

Superstars in two-sided markets: exclusives or not?

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This article studies incentives for a premium provider (Superstar) to offer exclusive contracts to competing platforms mediating the interactions between consumers and firms. When platform competition is intense, more consumers affiliate with the platform favored by Superstar exclusivity. This mechanism is self-reinforcing as firms follow consumer decisions and some join the favored platform only. Exclusivity always benefits firms and might eventually benefit consumers. A vertical merger (platform-Superstar) makes non-exclusivity more likely than if the Superstar was independent. The analysis provides novel insights for managers and policymakers and it is robust to several variations and extensions.

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1. Introduction

With the advent of digitization, most markets moved online and turned into two (or multi) sided platforms. These platforms enable valuable interactions between different groups of agents. When platforms compete, an agent usually faces a trade-off between singlehoming and multihoming. On the one hand, multihoming allows an agent to interact with a large mass of agents on the other side. On the other hand, the platform is a bottleneck for single-homing agents. When these agents generate a significant value for the other side, singlehoming guarantees a better bargaining position vis-à-vis the platform.¹

As pointed out by Biglaiser et al. (2019), the current understanding of two-sided markets falls short of explaining the role of premium agents (marquee users). Being more attractive than other (atomistic) agents, premium agents are able to group consumers and induce switching behavior. In turn, this ability might grant the pivotal agent market power allowing for lucrative deals. Examples exist in several digital markets. The music industry is populated by few top-rated artists (*e.g.*, *Beyoncé*, *Taylor Swift*) and a long tail of smaller, often unknown, artists. Similar evidence is also shared by the market for apps (*e.g.*, *Angry Birds*), open-source software (*e.g.*, *Pivotal*, *Red Hat*), videogames (*e.g.*, *Fortnite*), top gamers (*e.g.*, *Ninja*), audio-books (*e.g.*, *Robert Caro*, *Jeffery Deaver*), or by the market for investors and peer-to-peer (P2P) payment networks (Markovich & Yehezkel 2018). Even in more traditional markets, such as the shopping mall industry, consumers might have a strong preference for anchor stores (*e.g.*, prestige and fashion stores, departmental stores). For simplicity, we call these premium agents “Superstars”.

In all these markets, one can observe common behaviors that deserve further understanding. First, Superstars make different contractual choices. As reported in Table 1, there are many examples of exclusive dealing (*e.g.*, windowed releases, radius clauses) and these decisions co-exist with the choice of other Superstars to multihome. Second, often a premium product is self-produced by one competing platform or integrated following an acquisition. A fundamental question arises on the rationale behind these choices and their impact on platform competition, welfare, and surplus distribution.

We develop a general yet tractable model with two competing platforms acting as intermediaries between consumers and firms. Unlike previous studies dealing with exclusive contracts (Hagiu 2006, Hagiu & Lee 2011), we introduce heterogeneity in bargaining power on the firm side of the market. This side is composed of a fringe of atomistic firms and a Superstar who acts as a monopolist supplier of her product. We explore two variations of this model. When the platform and the Superstar are vertically separated, the decision of the Superstar is between offering her product to one (exclusive contract) or both platforms (non-exclusive contract). In the same vein, when the platform and the Superstar behave as a merged entity, the decision becomes whether (or not) to give access to the product also to the rival platform.

In order to understand the ability of exclusive dealing to reshape platform competition, we focus on an ex-ante symmetric market configuration.² A non-exclusive contract is neutral to the market competition: platforms are symmetric, all firms on the fringe multihome, and the consumer market is equally split between the two platforms. On the contrary, exclusive dealing induces demand asymmetries and externalities which are, then, further capitalized

¹For a textbook appraisal, see Belleflamme & Peitz (2015, 2018).

²In markets with network externalities, a coordination problem typically leads to a multiplicity of equilibria (*e.g.*, Caillaud & Jullien 2003, Hagiu 2006, Jullien 2011). We discuss this issue in Section 5.1.

Market	Exclusives	Type	Vertical integration
Music on-demand	Drake, F. Ocean on Tidal; Rihanna, Beyoncé on Apple; Taylor Swift on Spotify	Full or “windowed release”	Spotify acquired Gimlet and Parcast
Gaming	Spider Man, Gran Turismo Sport, The Last of Us, God of War on PS; Super Mario Odyssey and Pokemon: Sword and Shield on Switch; Fortnite on Epic Game Store	Console-specific, feature-specific, often limited in time	Historical feature of the industry (Lee 2013)
E-sport	Ninja (top gamer) left Twitch for Mixer	Exclusive streaming of games	No(t yet)
Audio books	Garzanti, Loganesi, “Originals”, Robert Caro, Jeffery Deaver, Michael Lewis on Amazon Audible; Bompiani on Storytel	Full	“Originals”
Apps	Bear, Timepage, Overcast on iOS; Steam Link, Tasker on Android	Full	Apple’s Arcade and Shazam, Google’s Suite
Shopping Malls	Anchor store	Often radius clauses	Departmental store
⋮	⋮	⋮	⋮

Table 1: Industry Background

by the Superstar when designing the contract. Specifically, exclusive dealing brings about a business stealing effect which is more intense when competition intensifies. This is because a large number of consumers will then affiliate with the platform *favored* by the presence of the Superstar. In equilibrium, this generates positive spillovers on the fringe firms, with two interesting effects. First, aggregate variety increases as more firms join the market and affiliate with the *favored* platform relative to when a non-exclusive contract is signed. In practice, some zerohomers become singlehomers. Second, some firms who were previously active on both platforms find it profitable to join only the *favored* platform. In practice, some multihomers turn into singlehomers.

Although business stealing emphasizes the gain from exclusivity, this choice requires the Superstar to give up a share of the potential customer base. This implies that the surplus extracted from the *favored* platform needs to be sufficiently large to compensate for the foregone revenues otherwise obtained under non-exclusivity. We find that the optimal choice ultimately depends on the fierceness of platform competition. When competition is sufficiently intense, a large mass of consumers and small firms would migrate to the *favored* platform, and then she extracts more surplus by exploiting the endogenous asymmetry in the market.

Under vertical integration, exclusivity becomes less likely than under vertical separation. Indeed, exclusivity puts downward pressure on prices, because the merged entity internalizes the benefit of the Superstar subsidiary and thus becomes less appealing. Under non-exclusivity, instead, being vertically integrated or separated does not change the platform competition.

A contractual arrangement featuring an exclusive deal is often deemed as anti-competitive. Typically, these concerns stand on the possibility of foreclosure or entry deterrence of efficient rivals arising in markets without network effects, as already discussed by previous pioneering studies (see *e.g.*, Rasmusen et al. 1991, Bernheim & Whinston 1998, Fumagalli & Motta 2006, *inter alia*). Our model shows that the presence of indirect network externalities may overturn the common conclusion in the one-sided literature that exclusivity is anti-competitive. The value creation stemming from exclusivity is intrinsically linked to the two-sidedness of the market, as the entry of new firms creates a surplus for consumers. As a result, when indirect network externalities are powerful, exclusivity may eventually lead to a scenario that is welfare-enhancing for both sides.

The variation of the model that features a merged entity (platform-Superstar) provides a framework to understand other potentially anti- or pro-competitive effects created by the recent stream of mergers and acquisitions in markets with network externalities (*e.g.*, RedHat-IBM, Pivotal-WMware, Gimlet-Spotify). There is extensive literature in one-sided markets discussing these effects (see Rey & Tirole 2007 for a review) and an overwhelming consensus on the increased post-merger incentive to foreclose a rival of an essential input. Our results suggest due diligence for antitrust enforcers when scrutinizing vertical mergers as overlooking the presence of network externalities might lead to an overestimation of the harm and, thus, excessive bans. In the article, we discuss these results in light of the European Commission’s guidelines on merger control and provide a comparison of the competitive outcome arising in markets with and without network externalities.

Finally, we demonstrate that our results hold when including multihoming consumers, the presence of platforms’ bargaining power, and when accounting for elastic demand.

The road map is as follows. In Section 2, we provide descriptive evidence of contractual arrangements in different industries and we survey the relevant and related literature. In Section 3, we present the preliminaries of the model. Section 4 studies the optimal contractual arrangements, the welfare impact of exclusive dealing, and the effect of a vertical merger. Section 5 presents a series of extensions and we discuss the generality of our setting alongside with the problem of coordination and multiplicity of equilibria. Section 6 discusses the main results and their policy relevance.

2. Background

2.1. Industry Background

In this section, we present circumstantial evidence of some industry practices which may feature and motivate our model. Although practices may differ on a case-by-case basis, these industries are all characterized by interactions between different sides of the market, network externalities, Superstars, exclusive dealing, and some degree of vertical integration.

Music on-demand industry. In the music streaming market, the global growth rate reached 34% in 2019 and accounted for almost half of music revenues (IFPI 2019). Since 2016, music has experienced an exclusive war. Starting with Apple Music and Tidal, several

artists signed exclusive contracts, often in the form of windowed release.³ Notable examples refer to Drake (*Views*, *Hotline Bling*, *Summer Sixteen*), Frank Ocean (*Blonde*, followed by his album *Endless*), Chance the Rapper (*Coloring Book*), and more recently PNL (with the *Deux Frères* album) on Apple Music, Kanye West (*The Life of Pablo*), Rihanna (*Anti*) or Beyoncé (*Lemonade* and *Die With You*) on Tidal. Revenues from exclusive deals can be highly lucrative, ranging from \$ 500,000 obtained by Chance The Rapper to \$ 20 millions of Drake, and equity stakes obtained by Rihanna, Kanye, Beyoncé.

Whereas these artists opted for exclusives, others continued to offer their records to their largest possible audience.⁴ In 2018, Spotify turned into exclusives as well (*e.g.*, with Taylor Swift's *Delicate* and the acoustic version of *Earth, Wind & Fire's September*) and, in 2019, struck a multi-year deal with Higher Ground Audio, a podcast production company, to produce a series of podcasts with Barack and Michelle Obama. Moreover, the industry features several cases of vertical integration (*e.g.*, Tidal was launched as an artist-owned streaming platform)⁵ and acquisitions (*e.g.*, Spotify acquired podcast producers Gimlet and Parcast).

Gaming industry. The gaming industry, which expects to hit \$300 billion by 2025,⁶ has been historically characterized by a large proportion of exclusive agreements, negotiations, and a high degree of vertical integration (Lee 2013). In this context, exclusivity may be console- or/and PC-specific, permanent or limited in time, or only related to some features of the videogame. In 2019, Epic Store, the gaming house producing the popular *Fortnite*, announced that “*store exclusives are the only way to improve Steam and the PC market*”. Thanks to that game, Epic Games Store was able to attract as much as 85 million users on the platform and additional exclusive developers due to generous revenue split (*e.g.*, *Metro Exodus*, initially planned to be released on Steam).⁷ In the same year, several small indie games, including *Ooblets*, were announced exclusively on that platform and an agreement was signed with Ubisoft, a major games publisher, on selected exclusive titles.

Most titles are also developed in-house as first-party content, *e.g.*, Epic's *Fortnite* was a publisher turned into a distributor. In the home console market, *MLB The Show 19*, *Gran Turismo Sport*, *The Last of Us*, *God of War*, amongst others, are developed by Sony and only

³Windowed releases represent a recent practice in the music industry market under which a content is released exclusively on a platform for a limited period.

⁴Lady Gaga expressed her strong opinion against exclusive contracts. The opposition against exclusive contracts also mounted on the platform side, with Spotify claiming in 2016 that Superstar exclusives were bad for artists, consumers, and platforms. See *e.g.*, RollingStone, October 5, 2016. 'How Apple Music, Tidal Exclusives Are Reshaping Music Industry': <http://www.rollingstone.com/music/news/inside-the-war-over-album-exclusives-w443385>. See also Digitalmusicnews, September 18, 2017. 'The Chinese Government Says Streaming Music Exclusives Suck' <https://www.digitalmusicnews.com/2017/09/18/sappprft-streaming-music-exclusives/>

⁵Current owners are Jay-Z, who also offers exclusive contents on the platform and the US mobile carrier Sprint. According to the platform's website, the artists-owners also include Alicia Keys, Arcade Fire's Win Butler and Regine Chassagne, Beyoncé, Calvin Harris, Coldplay's Chris Martin, Daft Punk, Damian Marley, deadmau5, Indochine, J. Cole, Jack White, Jason Aldean, Kanye West, Lil Wayne, Madonna, Nicki Minaj, Rihanna, T.I., and Usher.

⁶See Variety, 'Video Games Could Be a \$300 Billion Industry by 2025 (Report)' <https://variety.com/2019/gaming/news/video-games-300-billion-industry-2025-report-1203202672/>.

⁷See *e.g.*, The Verge "Epic Games Store chief says they'll eventually stop paying for exclusive PC games" <https://www.theverge.com/2019/3/21/18276181/epic-games-store-exclusives-pc-gaming-fortnite-steve-allison-gdc-2019>

available on Sony’s own console PlayStation (PS) 4. Nintendo released exclusively *Super Mario Odyssey* and *Pokemon: Sword and Shield* for its Switch. Third-party developers are mostly heterogeneous in their homing decisions, with some available exclusively on some consoles (e.g., Marvel’s *Spider Man* on PS), and others available non-exclusively (e.g., *Grand Theft Auto V* on Xbox and PS or Electronic Arts’s *FIFA 2019* on Xbox, Switch, and PS). Similar trends might be envisioned in the emerging cloud streaming market, with Google Stadia and Apple Arcade launched in 2019.

E-sport Market. This market is worth \$10.1 billion by the end of 2019 and consists of streaming live games or pre-recorded games. Two platforms (YouTube Gaming and Amazon’s Twitch) dominate the market, followed by fast-growing platforms such as Facebook Live and Microsoft’s Mixer (StreamLab 2018). The most played game is *Fortnite*. Platforms compete by attracting game streamers and users paying a monthly subscription fee to have access to the platform. In 2019, a significant change in the industry concerned the decision of the most followed player (with more than 14 million followers), Ninja, to leave Twitch for an exclusive contract with Mixer. Following Ninja’s decision, the number of downloads of the app increased by 650,000 in five days and several streamers were thought to follow Ninja’s move.⁸

Publishing Industry. In the publishing industry, audio-books are on the rise, with revenues growing by 24.5% and more than 44,685 titles published in the US in 2018 (APA 2019). Platforms such as Amazon’s Audible and Storytel charge consumers a fixed monthly fee for access to their audio-book catalog. More importantly, this market is characterized by several exclusive titles. For instance, Audible has an exclusive agreement with Italian publishers (e.g., *Garzanti*, *Loganesi*) and so Storytel (with Gruppo Giunti’s *Disney/Bompiani*). The former has also launched “Originals”, a series of exclusives produced in-house by the platform and narrated by celebrated storytellers. In the US, Audible struck a deal directly with some best-selling authors by-passing major publishers (e.g., *Robert Caro*, *Jeffery Deaver*, *Michael Lewis*)⁹ and Amazon’s own distribution channel, ACX, allows right-holders (e.g., authors, publishers) to distribute their rights exclusively to its network or non-exclusively also to other retailers.

Apps and Developers Industry. The app market is characterized by two dominant platforms, Apple iOS, and Android, which allow interactions between developers and users. Whereas most apps are available on both platforms, there are several others which are either exclusive on Apple iOS (e.g., *Bear*, *Timepage*, *Overcast*) or on Android (e.g., *Steam Link*, *Tasker*). Both platforms charge a fee to developers to get an account and publish their apps (e.g., Google charges a one-time fee, whereas Apple a yearly fee) but the former scrutinizes

⁸See TheVerge, ‘What is Mixer, Ninja’s new exclusive streaming home?’, August 1, 2019: <https://www.theverge.com/2019/8/1/20750432/mixer-ninja-microsoft-twitch-youtube-streaming-fortnite>. Also, see e.g. ‘Twitch Streamers React to Ninja’s Exclusive Move to Mixer’ <https://www.ign.com/articles/2019/08/01/twitch-streamers-react-to-ninjas-exclusive-move-to-mixer> and The Business Insider, ‘Ninja became the first Mixer streamer to reach 1 million subscribers, less than a week after announcing he was ditching Twitch for Microsoft’, August 7, 2019, <https://www.businessinsider.com/ninja-mixer-top-streamer-one-million-followers-2019-8?>

⁹See e.g., The New York Times, ‘Want to Read Michael Lewis’s Next Work? You’ll Be Able to Listen to It First’, June 2, 2019, <https://www.nytimes.com/2018/06/02/books/audible-michael-lewis-audiobooks.html>.

apps based on their content and safety. Developers can offer their apps for free and earn from in-app ads, ask for an upfront payment, or have in-app purchases features. In the latter two cases, the platform obtains a share.¹⁰ This market features a long tail of apps and few tops and best-sellers (*e.g.*, *Angry Birds*, *WhatsApp*) whose appearance might generate more entry in the market by similar apps (Ershov 2018) and act as discovery facilitators. Moreover, several apps are also built in-house or acquired by platforms, so featuring a certain degree of vertical integration (*e.g.*, *Apple's Arcade* and *Shazam*, *Google's Suite*).

Shopping Mall Industry. Shopping malls are an example of non-digital platforms characterized by externalities. Consumers decide to which mall to shop based on the number of retailers and their preferences (*e.g.*, distance), and retailers may sign an exclusive or non-exclusive contract with the mall. This market features the presence of anchor stores which can benefit from more favorable contractual terms. Previous research has shown that anchor stores generate demand externalities to non-anchor stores which experience higher sales (Pashigian & Gould 1998, Gould et al. 2005).

Moreover, exclusive dealing in the industry is common, and lease agreements often feature radius clauses, that is, contractual arrangements which prohibit the opening of the same shopping activity within a given distance (Lentzner 1977). The presence of contracts featuring radius might hinder market competition and, therefore, attracted the attention of several competition agencies (*e.g.*, the German Federal Cartel Office, the UK Competition and Markets Authority) and the Austrian Supreme Court.¹¹

2.2. Related Literature

Our article relates to several streams of industrial organization literature. Above all, it relates to a number of papers dealing with two-sided markets (Caillaud & Jullien 2003, Rochet & Tirole 2003, 2006, Armstrong 2006, Jullien 2011).

Traditionally, the literature of the two-sided markets considers exclusive contracts as a tool in the hands of platforms to manage the homing decisions of the two sides of the market. For instance, Armstrong & Wright (2007) show that when platforms offer an exclusive contract to some sellers, they do so by charging a prohibitively high price to multihomers and a discount to singlehomers. As a result, there is a partial (complete) foreclosure as all users on this side (both sides) would prefer to singlehome.¹²

Our article takes a novel perspective: it is an agent, the Superstar, and not the platform(s), that has bargaining power. The contractual design assumes the form of a take-it-or-leave-it (TIOLI) offer, which is equivalent to a second-price sealed auction with negative externalities, as in Jehiel & Moldovanu (2000). In their model, the optimal bid equals the difference in payoffs when winning and losing the auction. In our framework, the tariff optimally designed

¹⁰However, whereas exclusivity is common practice in this market, we are not currently aware of exclusive *contracts* between developers and platforms. Albeit these may arise in the future.

¹¹See *e.g.*, Kluwer Competition Law Blog, 'Property leases and competition law: Some clarity on restrictions in leases' <http://competitionlawblog.kluwercompetitionlaw.com/2015/12/08/property-leases-and-competition-law-some-clarity-on-restrictions-in-leases/>

¹²Recently, Belleflamme & Peitz (2019*b*) have shown that when platforms prefer to impose exclusivity to both sides of the market, at least one side is likely to be harmed, whereas allowing multihoming may accomplish the purpose of having all sides of the market and the platforms better off.

by the Superstar equals the difference in profits obtained by the platform when winning the contract and when only the rival wins the contract. A similar model is also used by Montes et al. (2019) to study data sales in the data broker industry. In their framework, only exclusive contracts between a data broker and a downstream firm arise. Differently, in ours, the presence of indirect network externalities for the Superstar renders a non-exclusive contract also attractive in equilibrium when market competition is not sufficiently intense. Hence, both exclusive and non-exclusive contracts can arise.

This article also adds to this literature by explicitly modeling heterogeneity in market power on one side of the market. Up until recently, most of the literature has considered markets populated by small agents whose decisions are not likely to stimulate enough switching behaviors (see e.g., discussion in Biglaiser et al. 2019). However, several industries show the presence of agents able to attract and coordinate consumers. The only study dealing with this issue is by Markovich & Yehezkel (2018), who present a model of platform competition, with direct rather than indirect externalities. The authors study how grouping users may facilitate the migration from a less efficient focal platform to a more efficient non-focal one. In our paper, it is the Superstar that, by joining exclusively one platform, may coordinate some users and, in turn, some fringe firms towards one platform.¹³

Contrary to the efficiency argument put forward by the “Chicago critique”, the most recent economic literature has highlighted how exclusivity might entail anti-competitive effects by deterring entry or leading to foreclosure of more efficient rivals (Aghion & Bolton 1987, Rasmusen et al. 1991, Fumagalli & Motta 2006, Abito & Wright 2008, Fumagalli et al. 2009, 2012). Similar anti-competitive practices can also arise in the presence of network externalities when an incumbent can make exclusive introductory deals and prevent more efficient platforms from entering the market (Doganoglu & Wright 2010) or in the presence of interlocking bilateral relationships between upstream and downstream firms (Nocke & Rey 2018). Nevertheless, exclusive dealing might also entail pro-competitive effects such as effort provision (Segal & Whinston 2000, De Meza & Selvaggi 2007) or entry deterrence by inefficient firms (Innes & Sexton 1994). In supporting this view, the empirical evidence from the videogame industry has shown that exclusive deals between platforms and videogame producers might help small platforms to enter the market and challenge the incumbent (Lee 2013). A major difference between our framework and the discussed studies is the presence of indirect network externalities, which amplifies the impact of exclusive dealing, generating, in equilibrium, a market entry on the fringe and a possible benefit for consumers.¹⁴ This is a new result linked to the two-sidedness of the market.¹⁵ If applying our model to a one-sided market, free of network

¹³In a different model, Ishihara & Oki (2017) consider platform competition in a market where a monopolist multi-product content provider decides on how much content to provide exclusively to each platform and how this affects its bargaining power relative to the platform(s).

¹⁴In our framework, this benefit stems from the fact that the Superstar and the firms on the fringe do not compete with each other. This is a credible assumption if sellers are enough differentiated, as frequently arising in the shopping mall industry, in the market for videogames or artists. For instance, Lee (2013) found that the video game industry features poor substitutability of videogames and the presence of a handful number of Superstar titles which can impact console sales by as much as 5%. For a model with competing firms on the seller side, see e.g., Belleflamme & Toulemonde (2009), Belleflamme & Peitz (2019a), Karle et al. (2019).

¹⁵This is partly similar to what presented by Kourandi et al. (2015), who study the contractual decision made by Internet Service Providers to content providers. Differently from us, they show that exclusivity can be welfare enhancing when the competition of content providers over informative ads is sufficiently intense.

externalities, exclusivity would always emerge, causing harm to consumers.¹⁶

Further, our article makes several contributions to the literature on vertical integration in two-sided markets. As discussed, the presence of first-party content, in-house production, and several acquisitions have rendered platforms vertically integrated. Starting from similar motivating examples, Pouyet & Trégouët (2018) focus on the impact that vertical integration in two-sided markets may have on competition, showing that the relative size of the indirect network externalities is key to assess the pro- or anti-competitiveness of a vertical merger. D’Annunzio (2017) presents one of the first studies dealing with competing platforms and the decision to provide premium content. She shows that whereas a premium content is always offered exclusively, vertical integration between the provider and one platform may change incentives to invest in quality. In ours, instead, non-exclusivity arises more prominently in the presence of vertical integration as it is a way to soften market competition and avoid aggressive pricing strategies that the presence of indirect network externalities triggers.

Unlike the above-discussed studies, in our model, the Superstar faces a trade-off between exclusivity and non-exclusivity, and this choice depends on how intense the platform competition is. These results partly resemble those of Weeds (2016).¹⁷ She studies the incentives of a vertically integrated TV to offer its premium programming to a rival distributor. She finds that when the competition is dynamic, exclusivity might be the best solution, thereby contrasting traditional findings in static markets. Because of switching costs, the future market-share advantage might outweigh the opportunity cost of giving up to some current audience. Similar to Weeds (2016), in our model, the emergence of exclusivity is linked to the strength of the downstream competition. However, our result depends on the static competition and rent extraction effects rather than on the dynamic aspects stemming from switching costs.

3. The Model

We consider a two-sided market in which consumers singlehome and firms can either multihome or singlehome. The firms’ side is composed by a premium provider that we label as Superstar and a fringe of small providers. There are two platforms $i = 1, 2$ which compete for the two sides.

Consumers. There is a continuum of consumers, whose preferences are quasi-linear in money and are indexed by $m \in [\underline{m}, \bar{m}]$, which is symmetric around 0 with $\underline{m} = -\bar{m} < 0$. The parameter m denotes the measure of the relative preference for 2 against 1 and it is distributed according to a cumulative density function $F(\cdot)$ with density $f(\cdot)$. Hereafter, we will refer to m as the consumer type.

When consumers join a given platform, they obtain a value (v) independent of externalities. They also receive positive externalities due to the presence of firms on the other side. The

¹⁶Similarly, Armstrong (1999) shows that, in a traditional one-sided market, premium content is always offered exclusively. Moreover, in comparing different types of contracts, he also shows that, with exclusivity, a lump-sum contract is revenue-maximizing relative to a royalty-based one.

¹⁷In Abito & Wright (2008), exclusive dealing between a buyer and an input supplier is more likely to arise when the competition between downstream firms is sufficiently intense. Their result differs from that of Fumagalli & Motta (2006), who, instead, found that exclusivity is more likely to arise when the competition is softened. As discussed, we differ from these papers by considering a two-sided market and the fact that an exclusive deal between a platform and a Superstar does not lead to market tipping scenarios.

Superstar generates a value ϕ for the consumer, whereas each firm affiliated with platform i entails an indirect network benefit θ . We assume v to be sufficiently high so as that all consumers subscribe to either platform. We denote by $g_i = \{0, 1\}$ the indicator function expressing the presence/absence of the Superstar and by N_i the number of other small providers on platform i . Hence, the total utility of a type- m consumer from joining platform 1 at price p_1 is:

$$u_1(g_1) = v + \phi g_1 + \theta N_1 - p_1 - m/2,$$

where p_1 is the price paid by the consumers. Similarly, the utility a consumer m enjoys by patronizing platform 2 is

$$u_2(g_2) = v + \phi g_2 + \theta N_2 - p_2 + m/2.$$

Consumers prefers to join platform 1 over platform 2 whenever $u_1(g_1) > u_2(g_2)$ or

$$m < m^*(g_1, g_2) := \phi(g_1 - g_2) - (p_1 - p_2) + \theta(N_1 - N_2). \quad (1)$$

The demand for platform 1 is represented by all consumers with $m < m^*$ and the remaining consumers will join platform 2. Hence,

$$D_1(g_1, g_2) = F(m^*(g_1, g_2)) = F(g_1, g_2), \quad D_2(g_2, g_1) = 1 - F(g_1, g_2). \quad (2)$$

Note that the first argument in the demand function relates to the associated player's exclusivity choice and the second argument to that of the rival. For instance, $D_i(g_i, g_j)$, is the demand at platform i where g_i is the indicator function of the Superstar's choice to join platform i and g_j is the Superstar's choice to join its rival, platform j . We follow the same reasoning throughout the article. Moreover, we also assume regularity conditions. As in Fudenberg & Tirole (2000), we let the demand on platform 1 be a cumulative distribution function (cdf), $F(m)$, with the following properties.

Assumption 1. $F(\cdot)$ is smooth, with strictly positive density function $f(\cdot)$. $F(\cdot)$ is symmetric around zero and is such that the monotone hazard rate $\frac{f(m)}{1-F(m)}$ is increasing with m .

Firms. On this side of the market, there are a fringe of small firms s and the Superstar, S . Both types benefit from interactions with consumers. The value of such interactions is denoted by γ and γ^S for the small firms and the Superstar, respectively. These represent a measure of the indirect network externalities in this side of the market.

The fringe firms are a continuum denoted by their entry costs $k \in [0, \infty]$, with a cdf $\Lambda(\cdot)$ and density $\lambda(\cdot)$. Their utility when joining platform i is $u_i^s = \gamma D_i(g_i) - k$, and they do so for any $u_i^s > 0$, that is for any $k < \gamma D_i$. The mass of firms on platform i is

$$N_i = \Lambda(\gamma D_i).$$

As for the consumers, the following properties apply.

Assumption 2. $\Lambda(\cdot)$ is smooth, with strictly positive density function $\lambda(\cdot)$ and $\lambda'(\cdot) > 0$. The monotone hazard rate $\frac{\lambda(k)}{1-\Lambda(k)}$ is increasing with k .

The economic intuition behind $\lambda' > 0$ is that there are larger number of high-cost firms than low-cost ones. Also, this is a sufficient condition for monotone hazard rate to be increasing.

Unlike the small firms, the Superstar has all the bargaining power over her product and makes a take-it-or-leave-it (TIOLI) offer to the platform(s), with $T_i(g_i, g_j)$ representing the tariff she sets. $T_i(1, 0)$ ($T_j(1, 0)$) implies that platform i (j) has an exclusive contract with the Superstar, whereas $T_i(1, 1)$ and $T_j(1, 1)$ imply a non-exclusive contract. Other than tariffs, the Superstar makes ancillary revenues when interacting with consumers. Hence, the Superstar cares about her total market coverage.

The profit of the Superstar when offering an exclusive contract to platform i is given as:

$$\Pi^S(1, 0) = \gamma^S \cdot D_i(1, 0) + T_i(1, 0),$$

whereas the profit when offering non-exclusive contracts is:

$$\Pi^S(1, 1) = \gamma^S \cdot (D_i(1, 1) + D_j(1, 1)) + T_i(1, 1) + T_j(1, 1).$$

Platforms. Platforms collect revenues from the consumer side of the market. Given the offer of the Superstar, platform(s) decide whether to accept the offer and then they compete in prices. Specifically, platforms set prices to maximize their profits which are given by:

$$\Pi_i(g_i, g_j) - T_i(g_i, g_j) = p_i \cdot D_i(g_i, g_j) - g_i T_i(g_i, g_j). \quad (3)$$

Note that the tariff T_i depends on g_1 and g_2 , since the payment to have the Superstar on board differs under exclusivity and non-exclusivity.

Timing. The timing of the game is as follows. In the first stage, the Superstar offers a contract to one platform (exclusivity) or both platforms (non-exclusive). In the second stage, platforms accept or reject the contracts. After observing contracts acceptance or rejection, platforms then set prices to consumers simultaneously. Finally, firms and consumers decide whether to enter the market and which platform to join. The equilibrium concept is subgame perfect Nash equilibrium.

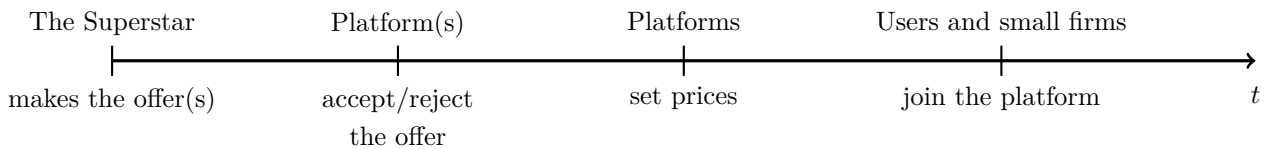


Figure 1: Timing of the model

4. Analysis

In this section, the model is analyzed by backward induction. By using the demands in equation (2), we first present the price competition on the consumer side for a given presence of the Superstar in each platform. Then, we study the optimal contractual choice of the Superstar.

4.1. Price competition

When prices are chosen, platform i has already received and accepted (or not) the offer of the Superstar. Thus, the price p_i only impacts the first term in the platforms' profits. By differentiating equation (3) with respect to p_i , the first-order conditions are given as follows:

$$\begin{aligned}\frac{\partial \Pi_1}{\partial p_1} &= D_1(g_1, g_2) + p_1 \left(- \frac{f(g_1, g_2)}{1 - f(g_1, g_2)\gamma\theta[\lambda(D_1) + \lambda(D_2)]} \right), \\ \frac{\partial \Pi_2}{\partial p_2} &= D_2(g_2, g_1) + p_2 \left(- \frac{f(g_1, g_2)}{1 - f(g_1, g_2)\gamma\theta[\lambda(D_1) + \lambda(D_2)]} \right).\end{aligned}$$

The following lemma provides the conditions of the equilibrium prices given g_1 and g_2 .

Lemma 1. *The optimal prices are characterized by the intersection of the following best responses:*

$$\begin{aligned}p_1(m^*) &= \frac{F(m^*)}{f(m^*)} - F(m^*)\gamma\theta[\lambda(D_1) + \lambda(D_2)], \\ p_2(m^*) &= \frac{1 - F(m^*)}{f(m^*)} - (1 - F(m^*))\gamma\theta[\lambda(D_1) + \lambda(D_2)].\end{aligned}$$

Proof. See Appendix A.1. □

As in Armstrong (2006), the optimal prices account for both the platform differentiation, through $F(m^*)$, and indirect network externalities, through $\gamma\theta$. Indeed, consumers are rewarded for the positive externality created on the other side. The cutoff m^* , which is a function of (g_1, g_2) , captures the impact of the Superstar on prices. If $g_i = g_j$, platforms are symmetric and the price is equal to that one in the standard competitive-bottleneck models (Armstrong 2006, Rasch & Wenzel 2013). This case is summarized by the following lemma.

Lemma 2. *If $g_i = g_j = g \in \{0, 1\}$, the two platforms charge the same price*

$$\begin{aligned}p^* &= \frac{F(0)}{f(0)} \left(1 - f(0)\gamma\theta(\lambda(D_1) + \lambda(D_2)) \right) \\ &= \frac{1}{2f(0)} - \gamma\theta\lambda(\gamma/2).\end{aligned}$$

The platforms split the market equally. All active firms given by $N_1^ = N_2^* = \Lambda(\gamma/2)$ multi-home, whereas all the others zero-home.*

Proof. See Appendix A.2. □

Lemma 2 describes a symmetric scenario where neither platform enjoys the competitive advantage of the premium content. Two cases are subsumed in this lemma. In the first case, $g = 0$ and no platform hosts the Superstar. In the second case, $g = 1$ and both platforms host the Superstar. We obtain that the final consumer demand is equal in the two cases, $F(0, 0) = F(1, 1) = F(m^* = 0)$. Figure 2 provides a graphical representation of consumers' and firms' participation.

Assume that the Superstar offers an exclusive contract to platform 1. The equilibrium outcomes is reported in the following lemma.

Lemma 3. *With an exclusive contract, e.g., $g_1 = 1$ and $g_2 = 0$, equilibrium prices are*

$$p_1^*(1, 0) > p^* > p_2^*(0, 1).$$

Platform 1 has a higher consumer demand

$$D_1^*(1, 0) > 1/2 > D_2^*(0, 1).$$

Proof. See Appendix A.3. □

Lemma 3 highlights important differences with the symmetric case described above. First, one can observe that an exclusive contract renders the final prices asymmetric: the platform *avored* by Superstar exclusivity sets a higher price than the rival. Note that this price is higher than in the case with non-exclusive contracts, *i.e.*, $p_1^* > p^*$. By contrast, the rival's price decreases, *i.e.*, $p_2^* < p^*$. Importantly, the magnitude of the price change is lower than the value generated by the Superstar, $\frac{\partial p_1}{\partial \phi} - \frac{\partial p_2}{\partial \phi} \in [0, 1]$. As such, there is some surplus left over to the final consumers and this triggers a demand expansion for the *avored* platform. This is the typical (first-order) *business-stealing effect*, which further gives rise to positive indirect network externalities on the other side of the market (fringe firms' side).

This is an eye-catching result: the asymmetry generated by an exclusive contract on the demand side is further magnified by a large number of firms on the fringe joining the platform with the Superstar as well. Indeed, the Superstar agglomerates consumers and firms on the *avored* platform. Formally, the feedback loop on the fringe side is such that:

$$N_1 = \Lambda(\gamma D_1^*) > \Lambda(\gamma/2) > N_2 = \Lambda(\gamma D_2^*).$$

The following proposition summarizes our findings, discussing the impact of an exclusive contract offered by the Superstar on the homing decision of the other fringe firms.

Proposition 1. *Superstar exclusivity fosters entry in the market and induces singlehoming of some fringe firms. Specifically,*

- *fringe firms with cost $k \in (0, \gamma D_2^*]$ multihome;*
- *fringe firms with cost $k \in (\gamma D_2^*, \gamma D_1^*]$ singlehome on platform 1;*
- *fringe firms with cost $k \in (\gamma D_1^*, \infty)$ zero-home.*

The intuition of the above results is as follows. Under exclusive dealing, the impact on the fringe side is twofold relative to non-exclusivity. First, a larger mass of fringe firms are active in the market. Second, some firms become exclusively active on the *avored* platform without the need for an exclusive contract. This is an interesting result in itself as the exclusive presence of the Superstar affords some fringe firms with high costs to be active in the market. This creates more exclusivity as some firms singlehome.

To see this mechanism graphically, consider Figure 2, which depicts the case when the Superstar offers a non-exclusive contract. The consumer side is equally split between the two platforms, all firms with low production costs $k < \gamma/2$ multihome, whereas those firms with high production costs remain inactive. With Superstar exclusivity on platform 1, a larger mass of consumers are active on that platform with respect to the rival ($D_1^* \equiv F(1, 0) > 1/2$).

As the size of fringe active on a platform depends on the beliefs regarding the number of consumers on that platform, some firms that were zerohomers in the non-exclusive case are now singlehomers. Moreover, some of the high-cost multihomers (in the non-exclusive contracts case) now singlehome on the *favoured* platform. Figure 3 provides a graphical representation of the effect of an exclusive contract on both sides.

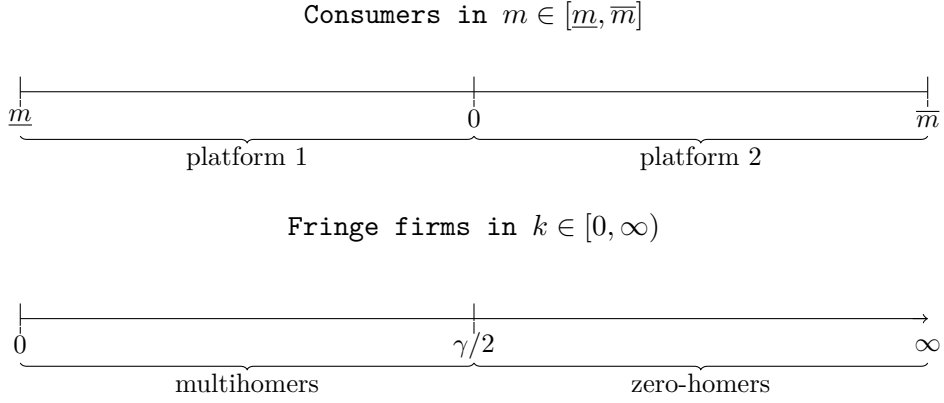


Figure 2: Non-exclusive contract.

Under non-exclusivity, the consumer side is equally split and symmetric around 0. All fringe firms with $k \leq \gamma/2$ are multihomers, whereas the others are zerohomers.

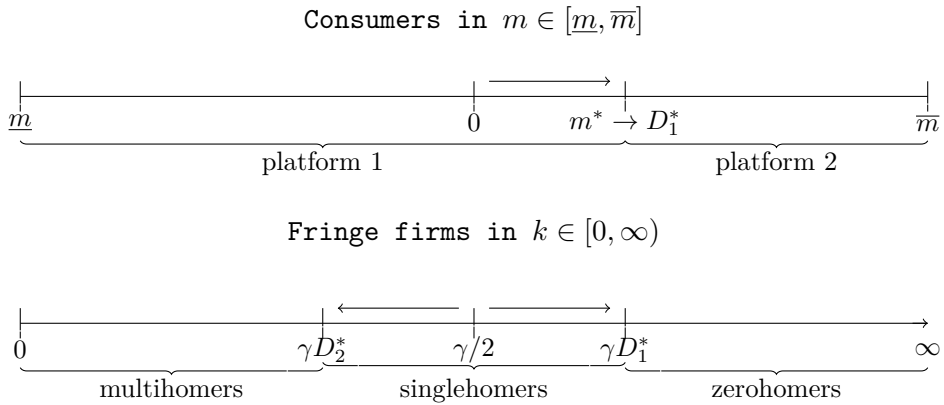


Figure 3: Exclusive contract with platform 1.

Under exclusivity with platform 1, more consumers affiliate with platform 1 ($\hat{m} \equiv D_1^* > 1/2$). Fringe firms with low production costs multihome, intermediate-cost firms (for $k \in (\gamma D_2^*, \gamma D_1^*]$) singlehome on platform 1 and high-cost firms zero-home.

4.2. Contractual stage: exclusive and non-exclusive contracts

To determine the optimal contractual design, we compare Superstar's profits under exclusivity with those under on-exclusivity. The contractual design is similar to an auction with negative

externalities, as in Jehiel & Moldovanu (2000).¹⁸ The Superstar offers the contract to platform 1 under the threat of providing the exclusive contract to the rival if platform 1 rejects the offer.

Exclusive contract. Suppose the Superstar offers an exclusive contract to platform 1. In this case, the Superstar solves the following problem:

$$\begin{aligned} \max_{T_1(1,0)} \Pi^S(1,0) &= \gamma^S \cdot D_1(1,0) + T_1(1,0) \\ \text{subject to } \Pi_1^*(1,0) - T_1(1,0) &\geq \Pi_1^*(0,1), \end{aligned}$$

where $\Pi_1^*(g_1, g_2)$ is the equilibrium gross profit of platform 1. Note that the participation constraint simply maps the value generated by the Superstar on the platform, *i.e.*, $\Pi_1^*(1,0) - \Pi_1^*(0,1)$. The larger this value, the larger the tariff that the Superstar can collect. As a result, the Superstar sets $T_1(1,0) = \Pi_1^*(1,0) - \Pi_1^*(0,1)$ such that platform 1's participation constraint is binding.

Non-exclusive contract. Suppose the Superstar offers a contract to both platforms and obtains ancillary revenues over the entire market. To be incentive-compatible, each platform should prefer profits with the non-exclusive contract, $\Pi_i^*(1,1) - T_i(1,1)$, to the outside option $\Pi_i^*(0,1)$. Formally, the Superstar solves the following problem:

$$\begin{aligned} \max_{T_1(1,1), T_2(1,1)} \Pi^S(1,1) &= \gamma^S \cdot (D_1(1,1) + D_2(1,1)) + T_1(1,1) + T_2(1,1) \\ \text{subject to } \Pi_1^*(1,1) - T_1(1,1) &\geq \Pi_1^*(0,1), \\ \Pi_2^*(1,1) - T_2(1,1) &\geq \Pi_2^*(0,1). \end{aligned}$$

Also in this case, $\Pi_1^*(1,1) - \Pi_1^*(0,1)$ and $\Pi_2^*(1,1) - \Pi_2^*(0,1)$ represent the extra-value created by the Superstar. In equilibrium, the contractual tariffs are $T_i(1,1) = \Pi_i^*(1,1) - \Pi_i^*(0,1)$, for $i \in \{1, 2\}$, and the participation constraints of the platforms are binding. Exploiting symmetry, we have $\Pi_1^*(0,1) = \Pi_2^*(0,1) = \Pi^*(0,1)$, $\Pi_1^*(1,0) = \Pi_2^*(1,0) = \Pi^*(1,0)$, and $D_1(1,0) = D_2(1,0) = D^*(1,0)$. Hence, the Superstar's profits in the two contractual regimes are:

$$\Pi^S(1,0) = \gamma^S D^*(1,0) + \Pi^*(1,0) - \Pi^*(0,1),$$

$$\Pi^S(1,1) = \gamma^S + 2[\Pi^*(1,1) - \Pi^*(0,1)] = \gamma^S + 2[\Pi^*(1,1) - \Pi^*(0,1)].$$

Contract design. The contractual design directly follows from the comparison of the profits in the two regimes. The next proposition presents the main result.

Proposition 2. *There exists a cutoff*

$$\tilde{\gamma}^S = \frac{\Pi^*(1,0) + \Pi^*(0,1) - 2\Pi^*(1,1)}{1 - D^*(1,0)}$$

¹⁸Note that this is as if the platform and the Superstar strike a public contract. However, in our framework, also a secret contract would not change the main results.

such that non-exclusive contracts are chosen in equilibrium if, and only if, $\gamma^S \geq \tilde{\gamma}^S$. Else, an exclusive contract is chosen.

When the Superstar offers an exclusive contract, two forces are at stake. First, a *rent extraction effect*, which is captured by the numerator of $\tilde{\gamma}^S$. This simply represents the difference between the tariffs collected under the two regimes, *i.e.*, $T_1^*(1, 0) - 2T_1^*(1, 1)$. Second, a *competition effect*, which results from the demand expansion of the customer base of the *favoured* platform. This is captured by the denominator of $\tilde{\gamma}^S$. Such an effect gets stronger as the degree of differentiation between platforms decreases. When competition is very intense, consumers are more responsive to the presence of the Superstar, which therefore increases $D_1^*(1, 0)$. In turn, as $D_1^*(1, 0)$ increases, the denominator of $\tilde{\gamma}$ shrinks, thereby increasing the room for exclusivity.

Differently, when γ^S is sufficiently large, the Superstar finds it optimal to offer a non-exclusive contract. In this case, reaching the entire market, $D_1^*(1, 1) + D_2^*(1, 1) = 1$, is more profitable when indirect network externalities are significant and, indeed, this outweighs any rent extraction effect.

In the following corollary, we show how the value generated by the Superstar impacts the already discussed critical cutoff γ^S .

Corollary 1. *A Superstar generating a higher value is more likely to prefer exclusivity, as $\frac{\partial \tilde{\gamma}^S}{\partial \phi} > 0$.*

Proof. See Appendix A.4. □

The above corollary states that when the Superstar brings about value to consumers, then the latter are more responsive to it, *ceteris paribus*. As a result, the rent extraction and competition effect drive upwards the critical value $\tilde{\gamma}^S$, which makes the emergence of exclusivity more likely.

Bargaining. So far, we have assumed that all the bargaining power is with the Superstar who makes a TIOLI offer. This is credible in many industries in which top-rated firms are very valuable and can auction their availability. In other cases, agreements are reached following negotiations. Adachi & Tremblay (2019) report evidence of negotiations between Apple Music and labels on the share of revenues to go on the latter, and similar negotiations are also common in the cable television industry, in the video-on-demand streaming market, and between large retailers and credit card companies.¹⁹

Nevertheless, a mechanism similar to the one described by Proposition 2 also applies in these cases. The tariffs that each platform would be willing to pay necessarily map the value created by the Superstar. This is true for any bargaining power. Indeed, exclusivity will prevail whenever the Superstar is able to profitably exploit the demand asymmetries. On the contrary, non-exclusive contracts will be signed as long as the audience becomes more critical (*i.e.*, large enough γ^S).

Formally, suppose the Superstar to have a bargaining power *vis-à-vis* platforms denoted by $\beta \in [0, 1]$. In Appendix A.5, we show that when the Superstar offers a non-exclusive contract, say to platform 1, the optimal tariff is identical to the one when she has full bargaining power,

¹⁹See *e.g.*, Crawford & Yurukoglu (2012), Crawford et al. (2018) for empirical applications of bilateral negotiations in the cable TV industry.

i.e., $T_1^*(1,0) = \Pi_1^*(1,0) - \Pi_1^*(0,1)$. As a result, the Superstar's profit remains unchanged regardless of the value of β . This is because exclusive contracts inherently imply control and market power for the supplier and, hence, the rent extraction is full. On the other hand, when the Superstar offers a non-exclusive contract, she internalizes her bargaining power and sets $T_i^*(1,1) = \beta(\Pi_i^*(1,1) - \Pi_j^*(0,1)) - (1 - \beta)(\gamma^S(1 - D_j(0,1)))$.

We note that contractual tariffs increase with β . In the limit case in which $\beta = 1$, tariffs are at their maximum. This implies that her profits under non-exclusivity are discounted when her bargaining power is very low as the possibility to extract rents is dampened when β decreases. Hence, exclusivity becomes relatively more profitable. Therefore, we can conclude the following.

Proposition 3. *As the bargaining power shifts away from the Superstar to the platform(s), there is a higher incentive to offer an exclusive contract.*

Proof. See Appendix A.5. □

This result has an interesting implication for policy makers. If exclusive dealing is seen with concern, then balancing the bargaining power of the Superstar with that of the platforms might turn out increasing the likelihood of these exclusive conducts.

4.3. Welfare Analysis

In what follows, we show how the contractual choice of a Superstar affects surplus of fringe firms and consumers.

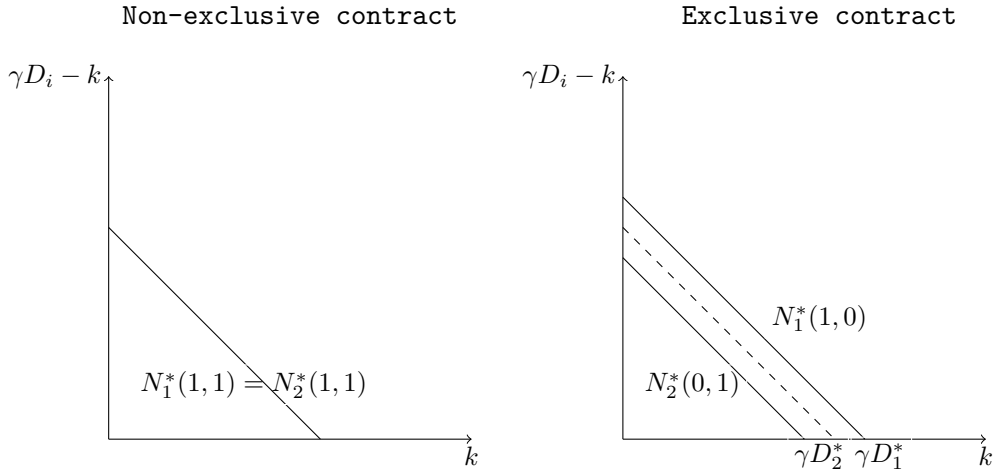


Figure 4: Profit Surplus on the Fringe

The figure depicts the surplus on the fringe side of the market under both regimes. Each triangle represents the surplus of the fringe. The exclusive case always achieves higher total surplus.

Impact on Fringe Firms. When the Superstar is either absent or offers a non-exclusive contract, welfare of small firms is unchanged due to the symmetry of platforms. This result arises from the inelastic consumer demand specification, which we relax in Section 5. In contrast, in the case of exclusive dealing, the mass and the homing decision of the active fringe

firms change. This is because the Superstar grants to the *avored* platform an advantage in the number of active firms on the fringe. By comparing the surplus of the fringe under the two regimes, we state the following.

Proposition 4. *The fringe firms' surplus is higher under an exclusive contract than under non-exclusive contracts.*

Proof. See Appendix A.6. □

The result in Proposition 4 is determined by the gains enjoyed by the zero-homers and multihomers who become singlehomers. It suggests that those small firms (*e.g.*, emerging artists, startups, retailers) with sufficiently high costs and who otherwise would have struggled to be active on the market should welcome Superstar exclusivity. This result can also be explained graphically as in Figure 4, which plots the surplus of each firm according to the cost faced. The triangles represent the mass of active firms in each platform. Moving from the case of non-exclusivity (left panel) to the case of exclusivity (right panel), the intercept increases for firms on platform 1 and decreases for firms in platform 2 and the net effect is positive.

Impact on Consumers' Surplus. In what follows we present the effect that exclusivity has on consumer surplus.

Denote $\Delta CS = CS(1,0) - CS(1,1)$, the net gain (loss) from exclusivity for consumers in platform 1, where $CS(1,0)$ and $CS(1,1)$ represent the consumer surplus under exclusivity and non-exclusivity, respectively. Direct computation is reported in the Appendix. After some arithmetic manipulation, we have the following expression:

$$\Delta CS = \underbrace{\theta[\bar{N} - \Lambda(\gamma/2)]}_{\Delta \text{ externalities}} - \underbrace{\phi(1 - F(m^*))}_{\text{prevented access}} - \underbrace{[\bar{p} - p^*(1,1)]}_{\Delta \text{ prices}} - \underbrace{\int_0^{m^*} mf(m)dm}_{\text{preference mismatch}} \quad (4)$$

where $\bar{N} = F(m^*)N_1(1,0) + (1 - F(m^*))N_2(0,1)$ and $\bar{p} = F(m^*)p_1(1,0) + (1 - F(m^*))p_2(0,1)$ are the expected mass of firms and the expected prices under exclusivity, respectively. We can then state the following proposition.

Proposition 5. *The total effect of exclusivity on consumer surplus is ambiguous for any $\theta > 0$. It increases for consumers on the favored platform and decreases for consumers on the rival platform. If $\theta = 0$, the total consumer surplus is unambiguously lower under non-exclusivity.*

Proof. See Appendix A.7. □

There are two opposite forces at stake. On the one hand, consumers joining the platform without the Superstar suffer. This is because the presence of the Superstar only on platform 1 prevents access to the Superstar product to those consumers affiliating with platform 2. On the other hand, there is a surplus generation for those consumers on the *favored* platform. These effects are further amplified by the network externalities.

Equation (4) provides insights on the effects of exclusivity in our setup. One can observe four elements. The first one is about the externalities that the presence of firms generates on consumers. This is higher under exclusivity as consumers on the *favored* platform are exposed to a large variety of firms than under non-exclusivity. However, exclusivity also entails

three negative effects. These are represented by (i) the prevented access to the Superstar to consumers on the *non-favored* platform; (ii) the increase in the expected price consumers pay ; and (iii) the augmented preference mismatch, as there are consumers who inefficiently buy from their less preferred platform.

As (i), (ii), (iii) enter negatively in ΔCS , one can understand the paramount relevance of indirect network externalities in driving up consumer surplus under exclusivity. In the limit case in which consumers do not benefit from the presence of the fringe ($\theta = 0$), and which would be a one-sided market, the net effect of exclusivity is unambiguously negative. Indeed, it is the presence of indirect network externalities that might create value from exclusivity. What is critical in determining the final net effect when $\theta > 0$ is how many consumers the Superstar moves toward the *favored* platform and this depends on the distribution of consumer preferences. If a large mass of consumers are concentrated around zero, the market is very competitive, and then many consumers would follow the Superstar on the *favored* platform. This lowers the extent of the prevented access associated with exclusivity. Moreover, there are also strong network externalities as many firms would follow these consumers, thereby creating additional value for them.²⁰

4.4. Vertical Integration

This section aims to understand the impact of a vertical merger (platform-Superstar) in markets with network effects and provide policy-relevant insights. Typically, vertical mergers are viewed with suspicion by policy makers as they increase the likelihood of anti-competitive conducts by the merged entity. We focus in this article on the input foreclosure theory of harm espoused by the antitrust agencies. The incentives to foreclose have been widely discussed by the previous literature (see *e.g.*, Salinger 1988, Ordover et al. 1990, Bourreau et al. 2011 and for a survey Rey & Tirole 2007).

The European Commission, in its merger control, follows the Non-Horizontal Merger Guidelines (NHMG) for assessing a vertical merger. The commission looks at the ability and incentive of a vertically integrated entity to foreclose rivals and the ensuing impact of such a strategy on the effective competition. According to the above guidelines, foreclosure is a concern when the upstream firm (*i*) has a significant degree of market power, (*ii*) is an important supplier of input, *e.g.*, it represents a significant cost factor for the downstream firm (NHMG 2008, para 35), and (*iii*) the merged entity would be able to negatively affect the availability of inputs to its rivals, (NHMG 2008, para 36). In our case, the three conditions are fulfilled by the Superstar. She is bestowed with a significant degree of market power vis-à-vis the platforms and her premium product is an important but not indispensable input in the downstream market. Small firms are still active on the non-integrated platform. Moreover, she has the power to negatively affect the rival by excluding access to this input. Thus, according to the NHMG (2008), a vertical merger in our setup presents overwhelming evidence on the ability to (input) foreclose a rival platform.²¹ In the following, we discuss the incentives to

²⁰In the Appendix, we provide an example of how consumer surplus changes with exclusivity in the most conservative case, that is, with a uniform distribution of consumer preferences. We show that there exists an area in which exclusivity is welfare-enhancing if indirect network externalities (θ) are sufficiently large.

²¹The rationale for these incentives is nicely captured by NHMG: “*The incentive to foreclose depends on the degree to which foreclosure would be profitable. The vertically integrated firm will take into account how its*

foreclose a rival when there are network externalities and the proposed merged-entity presents an overwhelming ability to foreclose supply of inputs to rivals.

Incentive to foreclose. We modify our baseline model as follows. Without loss of generality, let us assume that the Superstar is integrated with platform 1. This merged entity has two alternatives: be the sole distributor of a premium product, or distribute the premium product to the rival, platform 2. In the latter case, the merged entity sets $T_2(g_2)$. The profits of the merged entity are denoted by Π_1^S , which comprise of the revenues made in the downstream market, Π_1 , and those made from the subsidiary. The revenues from the Superstar are the ancillary revenues of the Superstar and the tariff charged to the rival in case of a non-exclusive contract, $T_2(g_2)$. Indeed, the merged entity can now choose both p_1 and T_2 . Formally, the profits of the merged entity are:

$$\Pi_1^S(g_2) = \underbrace{p_1 D_1(g_2)}_{\Pi_1} + \gamma^S (D_1(g_2) + g_2 \cdot D_2(g_2)) + g_2 T_2(g_2).$$

Under exclusivity ($g_2 = 0$), the first-order conditions are:

$$\frac{\partial \Pi_1^S(0)}{\partial p_1} = D_1(0) + (p_1(0) + \gamma^S) \frac{\partial D_1(0)}{\partial p_1}.$$

We can now state the following lemma.

Lemma 4. *In the exclusive regime, prices under vertical integration are lower than the prices under vertical separation. If $g_2 = 0$, equilibrium prices are:*

$$\frac{\partial \Pi_1^S(0)}{\partial p_1} \Big|_{p_1^*(1,0)} = \gamma^S \frac{\partial D_1(0)}{\partial p_1} \Big|_{p_1^*(1,0)} < 0.$$

The above lemma provides an interesting result. The merged entity internalizes the externality of downstream prices on the Superstar’s ancillary revenues. This makes the merged entity price more aggressively to attract demand. As prices are strategic complements, also the price of the rival platform falls. As the reduction in prices is larger than the demand expansion for the merged entity, the downstream profits are lower as compared to those arising with exclusivity under vertical separation, *i.e.*, $\Pi_1(0) < \Pi_1(1, 0)$.

This is a relevant result for antitrust enforcers. The NHMG suggests that under input foreclosure softens downstream competition with resulting higher prices.²² Against this backdrop, Lemma 4 shows that, when accounting for network externalities γ^S , exclusivity entails a pro-competitive effect for consumers, with lower prices.

supplies of inputs to competitors downstream will affect not only the profits of its upstream division, but also of its downstream division. Essentially, the merged entity faces a trade-off between the profit lost in the upstream market due to a reduction of input sales to (actual or potential) rivals and the profit gain, in the short or longer term, from expanding sales downstream or, as the case may be, being able to raise prices to consumers. (NHMG 2008, para 40, p.7)

²²For instance, the NHMG reports that “a decision of the merged entity to restrict access to its inputs reduces the competitive pressure exercised on remaining input suppliers, which may allow them to raise the input price they charge to non-integrated downstream competitors (NHMG 2008, para 38)”. As a consequence, final consumer price rises in the market.

Under non-exclusivity, instead, market competition follows Section 4: platforms' profits do not react to the presence of a merged entity as the total consumer demand is a constant and does not depend on prices. In turn, the indirect network externality, γ^S , does not play any role. Consistently with the baseline model, the decision to offer a contract to a rival platform hinges upon both the rent extraction and the fierceness of the platform competition. Notably, this happens if $\gamma^S \leq \tilde{\gamma}^{vi}$, whereby $\tilde{\gamma}^{vi}$ is implicitly characterized by the following expression:

$$\tilde{\gamma}^{vi} = \frac{\Pi_1^*(0) + \Pi_2^*(0) - 2\Pi^*(1, 1)}{1 - D_1(0)}.$$

By comparing the critical values of γ^S , we can state the following proposition.

Proposition 6. *A merged entity (platform-Superstar) has more incentives to offer non-exclusive contracts than under vertical separation, as $\tilde{\gamma}^{vi} < \tilde{\gamma}$.*

Proof. See Appendix A.8. □

The result in Proposition 6 stems from the increased competitive pressure on prices that indirect network externalities exert under exclusivity. Importantly, this result contrasts the traditional understanding that vertical mergers increase the risk of input foreclosure.

Note that these results apply in the presence of two essential factors, which require due diligence by antitrust enforcers. First, the *identification of indirect network externalities* when defining a market, as these exert competitive pressure on both platforms. Second, *exclusivity should not lead to market tipping* and, indeed, input foreclosure should not prevent the rival platform from attracting consumers and some small firms.

4.5. One-sided market vs. Two-sided market

Current theories of harms related to input foreclosure stemming from exclusive dealing and vertical mergers were originated in a framework without indirect network externalities. In this subsection, we compare our main results with those emerging in a traditional one-sided market.

Table 2 presents a summary of the main results when indirect network externalities are absent ($\gamma^S = 0$ and $\theta = 0$) and when present ($\gamma^S > 0$, and $\theta > 0$). The former case characterizes a one-sided market model in which the Superstar acts as an input supplier to either one or both platforms. Fringe firms are absent and the Superstar does not make any ancillary revenues ($\gamma^S = 0$).

First, consider the optimal contracts in Proposition 2. It is immediate to see how much the equilibrium contract choice crucially depends on γ^S . When sufficiently large, a non-exclusive contract drives the platform to reach the entire market as ancillary revenues become prominent. Suppose, instead, $\gamma^S = 0$, and so the Superstar only makes revenues through the contract. It is, then, straightforward to see that $\gamma^S = 0 < \tilde{\gamma}^S$ always. As a result, the Superstar always sets an exclusive contract, through which she can extract all surplus generated by the asymmetry in the market. We note that this result is reminiscent of that of Montes et al. (2019), who found that exclusive contracts always arise in the data broker industry.

Second, consider the welfare impact of exclusive dealing in a traditional framework in which platforms are sole distributors of the Superstar. In this case, as fringe firms are absent, we consider only the effect on consumer surplus. We note that exclusivity entails a higher

price than under non-exclusivity. This is also the case in our two-sided market framework. However, the price set on the consumer side is discounted by the strength of the indirect network externalities. In a one-sided market, consumers do not get such a discount and do not enjoy higher surplus due to the entry of additional fringe firms ($\theta = 0$). As a consequence, an exclusive contract turns out to be detrimental to consumers. This result is of paramount relevance for policy-makers and justifies the general reluctance and skepticism associated with exclusive dealing. However, when introducing indirect network externalities, these conclusions might not always be supported.

Third, absent indirect network externalities, a vertical merger always leads to input foreclosure of the premium product to the rival platform. This is because, when $\gamma^S = 0$, downstream profits under vertical integration and vertical separation are the same, so that exclusivity always arises. Moreover, relative to non-exclusivity, the price charged by the merger entity is higher. These results also suggest that the current understanding of the European Commission's NHMG (2008) are suitable for one-sided markets in which exclusivity (and input foreclosure) always arises.

Setting	Contract choice	Impact on FS	Impact on CS	Contract choice with a vertical merger
One-sided market	Exclusive, always	Absent	Negative	Exclusive, always
Two-sided market	Exclusive for $\gamma^S < \tilde{\gamma}^S$	Exclusive increases fringe surplus	Ambiguous	Exclusive for $\gamma^S < \tilde{\gamma}^{vi}$, with $\tilde{\gamma}^{vi} < \tilde{\gamma}^S$

Table 2: Comparison

For one-sided market, we consider the case in which the firms on the fringe are absent and the Superstar and consumers do not benefit from positive indirect network externalities when interacting with consumers ($\theta, \gamma^S = 0$). For two-sided market, we refer to the case in which firms on the fringe are present, consumers and the Superstar benefit from indirect network externalities ($\theta, \gamma^S > 0$). FS stands for the total surplus for fringe firms. CS stands for total consumer surplus.

5. Extensions

The above results are robust to several extensions and more complex scenarios. We now consider several alternative model specifications and relax those assumptions which may seem too stringent. First, we discuss the coordination problem, which is typical of two-sided markets. Then, we introduce the presence of multihoming consumers and we argue that our results do not change when accounting for an elastic demand function with differentiated platforms. Finally, we also discuss how the trade-off holds when platforms set a price to both sides of the market.

5.1. Coordination Problem

The scenario proposed so far is also well suited to explain the contractual decisions of a Superstar entering a market in which there are already inherited and symmetric market shares.

In the latter case, exclusive dealing would stimulate switching decisions on *favoured* platform. As in any model with network externalities (*e.g.*, Caillaud & Jullien 2003, Hagiu 2006, Damiano & Li 2007, Jullien 2011, Markovich & Yehezkel 2018), this also entails a coordination problem. If consumers believe that a sufficiently large number of other consumers and firms will follow the Superstar, then the market can tip. For instance, a device to solve this coordination problem is grouping homogeneous users as in Markovich & Yehezkel (2018). This would help an efficient platform to drive a less efficient focal rival out of the market. In our framework, the pivotal agent is the Superstar, which is on the firm side of the market. Clearly, market tipping does not emerge in our framework when a mass of consumers have strong preferences for one platform over the other, regardless of network externalities.

Consider an asymmetric inherited market, in which some consumers were not joining their preferred platform if everything else was symmetric. In this case, the Superstar has the option to join the large (or dominant) platform and more likely to lead to market tipping. Indeed, the Superstar's exclusive contract with the dominant platform works as a coordination device, with the rival driven out of the market. By contrast, if the Superstar is exclusive on the small platform, our problem resembles that of Markovich & Yehezkel (2018) but with heterogeneous consumers. Heterogeneity implies that the Superstar might be able to overturn market dominance but hardly would generate market tipping. This is similar to what discussed by Lee (2013) in which exclusive contracts in the videogame industry helped entrants to gain market shares and foster competition.

The crucial role of consumer expectations and coordination can be better grasped by relaxing the assumption of differentiated platforms. Consider a continuum of consumers distributed according to their cost of joining a platform $m \in [0, \infty)$, with a distribution function $\Upsilon(m)$ and density function $v(m)$. The utility of a consumer joining platform i is $u_i(g_i) = v + \phi g_i + \theta N_i - p_i - m$. It is important here to clarify that platforms compete *à la* Bertrand: consumers join a platform i if $u_i > u_j$ and $m < M_i^*(g_i) := v + \phi g_i + \theta N_i - p_i$. Hence, the mass of consumers active on a platform is given as:

$$D_i(g_i) = \begin{cases} \Upsilon(M_i^*(g_i)) & \text{if } p_i < \bar{p} \equiv p_j + \phi(g_i - g_j) + \theta(N_i - N_j) \\ \Upsilon(M_i^*(g_i))/2 & \text{if } p_i = \bar{p} \\ 0 & \text{if } p_i > \bar{p} \end{cases}$$

It is clear that, regardless of contract choice, expectations of consumers about the other side participation are crucial to determine the outcomes of the competition. In this regard, the most meaningful scenario is the one in which consumers believe that a non-exclusive contract ($g_1 = g_2$) implies $N_1 = N_2$ and that an exclusive contract struck with platform 1 ($g_1 = 1, g_2 = 0$) implies $N_1 > N_2$. Under these beliefs, we can state the following proposition.

Proposition 7. *There exists a cutoff*

$$\tilde{\gamma}^{nd} = \frac{\Pi_1^*(1, 0)}{2D_1^*(1, 1) - D_1^*(1, 0)}$$

such that non-exclusive contracts are chosen in equilibrium if, and only if, $\gamma^S \geq \tilde{\gamma}^{nd}$. Else, exclusive contracts are chosen.

Proof. See Appendix A.9. □

The cutoff in Proposition 7 is even simpler than the one in Proposition 2. Note that the numerator $\tilde{\gamma}^{nd}$ is simply the profit of the *favored* platform under exclusivity. Under the favorable expectations assumed above, exclusive dealing always leads to market tipping and this profit can be fully extracted by the Superstar. Indeed, the platform receiving an exclusive offer can either accept (and get a net profit of zero) or reject (and exit the market). A non-exclusive contract, instead, can never lead to a positive payment to the Superstar, as the Bertrand-like competition leads to zero profits. Under these conditions, the only reason for the Superstar to sign a non-exclusive contract lies in the very large total demand she can eventually access. Formally, this is captured by the denominator of $\tilde{\gamma}^{nd}$.

Nevertheless, beliefs are crucial. For instance, suppose that, under non-exclusivity, all consumers believe that $N_1 > N_2$ and then coordinate on platform 1. As a result, consumers would believe exclusivity on 1 to be less likely. The reason is the following: under such beliefs, the outside option of platform 1, $\Pi_1(0, 1)$, is not necessarily zero. As a result, the Superstar makes lower profits and $\tilde{\gamma}^{nd}$ is lower as the numerator becomes $\Pi_1(1, 0) - \Pi_1(0, 1)$. As in Markovich & Yehezkel (2018), exclusivity could become a device to move consumers from the *focal* platform to the rival.

5.2. Multihoming Consumers

In most markets, consumer multihoming is quite common, and platforms have overlapping market shares. However, if the same consumers and firms are on the same platform, the previous equilibrium contract choices might not survive: a non-exclusive contract might turn out being more likely as, when consumers multihome, their switching behavior gets less relevant for both platforms and fringe firms.²³ In this section, we provide an analysis of this case and show that results are not so straightforward.

In this variation, we characterize the utility, u^m , of a multihoming consumer as follows:

$$u^m = v + \phi \max\{g_1, g_2\} + \theta \max\{N_1, N_2\} - (p_1 + p_2). \quad (5)$$

Note that when consumers multihome, there is no longer a preference mismatch ($m = 0$). This is because they receive an extra-value for going to their preferred platform that adequately compensates the cost of going to the rival one. Moreover, we assume that the benefit of consumer joining a platform, v , is only obtained once and not duplicated. The same happens when interacting with the same firms or with the Superstar. This implies having access to $\max\{N_1, N_2\}$ firms.²⁴

Thus, consumers' decision is between multihoming and singlehoming. Regardless of the Superstar's exclusivity, consumers make their decision based on their preference $m/2$. When they affiliate to their preferred platform, $m/2$ is a benefit rather than the cost, and its absolute value enters positively in the utility function. Therefore, we compare u_1 to u^M for any $m < 0$

²³This is also discussed by recent studies, in which multiple interactions with the same consumers generate decreasing returns for the opposite side of the market, where there are advertisers, content providers, or sellers (Ambrus et al. 2016, Athey et al. 2016, Calvano & Polo 2019, D'Annunzio & Russo 2019, Anderson et al. 2018). Multihoming on both the consumer and content provider side is also discussed by Choi (2010).

²⁴This is a plausible assumption as consumers do not benefit differently from interacting with the same firms twice. However, when considering a less restrictive case, in which multihoming potentially gives rise to as much as 2ϕ and $2\min\{N_1, N_2\}$, this will only be a scale effect on the utilities and will not change qualitatively results and intuitions.

and u_2 to u^M for any $m > 0$. It follows that an agent with a relative preference to platform i is never indifferent between joining platform j and multihoming. This implies that $u_1^{sh} > u^M$ when

$$m < m_1^*(g_1, g_2) := 2(\phi \min\{0, g_1 - g_2\} + \theta \min\{N_1 - N_2, 0\} + p_2), \quad (6)$$

and, similarly, $u_2^{sh} > u^M$ when

$$m > m_2^*(g_2, g_1) := 2(\phi \max\{0, g_1 - g_2\} + \theta \max\{N_1 - N_2, 0\} - p_1). \quad (7)$$

The total demand for platform 1 is $D_1 = F(m_2^*)$, and this is constituted of $D_1^{sh} = F(m_1^*)$ singlehomers, whereas the remaining consumers multihome. In the same manner, we determine the total demand of platform 2, which is $D_2 = 1 - F(m_1^*)$, of which $D_2^{sh} = 1 - F(m_2^*)$ is the demand from singlehoming consumers.

It is important to note that the demand for a platform does not change moving from a situation of non-exclusivity to one of exclusivity on the rival platform. Indeed, the demand for platform i is determined by the decision of the consumer indifferent between multihoming and singlehoming on the rival. The latter is not affected by the presence of the Superstar, which is guaranteed in both options. What changes, instead, is the demand of the rival platform, as multihoming always guarantees access to the Superstar, whereas singlehoming ensures it only in the case of non-exclusivity.

To fix ideas, consider a situation in which the Superstar opts for non-exclusivity ($g_1 = g_2$) and a symmetric scenario in which $N_1 = N_2$. Then, the two cutoffs m_1^* and m_2^* depend only on prices, as they boil down to:

$$m_1^*(1, 1) = 2p_2 \text{ and } m_2^*(1, 1) = -2p_1. \quad (8)$$

Now, consider the case in which the Superstar goes exclusively on 1 (*i.e.*, $g_1 = 1$ and $g_2 = 0$) and consequently $N_1 > N_2$. Plugging them into the cutoffs, we finally obtain:

$$m_1^*(1, 0) = 2p_2 \text{ and } m_2^*(0, 1) = 2(\phi + \theta(N_1 - N_2) - p_1). \quad (9)$$

This determines the following result.

Proposition 8. *There exists a cutoff*

$$\tilde{\gamma}^M = \frac{\Pi_1^{M*}(1, 0) - \Pi_1^{M*}(0, 1)}{1 - D_1^M(1, 0)}$$

such that non-exclusive contracts are chosen in equilibrium if, and only if, $\gamma^S \geq \tilde{\gamma}^M$. Else, exclusive contracts are chosen.

Proof. See Appendix A.10. □

Proposition 8 shows that the central insights of the baseline model also hold when consumers multihome. In this case, however, the fact that the Superstar goes exclusively on platform i only affects the consumer choice between multihoming and singlehoming on platform j . This is a relevant difference relative to the baseline model. Platform i has less incentive to accept an exclusive contract as the resulting gain generates less demand expansion making the threat of exclusivity with the rival less severe. However, under non-exclusivity, the threat of exclusivity

with the rival is absent. To this end, the non-exclusive agreement must be reached for free. These two forces go towards opposite directions and the cutoff under which exclusive dealing arises moves accordingly.

5.3. Two-sided Pricing

In most cases, platforms set a price on both sides of the market. In the music industry, artists are remunerated by platforms like Spotify and Tidal. In the app market, developers pay an annual fee to have their account, and so in many other markets. In what follows, we consider a scenario in which platform i sets a duple of prices $\{l_i, p_i\}$ to maximize profits, where p_i is the price set on the consumer side and l_i is the one on the fringe.

To shed some light in this respect, we make an example with uniform distributions of functions $F(\cdot)$ and $\Lambda(\cdot)$. We focus on the case in which consumers pay a positive price, whereas the price on the fringe can either be positive or negative depending on the relative size of indirect network externalities. Even though the small firms are now influenced by the price/subsidy when joining the market, the Superstar's decision remains affected only by the fierceness of the downstream competition. When ancillary revenues on the fringe are relatively small relative to consumers' indirect network externalities, small firms are subsidized ($l_i < 0$) for the externality they create. Under exclusivity, the response of any small firm to additional consumer switching from platform j to i is less reactive. So, the platform hosting the Superstar subsidizes the fringe even more. In the opposite case, consumers are more valuable for the fringe. As a result, the platform extracts more surplus by charging them a higher price under exclusivity.

Hence, exclusivity entails a direct effect on the consumer side and an indirect one on the fringe. By subsidizing or charging the latter, the platform mainly manages the size of the feedback effect due to indirect network externalities. The direct effect on the consumer side instead continues to only hinge upon platform differentiation. So, exclusivity emerges in equilibrium when consumers are more responsive, and non-exclusivity emerges otherwise. For details, see Appendix A.11.

6. Concluding Remarks

Exclusive contracts are commonly observed in different markets. This article studies the rationale behind the emergence of these contracts in markets with network externalities and the potential anti- or pro-competitive effects of such choices.

We find that exclusivity emerges as a profitable contractual choice when platform competition is more severe because consumers are very responsive to the presence of the Superstar. This effect is further magnified by the two-sidedness of the market as the *favoured* platform becomes more appealing for a large mass of firms, with some zero-homers and multihomers becoming singlehomers. These results are robust to several extensions and variations and allow for a comparison of market interactions in the presence and the absence of indirect network externalities.

There is supportive evidence of our mechanism in different industries in which platforms deal with top-rated agents. In the music industry, the presence of Superstars can generate positive and remunerative spillovers on small artists in the form of content discovery through playlists.

In the same vein, the mobile app market can be exposed to the positive spillover generated by the Superstars. For instance, Ershov (2018) provide evidence of demand-discovery and entry of new developers triggered by Superstar apps. In the e-sport, anecdotal evidence shows that when Richard Tyler Blevins (a.k.a. Ninja) left Twitch for Mixer, the latter experienced a boost in its app downloads.

In the supply chain industry, an agent offering patent rights for a technology that enhances consumer experience may either sign an exclusive licensing contract or non-exclusive licensing contracts. We conjecture that the manufacturer winning the exclusive right would attract more consumers as well as a larger cluster of ancillary suppliers to that product. This may result in cheaper production costs enhancing further a manufacturing firm's market power *vis-à-vis* the rival. The contractual choice will again depend on the possibility for the Superstar to extract surplus and generate sufficient demand expansion.

In the same vein, this article can also provide insights for the cloud platform market with open-source developers. This market features the co-existence of large firms (*e.g.*, VMware and Red Hat) and smaller open-source software developers. We conjecture that an exclusive deal between a big player and one clouding platform (*e.g.*, Amazon, IBM) may induce more small developers to offer exclusives as well.

Policy Lessons. This article yields several policy lessons for markets characterized by (positive) indirect network externalities. These are discussed below.

Policy Lesson no.1: Theory of harm needs adaptation to two-sided markets.

Indirect network externalities do matter. Our results suggest that exclusive dealing between a premium agent and platform(s) is not necessarily bad for welfare. Because of network effects, an exclusive contract might become the first-best choice in the industry, thereby benefiting fringe firms and final consumers. These results, hence, suggest that antitrust enforcers should be cautious when applying traditional one-sided theories of harm not initially designed for two-sided markets. In particular, current policymaking should be not overhauled. Instead, we suggest that policymakers should strongly consider the impact of indirect network externalities. Our results only hold in the presence of competition in the platform market.

Policy Lesson no.2: Banning exclusive dealing may lead to unintended effects.

Policy measures leading to a ban on exclusive dealing are undoubtedly detrimental to fringe firms who benefit from positive spillover of exclusivity from large firms. Potentially, consumers might be negatively impacted by a ban on exclusive contracts. We thus recommend policymakers to be circumspect when making market interventionist policies to correct for the apparent harm in the market caused by exclusivity.

Policy Lesson no.3: Competition drives exclusivity.

Our results suggest that the *raison d'être* of exclusive dealing in two-sided markets does not necessarily stem from anti-competitive conducts. Instead, it is the intense competition that ensures more responsive consumers and, indeed, a surplus to be extracted by the Superstar with an exclusive contract. Thus, policies devoted to sustaining fiercer competition in the market may strike with eventual policy goals of limiting the extent of exclusive arrangements. For instance, facilitating switching behaviors, through data portability, eventually may increase competitive pressure by lowering consumer attachment to their preferred platform and, in turn, lead to more exclusivity.

Policy Lesson no.4: More power to the platforms leads to more exclusivity.

Platforms such as Google, Facebook, Apple, and Amazon (a.k.a. GAFA) have immense market

power, due to strong network effects and data collection. Our results suggest that, in a contractual framework in which platforms retain some market power vis-à-vis the premium agents, there are more incentives for exclusivity relative to when the Superstar has full control over the contract. Again if exclusive deals are perceived to be welfare-diminishing, policy-makers should be aware that increasing bargaining power of platforms to balance that of the premium agent might be dangerous and create more exclusivity in the market as well.

Policy Lesson no.5: Due diligence when assessing vertical mergers with network effect.

Traditional theory associates two potential harmful effects of vertical mergers on the competition. First, a vertical merger increases the likelihood of anti-competitive conducts, such as input foreclosure. Second, when input foreclosure occurs, consumer prices are expected to increase. These concerns are discussed in the NHMG of the European Commission. In contrast, this work shows that, when network effects are at stake, the opposite holds. First, the likelihood of input foreclosure (through exclusive supply) is lower under vertical integration than under vertical separation. Second, when input foreclosure occurs, consumer prices are reduced. These results are in direct conflict with the current understanding based on traditional theory originated in one-sided markets. Indeed, this suggests that policymakers have to do their due diligence when assessing mergers with indirect network effects. It is important here to note that these results are sensitive to the existence of market competition. When scrutinizing vertical mergers in markets with network effects, policymakers should ensure that the competition remains sustainable and market tipping is ruled out. Specifically, if vertical integration or exclusivity lead the market to tip, the final outcome would be identical to that of foreclosure and should be deemed as anti-competitive.

References

- Abito, J. M. & Wright, J. (2008), 'Exclusive dealing with imperfect downstream competition', *international Journal of industrial organization* **26**(1), 227–246.
- Adachi, T. & Tremblay, M. J. (2019), 'Bargaining in two-sided markets', *Available at SSRN 3388383*.
- Aghion, P. & Bolton, P. (1987), 'Contracts as a barrier to entry', *American Economic Review* **77**(3), 388–401.
- Ambrus, A., Calvano, E. & Reisinger, M. (2016), 'Either or both competition: A "two-sided" theory of advertising with overlapping viewerships', *American Economic Journal: Microeconomics* **8**(3), 189–222.
- Anderson, S. P., Foros, Ø. & Kind, H. J. (2018), 'Competition for advertisers and for viewers in media markets', *The Economic Journal* **128**(608), 34–54.
- APA (2019), 'Annual Sales Survey. Audio Publishers Association'.
- Armstrong, M. (1999), 'Competition in the pay-tv market', *Journal of the Japanese and International Economies* **13**(4), 257–280.
- Armstrong, M. (2006), 'Competition in two-sided markets', *The RAND Journal of Economics* **37**(3), 668–691.

- Armstrong, M. & Wright, J. (2007), ‘Two-sided markets, competitive bottlenecks and exclusive contracts’, *Economic Theory* **32**(2), 353–380.
- Athey, S., Calvano, E. & Gans, J. S. (2016), ‘The impact of consumer multi-homing on advertising markets and media competition’, *Management Science* **64**(4), 1574–1590.
- Belleflamme, P. & Peitz, M. (2015), *Industrial organization: markets and strategies*, Cambridge University Press.
- Belleflamme, P. & Peitz, M. (2018), ‘Platforms and network effects’, *Handbook of Game Theory and Industrial Organization, Volume II: Applications* **2**, 286.
- Belleflamme, P. & Peitz, M. (2019a), ‘Managing competition on a two-sided platform’, *Journal of Economics & Management Strategy* **28**(1), 5–22.
- Belleflamme, P. & Peitz, M. (2019b), ‘Platform competition: Who benefits from multihoming?’, *International Journal of Industrial Organization* **64**, 1–26.
- Belleflamme, P. & Toulemonde, E. (2009), ‘Negative intra-group externalities in two-sided markets’, *International Economic Review* **50**(1), 245–272.
- Bernheim, B. D. & Whinston, M. D. (1998), ‘Incomplete contracts and strategic ambiguity’, *American Economic Review* pp. 902–932.
- Biglaiser, G., Calvano, E. & Crémer, J. (2019), ‘Incumbency advantage and its value’, *Journal of Economics & Management Strategy* **28**(1), 41–48.
- Bourreau, M., Hombert, J., Pouyet, J. & Schutz, N. (2011), ‘Upstream competition between vertically integrated firms’, *The Journal of Industrial Economics* **59**(4), 677–713.
- Caillaud, B. & Jullien, B. (2003), ‘Chicken & egg: Competition among intermediation service providers’, *RAND Journal of Economics* pp. 309–328.
- Calvano, E. & Polo, M. (2019), ‘Strategic differentiation by business models: Free-to-air and pay-tv’, *Economic Journal. Forthcoming* .
- Carroni, E., Madio, L. & Shekhar, S. (2019), ‘Superstars in two-sided markets: exclusives or not?’, *CESifo Working Papers No. 7853* .
- Choi, J. P. (2010), ‘Tying in two-sided markets with multi-homing’, *The Journal of Industrial Economics* **58**(3), 607–626.
- Crawford, G. S., Lee, R. S., Whinston, M. D. & Yurukoglu, A. (2018), ‘The welfare effects of vertical integration in multichannel television markets’, *Econometrica* **86**(3), 891–954.
- Crawford, G. S. & Yurukoglu, A. (2012), ‘The welfare effects of bundling in multichannel television markets’, *American Economic Review* **102**(2), 643–85.
- Damiano, E. & Li, H. (2007), ‘Price discrimination and efficient matching’, *Economic Theory* **30**(2), 243–263.

- D'Annunzio, A. (2017), 'Vertical integration in the tv market: Exclusive provision and program quality', *International Journal of Industrial Organization* **53**, 114–144.
- D'Annunzio, A. & Russo, A. (2019), 'Ad networks, consumer tracking, and privacy', *Management Science*, *Forthcoming* .
- De Meza, D. & Selvaggi, M. (2007), 'Exclusive contracts foster relationship-specific investment', *The RAND Journal of Economics* **38**(1), 85–97.
- Doganoglu, T. & Wright, J. (2010), 'Exclusive dealing with network effects', *International Journal of Industrial Organization* **28**(2), 145–154.
- Ershov, D. (2018), 'Competing with superstars in the mobile app market', *NET Institute* .
- Fudenberg, D. & Tirole, J. (2000), 'Customer poaching and brand switching', *RAND Journal of Economics* pp. 634–657.
- Fumagalli, C. & Motta, M. (2006), 'Exclusive dealing and entry, when buyers compete', *American Economic Review* **96**(3), 785–795.
- Fumagalli, C., Motta, M. & Persson, L. (2009), 'On the anticompetitive effect of exclusive dealing when entry by merger is possible', *The Journal of Industrial Economics* **57**(4), 785–811.
- Fumagalli, C., Motta, M. & Rønde, T. (2012), 'Exclusive dealing: investment promotion may facilitate inefficient foreclosure', *The Journal of Industrial Economics* **60**(4), 599–608.
- Gould, E. D., Pashigian, B. P. & Prendergast, C. J. (2005), 'Contracts, externalities, and incentives in shopping malls', *Review of Economics and Statistics* **87**(3), 411–422.
- Hagiu, A. (2006), 'Pricing and commitment by two-sided platforms', *The RAND Journal of Economics* **37**(3), 720–737.
- Hagiu, A. & Lee, R. S. (2011), 'Exclusivity and control', *Journal of Economics & Management Strategy* **20**(3), 679–708.
- IFPI (2019), 'IFPI Global Music report 2019. State of the industry'.
- Innes, R. & Sexton, R. J. (1994), 'Strategic buyers and exclusionary contracts', *The American Economic Review* pp. 566–584.
- Ishihara, A. & Oki, R. (2017), 'Exclusive content in two-sided markets', *Mimeo* .
- Jehiel, P. & Moldovanu, B. (2000), 'Auctions with downstream interaction among buyers', *The RAND Journal of Economics* pp. 768–791.
- Jullien, B. (2011), 'Competition in multi-sided markets: Divide and conquer', *American Economic Journal: Microeconomics* **3**(4), 186–220.
- Karle, H., Peitz, M. & Reisinger, M. (2019), 'Segmentation versus agglomeration: Competition between platforms with competitive sellers', *Journal of Political Economy*. *Forthcoming*. .

- Kourandi, F., Krämer, J. & Valletti, T. (2015), ‘Net neutrality, exclusivity contracts, and internet fragmentation’, *Information Systems Research* **26**(2), 320–338.
- Lee, R. S. (2013), ‘Vertical integration and exclusivity in platform and two-sided markets’, *American Economic Review* **103**(7), 2960–3000.
- Lentzner, J. (1977), ‘The antitrust implications of radius clauses in shopping center leases’, *University of Detroit Journal of Urban Law* **55**, 1.
- Markovich, S. & Yehezkel, Y. (2018), ‘Group hug: Platform competition with user-groups’, *Available at SSRN 3259517*.
- Montes, R., Sand-Zantman, W. & Valletti, T. (2019), ‘The Value of Personal Information in Online Markets with Endogenous Privacy’, *Management Science* **65**(3), 1342–1362.
- NHMG (2008), ‘Guidelines on the assessment of non-horizontal mergers under the Council Regulation on the control of concentrations between undertakings. European Commission. Official Journal of the European Union. 2008/c 265/07’.
- Nocke, V. & Rey, P. (2018), ‘Exclusive dealing and vertical integration in interlocking relationships’, *Journal of Economic Theory* **177**, 183–221.
- Ordover, J. A., Saloner, G. & Salop, S. C. (1990), ‘Equilibrium vertical foreclosure’, *The American Economic Review* pp. 127–142.
- Pashigian, B. P. & Gould, E. D. (1998), ‘Internalizing externalities: the pricing of space in shopping malls’, *The Journal of Law and Economics* **41**(1), 115–142.
- Pouyet, J. & Trégouët, T. (2018), ‘Vertical mergers in platform markets’, *Unpublished manuscript*.
- Rasch, A. & Wenzel, T. (2013), ‘Piracy in a two-sided software market’, *Journal of Economic Behavior & Organization* **88**, 78–89.
- Rasmusen, E. B., Ramseyer, J. M. & Wiley Jr, J. S. (1991), ‘Naked exclusion’, *The American Economic Review* pp. 1137–1145.
- Rey, P. & Tirole, J. (2007), ‘A primer on foreclosure’, *Handbook of industrial organization* **3**, 2145–2220.
- Rochet, J.-C. & Tirole, J. (2003), ‘Platform competition in two-sided markets’, *Journal of the European Economic Association* **1**(4), 990–1029.
- Rochet, J.-C. & Tirole, J. (2006), ‘Two-sided markets: a progress report’, *The RAND Journal of Economics* **37**(3), 645–667.
- Salinger, M. A. (1988), ‘Vertical Mergers and Market Foreclosure*’, *The Quarterly Journal of Economics* **103**(2), 345–356.
- Segal, I. R. & Whinston, M. D. (2000), ‘Exclusive contracts and protection of investments’, *RAND Journal of Economics* pp. 603–633.

StreamLab (2018), 'Live Streaming Q3'18 Report, <https://blog.streamlabs.com/live-streaming-q318-report-40-of-twitch-using-slobs-pubg-popularity-on-the-decline-mixer-fb923cdbc70>'.

Weeds, H. (2016), 'Tv wars: Exclusive content and platform competition in pay tv', *The Economic Journal* **126**(594), 1600–1633.

Appendix A.

A.1. Proof of Lemma 1

We derive the optimal price and resulting demand for any given value of g_1 and g_2 . For ease of exposition, let us state the demand functions on both sides of the market:

$$D_1 = F(m^*(g_1, g_2)), \quad D_2 = 1 - F(m^*(g_1, g_2)). \quad (\text{A-1})$$

$$N_1 = \Lambda(\gamma D_1^*), \quad N_2 = \Lambda(\gamma D_2^*).$$

First, we derive the effect of price changes on demand for a given platform i , which is defined as follows:

$$\frac{\partial D_i}{\partial p_i} = f(g_i, g_j) \left[\theta \left(\frac{\partial N_i}{\partial p_i} - \frac{\partial N_j}{\partial p_i} \right) - 1 \right], \quad (\text{A-2})$$

where:

$$\begin{aligned} \frac{\partial N_i}{\partial p_i} - \frac{\partial N_j}{\partial p_i} &= \gamma \left[\lambda(\gamma D_i^*) \frac{\partial D_i}{\partial p_i} + \lambda(\gamma D_j^*) \frac{\partial D_i}{\partial p_i} \right] \\ &= \gamma \frac{\partial D_i}{\partial p_i} \left[\lambda(\gamma D_i^*) + \lambda(\gamma D_j^*) \right], \end{aligned} \quad (\text{A-3})$$

Substituting it into (A-3) and rearranging the right-hand side (RHS) and the left-hand side (LHS), the effect of a price change on demands is given by:

$$\frac{\partial D_i}{\partial p_i} = \frac{-f(g_i, g_j)}{[1 - f(g_i, g_j)\theta\gamma(\lambda(\gamma D_i^*) + \lambda(\gamma D_j^*))]} = -\frac{\partial D_j}{\partial p_j}. \quad (\text{A-4})$$

Next, consider the first-order conditions (FOCs) resulting from the platforms' profit maximization, then

$$\frac{\partial \Pi_i}{\partial p_i} = D_i + p_i \frac{\partial D_i}{\partial p_i} = 0 \Leftrightarrow p_i = -\frac{D_i}{\frac{\partial D_i}{\partial p_i}}. \quad (\text{A-5})$$

Plugging (A-4) and (A-1) into (A-5), we get the following best-responses, which characterize prices in Lemma 1:

$$\begin{aligned} p_1(m^*) &= \frac{F(m^*)}{f(m^*)} - F(m^*)\gamma\theta[\lambda(\gamma D_1) + \lambda(\gamma D_2)], \\ p_2(m^*) &= \frac{(1 - F(m^*))}{f(m^*)} - (1 - F(m^*))\gamma\theta[\lambda(\gamma D_1) + \lambda(\gamma D_2)]. \end{aligned}$$

A.2. Proof of Lemma 2

Consider the problem faced by platform i when the Superstar is absent or offers a non-exclusive contract, *i.e.*, $g_1 = g_2 = g \in \{1, 0\}$. From the FOCs, we have:

$$\begin{aligned}\frac{\partial \Pi_1}{\partial p_1} &= D_1(g, g) + p_1 \left[- \frac{f(g, g)}{1 - f(g, g)\gamma\theta(\lambda(\gamma D_1^*) + \lambda(\gamma D_2^*))} \right], \\ \frac{\partial \Pi_2}{\partial p_2} &= D_2(g, g) + p_2 \left[- \frac{f(g, g)}{1 - f(g, g)\gamma\theta(\lambda(\gamma D_1^*) + \lambda(\gamma D_2^*))} \right].\end{aligned}$$

If consumers' beliefs are correct and symmetric with respect to the platforms, then one of the equilibria is fully symmetric. Hence, by imposing $p_1 = p_2$, then $D_1^* = D_2^* = F(g, g) = F(m^* = 0) = 1/2$ and, as a result, $N_1^* = N_2^*$. The equilibrium prices are:

$$\begin{aligned}p_1^* = p_2^* &= \frac{F(0)}{f(0)} [1 - f(0)\gamma\theta(\lambda(\gamma D_1^*) + \lambda(\gamma D_2^*))] \\ &= \frac{1}{2f(0)} - \gamma\theta\lambda(\gamma/2),\end{aligned}$$

given that $F(0, 0) = 1/2$ and $\lambda(\gamma D_1^*) + \lambda(\gamma D_2^*) = 2\lambda(\gamma/2)$. To conclude the proof, note that all active firms multihome, $N_1^* = N_2^* = \Lambda(\gamma/2)$. All the others, zero-home.

A.3. Proof of Lemma 3

Consider an exclusive contract between the Superstar and platform 1 ($g_1 = 1, g_2 = 0$). From the FOCs of the platforms, we have:

$$\begin{aligned}\frac{\partial \Pi_1}{\partial p_1} &= D_1(1, 0) + p_1 \left[- \frac{f(1, 0)}{1 - f(1, 0)\gamma\theta(\lambda(\gamma D_1) + \lambda(\gamma D_2))} \right], \\ \frac{\partial \Pi_2}{\partial p_2} &= D_2(0, 1) + p_2 \left[- \frac{f(1, 0)}{1 - f(1, 0)\gamma\theta(\lambda(\gamma D_1) + \lambda(\gamma D_2))} \right].\end{aligned}$$

To fully study the effect on demands and prices, we should consider that a consumer with preference m is willing to go to platform 1 if $u_1(1, 0) > u_2(0, 1)$. To see whether $p_1(1, 0) > p_2(0, 1)$, and, as a consequence, $D_1(1, 0) > D_2(0, 1)$ for any $\phi > (p_1 - p_2)$, we must look at the effect of ϕ on prices. Hence, taking the total derivative of the FOCs, we get:

$$\begin{aligned}\frac{d^2 \Pi_1}{dp_1 d\phi} &= \frac{\partial^2 \Pi_1}{\partial p_1^2} \frac{\partial p_1}{\partial \phi} + \frac{\partial^2 \Pi_1}{\partial p_1 \partial \phi} + \frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} \frac{\partial p_2}{\partial \phi} = 0 \\ \frac{d^2 \Pi_2}{dp_2 d\phi} &= \frac{\partial^2 \Pi_2}{\partial p_2 \partial p_1} \frac{\partial p_1}{\partial \phi} + \frac{\partial^2 \Pi_2}{\partial p_2 \partial \phi} + \frac{\partial^2 \Pi_2}{\partial p_2^2} \frac{\partial p_2}{\partial \phi} = 0\end{aligned}$$

By solving the above system of equations, we get:

$$\begin{aligned}\frac{\partial p_1}{\partial \phi} &= \left\{ \frac{\partial^2 \Pi_1}{\partial p_1 \partial \phi} \frac{\partial^2 \Pi_2}{\partial p_2^2} - \frac{\partial^2 \Pi_2}{\partial p_2 \partial \phi} \frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} \right\} \Phi^{-1} \\ \frac{\partial p_2}{\partial \phi} &= \left\{ \frac{\partial^2 \Pi_1}{\partial p_1^2} \frac{\partial^2 \Pi_2}{\partial p_2 \partial \phi} - \frac{\partial^2 \Pi_1}{\partial p_1 \partial \phi} \frac{\partial^2 \Pi_2}{\partial p_2 \partial p_1} \right\} \Phi^{-1}\end{aligned}\tag{A-6}$$

where $\Phi \equiv \frac{\partial^2 \Pi_2}{\partial p_2 \partial p_1} \frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} - \frac{\partial^2 \Pi_1}{\partial p_1^2} \frac{\partial^2 \Pi_2}{\partial p_2^2} < 0$ and $\text{sign}\left(\frac{\partial^2 \Pi_2}{\partial p_2^2}\right) = \text{sign}\left(\frac{\partial^2 \Pi_1}{\partial p_1^2}\right) < 0$ to ensure concavity. Further investigation is, instead, required for the terms in the numerator. We proceed as follows.

- (i) Show that $\frac{\partial p_1}{\partial \phi} > 0$ and $\frac{\partial p_2}{\partial \phi} < 0$. Let us consider $\frac{\partial^2 \Pi_1}{\partial p_1 \partial \phi}$ and $\frac{\partial^2 \Pi_2}{\partial p_2 \partial \phi}$, which are determined as follows:

$$\begin{aligned} \frac{\partial^2 \Pi_1}{\partial p_1 \partial \phi} &= f(m^*) \frac{\partial m^*}{\partial \phi} + p_1 \frac{\partial X}{\partial m} \frac{\partial m^*}{\partial \phi}, \\ \frac{\partial^2 \Pi_2}{\partial p_2 \partial \phi} &= -f(m^*) \frac{\partial m^*}{\partial \phi} + p_2 \frac{\partial X}{\partial m} \frac{\partial m^*}{\partial \phi}, \end{aligned} \quad (\text{A-7})$$

where

$$X = -\frac{f(m^*)}{1 - f(m^*)\gamma\theta[\lambda(\gamma D_1) + \lambda(\gamma D_2)]} < 0$$

and X increases in m by Assumption 1. By considering the effect of ϕ on m^* for given prices p_1 and p_2 , we obtain the following results:

$$\frac{\partial m^*}{\partial \phi} = 1 + \theta \left(\frac{\partial N_1}{\partial \phi} - \frac{\partial N_2}{\partial \phi} \right),$$

and

$$\frac{\partial N_1}{\partial \phi} = \lambda(\gamma D_1)\gamma f(m^*) \frac{\partial m}{\partial \phi}, \quad \frac{\partial N_2}{\partial \phi} = -\lambda(\gamma D_2)\gamma f(m^*) \frac{\partial m}{\partial \phi}.$$

Solving the above system of equations, we then get:

$$\frac{\partial m^*}{\partial \phi} = \frac{1}{1 - \theta\gamma f(m^*)[\lambda(\gamma D_1) + \lambda(\gamma D_2)]}$$

and

$$\frac{\partial N_1}{\partial \phi} = \frac{\lambda(\gamma D_1)\gamma f(m^*)}{1 - \theta\gamma f(m^*)[\lambda(\gamma D_1) + \lambda(\gamma D_2)]}, \quad \frac{\partial N_2}{\partial \phi} = -\frac{\lambda(\gamma D_2)\gamma f(m^*)}{1 - \theta\gamma f(m^*)[\lambda(\gamma D_1) + \lambda(\gamma D_2)]}.$$

It follows that $\frac{\partial m^*}{\partial \phi} > 0$, $\frac{\partial N_1}{\partial \phi} > 0$, and $\frac{\partial N_2}{\partial \phi} < 0$. This proves that $\frac{\partial^2 \Pi_1}{\partial p_1 \partial \phi} > 0$ by simply substituting $\frac{\partial m^*}{\partial \phi} > 0$ into (A-7). Moreover, $\frac{\partial^2 \Pi_2}{\partial p_2 \partial \phi} < 0$ if $-f(m^*) + p_2 \frac{\partial X}{\partial m} < 0$, which is a sufficient condition to ensure concavity of Π_2 .

By exploiting $\frac{\partial m^*}{\partial \phi} = \frac{\partial m^*}{\partial p_2} = -\frac{\partial m^*}{\partial p_1}$, we also have:

$$\frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} = \frac{\partial^2 \Pi_1}{\partial p_1 \partial \phi} > 0, \quad \frac{\partial^2 \Pi_2}{\partial p_1 \partial p_2} = -\frac{\partial^2 \Pi_2}{\partial p_2 \partial \phi} > 0.$$

Using the above expressions, we also have:

$$\begin{aligned}\frac{\partial^2 \Pi_1}{\partial p_1^2} &= \frac{\partial^2 \Pi_1}{\partial p_1 \partial \phi} + X = \frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} + X, \\ \frac{\partial^2 \Pi_2}{\partial p_2^2} &= \frac{\partial^2 \Pi_2}{\partial p_2 \partial \phi} + X = -\frac{\partial^2 \Pi_2}{\partial p_1 \partial p_2} + X.\end{aligned}\tag{A-8}$$

Plugging into (A-6), we then have:

$$\begin{aligned}\frac{\partial p_1}{\partial \phi} &= \frac{\partial^2 \Pi_1}{\partial p_1 \partial \phi} X \Phi^{-1} > 0, \\ \frac{\partial p_2}{\partial \phi} &= \frac{\partial^2 \Pi_2}{\partial p_2 \partial \phi} \underbrace{\left(\frac{\partial^2 \Pi_1}{\partial p_1^2} + \frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} \right)}_{< 0} \Phi^{-1} < 0.\end{aligned}\tag{A-9}$$

which then implies that $\frac{\partial p_1}{\partial \phi} - \frac{\partial p_2}{\partial \phi} > 0$, as we show below.

$$\frac{\partial p_1}{\partial \phi} - \frac{\partial p_2}{\partial \phi} = \Phi^{-1} \left[\frac{\partial^2 \Pi_1}{\partial p_1 \partial \phi} X - \frac{\partial^2 \Pi_2}{\partial p_2 \partial \phi} \underbrace{\left(\frac{\partial^2 \Pi_1}{\partial p_1^2} + \frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} \right)}_{< 0} \right] > 0\tag{A-10}$$

- (ii) To see whether also the following condition is satisfied, $1 > \frac{\partial p_1}{\partial \phi} - \frac{\partial p_2}{\partial \phi} > 0$, we must check whether the denominator Φ is larger in absolute values than the numerator. Hence,

$$\frac{\partial^2 \Pi_1}{\partial p_1^2} \frac{\partial^2 \Pi_2}{\partial p_2^2} - \frac{\partial^2 \Pi_2}{\partial p_2 \partial p_1} \frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} > \frac{\partial^2 \Pi_2}{\partial p_2 \partial \phi} \left(\frac{\partial^2 \Pi_1}{\partial p_1^2} + \frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} \right) - \frac{\partial^2 \Pi_1}{\partial p_1 \partial \phi} X,$$

As $\frac{\partial^2 \Pi_2}{\partial p_2 \partial \phi} = -\frac{\partial^2 \Pi_2}{\partial p_2 \partial p_1}$, then the above expression simplifies to:

$$\frac{\partial^2 \Pi_1}{\partial p_1^2} \left(\frac{\partial^2 \Pi_2}{\partial p_2^2} - \frac{\partial^2 \Pi_2}{\partial p_2 \partial \phi} \right) > -\frac{\partial^2 \Pi_1}{\partial p_1 \partial \phi} X,$$

Moreover, from (A-8), we have $\frac{\partial^2 \Pi_2}{\partial p_2^2} - \frac{\partial^2 \Pi_2}{\partial p_2 \partial \phi} = X < 0$, which implies that:

$$X \left[\frac{\partial^2 \Pi_1}{\partial p_1^2} + \frac{\partial^2 \Pi_1}{\partial p_1 \partial \phi} \right] > 0,$$

which is always true as $X < 0$ and the term within the squared brackets is negative. Hence, in (A-10), the denominator is larger than the numerator and it proves that $0 < \frac{\partial p_1}{\partial \phi} - \frac{\partial p_2}{\partial \phi} < 1$.

- (iii) Given (i) and (ii), the equilibrium demand $D_1(m^*)$ increases. Formally, we have:

$$\frac{\partial D_1(m^*)}{\partial \phi} = f(m^*) \left[1 + \theta \gamma \frac{\partial D_1(m^*)}{\partial \phi} [\lambda(\gamma D_1) + \lambda(\gamma D_2)] - \left(\frac{\partial p_1}{\partial \phi} - \frac{\partial p_2}{\partial \phi} \right) \right],$$

which can be rearranged as:

$$\frac{\partial D_1(m^*)}{\partial \phi} \left[1 - f(m^*)\theta\gamma[\lambda(\gamma D_1) + \lambda(\gamma D_2)] \right] = f(m^*) \left[1 - \left(\frac{\partial p_1}{\partial \phi} - \frac{\partial p_2}{\partial \phi} \right) \right].$$

Hence, the effect of ϕ on the demand of the platform exclusively hosting the Superstar is:

$$\frac{\partial D_1(m^*)}{\partial \phi} = \frac{f(m^*) \left[1 - \left(\frac{\partial p_1}{\partial \phi} - \frac{\partial p_2}{\partial \phi} \right) \right]}{1 - f(m^*)\theta\gamma[\lambda(\gamma D_1) + \lambda(\gamma D_2)]} > 0.$$

Note that, under non-exclusivity, demands do not respond to ϕ and $\frac{\partial D_1(1,1)}{\partial \phi} = 0$.

A.4. Proof of Corollary 1

Consider how the critical value $\tilde{\gamma}^S$ in Proposition 2 changes with ϕ ,

$$\frac{\partial \tilde{\gamma}^S}{\partial \phi} = \frac{\partial F(1,0)}{\partial \phi} \left[\frac{(p_1(1,0) - p_2(0,1)(1 - D_1(1,0)) + (\Pi_1^*(1,0) + \Pi_1^*(0,1) - 2\Pi_1^*(1,1)))}{(1 - D_1(1,0))^2} \right] > 0$$

The above comparative static is positive. Note that $D_i(0,1) = 1 - D_i(1,0)$ for $i \in 1,2$. Moreover, $p_1(1,0) > p_1(0,1)$ and $\frac{\partial F(1,0)}{\partial \phi} > 0$. Hence, in the relevant parameter space, $\tilde{\gamma}^S$ increases as ϕ increases. This implies that the probability of an exclusive contract increases with ϕ .

A.5. Proof of Proposition 3

Consider the bargaining problem of the Superstar when offering an exclusive contract to platform 1. This is formally expressed as follows:

$$\max_{T_1(1,0)} [(\gamma^S D_1(1,0) + T_1(1,0)) - (\gamma^S D_2(1,0) + T_2(1,0))]^\beta [\Pi_1^*(1,0) - T_1(1,0) - \Pi_1^*(0,1)]^{1-\beta}.$$

Note that the first square bracket is composed of two terms, which denote the inside and outside option of the Superstar, respectively. The former represents revenues from offering the contract to platform 1, whereas the latter represents revenues from offering the contract to platform 2. The second square bracket comprises of the difference between the inside and outside option of platform 1. From the FOC, we then obtain that:

$$T_1(1,0) = \beta[\Pi_1^*(1,0) - \Pi_1^*(0,1)] - (1 - \beta)[\gamma^S(D_1(1,0) - D_2(0,1)) - T_2(0,1)].$$

As $T_1(1,0) = T_2(1,0)$ and $D_1(1,0) = D_2(1,0)$, the optimal contractual tariff under exclusivity is equal to:

$$T_1^*(1,0) = \Pi_1^*(1,0) - \Pi_1^*(0,1).$$

Note that the above expression is the same as in the baseline framework with $\beta = 1$. Hence, when designing exclusive contracts, β does not play any role.

Consider now a non-exclusive contract and the decision to offer it to platform i . Formally, the bargaining problem is to maximize the following by choosing $T_i(1, 1)$,

$$[(\gamma^S + T_i(1, 1) + T_j(1, 1)) - (\gamma^S D_j(1, 0) + T_j(1, 1))]^\beta [\Pi_i^*(1, 1) - T_i(1, 1) - \Pi_i^*(0, 1)]^{1-\beta}.$$

From the FOCs, and solving for the optimal (symmetric) tariffs, we obtain that:

$$T_i^*(1, 1) = \beta(\Pi_i^*(1, 1) - \Pi_i^*(0, 1)) - (1 - \beta)(\gamma^S(1 - D_j(1, 0))),$$

and $T_i^*(1, 1) = T_j^*(1, 1)$. We also note that $D_j(1, 0) = D_i(1, 0)$. The total profit of the Superstar is:

$$\begin{aligned} \Pi^S(1, 1) &= \gamma^S + T_1^*(1, 1) + T_2^*(1, 1) \\ &= \gamma^S + 2[\beta(\Pi_i^*(1, 1) - \Pi_i^*(0, 1)) - (1 - \beta)(\gamma^S(1 - D_i(1, 0)))], \end{aligned}$$

It is straightforward to see that the non-exclusive profit of the Superstar is increasing in β . By comparing profits under the two contractual designs, we can see that:

- If $0 < \beta \leq 1/2$, then $\Pi^S(1, 0) \geq \Pi^S(1, 1)$. Hence, exclusivity always emerges.
- If $1/2 < \beta \leq 1$, then $\Pi^S(1, 0) \geq \Pi^S(1, 1)$ if, and only if, $\gamma^S < \bar{\gamma}^S$, where

$$\bar{\gamma}^S =: \frac{\Pi_1^*(1, 0) + (2\beta - 1)\Pi_1^*(0, 1) - 2\beta\Pi_1^*(1, 1)}{(2\beta - 1)(1 - D_1(1, 0))}$$

Note that in $[1/2, 1)$, $\bar{\gamma}^S \geq \tilde{\gamma}^S$. As a result, exclusivity is more likely to arise than with full bargaining power for the Superstar.

A.6. Proof of Proposition 4

Consider now the surplus of small firms in the two regimes. Under non-exclusivity, $FS_1(1, 1) = FS_2(1, 1)$, and total firm surplus is:

$$FS(1, 1) = \int_0^{\gamma/2} [(\gamma - 2k)\lambda(k)]dk.$$

Consider now the fringe surplus with an exclusive contract on platform 1. First, we consider the surplus of firms who join the platform without the exclusive contract (*i.e.*, $k \in [0, \gamma D_2^*]$):

$$FS_2(0, 1) = \int_0^{\gamma D_2^*} [(\gamma * D_2^* - k)\lambda(k)]dk.$$

Consider now the surplus of firms joining the platform with the exclusive contract. Some of these firms singlehome, whereas others multihome, so their surplus is as follows:

$$FS_1(1, 0) = \int_0^{\gamma D_1^*} [(\gamma D_1^* - k)\lambda(k)]dk.$$

Total surplus under exclusivity is the sum of the two above expressions:

$$FS(1, 0) = \int_0^{\gamma D_2^*} [(\gamma D_2^* - k)\lambda(k)]dk + \int_0^{\gamma D_1^*} [\lambda(k)(\gamma D_1^* - k)]dk.$$

Compare now the gain/loss generated by exclusivity, then $\Delta FS = FS(1, 0) - FS(1, 1)$, which can then be rewritten as:

$$\Delta FS = \int_0^{\gamma D_2^*} [(\gamma D_2^* - k)\lambda(k)]dk + \int_0^{\gamma D_1^*} [(\gamma D_1^* - k)\lambda(k)]dk - \int_0^{\gamma/2} [(\gamma - 2k)\lambda(k)]dk. \quad (\text{A-11})$$

To show that there is a gain from exclusivity, it suffices to show that $\Delta FS > 0$. We note that (A-11) can be rewritten as follows:

$$\begin{aligned} \Delta FS &= \int_0^{\gamma D_2^*} [(\gamma - 2k)\lambda(k)]dk + \int_{\gamma D_2^*}^{\gamma D_1^*} [(\gamma D_1^* - k)\lambda(k)]dk - \int_0^{\gamma/2} [(\gamma - 2k)\lambda(k)]dk, \\ &= \int_{\gamma D_2^*}^{\gamma D_1^*} [(\gamma D_1^* - k)\lambda(k)]dk - \int_{\gamma D_2^*}^{\gamma/2} [(\gamma - 2k)\lambda(k)]dk, \end{aligned}$$

which can be further simplified as:

$$\Delta FS = \int_{\gamma/2}^{\gamma D_1^*} [(\gamma D_1^* - k)\lambda(k)]dk - \int_{\gamma D_2^*}^{\gamma/2} [(\gamma D_2^* - k)\lambda(k)]dk.$$

Integration by parts imply:

$$\Delta FS = \int_{\gamma/2}^{\gamma D_1^*} \Lambda(k)dk - \int_{\gamma D_2^*}^{\gamma/2} \Lambda(k)dk - \left(\Lambda(k)(\gamma D_2^* - k) \right)_{\gamma D_2^*}^{\gamma/2} + \left(\Lambda(k)(\gamma D_1^* - k) \right)_{\gamma/2}^{\gamma D_1^*},$$

and, hence,

$$\Delta FS = \int_{\gamma/2}^{\gamma D_1^*} \Lambda(k)dk - \int_{\gamma D_2^*}^{\gamma/2} \Lambda(k)dk. \quad (\text{A-12})$$

Note that $\Delta FS > 0$ if, and only if, $\int_{\gamma/2}^{\gamma D_1^*} \Lambda(k)dk - \int_{\gamma D_2^*}^{\gamma/2} \Lambda(k)dk > 0$. Using the Simpson's Rule to approximate the value of the integrals, it follows

$$\int_{\gamma/2}^{\gamma D_1^*} \Lambda(k)dk \simeq \frac{\Delta x}{3}[y_0 + 4y_1 + 2y_2 + 4y_3 + \dots + y_n],$$

where n be an even number, $\Delta x = (\gamma D_1^* - \gamma/2)/n$, $y_0 = \Lambda(\gamma/2 + \Delta x)$, $y_1 = \Lambda(\gamma/2 + 2\Delta x)$, $y_2 = \Lambda(\gamma/2 + 3\Delta x)$, and so on and so forth. Similarly,

$$\int_{\gamma D_2^*}^{\gamma/2} \Lambda(k)dk \simeq \frac{\Delta x'}{3}[y'_0 + 4y'_1 + 2y'_2 + 4y'_3 + \dots + y'_n],$$

with $\Delta x' = (\gamma/2 - \gamma D_2^*)/n$, $y'_0 = \Lambda(\gamma D_2^* + \Delta x')$, $y'_1 = \Lambda(\gamma D_2^* + 2\Delta x')$, $y'_2 = \Lambda(\gamma D_2^* + 3\Delta x')$, and so on and so forth. Indeed, we can rewrite (A-12) as:

$$\frac{\Delta x}{3}[y_0 + 4y_1 + 2y_2 + 4y_3 + \dots + y_n] - \frac{\Delta x'}{3}[y'_0 + 4y'_1 + 2y'_2 + 4y'_3 + \dots + y'_n] \quad (\text{A-13})$$

Knowing that $\gamma D_1^* - \gamma/2 = \gamma/2 - \gamma D_2^*$ as $\frac{\partial N_1^*}{\partial \phi} = -\frac{\partial N_2^*}{\partial \phi}$, we then have:

$$\frac{\Delta x'}{3} \left[(y_0 - y'_0) + 4(y_1 - y'_1) + 2(y_2 - y'_2) + 4(y_3 - y'_3) + \dots + (y_n - y'_n) \right],$$

where $(y_0 - y'_0) = \Lambda(\gamma/2 - \gamma D_2^*) > 0$, and

$$(y_1 - y'_1) = \Lambda \left[\frac{\gamma}{2} + \frac{2}{n} \left(\gamma D_1^* - \frac{\gamma}{2} \right) \right] - \Lambda \left[\gamma D_2^* + \frac{2}{n} \left(\frac{\gamma}{2} - \gamma D_2^* \right) \right] > 0,$$

$$(y_2 - y'_2) = \Lambda \left[\frac{\gamma}{2} + \frac{3}{n} \left(\gamma D_1^* - \frac{\gamma}{2} \right) \right] - \Lambda \left[\gamma D_2^* + \frac{3}{n} \left(\frac{\gamma}{2} - \gamma D_2^* \right) \right] > 0,$$

and so on and so forth. As a result, (A-13) is positive and this proves that $\Delta FS > 0$.

A.7. Proof of Proposition 5

First, consider when the Superstar is non-exclusive, *i.e.*, $g_1 = g_2 = 1$. Consumer surplus on platform 1 is:

$$CS_1(1, 1) = \int_{\underline{m}}^0 [(v + \phi + \theta N_1^*(1, 1) - p_1^*(1, 1) - m/2)] f(m) dm.$$

Integration by parts imply that the above expression is equivalent to

$$CS_1(1, 1) = \frac{1}{2} [v + \phi + \theta N_1^*(1, 1) - p_1^*(1, 1)] + \int_{\underline{m}}^0 \frac{F(m)}{2} dm,$$

as $F(0) = 1/2$ and $F(\underline{m}) = 0$. Using the same reasoning, consumer surplus on platform 2 is:

$$CS_2(1, 1) = \frac{1}{2} [v + \phi + \theta N_2^*(1, 1) - p_2^*(1, 1) + \bar{m}] - \int_0^{\bar{m}} \frac{F(m)}{2} dm.$$

Knowing that, under symmetry, $p_1^*(1, 1) = p_2^*(1, 1)$, $N_2^* = N_1^* = \Lambda(\gamma/2)$, $F(m^*) = F(0) = 1/2$, and $\int_0^{\bar{m}} \frac{F(m)}{2} dm = -\int_{\underline{m}}^0 \frac{F(m)}{2} dm$, then total consumer is equal to

$$CS(1, 1) = v + \phi + \theta \Lambda(\gamma/2) - p^*(1, 1) + \bar{m}/2 \quad (\text{A-14})$$

where $p^*(1, 1) := p_1^*(1, 1) = p_2^*(1, 1)$. It is immediate to note that the presence of the Superstar increases consumer welfare, as $CS(1, 1)$ increases with ϕ .

Second, consider how consumer surplus varies with exclusivity, say in platform 1. The

consumer surplus on platform 1 is:

$$CS_1(1, 0) = \int_{\underline{m}}^0 [v + \phi + \theta N_1^*(1, 0) - p_1^*(1, 0) - m/2] f(m) dm + \int_0^{m^*} [v + \phi + \theta N_1^*(1, 0) - p_1^*(1, 0) - m/2] f(m) dm.$$

Simplifying, we get

$$CS_1(1, 0) = \int_{\underline{m}}^{m^*} \frac{F(m)}{2} dm + [v + \phi + \theta N_1^*(1, 0) - p_1^*(1, 0) - m^*/2] F(m^*).$$

Similarly, consumer surplus on platform 2 is equal to:

$$CS_2(0, 1) = \int_{m^*}^{\bar{m}} [v + \theta N_2^*(0, 1) - p_2^*(0, 1) + m/2] f(m) dm,$$

which can be expressed as follows:

$$CS_2(0, 1) = (1 - F(m^*)) [v + \theta N_2^*(0, 1) - p_2^*(0, 1)] + \frac{1}{2} (\bar{m} - m^* F(m^*)) - \int_{m^*}^{\bar{m}} \frac{F(m)}{2} dm.$$

Summing up $CS_1(1, 0)$ and $CS_2(0, 1)$, total consumer surplus under exclusivity, which we denote by $CS(1, 0)$, is:

$$CS(1, 0) = \frac{1}{2} \left[\int_{\underline{m}}^{m^*} F(m) dm - \int_{m^*}^{\bar{m}} F(m) dm \right] + [v + \phi + \theta N_1^*(1, 0) - p_1^*(1, 0) - m^*/2] F(m^*) + (1 - F(m^*)) [v + \theta N_2^*(0, 1) - p_2^*(0, 1)] + \frac{1}{2} (\bar{m} - m^* F(m^*)). \quad (\text{A-15})$$

As $F(\cdot)$ is symmetric around 0, the value of $\bar{m} - m^*$ is equal to the value of $m^* - \bar{m}$. By exploiting this argument, we have

$$\int_{\underline{m}}^{m^*} F(m) dm - \int_{m^*}^{\bar{m}} F(m) dm \equiv \int_{\underline{m}}^{m^* - \bar{m}} F(m) dm + \int_{m^* - \bar{m}}^{m^*} F(m) dm - \int_{m^*}^{\bar{m}} F(m) dm.$$

We note that the first and the third terms on the RHS cancel out. The second term can further be simplified by exploiting symmetry of $F(\cdot)$ around 0. Hence, the above expression can be rewritten as follows:

$$\int_{m^* - \bar{m}}^{m^*} F(m) dm = \int_{m^* - \bar{m}}^0 F(m) dm + \int_0^{m^*} F(m) dm = 2 \int_0^{m^*} F(m) dm.$$

Using the above simplification, consumer surplus under exclusivity in (A-15) can be rewritten as follows:

$$CS(1, 0) = \int_0^{m^*} F(m) dm + [v + \phi + \theta N_1^*(1, 0) - p_1^*(1, 0) - m^*/2] F(m^*) + (1 - F(m^*)) [v + \theta N_2^*(0, 1) - p_2^*(0, 1)] + \frac{1}{2} (\bar{m} - m^* F(m^*)). \quad (\text{A-16})$$

Impact of exclusivity on CS on platform 1. To compare the consumer surplus in the two regimes, denote $\Delta CS_1 = CS_1(1, 0) - CS_1(1, 1)$ the net gain (loss) from exclusivity for consumers in platform 1. This is as follows,

$$\begin{aligned}\Delta CS_1 &= \int_{\underline{m}}^{m^*} \frac{F(m)}{2} dm + [v + \phi + \theta N_1^*(1, 0) - p_1^*(1, 0) - \frac{m^*}{2}] F(m^*) \\ &\quad - \frac{1}{2} [v + \phi + \theta N_1^*(1, 1) - p_1^*(1, 1)] - \int_{\underline{m}}^0 \frac{F(m)}{2} dm.\end{aligned}$$

Simplifying, we get:

$$\begin{aligned}\Delta CS_1 &= \int_0^{m^*} \frac{F(m^*)}{2} dm + (F(m^*) - \frac{1}{2}) [v + \phi + \theta N_1^*(1, 0) F(m^*) - \frac{N_1^*(1, 1)}{2}] \\ &\quad - [p_1^*(1, 0) F(m^*) - \frac{p_1^*(1, 1)}{2}] - \frac{m^* F(m^*)}{2}.\end{aligned}$$

To study the sign ΔCS_i , we know that $\Delta CS_1 = 0$ at $\phi = 0$. In this case, there is no value for the Superstar and consumer surplus in the two regimes are equivalent. To show that consumers on platform 1 increase their surplus under exclusivity, *i.e.*, $\Delta CS_1 > 0$, it is sufficient to show that $\forall \phi > 0$ $\frac{\partial \Delta CS_1}{\partial \phi} > 0 \implies \frac{\partial CS_1(1, 0)}{\partial \phi} > \frac{\partial CS_1(1, 1)}{\partial \phi}$. We know that $\frac{\partial CS_1(1, 1)}{\partial \phi} = 1/2$. Similarly, we can verify the effect of ϕ on ΔCS_1 as follows:

$$\begin{aligned}\frac{\partial CS_1(1, 0)}{\partial \phi} &= \frac{\partial}{\partial \phi} \int_{\underline{m}}^{m^*} \frac{F(m^*)}{2} dm + [v + \phi + \theta N_1^*(1, 0) - p_1^*(1, 0) - \frac{m^*}{2}] f(m^*) \frac{\partial m^*}{\partial \phi} \\ &\quad + F(m^*) [1 + \theta \frac{\partial N_1^*(1, 0)}{\partial \phi} - \frac{\partial p_1^*(1, 0)}{\partial \phi} - \frac{1}{2} \frac{\partial m^*}{\partial \phi}]\end{aligned}\tag{A-17}$$

Note that, using the Leibniz's Rule, the first term is equal to $F(m^*) \frac{\partial m^*}{\partial \phi}$, which is larger than $1/2$ as $F(m^*) > 1/2$.

Moreover, as the remaining terms are positive and $\frac{\partial m^*}{\partial \phi} = \frac{1}{1 - \theta \gamma f(m^*) (\lambda (\gamma D_1^*) + \lambda (\gamma D_2^*))} > 1$, it follows that that $\frac{\partial \Delta CS_1}{\partial \phi} > 0$. Indeed, consumer surplus on the platform with an exclusive contract increases.

Impact of exclusivity on CS on platform 2. Consider now the consumer surplus on platform 2,

$$\begin{aligned}\Delta CS_2 &= (1 - F(m^*)) (v + \theta N_2^*(0, 1) - p_2^*(0, 1)) + \frac{1}{2} (\bar{m} - m^* F(m^*)) - \int_{m^*}^{\bar{m}} \frac{F(m^*)}{2} dm \\ &\quad - \frac{1}{2} (v + \phi + \theta N_2^*(1, 1) - p_2^*(1, 1) + \bar{m}) + \int_0^{\bar{m}} \frac{F(m)}{2} dm,\end{aligned}$$

which can be simplified as follows:

$$\begin{aligned}\Delta CS_2 &= \int_0^{m^*} \frac{F(m)}{2} dm - \frac{1}{2} (m^* F(m^*) - \bar{m}) - v (F(m^*) - \frac{1}{2}) - \frac{\phi}{2} + \\ &\quad \theta ((1 - F(m^*)) N_2^*(0, 1) - \frac{N_2^*(1, 1)}{2}) - ((1 - F(m^*)) p_2^*(0, 1) - \frac{p_2^*(1, 1)}{2}).\end{aligned}$$

Note that, at $\phi = 0$, we have $\Delta CS_2 = 0$. Indeed, if $\frac{\partial \Delta CS_2}{\partial \phi} < 0$, then consumer surplus on platform 2 under exclusivity decreases. Note that this happens as long as $\frac{\partial CS_2(0,1)}{\partial \phi} < 1/2$, as $\frac{\partial CS_2(1,1)}{\partial \phi} = 1/2$. Consider now the effect of ϕ on $CS_2(0,1)$ and, using the Leibniz's Rule for $\int_{m^*}^{\bar{m}} \frac{F(m)}{2} dm$, then:

$$\begin{aligned} \frac{\partial CS_2(0,1)}{\partial \phi} = & \left[\theta \frac{\partial N_2(0,1)}{\partial \phi} - \frac{\partial p_2^*(0,1)}{\partial \phi} \right] (1 - F(m^*)) \\ & - f(m^*) \frac{\partial m^*}{\partial \phi} \left[v + \theta N_2^*(0,1) - p_2^*(0,1) + \frac{m^*}{2} \right]. \end{aligned}$$

The second expression in the above equation is always negative. Suppose the first expression is negative, then it is straightforward that $\Delta CS_2 < 0$. Instead, suppose that the first expression is positive this implies that $|\frac{\partial p_2^*(0,1)}{\partial \phi}| > |\theta \frac{\partial N_2(0,1)}{\partial \phi}|$. Moreover, if $\theta \frac{\partial N_2(0,1)}{\partial \phi} - \frac{\partial p_2^*(0,1)}{\partial \phi} \in [0, 1]$, then the entire first term is lower than $1/2$ and, indeed, $\Delta CS_2 < 0$. Finally, taking into account that fact that $1 > |\frac{\partial p_1^*(1,0)}{\partial \phi}| > |\frac{\partial p_2^*(0,1)}{\partial \phi}|$. Because $1 - F(m^*) < 1/2$, the expression, $[\theta \frac{\partial N_2(0,1)}{\partial \phi} - \frac{\partial p_2^*(0,1)}{\partial \phi}] (1 - F(m^*))$ is lower than $1/2$. This proves that $\Delta CS_2 < 0$: as a result, consumers on platform 2 are worse-off with exclusivity on platform 1.

Impact of exclusivity on total CS. To provide a complete analysis of the impact of exclusivity on total consumer surplus, we proceed as follows. Denote $\Delta CS = CS(1,0) - CS(1,1)$, where $CS(1,0)$ and $CS(1,1)$ are determined by (A-15) and (A-14), respectively. Then, the net effect of exclusivity on consumer surplus is:

$$\begin{aligned} \Delta CS = & \int_0^{m^*} F(m) dm + [v + \phi + \theta N_1^*(1,0) - p_1^*(1,0) - m^*/2] F(m^*) \\ & + (1 - F(m^*)) [v + \theta N_2^*(0,1) - p_2^*(0,1)] + \frac{1}{2} (\bar{m} - m^* F(m^*)) \\ & - [v + \phi + \theta \Lambda(\gamma/2) - p^*(1,1)] - \bar{m}/2. \end{aligned}$$

We note that the above expression can also be rearranged as follows. Denote by $\bar{N} = F(m^*)N_1(1,0) + (1 - F(m^*))N_2(0,1)$ and $\bar{p} = F(m^*)p_1(1,0) + (1 - F(m^*))p_2(0,1)$, and define the preference mismatch as

$$\begin{aligned} pref_mism = & \int_{m^*}^{\bar{m}} \frac{m}{2} f(m) dm - \int_{\underline{m}}^{m^*} \frac{m}{2} f(m) dm + \int_{\underline{m}}^0 \frac{m}{2} f(m) dm - \int_0^{\bar{m}} \frac{m}{2} f(m) dm \\ = & \left(\int_{m^*}^{\bar{m}} \frac{m}{2} f(m) dm - \int_0^{\bar{m}} \frac{m}{2} f(m) dm \right) - \left(\int_{\underline{m}}^{m^*} \frac{m}{2} f(m) dm - \int_{\underline{m}}^0 \frac{m}{2} f(m) dm \right) \\ = & \left(- \int_0^{m^*} \frac{m}{2} f(m) dm \right) - \left(\int_0^{m^*} \frac{m}{2} f(m) dm \right) = - \int_0^{m^*} m f(m) dm \end{aligned}$$

Using the above, we then have the same expression we used in (4) and which helps the reader to understand the effects at stake.

$$\Delta CS = \underbrace{\theta[\bar{N} - \Lambda(\gamma/2)]}_{\Delta \text{ externalities}} - \underbrace{\phi(1 - F(m^*))}_{\text{prevented access}} - \underbrace{[\bar{p} - p^*(1, 1)]}_{\Delta \text{ prices}} - \underbrace{\int_0^{m^*} mf(m)dm}_{\text{preference mismatch}}.$$

Example with a uniform distribution of preferences.

To further corroborate our findings, we provide a short example with a uniform distribution of preferences.²⁵ We make following simplifying assumptions:

1. Consumers are uniformly distributed according to their preferences $m \sim \mathcal{U}(-1/2, 1/2)$ and the associated density function is 1.
2. The fringe firms are uniformly distributed according to their costs as $k \sim \mathcal{U}(0, 1)$ with the associated density of 1.

These two simplifications, along with $\gamma < 1$, help us provide intuitions.

Exclusivity on platform 1. Following the benchmark model, we find that platforms set a price equal to

$$p_1^*(1, 0) = \frac{1}{2} - \gamma\theta + \frac{1}{3}\phi > p_2^*(0, 1) = \frac{1}{2} - \gamma\theta - \frac{1}{3}\phi$$

and the associated demands are $D_1^*(1, 0) = \frac{\phi}{3(1-2\gamma\theta)}$ and $D_2^*(0, 1) = 1 - D_1^*(1, 0)$. The number of firms on each platform is $N_1^*(1, 0) = \gamma D_1^*$ and $N_2^*(0, 1) = \gamma D_2^*$, with $N_1^*(1, 0) > N_2^*(0, 1)$. The associated platform profits are

$$\Pi_1(1, 0)^* - T_1(1, 0) = \frac{(2\phi + 3(1 - 2\gamma\theta))^2}{36(1 - 2\gamma\theta)} - T_1(1, 0) \text{ and } \Pi_2^*(0, 1) = \frac{(2\phi - 3(1 - 2\gamma\theta))^2}{36(1 - 2\gamma\theta)}$$

with and without an exclusive contract. The optimal contract tariff is equal to $T_1^*(1, 0) = \frac{2\phi}{3}$ and the Superstar obtains $\Pi^S(1, 0) = \gamma^S D_1^* + T_1^*(1, 0)$. Total consumer surplus is:

$$CS(1, 0) = v + \frac{1}{8}\phi \left(9 + \frac{\phi}{(1 - 2\gamma\theta)^2} \right) + \frac{3\gamma\theta}{2} - \frac{3}{8}.$$

Non-exclusivity of the Superstar. The platforms set equilibrium prices equal to

$$p^* = p_1^*(1, 1) = p_2^*(1, 1) = \frac{1}{2} - \gamma\theta$$

and obtain $D_1^*(1, 1) = D_2^*(1, 1) = \frac{1}{2}$, with $N_1^*(1, 1) = N_2^*(1, 1) = \gamma/2$. Platform profits are given as $\Pi_i^* = \frac{1-2\theta}{4}$.

The non-exclusive tariff is given as $T^*(1, 1) = T_1^*(1, 1) = T_2^*(1, 1) = \phi \frac{3-5\gamma\theta-\phi}{9(1-2\gamma\theta)}$ and the

²⁵For detailed intuitions, there is an analogous model in an older version of the with a Hotelling set-up with consumers distributed uniformly on the interval $[0, 1]$. See Carroni et al. (2019)

Superstar obtains $\Pi^S(1, 1) = \gamma^S + 2T^*(1, 1)$. The resulting total consumer surplus is given as

$$CS(1, 1) = v + \phi + \frac{3\gamma\theta}{2} - \frac{3}{8}.$$

In what follows, we show that by comparing profits, $\Pi^S(1, 0) > \Pi^S(1, 1)$ if, and only if, $\gamma^S \leq \tilde{\gamma}$ where $\tilde{\gamma}^S =: \frac{4\phi^2}{3(3-2\phi-6\gamma\theta)}$. Thus, this resembles the critical value in Proposition 2. To ensure concavity and rule out market tipping, we assume that $\phi < \frac{3}{2}$ and $\theta < \frac{3-2\phi}{\gamma^S}$. Comparing consumer surplus in the two contractual regimes,

$$\Delta CS = CS(1, 0) - CS(1, 1) = \frac{1}{18}\phi \left(-9 + \frac{\phi}{(1-2\gamma\theta)^2} \right),$$

which is positive for $\theta > \frac{1}{2\gamma} - \frac{\sqrt{\frac{\phi}{\gamma^2}}}{6} > 0$ and $\phi < 1/4$. This result confirms (4). Indeed, there exists a parameter range for which when exclusivity is offered by the Superstar and this is welfare-enhancing. Hence, we confirm that our results hold under the uniform distribution case.

A.8. Proof of Proposition 6

As $\tilde{\gamma}(\gamma^S)$, we cannot directly compare the two cutoffs. Let $\Delta\pi^S(\cdot)$ the profit differential under non exclusivity relative to exclusivity. Denote $\Delta\pi^S = \Pi^S(1, 1) - \Pi^S(1, 0)$, the profit differential under vertical separation and $\Delta\pi^{S,vi} = \Pi_1^S(1) - \Pi_1^S(0)$, these terms can also be expressed as follows:

$$\begin{aligned} \Delta\pi^S &= \gamma^S(1 - D_1(1, 0)) + 2\Pi^*(1, 1) - \Pi_1^*(1, 0) - \Pi_1^*(0, 1) \\ \Delta\pi^{S,vi} &= \gamma^S(1 - D_1(0)) + 2\Pi^*(1, 1) - \Pi_1^*(0) - \Pi_2^*(0). \end{aligned}$$

Clearly, whenever $\Delta\pi^S < \Delta\pi^{S,vi}$, exclusivity is less likely to arise in the vertical integration. In what follows, we show that this is the case. Let us first introduce some remarks:

1. Notice that when $\gamma^S = 0$, $\Delta\pi^{S,vi} = \Delta\pi^S < 0$, because the downstream prices and the profits under vertical integration are equal to the ones under vertical separation (and there is no scope for non-exclusivity).
2. Point 1 implies that it is sufficient to show that $\frac{\partial\Delta\pi^S}{\partial\gamma^S} < \frac{\partial\Delta\pi^{S,vi}}{\partial\gamma^S}$ to prove our statement.
3. Notice also that $\frac{\partial\Delta\pi^S}{\partial\gamma^S} = 1 - D_1(1, 0)$, given that the downstream demands, prices and profits are not affected by γ^S in the vertical separation case.

4. Then notice that:

$$\begin{aligned}
\frac{\partial \Delta \pi^{S,vi}}{\partial \gamma^S} &= 1 - D_1(0) - \gamma^S \frac{\partial D_1(0)}{\partial \gamma^S} - \frac{\partial \Pi_1^*(0)}{\partial \gamma^S} - \frac{\partial \Pi_2^*(0)}{\partial \gamma^S} \\
&= 1 - D_1(0) - \left[\gamma^S \frac{\partial D_1(0)}{\partial p_2} + p_1 \frac{\partial D_1(0)}{\partial p_2} \right] \frac{\partial p_2}{\partial \gamma^S} + p_2 \frac{\partial D_1(0)}{\partial p_1} \frac{\partial p_1}{\partial \gamma^S} \\
&= 1 - D_1(0) + \frac{\partial D_1(0)}{\partial p_1} \left[(\gamma^S + p_1) \frac{\partial p_2}{\partial \gamma^S} + p_2 \frac{\partial p_1}{\partial \gamma^S} \right]. \tag{A-18}
\end{aligned}$$

5. Finally, the derivatives of optimal prices with respect to γ^S are:

$$\begin{aligned}
\frac{\partial p_1}{\partial \gamma^S} &= -\frac{\frac{\partial^2 \Pi_2^*(0)}{\partial p_2^2} \frac{\partial^2 \Pi_1^*(0)}{\partial p_1 \partial \gamma^S}}{\zeta} < \frac{\partial p_2}{\partial \gamma^S} = \frac{\frac{\partial^2 \Pi_1^*(0)}{\partial p_1 \partial \gamma^S} \frac{\partial^2 \Pi_2^*(0)}{\partial p_1 \partial p_2}}{\zeta} < 0, \\
\text{where } \zeta &= \frac{\partial^2 \Pi_1^*(0)}{\partial^2 p_1} \frac{\partial^2 \Pi_2^*(0)}{\partial^2 p_2} - \frac{\partial^2 \Pi_1^*(0)}{\partial p_1 \partial p_2} \frac{\partial^2 \Pi_2^*(0)}{\partial p_1 \partial p_2} > 0.
\end{aligned}$$

Plugging into (A-18), we get:

$$\frac{\partial \Delta \pi^{S,vi}}{\partial \gamma^S} = 1 - D_1(0) + \frac{\frac{\partial D_1(0)}{\partial p_1} \frac{\partial^2 \Pi_1^*(0)}{\partial p_1 \partial \gamma^S}}{\zeta} \left[(\gamma^S + p_1) \frac{\partial^2 \Pi_2^*(0)}{\partial p_1 \partial p_2} - p_2 \frac{\partial^2 \Pi_2^*(0)}{\partial^2 p_2} \right].$$

6. Exploiting that $\frac{\partial D_1}{\partial p_1} = \frac{\partial^2 \Pi_1^*(0)}{\partial p_1 \partial \gamma^S} := Y$, we can notice that:

$$\zeta = -2 \frac{\partial^2 \Pi_1^*(0)}{\partial p_1 \partial p_2} \frac{\partial^2 \Pi_1^*(0)}{\partial p_1 \partial p_2} + Y \left[p_1 \frac{\partial Y}{\partial m} + p_2 \frac{\partial Y}{\partial m} \right] \frac{\partial m}{\partial p_1} + Y^2 < Y^2,$$

and so:

$$\frac{\partial \Delta \pi^{S,vi}}{\partial \gamma^S} = 1 - D_1(0) - \underbrace{\frac{Y^2}{\zeta}}_{>1} \left[\underbrace{(\gamma^S + p_1 + p_2) \frac{\partial^2 \Pi_2^*(0)}{\partial p_1 \partial p_2}}_{>0} + 1 - D_1(0) \right] > 0$$

Therefore, we can conclude the following:

$$\frac{\partial \Delta \pi^{s,vi}}{\partial \gamma^S} - \frac{\partial \Delta \pi^s}{\partial \gamma^S} = D_1(1,0) - D_1(0) - \frac{X^2}{\zeta} \left[(\gamma^S + p_1 + p_2) \frac{\partial^2 \Pi_2^*(0)}{\partial p_1 \partial p_2} + 1 - D_1(0) \right]$$

given that $|D_1(1,0) - D_1(0)| < 1 - D_1(0)$.

A.9. Proof of Proposition 7

Consider first non-exclusivity $g_1 = g_2 = 1$. In this case platforms compete *à la* Bertrand competition, so that $p_1^* = p_2^* = 0$ because of the incentives of each platform to undercut the rival. Demand of each platform solves $D^*(1,1) = \Upsilon(v + \phi + \theta \Lambda(\gamma D^*)) / 2$ and profits are

$\Pi_1(1, 1) = \Pi_2(1, 1) = 0$. This also means that non-exclusivity can be incentive-compatible for platforms only if the Superstar gives its product away for free, i.e., $T_1(1, 1) = T_2(1, 1) = 0$. Indeed, platforms receive zero profits both when they both have the Superstar product (because of strong competition) and when the product is exclusive on the rival (because of market tipping). If the Superstar opts for this solution, she would get profit $2\gamma^S D_1^*(1, 1)$

Now consider the case in which Superstar is exclusive on platform 1. In this case, platform 1 can always find a price p_1^* that leads to market tipping and gain positive profits, $\Pi_1^*(1, 0) > 0$. Again, if platform 1 receives the offer of an exclusive product, the outside option would be zero, given that being exclusive on platform 2 would let the latter tip the market. The demand of platform 1 in this case solves $D_1^*(1, 0) = \Upsilon(v + \phi + \theta\Lambda(\gamma D_1^*(1, 0)) - p_1^*)$. Notice that $2D_1^*(1, 1) > D_1^*(1, 0)$ given that $p_1^* > 0$ and $\Upsilon(\cdot)$ is increasing. This solution would give to the Superstar a profit equal to $\gamma^S D_1^*(1, 0) + \Pi_1^*(1, 0)$. Comparing the two profits, we get the cutoff γ^{Hom} .

A.10. Proof of Proposition 8

Consider equations (8) and (9). It is easy to notice that the profit of platform 2 does not change in response to the decision of the Superstar, as for any p_2 , we have:

$$\Pi_2(1, 1) = p_2(1 - F(m_1^*(1, 1))) = p_2(1 - F(m_1^*(1, 0))),$$

This also implies that the optimal profits in the two cases are the same, i.e., $\Pi_2^{M*}(0, 1) = \Pi_2^{M*}(1, 1)$. This equality has an important implication for the Superstar, as the non-exclusive offer of its product to platform 2 is profit equivalent to the exclusivity on the rival platform. As a consequence, no positive payment can be asked, as the maximal tariff $T_2^M(1, 1)$ cannot be higher than $\Pi_2^{M*}(1, 1) - \Pi_2^{M*}(\cdot, \cdot)$ for platform 2 to accept the offer. Hence, $T_2^M(1, 1) = 0$ and, exploiting symmetry, also $T_1^M(1, 1) = 0$. As a result, the profit of the Superstar under non-exclusivity is:

$$\gamma^S + T_2^M(1, 1) + T_1^M(1, 1) = \gamma^S. \quad (\text{A-19})$$

Differently, platform 1 always gains a higher demand with $g_1 = 1$ and $g_2 = 0$. Moreover, for any price p_1 , we have:

$$\underbrace{p_1 F(m_2^*(1, 0))}_{\Pi_1(0,1)} = \underbrace{p_1 F(m_2^*(1, 1))}_{\Pi_1(1,1)} < \underbrace{p_1 F(m_2^*(0, 1))}_{\Pi_1(1,0)},$$

because $F(\cdot)$ is increasing and $m_2^*(1, 0) = m_2^*(1, 1) < m_2^*(0, 1)$. This also implies that the optimal profits in the two cases are such that $\Pi_1^{M*}(1, 0) > \Pi_1^{M*}(1, 1) = \Pi_1^{M*}(0, 1)$. This implies that the maximal tariff that can be charged to platform 1 is $T_1^M(1, 0) = \Pi_1^{M*}(1, 0) - \Pi_1^{M*}(0, 1)$. Therefore, the maximal Superstar profit in this case is:

$$\gamma^S D_1^M(1, 0) + T_1^M(1, 0) = \gamma^S D_1^M(1, 0) + \Pi_1^{M*}(1, 0) - \Pi_1^{M*}(0, 1). \quad (\text{A-20})$$

Comparing (A-19) with (A-20), we get the cutoff in Proposition 8.

A.11. Two-sided pricing

For the variation with two-sided pricing, we follow the same approach as in Section A.7, with a uniform distribution of consumers and firms and the associated density of 1. A singlehoming fringe firm on platform i obtains $\gamma \cdot D_i - k - l_i$, where l_i is the price paid by the small firms to access the platform. For $l_i < 0$, these firms are subsidized. A multihoming small firm gets $\gamma - 2k - l_i - l_j$. Platform i 's profits absent the Superstar are $\Pi_i(0, g_j) = p_i D_i(0, g_j) + l_i N_i$. When platform i hosts the Superstar, profits are $\Pi_i(1, g_j) = p_i D_i(1, g_j) + l_i N_i - T_i(1, g_j)$. To ensure concavity and rule out market tipping, we assume $0 < \phi < 1/2(3 - \gamma^2 - 2\gamma\theta - \theta^2)$, and $4 - \gamma^2 - 6\gamma\theta - \theta^2 > 0$. We then solve the game backwards. In the third stage, consumer demands become:

$$D_i(g_i, g_j) = \frac{1}{2} + \frac{\theta(l_i - l_j) + (p_j - p_i) + \phi(g_i - g_j)}{2(1 - \gamma\theta)}, \quad D_j(g_j, g_i) = 1 - D_i(g_i, g_j)$$

By anticipating future market shares, in the second stage, platforms have the following best replies for $i, j \in \{1, 2\}$, with $i \neq j$,

$$p_i(p_j, l_j) = \frac{(2 - \gamma(\gamma + 3\theta))(1 + 2(p_j + l_j\theta - \gamma\theta + \phi(g_i - g_j)))}{2(4 - \gamma^2 - 6\gamma\theta - \theta^2)},$$

$$l_i(l_j, p_j) = \frac{(\gamma - \theta)(1 - 2(p_j + l_j\theta - \gamma\theta + \phi(g_i - g_j)))}{2(4 - \gamma^2 - 6\gamma\theta - \theta^2)}$$

We now identify the equilibrium outcomes in the two contractual regimes. First, consider when the platform offers a non-exclusive contract ($g_i = g_j = g = 1$), platforms are symmetric and prices are symmetric as well, such that $p^*(1, 1) := p_1^*(1, 1) = p_2^*(1, 1) = 1/2 - \gamma(\gamma + 3\theta)/4$ for consumers and $l^*(1, 1) := l_1^*(1, 1) = l_2^*(1, 1) = (\gamma - \theta)/4$ for the fringe. Demands are given by $D_1^*(1, 1) = D_2^*(1, 1) =: 1/2$ and $N^*(1, 1) := N_1^*(1, 1) = N_2^*(1, 1) = (\gamma + \theta)/4$. Second, consider when the Superstar offers an exclusive contract to platform 1 ($g_1 = 1$ and $g_2 = 0$), equilibrium prices on the consumer side are:

$$p_1^*(1, 0) = p^*(1, 1) \left(1 + \frac{2\phi}{\eta}\right), \quad p_2^*(0, 1) = p^*(1, 1) \left(1 - \frac{2\phi}{\eta}\right).$$

Equilibrium prices on the fringe are:

$$l_1^*(1, 0) = l^*(1, 1) \left(1 + \frac{2\phi}{\eta}\right), \quad l_2^*(0, 1) = l^*(1, 1) \left(1 - \frac{2\phi}{\eta}\right),$$

where $\eta := 3 - \gamma^2 - 4\gamma\theta - \theta^2 > 0$. It can be easily seen that $p_1^*(1, 0) > p^*(1, 1) > p_2^*(0, 1) > 0$. When $\gamma > \theta$, the price for the firms on the fringe is positive and increases with the value generated by the Superstar. When $\gamma < \theta$, fringe firms are subsidized and the subsidy increases with the Superstar. Regardless of the pricing strategy, there is agglomeration of fringe firms on the *favoured* platform in the same vein as in the main paper. Specifically,

$$N_1^*(1, 0) = N^*(1, 1) \left(1 + \frac{2\phi}{\eta}\right), \quad N_2^*(0, 1) = N^*(1, 1) \left(1 - \frac{2\phi}{\eta}\right)$$

with $N_1^*(1, 0) > N^*(1, 1) > N_2^*(0, 1)$ as in Proposition 1. The contract in the two regimes follows the same reasoning as in the main model. Under exclusivity, Superstar's profit is $\Pi^S(1, 0) = \frac{\gamma^S}{2} + \frac{\phi(4-\gamma^2+2\gamma^S-6\gamma\theta-\theta^2)}{2\eta}$. In the case of non-exclusive deals, Superstar's profit is $\Pi^S(1, 1) = \gamma^S + \frac{\phi(4-\gamma^2-6\gamma\theta-\theta^2)(\eta-\phi)}{2\eta^2}$. Hence, the Superstar offers an exclusive deal whenever $\gamma^S < \tilde{\gamma}^S$, where:

$$\tilde{\gamma}^S \equiv \frac{(4 - \gamma^2 - 6\gamma\theta - \theta^2)\phi^2}{2\eta(\eta - \phi)}.$$

Else, she offers a non-exclusive contract. The mechanism behind this result is identical to that in Proposition 2.