ON TWO INTERESTING PROPERTIES OF PRIMES, P, WITH RECIPROCALS IN BASE 10 HAVING MAXIMUM PERIOD P-1.

Subramani K

Project Associate

International Institute of Information Technology, Bangalore
Email:subramani.k@iiitb.ac.in

Abstract

In this note we have studied some properties of decimal digits obtained by dividing 1 by primes which have maximum period property, namely, inverses of primes, p, whose decimal representation repeats after p-1. For such primes up to 1000, we have given a method for finding decimal digits of their inverses without using division. This works for many non-primes also. We have also given a construction method for obtaining magic squares for primes, 17, 29, 61 and 97 for which such magic squares are not known.

INTRODUCTION

Inverses of primes which have maximum period property, namely, inverses of primes, p, whose decimal representation repeats after p-1 are well studied in the mathematical literature [1]. Examples of such primes are 7 and 19 with,

1/7 = 0.142857142857…. or 1/19 = 0.052631578947368421 and so on. 1/11 or

1/13 don't have such property as it is easy to check. 1/11 =.0909090909.

1/13 = 0.076923,076923...

The Ekidhikena Purvena rule (by one more than the one before) from Bharati Krishna Tirtha's Vedic mathematics [5] works beautifully for finding decimal digits of inverse of numbers which end with 9.

For example, 1/19 =0.05263157894736842105263…. We start with 1 which is the last digit of recurring period, 052631578947368421 and multiply it by 2 repeatedly. When the result of multiplication is more than one digit, we carry it and add it to the result of next multiplication by 2 and so on. The process is shown below.

1 0 1 0 0 1 1 1 1 0 1 0 1 1 0 0 0 0

0 5 2 6 3 1 5 7 8 9 4 7 3 6 8 4 2 1 x 2

The result also can be obtained by dividing 1 by 2 and using the remainder in the next division as shown below.

1/2 052631578947368421 101001111010110000

Note that when we divide 1 by 2, we get 0 and remainder 1. Next time we use 1 and 0 as 10 and divide it by 2 to get 5. This process repeats.

Such methods of inverting numbers without the division operation is always desirable.

We have given a table to find inverses of primes such as 7, 17,19, 23,29, 41,59,61,97,109,113,131,149,167,179,181,193,223,229,233,257,263,269,313,337,367,379,383,389,419,433,461,487,491,499,503,509,541,571,577,593,619,647, 659,701,709,727,743,811,821,823,857,863,887,937,941,953,971,977,983.

It includes all such primes up to 1000.

We do this by finding a pair of numbers a and b which can be used for finding decimal digits of inverse of a prime without division. For example, decimal digits of 1/7 can be found using the pair 7 and 5 using the operation shown below. We call that operation extended multiplication of 7 by 5 to differentiate it from conventional multiplication rule used in arithmetic.

2 1 4 2 3 0

1 4 2 8 5 7 x 5

Note that one period is, 142857. We obtain it by starting with 7 and multiply it by 5 to get, 35. Use 5 as the preceding digit and multiply it by 5 and add carry 3 to get 28. Use 8 as the preceding digit and keep 2 as carry.

Table 1 gives multiplication factor to be used and the starting digit for all such primes up to 1000.

Prime Number, p	Last Digit of 1/p after which digits repeat	Multiplication factor
7	7	5
17	7	12
19	1	2
23	3	7
29	1	3
47	7	33
59	1	6
61	9	55
97	7	68
109	1	11

113	3	34
131	9	118
149	1	15
167	7	117
179	1	18
181	9	163
193	3	58
223	3	67
229	1	23
233	3	67
257	7	180
263	3	79
269	1	27
313	3	94
337	7	236
367	7	257
379	1	38
383	3	115
389	1	39
419	1	42
433	3	130
461	9	415
487	7	341
491	9	442
499	1	50
503	3	151
509	1	51
541	9	487

571	9	514
577	7	404
593	3	178
619	1	62
647	7	453
659	1	66
701	9	631
709	1	71
727	7	509
743	3	223
811	9	730
821	9	739
823	3	247
857	7	600
863	3	259
887	7	621
937	7	656
941	9	847
953	3	286
971	9	874
977	7	684
983	3	295

Table 1: Factors needed for obtaining decimal digits of inverse of primes

Comment: We wish to note that all the numbers presented in Table 1 can be obtained using application of Ekidhikena Purvena rule as illustrated, here, for 1/701. Note that 1/701 can be converted to 9/6309. That gives us 631 as the multiplication factor.

This approach works for non-primes also. We can obtain decimal digits of 1/27 = .037037037 using the following extended multiplication operation. We have used * to denote extended multiplication operations.

7 *19 = 730730730…

We can use the following extended multiplication operation for 1/39 =. 025641025641.

1*4 = 1465201465…

Decimal digits of a large number of fractions can be obtained without using division in this way.

Extended Multiplication and Extended Division operations

The method given by Ekidhikena Purvena which has been generalized to many more primes and other numbers leads to two interesting operations, which we call, extended multiplication and extended division operations. We have given some examples of extended multiplication operation. More study is needed to understand applications of such multiplication and division operations.

0 * 0 = 00000000000000000....

1*0 = 1000000000000000000000000.... (a very large number)

2*0 =200000000000000......

1 * 1 = 1111111111111111111111111111 (this is related to 1/9 =.11111111111…)

1*2 = 1248637498751362501248... (this is related to 1/19=.052631578947368421052631…)

7*5 =758241758241 ... (this is related to 1/7)

7 * 12 = 746711492532885074... (this is related to 1/17)

3 * 7 = 3193712565968062874340 ... (this is related to 1/2

New Inverse Prime Magic Squares

Magic squares obtained by decimal digits of 1/19 is very well known. Such magic squares [1,6] are possible for, 19, 383, 32327, 34061, 45341, 61967, 65699, 117541, 158771, 405817, 444287, 456503, 695389, 724781, 1102567, 1177859, 1498139, 2336989, 2695337, 3036857, 3249419, 3512749, 3571429, 4427299, 5141051, 7033823, 8324411, 9932179. We have given a method of constructing magic squares for primes such as 17, 29, 61 and 97.

Magic Square for primes less than 100 for 1/17, 1/29, 1/61 &1/97.

Method: Example for 1/17 = .0588235294117647…...

a) It has 16 digits, which is an even number. Therefore, we can create a magic square.

b) Split 0588235294117647, as, 05882352 & 94117647. Let A denote 05882352 and B denote 94117647.

c) Take second digit 4 from B. Start from 4 to form the first line as shown.

4117647058823529 (first line)

d) Take third digit 8 from A. Start from 8 to form the 2^{nd} line.

8823529411764705. This is the second line.

1764705882352941 (third line)

2352941176470588 (fourth line)

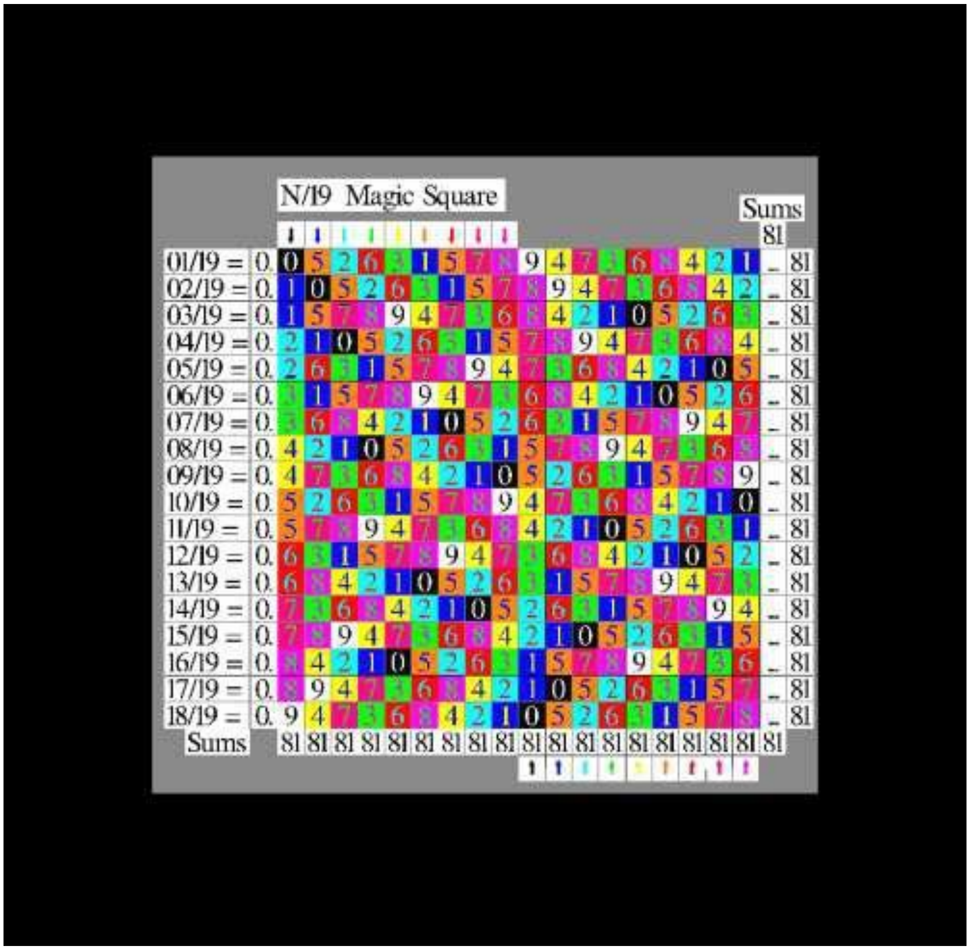
From third line onwards take 3^{rd} number from the 2^{nd} row above it as the starting number. Repeat this process till 16^{th} row.

1/17 =0.0588235294117647

4	1	1	7	6	4	7	0	5	8	8	2	3	5	2	9
8	8	2	3	5	2	9	4	1	1	7	6	4	7	0	5
1	7	6	4	7	0	5	8	8	2	3	5	2	9	4	1
2	3	5	2	9	4	1	1	7	6	4	7	0	5	8	8
6	4	7	0	5	8	8	2	3	5	2	9	4	1	1	7
5	2	9	4	1	1	7	6	4	7	0	5	8	8	2	3
7	0	5	8	8	2	3	5	2	9	4	1	1	7	6	4
9	4	1	1	7	6	4	7	0	5	8	8	2	3	5	2
5	8	8	2	3	5	2	9	4	1	1	7	6	4	7	0
1	1	7	6	4	7	0	5	8	8	2	3	5	2	9	4
8	2	3	5	2	9	4	1	1	7	6	4	7	0	5	8
7	6	4	7	0	5	8	8	2	3	5	2	9	4	1	1
3	5	2	9	4	1	1	7	6	4	7	0	5	8	8	2
4	7	0	5	8	8	2	3	5	2	9	4	1	1	7	6
2	9	4	1	1	7	6	4	7	0	5	8	8	2	3	5
0	5	8	8	2	3	5	2	9	4	1	1	7	6	4	7

Magic Square – Sum of numbers in all rows, all columns, and all diagonals is 72

The Famous 1/19 Magic Squares Very Well Known to Hindus (500BC) and others (taken from internet)



More Properties of Magic Squares Obtained by the Method Given Above

For 96*96 Magic square all columns, rows and diagonals are 432. If we cut 4 parts, say A B C and D. Each sub square's Diagonals are 216. Each Sub Square total is 10368. All 4 Sub squares total is 4*10368 =41472. This method works for 1/17, 1/97, 1/577, 1/3457, 1/20737 ......and has sum of all rows, columns, diagonals and inner Sub diagonals same. Example of 1/17 is given in Figure 2.

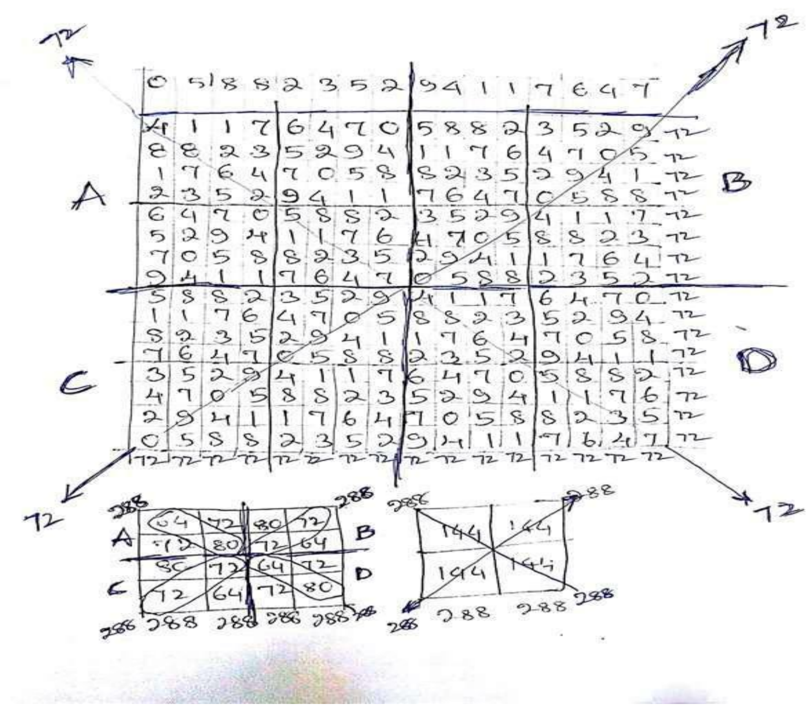


Fig. 2. Magic square for 1/17 shown with more properties

Acknowledgement:

Author thanks International Institute of Technology, Bangalore for providing support to continue explorations into numbers and their interesting properties.

REFERENCES

1. https://oeis.org/A072359 The On-Line Encyclopedia of Integer Sequences Founded in 1964 by N.J.A.Sloan

2. W. S. Andrews, Magic Squares and Cubes, pp. 176 Dover NY 1960.

3. J. Heleen, Journal of Recreational Mathematics, 30(1) 1999-2000 pp. 72-3 Soln. to Prob. 2394. Magic Reciprocals

4. M. J. Zerger, Journal of Recreational Mathematics, 30(2) 1999-2000 pp. 158-160 Soln. to Prob. 2420. Only 19?

5. https://en.wikibooks.org/wiki/Vedic_Mathematics/Sutras/Ekadhikena_Purvena

6.https://en.wikipedia.org/wiki/Prime_reciprocal_magic_square