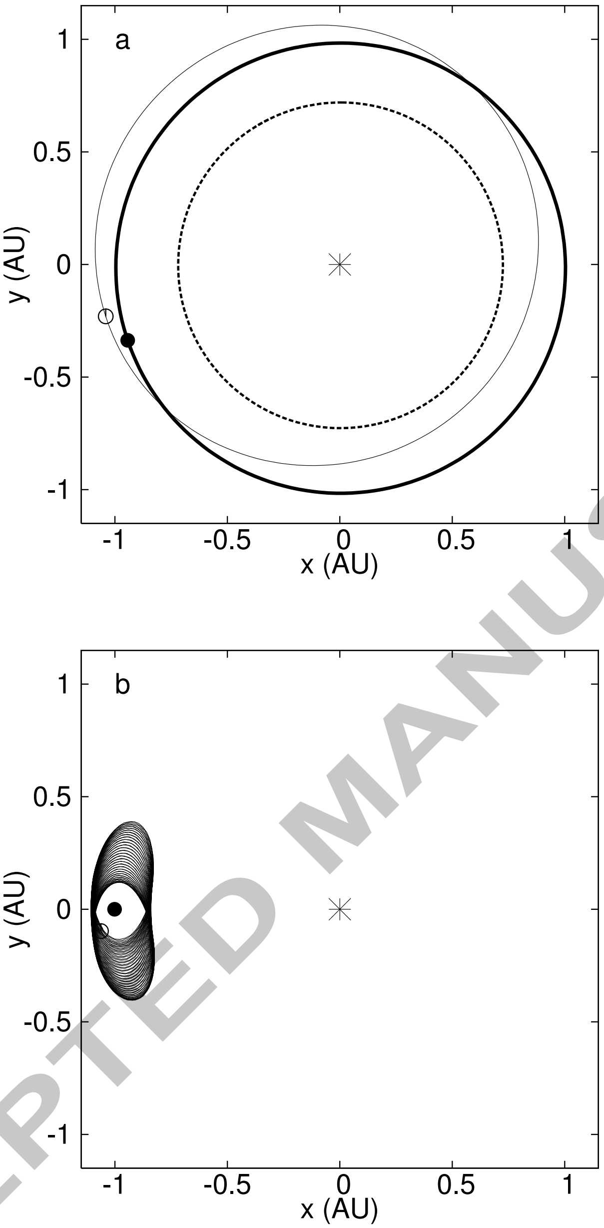

Fig. 2. Projection of the orbit of 2004 GU$_9$ onto the ecliptic plane: (a) Heliocentric orbit of 2004 GU$_9$ (thin line), Earth (thick line) and Venus (dashed line). (b) One QS loop in years 1954–2140 in the coordinate system corotating with Earth.

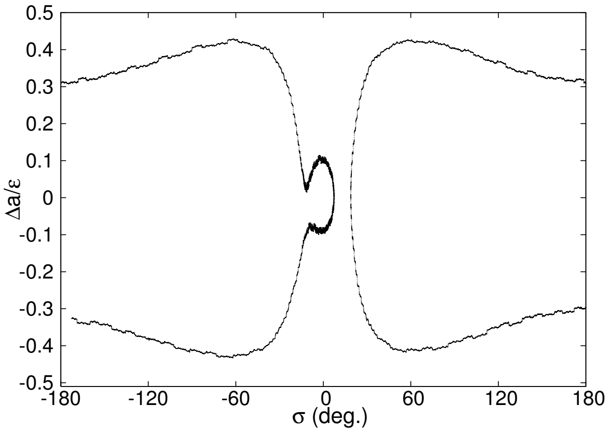

Fig. 3. One compound HS-QS loop of 2004 GU_9 during the years 1000–1476 viewed in a $(\Delta a, \sigma)$ plane.

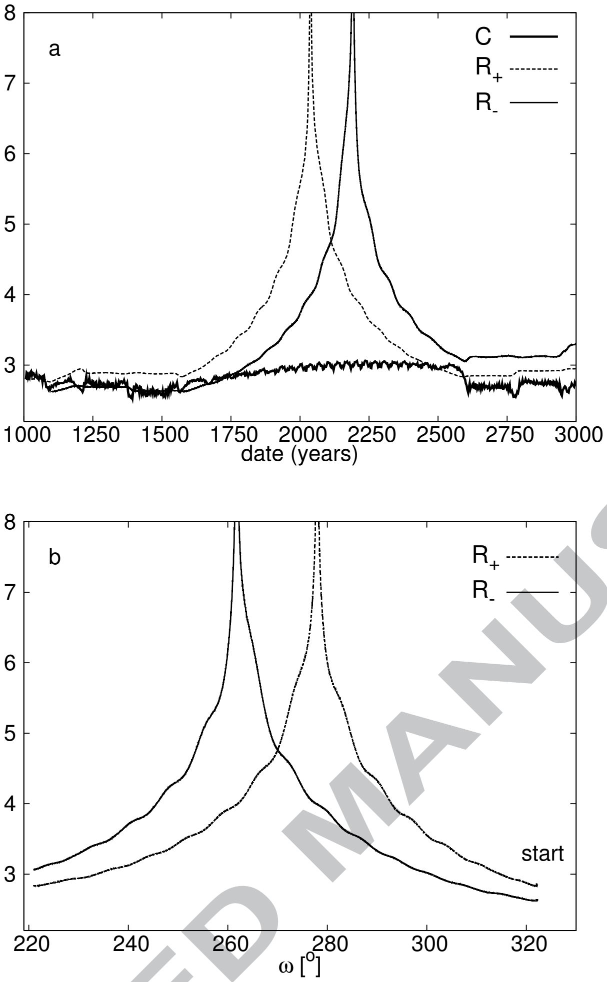

Fig. 4. (a) Time evolution of C (thick line), R_+ (dashed line) and R_- (thin line) for 2004 GU$_9$. (b) The maxima R_+ (dashed line) and R_- (thin line) as functions of the argument of pericenter during the QS state. The values of R_+ and R_- tend to infinity near 2039 and near 2189 respectively. In the former case we have $\omega = 277.8°$ and in the latter one $\omega = 261.7°$.

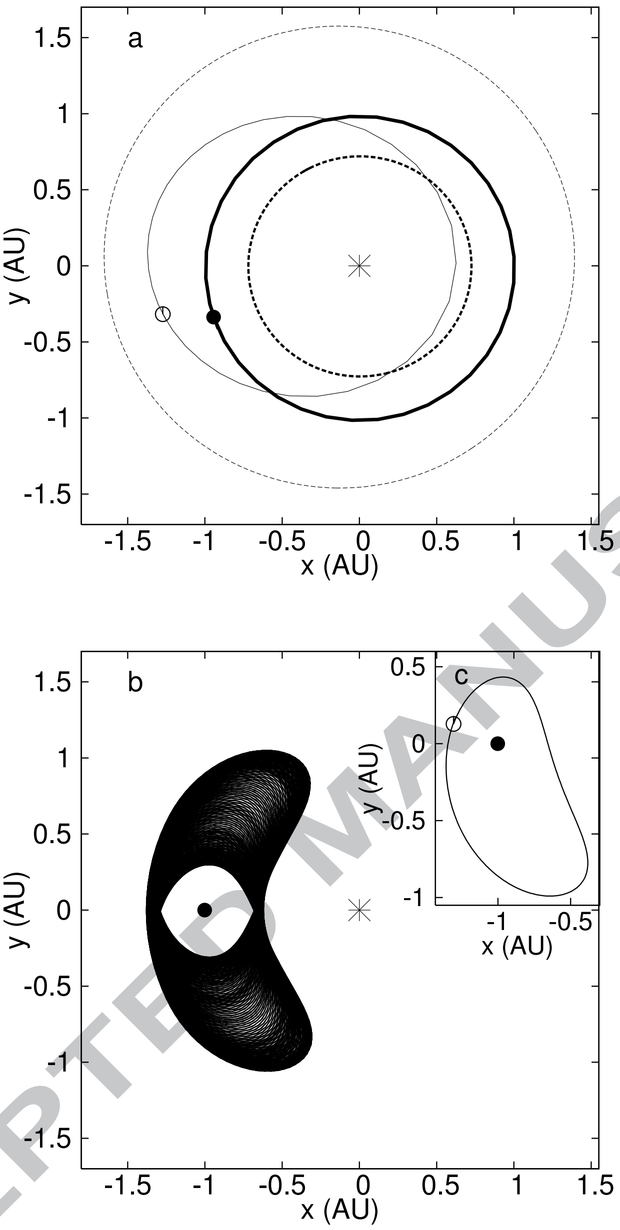

Fig. 5. Projection of the orbit of 2006 FV$_{35}$ onto the ecliptic plane: (a) Heliocentric orbit of 2006 FV$_{35}$ (thin line), Earth (thick line), Venus (thick dashed line) and Mars (thin dashed line). (b) One QS loop during the years 2000–2200 in the coordinate system corotating with Earth. (c) One epicyclic loop plotted from 2007 to 2008 in the frame revolving with Earth.

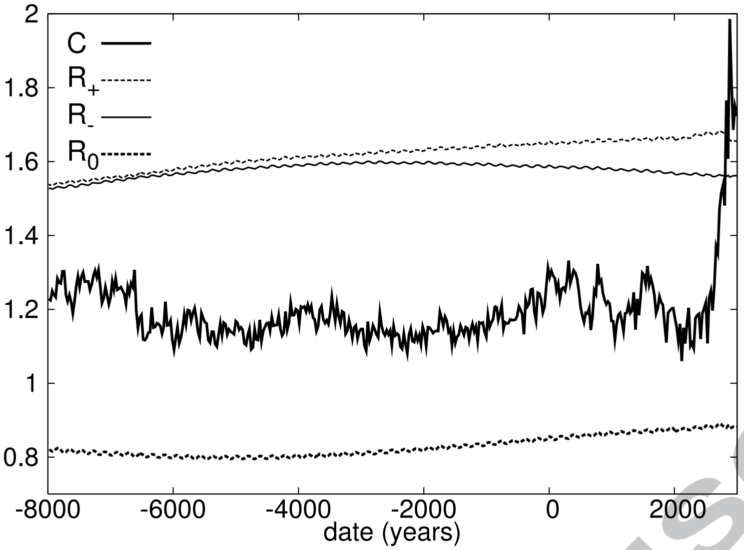

Fig. 6. Evolution of C (thick line), R_+ (dashed line), R_- (thin line) and R_0 (thick dashed line near bottom) for 2006 FV_{35}.