

ANALYSIS AND DESIGN OF A HYPERBOLIC COOLING TOWER

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Oceanic Technology, 1970

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A MASTER'S REPORT

submitted in partial fulfillment of the  
requirements for the degree

MASTER OF SCIENCE

Department of Civil Engineering

KANSAS STATE UNIVERSITY  
Manhattan, Kansas

1976

Approved by:



Major Professor

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## I. INTRODUCTION AND SCOPE

The hyperboloid of revolution can be generated by rotating a hyperbola about its directrix. Shells of this type are built throughout the world as cooling towers to lower the temperature of coolants used in electricity generating plants and chemical plants. This type of shell has proven to be efficient for use in reinforced concrete natural draught cooling towers for the conservation and reuse of the coolant (water).

The purpose of this report is to present the solutions for the stress resultants for the membrane and bending analysis and the corresponding displacements for cooling towers under dead load and wind load. Numerical results comparing solutions obtained by membrane theory and bending theory are presented.

## II. REVIEW OF LITERATURE

The first hyperbolic natural draught reinforced concrete cooling tower was designed by Prof. Van Iterson of the Dutch State Mines and installed at the Emma Colliery in 1916 (2). Towers of this type were installed at Lister Drive Power Station in Liverpool in 1925 and since then have become quite common and standard practice in Europe power stations where cooling towers are required. The typical size for old towers is shown Fig. 1. This type of tower has become a familiar sight in the United States with the first tower constructed in connection with a power station in Kentucky about fifteen years ago (1960). These structure sometimes reach over 350 feet in height and have base diameter often over 200 feet.

Immense quantities of water are required for the condensers of power stations, refineries, steel plants, etc. and sites with adequate cooling water are becoming rare; thus there is a need for the development of natural draught cooling towers for cooling and reusing large quantities of water.

The one-sheet hyperboloid is a convenient geometry for cooling towers with its straight-line generators for both structural and thermal reasons:

1. It has been proven (2) that the shear and vertical stresses are reduced by over 50% due to the "hyperbolic" shape of the shell compared with a cylinder of the same height and base diameter. Also this type stiffens the shell against wind force.
2. The momentum of the air entering the shell carries it into the center to form a vena contracta whose diameter depends on the

ratio of tower diameter to height of air inlet.

The other advantages of this hyperbolic concrete tower are (3):

1. Concrete towers are permanent.
2. There are no fans or similar equipment so there is lack of vibration due to resonance of fans and the tower. Therefore, the only power consumption is needed for pumping the water to the distribution pipes.
4. The natural draft towers minimize hazards such as fire, mist, and frozen spray.

Rish and Steel (2) discussed the treatment of the hyperboloid by assuming the shell to be made up of two truncated cones with a cylinder in between. Martin and Scriven (4) and Martin, Maddock, and Scriven (5) presented numerical solutions for dead load and wind load stresses and displacements in a particular shell. Gould and Lee (6,7,8,9) presented the membrane solutions and bending solutions for the stress resultants and displacements in hyperbolic cooling towers subjected to dead load, earthquake load, and wind load. The influence of the various shell parameters on the magnitude of the stress resultants and displacements is studied by these researchers and design tables are given to facilitate the design of such structures.



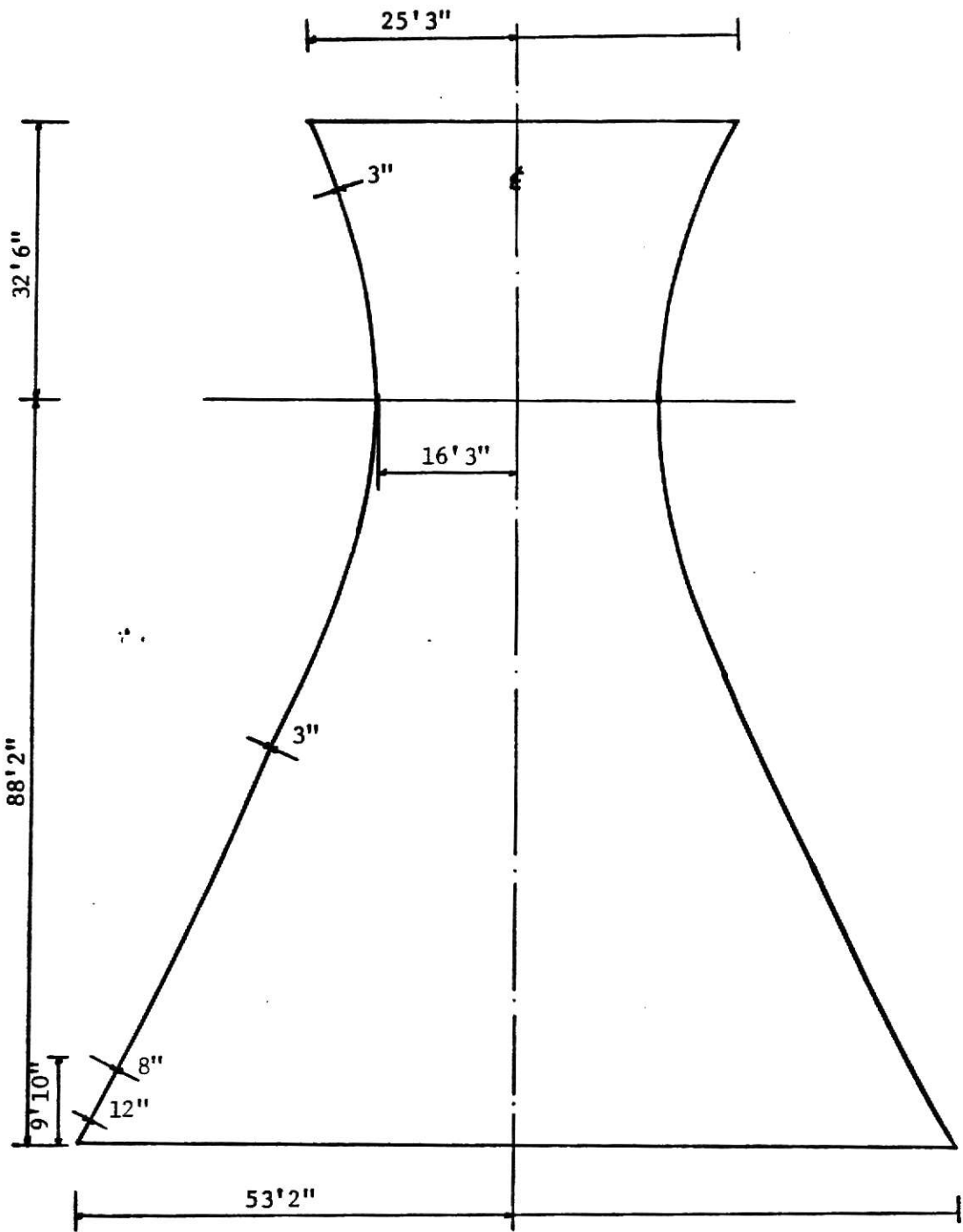


FIG. 1. Typical Old Natural Draught Cooling Tower

### III. DESIGN CONSIDERATIONS FOR HYPERBOLIC COOLING TOWERS (11)

#### 1. Size selection

Chilton (10) gave a formula to enable the size of cooling towers to be determined for a given cooling duty. This was

$$D = \frac{A_b (H)^{\frac{1}{2}}}{C (C)^{\frac{1}{2}}} \text{ --- (3.1)}$$

where

$A_b$  = the base area of the tower measured at pond sill;

$H$  = the height of the tower measured above sill level;

- $C$  is an efficiency factor known as the performance coefficient. In the past values of this have been in the region of 5.2 where water loadings were over 750 lb/hr/sq.ft, but new types of packing bring this down to give a  $C$  value of 5.0.

The Duty Coefficient  $D$  may be worked out from the formula:

$$\frac{W_L}{D} = 90.59 \frac{\Delta h}{\Delta T} (\Delta t + 0.3124 \Delta h)^{\frac{1}{2}} \text{ --- (3.2)}$$

where

$\Delta h$  = the change in total heat of the air passing through the tower;

$\Delta T$  = the change of temperature of the water passing through the tower;

$W_L$  = the water load in lb/hr.

$\Delta t$  = the change between the dry bulb air temperature and aspirated wet bulb air temperature.

## 2. General consideration for loading and analysis (11)

- a. The cooling tower shell should be considered to resist forces resulting from gravity loading, thermal gradient and icing, wind and earthquake, and foundation settlement. Also, temporary construction loading should be considered.
- b. When interior or exterior fill is supported on concrete, the effects of their loading should be considered in the design of the shell.
- c. Adequate stiffening of the top and the base of the shell should be provided.
- d. Cooling towers should be analyzed in accordance with recognized theories for thin elastic shell which for concrete are assumed to be uncracked, homogeneous and isotropic.
- e. The actual geometric profile, thickness variations and support conditions of the shell should be considered in the structural analysis.
- f. Equilibrium checks of internal forces and external loads should be performed regardless of the analysis method used.
- g. Results from model studies or full-scale tests may be used as a basis for the design and to check the validity of assumption involved in a mathematical analysis.

## 3. Method of analysis (11)

- a. An analysis which is based on a recognized bending theory for thin elastic shells is considered to be the most appropriate basis for the design of the tower and supporting structure. An analysis based on the membrane theory of thin shells may be satisfactory

for design provided that local bending in critical regions is accounted for by an appropriate method.

- b. Realistic boundary conditions should be considered in the analysis.
- c. Deformations which are computed from the elastic analysis should be checked to verify that they fall within the assumed limits of the applied theory.

#### 4. Stability (11)

- a. For wind load the critical shell buckling pressure may be estimated from test results. A wind buckling analysis should be made using the correct tower geometry and boundary conditions, and including the influence of dead weight. When made, the analysis should account for the influence of any anticipated shell hairline cracking.
- b. For dead load alone the critical shell buckling may be estimated by a simplified procedure which accounts for the dead load stresses in both the meridional and circumferential directions or by a dead load buckling analysis using the correct tower geometry and boundary conditions.
- c. Imperfections, which will reduce the buckling capacity are measured by deviation  $w_t$  in thickness over the arc length  $l$  where buckling capacity decreases as  $w_t/l$  increases. When imperfections larger than field tolerances occur, the engineer should make an estimate of the reduction in  $q_c$  to assure that adequate buckling capacity remains. The term  $q_c$  is the critical buckling pressure in psi along the windward meridian.

From an analysis of the wind tunnel test results reported (11), the following equation was obtained for estimating the critical shell buckling pressure.

$$q_c = C_c E (h/a)^{\alpha_c} \text{ - - - - - (3.3)}$$

in which

$C_c$  = an empirical coefficient taken to be 0.052;

$E$  = the modulus of elasticity of the concrete in psi;

$h$  = the shell thickness at the throat;

$a$  = the radius of the shell parallel circle at the throat ( the same unit as  $h$  );

$\alpha_c$  = an empirical coefficient taken to be 2.3.

The value  $q_c$  computed from this equation should be compared to the design wind pressure at the top of the tower to insure an adequate safety factor against buckling. It is intended that the design of the tower not be controlled by stability.

##### 5. Strength and serviceability requirements (11)

The cooling tower should be designed using the Strength Method according to the provisions of ACI 318-71 (16). Serviceability under working loads should be considered to insure that neither cracking nor deflections are excessive under the conditions of unfactored loading.

6. Reinforcement (11)

- a. The shell reinforcing in each direction should not be less than 0.35% of the cross sectional area of concrete.
- b. It is preferable to provide two layers of reinforcement in each direction.
- c. The maximum spacing of bars in each layers should not exceed twice the shell thickness, or no more than 18 inches.
- d. Reinforcement interrupted by openings should be replaced by not less than one and one half times the interrupted amount of reinforcement placed adjacent to the opening plus additional diagonal bars at corners of opening. For larger openings, the designer should take particular care to reinforce the opening to resist the design loads.

7. Splices in reinforcement (11)

- a. Splices in reinforcement should be designed according to the provisions of ACI 318-71 (16) except as provided in following.
- b. In case of a lap splice in tension, care should be taken to ensure the transfer of the design force without jeopardizing the integrity of the confining concrete.
- c. Splicing of reinforcing bars above a level not to exceed one-half the column spacing from the bottom should be distributed around the shell wall. No more than 1/3 of the vertical reinforcing should be spliced at one level.
- d. If splices in reinforcement are designed by a method not covered by ACI 318-71, the strength of the splices should be ensured.

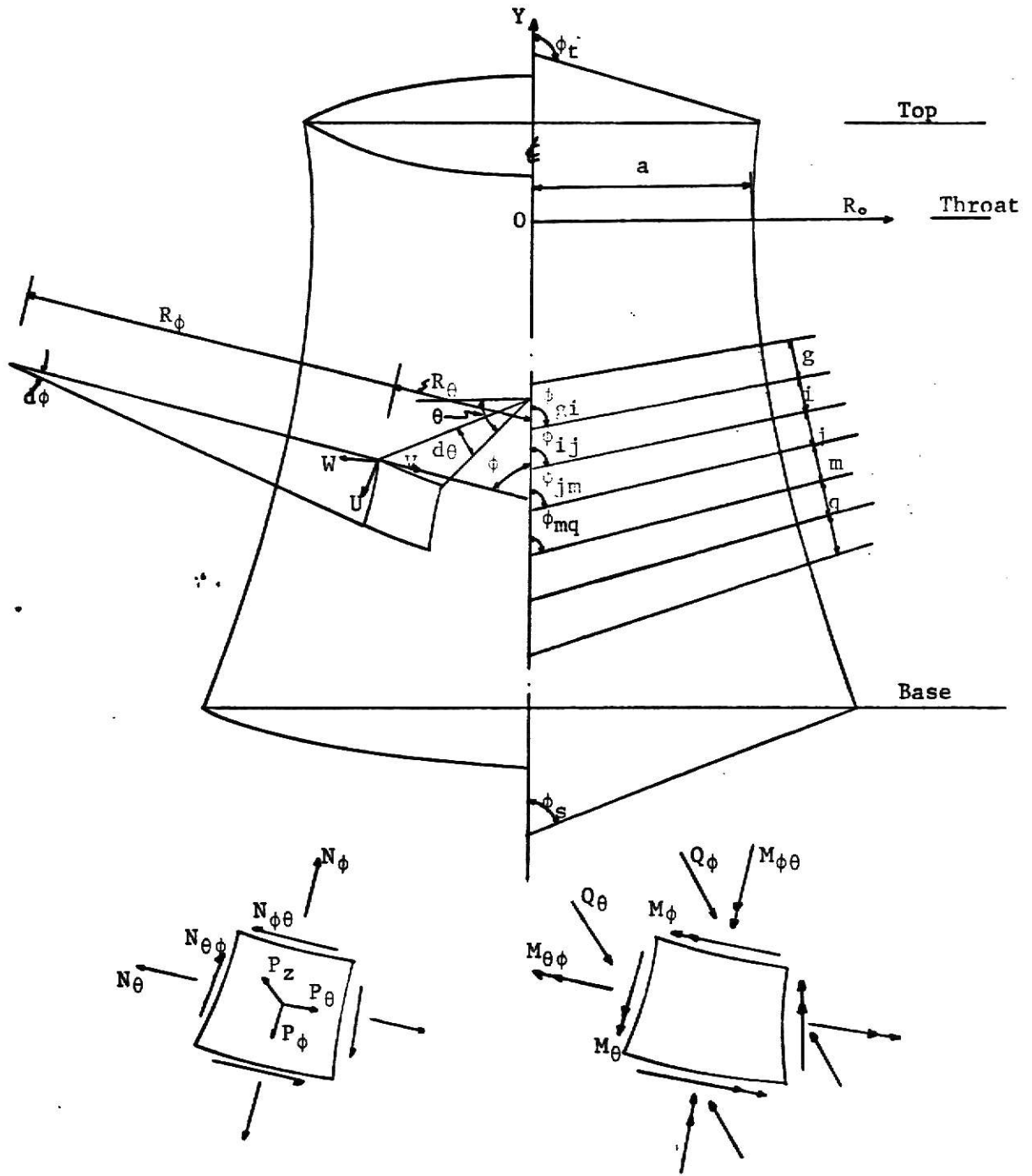


FIG. 2. Hyperboloid of Revolution

## IV. ANALYSIS OF HYPERBOLIC SHELL OF REVOLUTION

## 1. Surface geometry

The geometry of the shell surface (Fig. 2) is defined by

$$\frac{R_o^2}{a^2} - \frac{y^2}{b^2} = 1, \quad (4.1a)$$

in which  $R_o$  = the horizontal radius,  $y$  = the vertical coordinate,  
 $a$  = the throat radius and

$$b = \frac{aT}{(t^2 - a^2)^{1/2}} = \frac{bS}{(s^2 - a^2)^{1/2}}, \quad (4.1b)$$

in which  $s$  = the base radius,  $t$  = the top radius, and  $S$  and  $T$  = the  
 vertical distances from the throat to the base and the top of the shell  
 respectively.

The coordinate system is shown in Fig. 2 where the positive  
 directions of  $\phi$  and  $\theta$  as well as the load components per unit area of  
 middle surface  $P_\phi$ ,  $P_\theta$ ,  $P_z$  are indicated. The principal radii of  
 curvature  $R_\phi$  and  $R_\theta$  are given by

$$R_\phi = -a^2 b^2 \left( \frac{R_o^2}{a^4} + \frac{y^2}{b^4} \right)^{3/2}$$

$$= \frac{-a^2 b^2}{(a^2 \sin^2 \phi - b^2 \cos^2 \phi)^{3/2}}, \quad (4.1c)$$



$$R_{\theta} = a \left[ 1 + \frac{y^2}{a^2} (\bar{\alpha} + \bar{\alpha}^2) \right]^{\frac{1}{2}}$$

$$= \frac{a^2}{(a^2 \sin^2 \phi - b^2 \cos^2 \phi)^{\frac{1}{2}}} \quad (4.1d)$$

where  $\bar{\alpha} = \frac{b^2}{a^2}$ ,

and

$$R_{\phi} = \frac{a^2 \sin^2 \phi}{(a^2 \sin^2 \phi - b^2 \cos^2 \phi)^{\frac{1}{2}}} = R_{\theta} \sin \phi. \quad (4.1e)$$

## 2. Membrane theory

The differential equations of equilibrium of a shell of revolution based on membrane theory are well known (13) and given by

$$\frac{1}{R_{\phi}} \frac{\partial N_{\phi\theta}}{\partial \phi} + 2 \frac{\cot \phi}{R_{\theta}} N_{\phi\theta} + \frac{1}{R_{\theta} \sin \phi} \frac{\partial N_{\theta}}{\partial \theta} + P_{\theta} = 0, \quad (4.2a)$$

$$\frac{1}{R_{\phi}} \frac{\partial N_{\phi}}{\partial \phi} + \frac{\cot \phi}{R_{\theta}} (N_{\phi} - N_{\theta}) + \frac{1}{R_{\theta} \sin \phi} \frac{\partial N_{\phi\theta}}{\partial \theta} + P_{\phi} = 0, \quad (4.2b)$$

$$\frac{N_{\phi}}{R_{\phi}} + \frac{N_{\theta}}{R_{\theta}} - P_z = 0. \quad (4.2c)$$

The expressions relating stress resultants, strains and displacements are

$$\epsilon_{\phi} = \frac{1}{Eh} (N_{\phi} - \mu N_{\theta}), \quad (4.3a-1)$$

$$= \frac{1}{R_{\phi}} \left( \frac{\partial U}{\partial \phi} - W \right), \quad (4.3a-2)$$

$$\epsilon_{\theta} = \frac{1}{Eh} (N_{\theta} - \mu N_{\phi}), \quad (4.3b-1)$$

$$= \frac{1}{R_{\theta}} \left( \frac{1}{\sin \phi} \frac{\partial V}{\partial \theta} + U \cot \phi - W \right), \quad (4.3b-2)$$

$$\omega = \frac{2(1 + \mu)}{Eh} N_{\phi\theta}, \quad (4.3c-1)$$

$$= \frac{-1}{R_{\phi}} \frac{\partial U}{\partial \phi} + \frac{1}{R_{\theta}} U \cot \phi - \frac{1}{R_{\theta} \sin \phi} \frac{\partial V}{\partial \theta} = 0. \quad (4.3c-2)$$

#### Dead weight (12)

For shells of constant thickness, the components of the dead load are given by

$$P_{\theta} = 0, \quad P_{\phi} = g \sin \phi, \quad P_z = -g \cos \phi, \quad (4.4)$$

in which  $g$  is the dead weight per unit area of the surface. Due to symmetry of the loads,  $N_{\phi\theta} = N_{\theta\phi} = 0$ , and all terms involving derivatives with respect to  $\theta$  vanish. Upon inserting these and Eq. (4.4) into Eq. (4.2) the following equations of equilibrium will be obtained.

$$N_{\theta} R_{\phi} \cos \phi - \frac{d}{d\phi} (N_{\phi} R_{\theta}) = g \sin \phi R_{\theta} R_{\phi}, \quad (4.5a)$$

$$\frac{N_{\phi}}{R_{\phi}} + \frac{N_{\theta}}{R_{\theta}} = -g \cos \phi. \quad (4.5b)$$

Solving the differential equation (4.5a),

$$N_{\phi} = - \frac{W_d}{2\pi R_{\theta} \sin^2 \phi} \quad , \quad (4.6a)$$

in which  $W_d$  is the total vertical load above the level  $\phi$  found by integration as followed.

$$W_d = g \int 2\pi R_{\theta} R_{\phi} d\phi = 2\pi g a^4 b^2 \int_{\phi}^{\phi_t} \frac{\sin \phi d\phi}{(a^2 \sin^2 \phi - b^2 \cos^2 \phi)^2} \quad (4.6b)$$

Introducing the auxiliary variable  $\xi$ ,

$$\text{let } \cos \phi = \frac{a}{(a^2 + b^2)^{1/2}} \xi \quad (4.6c)$$

Therefore,

$$\begin{aligned} W_d &= 2\pi g a^4 b^2 \int_{\xi}^{\xi_t} \frac{a(a^2 + b^2)^{3/2}}{(a^4 - a^4 \xi^2 + a^2 b^2 - a^2 b^2 \xi^2)^2} d\xi \\ &= \frac{\pi g}{2} \frac{ab^2}{(a^2 + b^2)^{1/2}} \left| \frac{2\xi}{1 - \xi^2} + \ln \frac{1 + \xi}{1 - \xi} \right|_{\xi}^{\xi_t} \quad (4.6d) \end{aligned}$$

$$\text{Let } f(\xi) = \frac{2\xi}{1 - \xi^2} + \ln \frac{1 + \xi}{1 - \xi} \quad (4.6e)$$

$$\text{Then } N_{\phi} = - \frac{\pi g}{2} \frac{ab^2}{(a^2 + b^2)^{1/2}} \frac{1}{2\pi R_{\theta} \sin^2 \phi} [ f(\xi) - f(\xi_t) ] \quad (4.6f)$$

Simplifying,

$$N_{\phi} = -\frac{g}{4} b^2 (a^2 + b^2)^{\frac{1}{2}} \frac{(1 - \xi^2)^{\frac{1}{2}}}{(a^2 + b^2 - a^2 \xi^2)} [f(\xi) - f(\xi_t)], \quad (4.6g)$$

and using Eq. (4.5b),

$$N_{\theta} = -\frac{ga^2}{(a^2 + b^2)^{\frac{1}{2}}} \frac{\xi}{(1 - \xi^2)^{\frac{1}{2}}} + N_{\phi} \frac{a^2}{b^2} (1 - \xi^2). \quad (4.6h)$$

The displacement components U, V and W are positive as shown in Fig. 2.

Eliminating W between Eq. (4.3a-2) and Eq. (4.3b-2) and dropping the term involving V because of symmetry one obtains

$$\frac{\partial U}{\partial \phi} - U \cot \phi - R_{\phi} \epsilon_{\phi} + R_{\theta} \epsilon_{\theta} = 0. \quad (4.7a)$$

Substituting Eqs. (4.3a-1) and (4.3b-1) into (4.7a) and from Eq. (4.2c)  $N_{\theta}$  can be written in terms of  $N_{\phi}$  and Eq. (4.7a) becomes

$$\begin{aligned} \frac{\partial U}{\partial \phi} - U \cot \phi - \frac{N_{\phi}}{Eh} (R_{\phi} + \mu R_{\theta} + \frac{R_{\theta}^2}{R_{\phi}} + \mu R_{\theta}) + \frac{1}{Eh} (R_{\theta}^2 + \mu R_{\phi} R_{\theta}) g \cos \phi \\ = 0. \end{aligned} \quad (4.7b)$$

By applying the boundary condition,  $U = 0$  at  $\phi = \phi_s$ , the differential equation (4.7b) can be solved and

$$U = \frac{\sin \phi}{Eh} \int_{\phi_s}^{\phi} \left[ \frac{(R_{\phi}^2 + R_{\theta}^2 + 2\mu R_{\phi} R_{\theta})}{R_{\phi} \sin \phi} N_{\phi} - g R_{\theta} (R_{\theta} + \mu R_{\phi}) \cot \phi \right] d\phi. \quad (4.7c)$$

$$\begin{aligned}
W &= U \cot \phi - R_{\theta} \varepsilon_{\theta} \\
&= U \cot \phi - \frac{R_{\theta}}{Eh} (N_{\theta} - \mu N_{\phi}) \\
&= U \cot \phi + \frac{R_{\theta} N_{\phi}}{R_{\phi} Eh} (R_{\theta} + \mu R_{\phi}) - \frac{R_{\theta}^2 g \cos \phi}{Eh} .
\end{aligned} \tag{4.7d}$$

### Wind load

The equilibrium equations of the membrane theory, in the case of a hyperboloid of revolution can be reduced to the form (7),

$$\frac{r_{\theta} \sin \phi}{r_{\phi}} \frac{\partial \psi}{\partial \phi} + \frac{\partial \eta}{\partial \theta} = (p_z \cos \phi - p_{\phi} \sin \phi) r_{\theta} \sin^2 \phi, \tag{4.8a}$$

$$\frac{\partial \eta}{\partial \phi} - \frac{r_{\theta}}{\sin \phi} \frac{\partial \psi}{\partial \theta} = - (p_{\theta} \sin \phi + \frac{\partial p_z}{\partial \theta}) r_{\phi} r_{\theta}^2 \sin \phi, \tag{4.8b}$$

$$\frac{n_{\phi}}{r_{\phi}} + \frac{n_{\theta}}{r_{\theta}} - p_z = 0, \tag{4.8c}$$

in which

$$\psi = n_{\phi} r_{\theta} \sin^2 \phi, \quad \eta = n_{\phi \theta} r_{\theta}^2 \sin \phi, \tag{4.9}$$

$$\text{and } r_{\phi} = \frac{R_{\phi}}{a}, \quad r_{\theta} = \frac{R_{\theta}}{a},$$

$$n_{\phi} = \frac{N_{\phi}}{Pa}, \quad n_{\theta} = \frac{N_{\theta}}{Pa}, \quad n_{\phi \theta} = \frac{N_{\phi \theta}}{Pa}, \tag{4.10}$$

$$p_{\phi} = \frac{P_{\phi}}{Pa}, \quad p_{\theta} = \frac{P_{\theta}}{Pa}, \quad p_z = \frac{P_z}{Pa}.$$

In the above,

$P$  = a constant reference load intensity per unit area of the middle surface.

The load components may be expressed in the following form with  $n > 1$ :

$$p_\phi = p_{\phi n}(\phi) \cos n\theta, \quad p_\theta = p_{\theta n}(\phi) \sin n\theta, \quad p_z = p_{zn}(\phi) \cos n\theta, \quad (4.11a)$$

$$\text{and } \psi = \psi'(\phi) \cos n\theta. \quad (4.11b)$$

Eliminating  $n$  between Eqs. (4.8a) and (4.8b) yields

$$\frac{d^2\psi'}{d\phi^2} + \chi_1(\phi) \frac{d\psi'}{d\phi} + \chi_2(\phi)\psi' = \chi_3(\phi) \quad (4.12)$$

in which

$$\chi_1(\phi) = \left(2 \frac{r_\phi}{r_\theta} - 1\right) \cot\phi - \frac{1}{r_\phi} \frac{dr_\phi}{d\phi}, \quad (4.13a)$$

$$\chi_2(\phi) = \frac{-n^2 r_\phi}{r_\theta \sin^2\phi}, \quad (4.13b)$$

and

$$\begin{aligned} \chi_3(\phi) = & \frac{r_\phi}{r_\theta^2 \sin^2\phi} \frac{d}{d\phi} [(p_{zn} \cos\phi - p_{\phi n} \sin\phi) r_\theta^3 \sin^2\phi] - r_\phi^2 n (n p_{zn} - \\ & - p_{\theta n} \sin\phi). \end{aligned} \quad (4.13c)$$

The solution to the homogeneous part of Eq. (4.12) is (7)

$$\psi'_{jh}(\phi) = N(\phi) [C_j \cos(\rho_j \beta_j) + D_j \sin(\rho_j \beta_j)] \quad (4.14)$$

in which  $i, j$  denote the segment designation (see Fig. 2).

Then,

$$N(\phi) = \left( \frac{-r\phi}{H' r_0^2 \sin\phi} \right)^{\frac{1}{2}}, \quad (4.15a)$$

$$H' = \sqrt{-G(\phi)}, \quad (4.15b)$$

$$G(\phi) = \frac{1}{2} \frac{dX_1}{d\phi} + \frac{1}{4} X_1^2 - X_2, \quad (4.15c)$$

$$\rho(\phi) = \sqrt{1 + \Delta(\phi)}, \quad (4.15d)$$

and

$$\Delta(\phi) = \frac{1}{2G} \left[ \frac{1}{2G} \frac{d^2G}{d\phi^2} - \frac{5}{8G^2} \left( \frac{dG}{d\phi} \right)^2 \right]. \quad (4.15e)$$

$\rho(\phi)$  will be regarded as a constant  $\rho_j$  for segment  $j$  of the shell corresponding to the  $n$ th harmonic. The value of  $\rho_j$  will be taken as the average value for the segment under consideration and

$$\beta_j = \int_{\phi_{1j}}^{\phi} H'(\phi) d\phi \quad (4.15f)$$

with  $C_j$  and  $D_j$  being the constants of integration.

The particular solution to Eq. (4.12) by means of the method of variation of parameter is given by (7)

$$\psi'_{jp} = N(\phi) [A_j(\phi) \cos(\rho_j \beta_j) + B_j(\phi) \sin(\rho_j \beta_j)], \quad (4.16)$$

in which

$$A_j(\phi) = \frac{-1}{\rho_j} \int_{\phi_{1j}}^{\phi} \frac{\chi_3(\phi) \sin(\rho_j \beta_j)}{H'(\phi) N(\phi)} d\phi, \quad (4.17a)$$

and

$$B_j(\phi) = \frac{1}{\rho_j} \int_{\phi_{1j}}^{\phi} \frac{\chi_3(\phi) \cos(\rho_j \beta_j)}{H'(\phi) N(\phi)} d\phi. \quad (4.17b)$$

The integrals in Eqs. (4.17a) and (4.17b) generally must be evaluated numerically due to the complexity of the integrand. Stress resultants for  $n > 1$  with  $\psi'_j$  are determined as the sum of Eqs. (4.14) and (4.16)

$$\psi'_j = \psi'_{jh} + \psi'_{jp}. \quad (4.18)$$

Substituting back into Eqs. (4.9) and (4.11b) yields

$$n_{\phi n} = \frac{\psi'(\phi) \cos n\theta}{r_\theta \sin^2 \phi}. \quad (4.19)$$

Integrating Eq. (4.8a) in view of Eq. (4.9) yields  $n_{\theta\phi n}$ ; and considering Eq. (4.8c) gives the stress resultant  $n_{\theta n}$ . The expressions for the stress resultants in segment  $j$  for the  $n$ th harmonic are

$$n_{\phi n} = n'_{\phi n}(\phi) \cos n\theta, \quad (4.20a)$$

$$n_{\theta\phi n} = n'_{\theta\phi n}(\phi) \sin n\theta, \quad (4.20b)$$

and

$$n_{\theta n} = n'_{\theta n}(\phi) \cos n\theta. \quad (4.20c)$$



In these equations

$$n'_{\phi n}(\phi) = \frac{N}{r_{\theta} \sin^2 \phi} \{ [(C_j + A_j(\phi)) \cos(\rho_j \beta_j) + [D_j + B_j(\phi)] \sin(\rho_j \beta_j)] \}, \quad (4.21a)$$

$$n'_{\theta \phi n}(\phi) = \frac{1}{nr_{\theta}^2 \sin^2 \phi} [(p_{zn} \cos \phi - p_{\phi n} \sin \phi) r_{\theta}^3 \sin^2 \phi - \frac{r_{\theta}^2 \sin \phi}{r_{\phi}} \frac{d\psi'_j}{d\phi}], \quad (4.21b)$$

and

$$n'_{\theta n} = r_{\theta} (p_{zn} - \frac{n'_{\phi n}}{r_{\phi}}). \quad (4.21c)$$

The integration constants  $C_j$ ,  $D_j$  are determined from the boundary conditions, applying the boundary conditions  $n_{\phi} = n_{\theta \phi} = 0$  at  $\phi = \phi_t$  for the uppermost segment. The calculation of stress resultants proceeds down the shell with decreasing  $\phi$ , and the constants  $C_j$ ,  $D_j$  for any segment are obtained by equating the values of  $n'_{\phi n}$  and  $n'_{\theta \phi n}$  at the upper edge of segment  $j$  with the values of the corresponding stress resultants at the lower edge of segment  $i$  as shown in Fig. 2.

From Eqs. (4.21b) and (4.21c) and assuming  $p_{\phi n}$  and  $p_{zn}$  are constant with respect to  $\phi$  one obtains

$$C_j = [ \frac{A_i(\phi_{ij})}{\rho_i} + C_i ] \cos[\rho_i \beta_i(\phi_{ij})] + [ \frac{B_i(\phi_{ij})}{\rho_i} + D_i ] \sin[\rho_i \beta_i(\phi_{ij})], \quad (4.22a)$$

and

$$D_j = \frac{1}{Y''_j(\phi_{ij})} \{ [ \frac{A_i(\phi_{ij})}{\rho_i} + C_i ] Y'_i(\phi_{ij}) + [ \frac{B_i(\phi_{ij})}{\rho_i} + D_i ] + Y''_i(\phi_{ij}) - C_j Y'_j(\phi_{ij}) \}. \quad (4.22b)$$

in which

$$Y'_j(\phi) = \frac{dN}{d\phi} \cos(\rho_j \beta_j) - \rho_j NH' \sin(\rho_j \beta_j), \quad (4.22c)$$

$$Y''_j(\phi) = \frac{dN}{d\phi} \sin(\rho_j \beta_j) + \rho_j NH' \cos(\rho_j \beta_j). \quad (4.22d)$$

The constants for the uppermost segment t simplify to

$$C_t = 0, \quad (4.22e)$$

$$D_t = \frac{1}{\rho_t} \left( \frac{r_{\theta} r_{\phi}}{H' N} \right)_{\phi=\phi_t} \sin \phi_t (p_{zn} \cos \phi_t - p_{\phi n} \sin \phi_t). \quad (4.22f)$$

Take the load component in the form

$$p_{\phi n} = 0, \quad p_{\theta n} = 0, \quad p_{zn} = -\alpha_n, \quad (4.23a)$$

where  $\alpha_n$  is the Fourier coefficient for a particular pressure function.

$$\text{Then, } p_z = \sum_{n=0}^{\infty} p_{zn} \cos n\theta. \quad (4.23b)$$

The wind pressure distribution on a hyperbolic cooling tower is obtained by R. F. Rish and T. F. Steel from wind tests (2). The recommended formulas are summarized as follows.

$$P_z = - 1.524 \cos(1.89\theta), \quad 0^\circ < \theta < 47.6^\circ, \quad (4.23c)$$

$$P_z = + 0.69 \sin[3.61(\theta - 47.6)], \quad 47.6^\circ < \theta < 100^\circ, \quad (4.23d)$$

$$P_z = - 0.21, \quad 100^\circ < \theta < 180^\circ. \quad (4.23e)$$

The wind pressure is assumed to be constant throughout the height of the shell.

The solutions for  $n = 0$  and  $n = 1$  are presented for a wind pressure distribution of the form.

$$P_{\phi n} = 0, \quad P_{\theta n} = 0, \quad P_{zn} = - 1. \quad (23.f)$$

For  $n = 0$  the solutions is similar to that of dead load, and for  $n = 1$  to that of seismic load (6).

$$n_\phi = \alpha_0 n_{\phi 0} + \sum_{n=1}^8 \alpha_n n'_{\phi n} \cos n\theta, \quad (4.24a)$$

$$n_\theta = \alpha_0 n_{\theta 0} + \sum_{n=1}^8 \alpha_n n'_{\theta n} \cos n\theta, \quad (4.24b)$$

$$n_{\theta\phi} = \sum_{n=1}^8 \alpha_n n'_{\theta\phi n} \sin n. \quad (4.24c)$$

The stress resultant - displacement relationship can be given by  
(1, 7)

$$\frac{\partial \zeta}{\partial \phi} - \frac{r_{\theta}}{\sin \phi} \frac{\partial \gamma}{\partial \theta} = \frac{1}{\sin \phi} \left[ \frac{r_{\phi}^2 + r_{\theta}^2 + 2\mu r_{\phi} r_{\theta}}{r_{\phi}} n_{\phi} - r_{\theta} (r_{\theta} + \mu r_{\phi}) p_z \right], \quad (4.25a)$$

$$\frac{r_{\theta}^2}{r_{\phi}} \sin \phi \frac{\partial \gamma}{\partial \phi} + \frac{\partial \zeta}{\partial \theta} = 2(1 + \mu) r_{\theta} n_{\theta \phi}, \quad (4.25b)$$

$$\frac{1}{\sin \phi} \frac{\partial v}{\partial \theta} + u \cot \phi + w = r_{\theta} (n_{\theta} - \mu n_{\phi}), \quad (4.25c)$$

in which

$$\zeta = \frac{u}{\sin \phi}, \quad \gamma = \frac{v}{r_{\theta} \sin \phi}, \quad (4.26a)$$

$$u = \frac{UEh}{Pa^2}, \quad v = \frac{VEh}{Pa^2}, \quad w = \frac{WEh}{Pa^2}. \quad (4.26b)$$

In these

$E$  = Young's modulus,

$h$  = shell thickness,

and

$\mu$  = Poisson's ratio.

Taking the variable  $\gamma$  in the form

$$\gamma = \gamma'(\phi) \sin n\theta \quad (4.27)$$

and eliminating  $\zeta$  between Eqs. (4.25a) and (4.25b) leads to

$$\frac{d^2 \gamma'}{d\phi^2} + \chi_1(\phi) \frac{d\gamma'}{d\phi} + \chi_2(\phi) \gamma' = \chi_4(\phi), \quad (4.28a)$$

where

$$\begin{aligned} \chi_4(\phi) = & \frac{r_\phi}{r_\theta^2 \sin\phi} \left[ 2(1 + \mu) \frac{d}{d\phi} (r_\theta n'_{\theta\phi n} + \frac{n(r_\phi^2 + r_\theta^2 + 2\mu r_\phi r_\theta)}{r_\phi \sin\phi} n'_{\phi n} \right. \\ & \left. - n p_{zn} \frac{r_\theta(r_\theta + \mu r_\phi)}{\sin\phi} \right]. \end{aligned} \quad (4.28b)$$

Displacements for  $n = 0$  and  $n = 1$  are similar to that under dead load and seismic load respectively (6). To obtain the displacements for  $n > 1$ , the same method of variation of parameter on stress analysis is applied yielding the solution (7).

$$\gamma'_j = \gamma'_{jh} + \gamma'_{jp}. \quad (4.29)$$

In this,

$$\gamma'_{jh} = N [C'_j \cos(\rho_j \beta'_j) + D'_j \sin(\rho_j \beta'_j)], \quad (4.30a)$$

$$\gamma'_{jp} = N [A'_j(\phi) \cos(\rho_j \beta'_j) + B'_j(\phi) \sin(\rho_j \beta'_j)], \quad (4.30b)$$

$$\beta'_j = \int_{\phi_{jm}}^{\phi} H'(\phi) d\phi, \quad (4.30c)$$

$$A'_j(\phi) = \frac{1}{\rho_j} \int_{\phi_{jm}}^{\phi} \frac{-\chi_4(\phi) \sin(\rho_j \beta'_j)}{H'(\phi) N(\phi)} d\phi, \quad (4.30d)$$

$$B'_j(\phi) = \frac{1}{\rho_j} \int_{\phi_{jm}}^{\phi} \frac{\chi_4(\phi) \cos(\rho_j \beta'_j)}{H'(\phi) N(\phi)} d\phi, \quad (4.30e)$$

and  $C'_j$  and  $D'_j$  denote the constants of integration. The lower limit of integration for  $\beta'_j$ ,  $A'_j$  and  $B'_j$  is taken as the angle  $\phi_{jm}$  corresponding the boundary between segments  $j$  and  $m$  as shown in Fig. 2. By combining Eqs. (4.25) and (4.29), the displacements for segment  $j$  and the  $n$ th harmonic are given by

$$u_n = u'_n(\phi) \cos n\theta, \quad v_n = v'_n(\phi) \sin n\theta, \quad w_n = w'_n(\phi) \cos n\theta \quad (4.31)$$

in which,

$$u'_n(\phi) = \frac{-\sin\phi}{n} \left[ 2(1 + \mu) r_\theta n'_{\theta\phi n} - \frac{r_\theta^2 \sin\phi}{r_\phi} \frac{d\gamma'_j}{d\phi} \right], \quad (4.32a)$$

$$v'_n(\phi) = N(\phi) r_\theta \sin\phi \{ [C'_j + A'_j(\phi)] \cos(\rho_j \beta'_j) + [D'_j + B'_j(\phi)] \sin(\rho_j \beta'_j) \}, \quad (4.32b)$$

and

$$w'_n(\phi) = r_\theta (n'_{\theta n} - \mu n'_{\phi n}) - n v'_n \csc\phi - u'_n \cot\phi. \quad (4.32c)$$

The boundary conditions for displacements are  $u = v = 0$  at  $\phi = \phi_s$  in which  $\phi_s$  defines the base of the shell.

The integration constants  $C'_j$ ,  $D'_j$  are determined from the boundary conditions for the base of the shell. Proceed upward with increasing

$\phi$  and obtain by equating the values of  $u'_n$  and  $v'_n$  at the lower edge of segment  $j$  with the values of the corresponding displacement components at the upper edge of the segment  $m$ . Thus from Eqs. (4.30a) and (4.30b),

$$C'_j = \left[ \frac{A'_m(\phi_{jm})}{\rho_m} + C'_m \right] \cos[\rho_m \beta'_m(\phi_{jm})] + \left[ \frac{B'_m(\phi_{jm})}{\rho_m} + D'_m \right] \sin[\rho_m \beta'_m(\phi_{jm})], \quad (4.33a)$$

and

$$D'_j = \frac{1}{Z''_j(\phi_{jm})} \left\{ \left[ \frac{A'_m(\phi_{jm})}{\rho_m} + C'_m \right] Z'_m(\phi_{jm}) + \left[ \frac{B'_m(\phi_{jm})}{\rho_m} + D'_m \right] Z''_m(\phi_{jm}) - C'_j Z'_j(\phi_{jm}) \right\}, \quad (4.33b)$$

in which

$$Z'_j(\phi) = \frac{dN}{d\phi} \cos(\rho_j \beta'_j) - \rho_j N H' \sin(\rho_j \beta'_j), \quad (4.33c)$$

and

$$Z''_j(\phi) = \frac{dN}{d\phi} \sin(\rho_j \beta'_j) + \rho_j N H' \cos(\rho_j \beta'_j). \quad (4.33d)$$

The constants for the lowest segment  $s$  simplify to

$$C'_s = 0, \quad (4.33e)$$

and

$$D'_s = \frac{2(1 + \nu)}{\rho_s} \left( \frac{r_\phi n'_\theta \phi n}{r_\theta H' N} \right)_{\phi=\phi_s} \csc \phi_s. \quad (4.33f)$$

The expression for wind load displacements for the loading given by Eqs. (4.23) are given by (7)

$$u = \alpha_o u_o + \sum_{n=1}^{\infty} \alpha_n u'_n \cos n\theta, \quad (4.34a)$$

$$v = \sum_{n=1}^{\infty} \alpha_n v'_n \sin n\theta, \quad (4.34b)$$

and

$$w = \alpha_o w_o + \sum_{n=1}^{\infty} \alpha_n w'_n \cos n\theta. \quad (4.34c)$$

### 3. Bending theory

The equations of equilibrium for the bending of a shell of revolution are given by (8)

$$\frac{1}{r_\phi r_\theta \sin\phi} \left[ \frac{\partial(r_\theta \sin\phi n_\phi)}{\partial\phi} + r_\phi \frac{\partial n_{\theta\phi}}{\partial\phi} - \frac{\partial(r_\theta \sin\phi)}{\partial\phi} n_\theta \right] + \frac{q_\phi}{r_\phi} + p_\phi = 0, \quad (4.35a)$$

$$\frac{1}{r_\phi r_\theta \sin\phi} \left[ \frac{\partial(r_\theta \sin\phi n_{\theta\phi})}{\partial\phi} + r_\phi \frac{\partial n_\theta}{\partial\theta} + \frac{\partial(r_\theta \sin\phi)}{\partial\phi} n_{\theta\phi} \right] + \frac{q_\theta}{r_\theta} + p_\theta = 0, \quad (4.35b)$$

$$\frac{1}{r_\phi r_\theta \sin\phi} \left[ \frac{\partial(r_\theta \sin\phi q_\phi)}{\partial\phi} + \frac{\partial(r_\phi q_\theta)}{\partial\theta} \right] - \frac{n_\phi}{r_\phi} - \frac{n_\theta}{r_\theta} + p_z = 0, \quad (4.35c)$$

$$\frac{1}{r_\phi r_\theta \sin\phi} \left[ \frac{\partial(r_\theta \sin\phi m_{\phi\theta})}{\partial\phi} + r_\phi \frac{\partial m_{\phi\theta}}{\partial\theta} - \frac{\partial(r_\theta \sin\phi)}{\partial\phi} m_\theta \right] - q_\phi = 0, \quad (4.35d)$$



$$\frac{1}{r_\phi r_\theta \sin\phi} \left[ \frac{\partial(r_\theta \sin\phi m_{\phi\theta})}{\partial\phi} + \frac{\partial(r_\phi m_\theta)}{\partial\theta} + \frac{\partial(r_\theta \sin\phi)}{\partial\phi} m_{\theta\phi} \right] - q_\theta = 0, \quad (4.35e)$$

$$n_{\phi\theta} - n_{\theta\phi} + \frac{m_{\phi\theta}}{r_\phi} - \frac{m_{\theta\phi}}{r_\theta} = 0 \quad (4.35f)$$

in which

$$m_\phi = \frac{M_\phi}{Pa^2}, \quad m_\theta = \frac{M_\theta}{Pa^2}, \quad m_{\phi\theta} = \frac{M_{\phi\theta}}{Pa^2}, \quad m_{\theta\phi} = \frac{M_{\theta\phi}}{Pa^2}, \quad (4.36)$$

$$q_\phi = \frac{Q_\phi}{Pa}, \quad \text{and} \quad q_\theta = \frac{Q_\theta}{Pa}.$$

Equation(4.35f) is identically satisfied for  $n_{\phi\theta} = n_{\theta\phi}$ ,  $m_{\phi\theta} = m_{\theta\phi}$ .

Using the complex formulation (8),

$$\bar{N}_\phi = n_\phi - \frac{i}{\nu} \frac{m_\theta - \mu m_\phi}{1 - \mu^2}, \quad (4.37a)$$

$$\bar{N}_\theta = n_\theta - \frac{i}{\nu} \frac{m_\phi - \mu m_\theta}{1 - \mu^2}, \quad (4.37b)$$

$$\bar{N}_{\theta\phi} = n_{\theta\phi} + \frac{i}{\nu} \frac{m_{\theta\phi}}{1 - \mu}, \quad (4.37c)$$

in which

$$\nu = \frac{h/a}{\sqrt{12(1 - \mu^2)}}, \quad (4.37d)$$

and combining Eqs. (4.35a)through (4.35e) and dropping higher order terms leads to

$$\frac{1}{r_\phi} \frac{\partial \widetilde{N}_\phi}{\partial \phi} + \frac{\cot \phi}{r_\theta} (\widetilde{N}_\phi - \widetilde{N}_\theta) + \frac{1}{r_\theta \sin \phi} \frac{\partial \widetilde{N}_{\theta\phi}}{\partial \theta} + \frac{i\nu}{r_\phi^2} \frac{\partial \widetilde{N}}{\partial \phi} = -p_\phi, \quad (4.38a)$$

$$\frac{1}{r_\phi} \frac{\partial \widetilde{N}_{\theta\phi}}{\partial \phi} + 2 \frac{\cot \phi}{r_\theta} \widetilde{N}_{\theta\phi} + \frac{1}{r_\theta \sin \phi} \frac{\partial \widetilde{N}_\theta}{\partial \theta} + \frac{i\nu}{r_\theta^2 \sin \phi} \frac{\partial \widetilde{N}}{\partial \phi} = -p_\theta, \quad (4.38b)$$

$$\frac{\widetilde{N}_\phi}{r_\phi} + \frac{\widetilde{N}_\theta}{r_\theta} - i\nu G_1(\widetilde{N}) = p_z. \quad (4.38c)$$

In the above

$$\widetilde{N} = \widetilde{N}_\phi + \widetilde{N}_\theta, \quad (4.38d)$$

and

$$G_1(\ ) = \frac{1}{r_\phi r_\theta \sin \phi} \left\{ \frac{\partial}{\partial \phi} \left[ \frac{r_\theta \sin \phi}{r_\phi} \frac{\partial}{\partial \theta} (\ ) \right] + \frac{\partial}{\partial \theta} \left[ \frac{r_\phi}{r_\theta \sin \phi} \right. \right. \\ \left. \left. \times \frac{\partial}{\partial \theta} (\ ) \right] \right\}. \quad (4.38e)$$

Consider a harmonic  $n$  and introduce

$$n_{\phi n} = n'_{\phi n}(\phi) \cos n\theta, \quad n_{\theta n} = n'_{\theta n}(\phi) \cos n\theta, \quad n_{\theta\phi n} = n'_{\theta\phi n}(\phi) \sin n\theta,$$

$$m_{\phi n} = m'_{\phi n}(\phi) \cos n\theta, \quad m_{\theta n} = m'_{\theta n}(\phi) \cos n\theta, \quad m_{\theta\phi n} = m'_{\theta\phi n}(\phi) \sin n\theta,$$

$$q_{\phi n} = q'_{\phi n}(\phi) \cos n\theta, \quad q_{\theta n} = q'_{\theta n}(\phi) \sin n\theta, \quad (4.39)$$

$$\widetilde{N}_\phi = \widetilde{N}'_{\phi n}(\phi) \cos n\theta, \quad \widetilde{N}_\theta = \widetilde{N}'_{\theta n}(\phi) \cos n\theta, \quad \widetilde{N}_{\theta\phi} = \widetilde{N}'_{\theta\phi n}(\phi) \sin n\theta,$$

$$\widetilde{N} = \widetilde{N}'_n(\phi) \cos n\theta.$$

Also introduce the auxiliary variables

$$\psi = \tilde{\psi}_n(\phi) \cos n\theta, \quad (4.40a)$$

$$\tilde{\eta} = \tilde{\eta}_n(\phi) \sin n\theta, \quad (4.40b)$$

in which

$$\tilde{\psi}_n(\phi) = \tilde{N}_{\phi n} r_\theta \sin^2 \phi + i\nu \frac{r_\theta}{r_\phi} \sin \phi \cos \phi \frac{d\tilde{N}_n}{d\phi} \quad (4.40c)$$

$$\tilde{\eta}_n(\phi) = \tilde{N}_{\theta \phi n} r_\theta^2 \sin^2 \phi. \quad (4.40d)$$

In view of Eqs. (4.39) and (4.40), Eqs. (4.38a) through (4.38c) can be expressed in terms of the three dependent variables  $\tilde{\psi}_n$ ,  $\tilde{\eta}_n$  and  $\tilde{N}_n$ . Eliminating  $\tilde{\eta}_n$  leads to

$$G_{2n}(\tilde{\psi}_n) + n^2 \left(1 + \frac{i\nu k^2}{r_\theta}\right) \tilde{N}_n = \frac{1}{r_\phi r_\theta \sin \phi} \left\{ \frac{d}{d\phi} [(p'_{zn} \cos \phi - p'_{\phi n} \sin \phi) r_\theta^3 \sin^2 \phi] + np'_{\theta n} r_\phi r_\theta \sin^2 \phi \right\} \quad (4.41a)$$

and

$$G_{2n}(\tilde{N}_n) + \frac{i}{\nu} \tilde{N}_n + \frac{i}{\nu} \left( \frac{1}{r_\phi} - \frac{1}{r_\theta} \right) \frac{1}{\sin^2 \phi} \tilde{\psi}_n = \frac{i}{\nu} p'_{zn} r_\theta, \quad (4.41b)$$

in which

$$G_{2n}(\ ) = \frac{1}{r_\phi r_\theta \sin \phi} \frac{d}{d\phi} \left[ \frac{r_\theta^2 \sin \phi}{r_\phi} \frac{d(\ )}{d\phi} \right] - \frac{n^2}{r_\theta \sin^2 \phi} (\ ). \quad (4.41c)$$

Refer to the solution of the governing equation given in reference (8). The governing Eq. (4.41b) can be shown to be in the following form

$$\frac{d^2 \bar{N}_n}{d\phi^2} + \left[ \left( 2 \frac{r_\phi}{r_\theta} - 1 \right) \cot\phi - \frac{1}{r_\phi} \frac{dr_\phi}{d\phi} \right] \frac{d\bar{N}_n}{d\phi} + \left[ \frac{n^2}{\sin^2\phi} \frac{r_\phi}{r_\theta} \left( 1 - 2 \frac{r_\phi}{r_\theta} + \frac{ir_\phi^2}{r_\theta v} \right) \right] \bar{N}_n = \frac{ir_\phi^2}{r_\theta v} (n_{\phi n}^* + n_{\theta n}^*), \quad (4.42)$$

in which  $n_{\phi n}^*$  and  $n_{\theta n}^*$  are used to indicate a stress resultant derived from the membrane theory. An approximate particular integral to Eq. (4.42) is obtained by equating the coefficient of  $ir_\phi^2/r_\theta v$ , which gives

$$\bar{N}_{np} = n_{\phi n}^* + n_{\theta n}^*. \quad (4.43)$$

- Using the same technique as that of solving the homogeneous part of differential equation of membrane theory under wind load (8),

$$\bar{N}_{nh} = \frac{1}{r_\theta^{3/4} \sin^{1/2}\phi} \left[ \bar{A} e^{-(1-i)\beta} + \bar{A}' e^{(1-i)\beta} \right], \quad (4.44)$$

in which

$$\beta = \int_{\phi_t}^{\phi} \frac{-r_\phi}{\sqrt{2r_\theta v}} d\phi \quad (4.45)$$

and  $\bar{A}$  and  $\bar{A}'$  are constants of integration. The complex solution is obtained from Eqs. (4.43) and (4.44) as

$$\bar{N}_n = \bar{N}_{nh} + \bar{N}_{np}. \quad (4.46)$$

Separating the complex resultants into real and imaginary parts and dropping higher order terms, yields the following explicit expressions for the stress resultants (8).

$$\begin{aligned}
 n'_{\phi n} = & \delta_4 \cot \phi \sqrt{\frac{\nu}{2r_\theta}} \{A_1 e^{-\alpha} [\cos \alpha - (\delta_2 - \delta_5) \sin \alpha] - A_2 e^{-\alpha} [\sin \alpha + (\delta_2 - \delta_5) \\
 & \cos \alpha] - A_3 e^\beta [\cos \beta + (\delta_1 + \delta_5) \sin \beta] + A_4 e^\beta [-\sin \beta + (\delta_1 + \delta_5) \cos \beta]\} \\
 & + n^*_{\phi n}, \tag{4.47a}
 \end{aligned}$$

$$\begin{aligned}
 n'_{\theta n} = & \delta_4 \{A_1 e^{-\alpha} [(1 - \delta_3) \cos \alpha + \delta_3 \sin \alpha] - A_2 e^{-\alpha} [(1 - \delta_3) \sin \alpha + \delta_3 \cos \alpha] \\
 & + A_3 e^\beta [(1 + \delta_3) \cos \beta + \delta_3 \sin \beta] + A_4 e^\beta [(1 + \delta_3) \sin \beta - \delta_3 \cos \beta]\} + n^*_{\theta n}, \\
 & \tag{4.47b}
 \end{aligned}$$

$$\begin{aligned}
 n'_{\theta \phi n} = & n \delta_4 \csc \phi \sqrt{\frac{\nu}{2r_\theta}} \{A_1 e^{-\alpha} [\cos \alpha - (\delta_2 - \delta_6) \sin \alpha] - A_2 e^{-\alpha} [\sin \alpha + (\delta_2 \\
 & - \delta_6) \cos \alpha] - A_3 e^\beta [\cos \beta + (\delta_1 + \delta_6) \sin \beta] + A_4 e^\beta [-\sin \beta + (\delta_1 + \delta_6) \\
 & \times \cos \beta]\} + n^*_{\theta \phi n}, \tag{4.47c}
 \end{aligned}$$

$$\begin{aligned}
 m'_{\phi n} = & -\nu \delta_4 \{A_1 e^{-\alpha} \{[1 - (1 - \mu) \delta_3] \sin \alpha - (1 - \mu) \delta_2 \delta_3 \cos \alpha\} + A_2 e^{-\alpha} [1 \\
 & - (1 - \mu) \delta_3] \cos \alpha + (1 - \mu) \delta_2 \delta_3 \sin \alpha\} - A_3 e^\beta [1 + (1 - \mu) \delta_3] \sin \beta
 \end{aligned}$$

$$\begin{aligned}
& + (1 - \mu)\delta_1\delta_3\cos\beta\} + A_4 e^\beta\{[1 + (1 - \mu)\delta_3]\cos\beta + (1 - \mu)\delta_1\delta_3\sin\beta\} \\
& + \delta_8\frac{d}{d\phi}(n_{\phi n}^* + n_{\theta n}^*) - \delta_9(n_{\phi n}^* + n_{\theta n}^*), \tag{4.47d}
\end{aligned}$$

$$\begin{aligned}
m'_{\theta n} & = -\nu\delta_4[A_1 e^{-\alpha}\{[\mu + \delta_3(1 - \mu)]\sin\alpha + [\delta_2\delta_3(1 - \mu) - \delta_7]\sin\alpha\} \\
& + A_2 e^{-\alpha}\{[\mu + \delta_3(1 - \mu)]\cos\alpha - [\delta_2\delta_3(1 - \mu) - \delta_7]\sin\alpha\} + A_3 e^\beta\{-[\mu \\
& - \delta_3(1 - \mu)]\sin\beta - [\delta_1\delta_3(1 - \mu) + \delta_7]\cos\beta\} + A_4 e^\beta\{[\mu - \delta_3(1 \\
& - \mu)]\cos\beta + [\delta_1\delta_3(1 - \mu) + \delta_7]\sin\beta\}] - \delta_8\frac{d}{d\phi}(n_{\phi n}^* + n_{\theta n}^*) + \delta_9(n_{\phi n}^* \\
& + n_{\theta n}^*), \tag{4.47e}
\end{aligned}$$

$$\begin{aligned}
m'_{\theta\phi n} & = \frac{\nu\delta_4(1 - \mu)}{\sin\phi} \sqrt{\frac{\nu}{2r_\theta}} \{-A_1 e^{-\alpha}[(1 + \delta_6)\sin\alpha + \delta_2\cos\alpha] + A_2 e^{-\alpha}[-(1 \\
& + \delta_6)\cos\alpha + \delta_2\sin\alpha] + A_3 e^\beta[-(1 - \delta_6)\sin\beta + \delta_1\cos\beta] + A_4 e^\beta \times \\
& [(1 - \delta_6)\cos\beta + \delta_1\sin\beta]\} + \nu\frac{r_\theta}{r_\phi} \delta_7\sin\phi \frac{d}{d\phi}(n_{\phi n}^* + n_{\theta n}^*) - \nu\delta_7\cos\phi \\
& \tag{4.47f}
\end{aligned}$$

$$\begin{aligned}
q'_{\phi n} & = \delta_4 \sqrt{\frac{\nu}{2r_\theta}} [A_1 e^{-\alpha}(\cos\alpha - \delta_2\sin\alpha) - A_2 e^{-\alpha}(\sin\alpha + \delta_2\cos\alpha) - A_3 e^\beta \\
& \times(\cos\beta + \delta_1\sin\beta) + A_4 e^\beta(-\sin\beta + \delta_1\cos\beta)] - \delta_{10}\frac{d}{d\phi}(n_{\phi n}^* + n_{\theta n}^*), \\
& \tag{4.47g}
\end{aligned}$$

$$q'_{\theta n} = \frac{nv}{r_{\theta} \sin \phi} [A_1 e^{-\alpha} \sin \alpha + A_2 e^{-\alpha} \cos \alpha - A_3 e^{\beta} \cos \beta + A_4 e^{\beta} \sin \beta] + \delta_{11} (n^*_{\phi n} + n^*_{\theta n}). \quad (4.47h)$$

In these equations

$$\delta_1 = 1 - \frac{1}{r_{\phi} \delta_4} \frac{d}{d\phi} (\delta_4) \sqrt{2\nu r_{\theta}}.$$

$$\delta_2 = 1 + \frac{1}{r_{\phi} \delta_4} \frac{d}{d\phi} (\delta_4) \sqrt{2\nu r_{\theta}}.$$

$$\delta_3 = \cot \phi \sqrt{\frac{\nu}{2r_{\theta}}}.$$

$$\delta_4 = \frac{1}{r_{\theta}^{3/4} \sin^{1/2} \phi}.$$

$$\delta_5 = \frac{n^2}{\sin \phi \cos \phi} \sqrt{\frac{2\nu}{r_{\theta}}}.$$

$$\delta_6 = \cot \phi \sqrt{\frac{2\nu}{r_{\theta}}}. \quad (4.48)$$

$$\delta_7 = \frac{(1 - \mu)\nu}{r_{\theta} \sin^2 \phi}.$$

$$\delta_8 = \frac{(1 - \mu)\nu^2}{r_{\phi}} \cot \phi.$$

$$\delta_9 = \frac{n^2(1 - \mu)\nu^2}{r_{\theta} \sin^2 \phi}.$$

$$\delta_{10} = (1 - \mu) \left( \frac{\nu}{r_{\theta}} \right)^2.$$

$$\delta_{11} = n \left( \frac{v}{r_\theta} \right)^2 \csc^2 \phi.$$

$$\alpha = \int_{\phi_s}^{\phi} \frac{-r_\theta}{\sqrt{2r_\theta v}} d\phi.$$

The constants of integration  $A_1$  to  $A_4$  are related to those defined in Eq. (4.44) by

$$A_1 = (\operatorname{Re} \bar{A} \cos \bar{\beta} + \operatorname{Im} \bar{A} \sin \bar{\beta}) e^{\bar{\beta}},$$

$$A_2 = (\operatorname{Im} \bar{A} \cos \bar{\beta} - \operatorname{Re} \bar{A} \sin \bar{\beta}) e^{\bar{\beta}},$$

(4.49a)

$$A_3 = \operatorname{Re} \bar{A}';$$

$$A_4 = \operatorname{Im} \bar{A}',$$

in which

$$\bar{\beta} = \int_{\phi_s}^{\phi} \frac{-r_\theta}{\sqrt{2r_\theta v}} = \alpha - \beta = \text{constant.} \quad (4.49b)$$

Only two of the four constants  $A_1$  to  $A_4$  will be present in any equation derived from the boundary conditions because for the shell  $\alpha = 0$  at the base and  $\beta = 0$  at the top.

The complete solutions for the stress resultants for dead load and earthquake load may be obtained by setting  $n = 0$  and  $n = 1$  respectively. The solutions for wind load requires a superposition of the expressions for  $n = 0$ ,  $n = 1$  and higher harmonics. Also the form of the stress



resultants  $n_{\phi n}^*$ ,  $n_{\theta n}^*$ ,  $n_{\theta \phi n}^*$  obtained from the membrane theory is contingent upon the value of  $n$ .

The displacement  $u$ ,  $v$ ,  $w$  would be in terms of the stress resultants  $n_{\phi}$ ,  $n_{\theta}$ , and  $n_{\theta \phi}$ . The derived procedure of solutions is identical with that followed in earlier studies from the membrane theory under wind load. Generally, for design purposes the membrane displacements for a wide range of shell parameter should be sufficiently accurate (6, 7, 8).

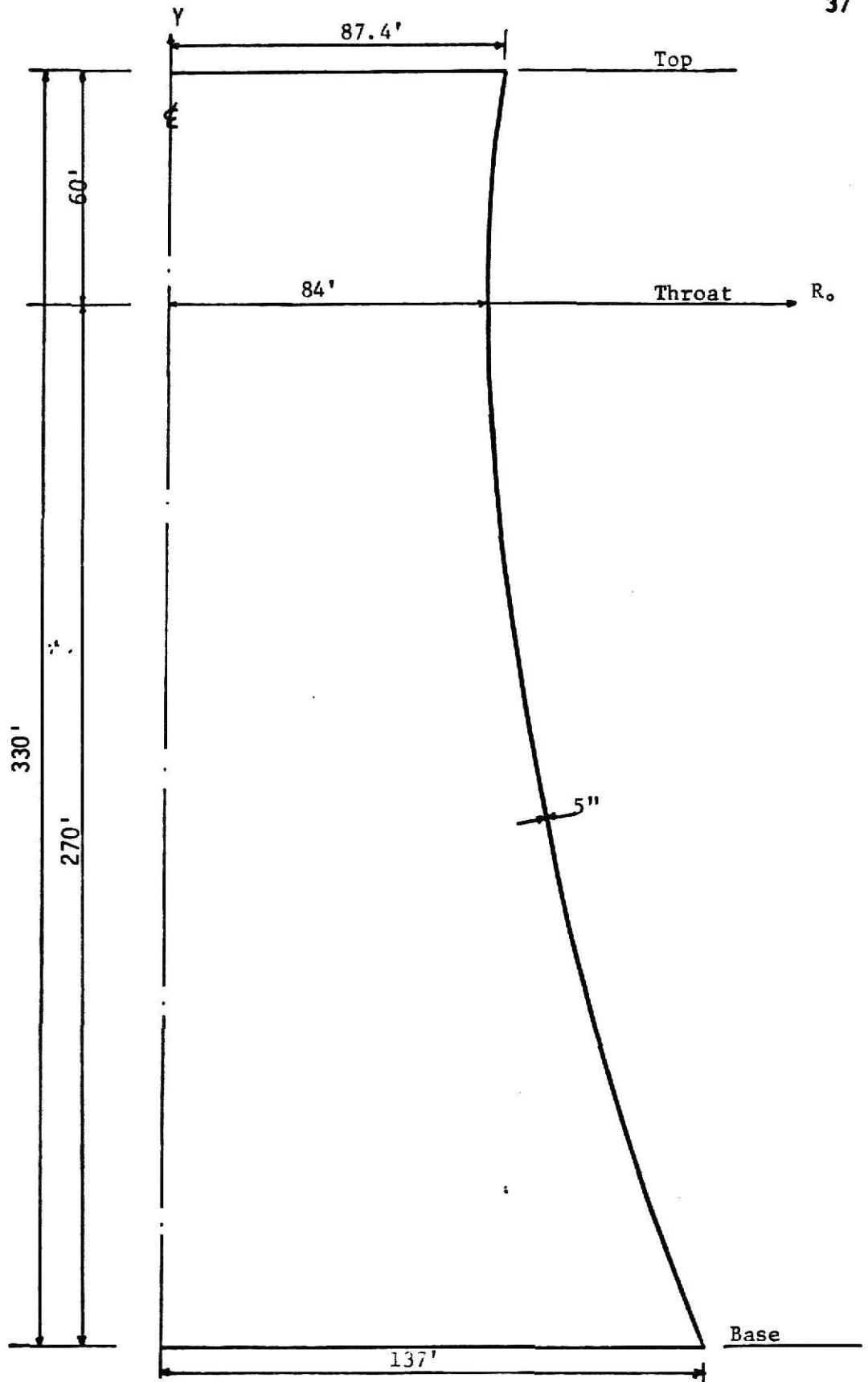


FIG. 3. Outline of Specimen Cooling Tower

## V. NUMERICAL SOLUTIONS AND DESIGN EXAMPLE

The hyperbolic cooling tower size used for this example is shown in Fig. 3.

### Loads

Dead weight  $g = 0.43403$  psi.

Wind pressure  $P_z = 0.35$  psi corresponding to a wind velocity of 140 MPH.

### Material parameters

Young's modulus  $E_\phi = E_\theta = 3000000$  psi.

Poisson's ratio  $\mu_\phi = \mu_\theta = 0.17$ .

1. Comparison between long hand membrane solutions and computer bending solutions for dead load.

The long hand numerical solutions for the membrane theory are based on Eqs. (4.1a), (4.1b), (4.1d), (4.1e), (4.6c), (4.6e), (4.6g) and (4.6h). The computer program (14) used in this Report calculates the bending solutions by means of the numerical integration method of analysis given in Reference (15).

The results of the calculation for the force resultants  $N_\phi$  and  $N_\theta$  and the comparisons between long hand membrane solutions and computer bending solutions are given in Table 1. Figs. 4, 5 and 6 illustrate the distribution of the force resultants.

Comments on comparison of these results.

- a. The values and variation for  $N_\phi$  are the same at all cross sections

between these two solutions.

- b. The values of  $N_\theta$  are different only near the base where the bending solutions are smaller than membrane solutions.
- c. In the membrane state bending forces are absent, equilibrium of the shell is maintained by the in-plane forces  $N_\phi$ ,  $N_\theta$  and  $N_{\phi\theta}$ .
- d. Due to symmetry of the dead loads, only a small bending moment  $M_\phi$  occurs at the bottom. The in-plane forces  $N_{\phi\theta}$  are all equal to zero. There is almost no bending moment  $M_\theta$  in the structure due to dead load.

In general, it has been found that the in-plane forces obtained using a bending theory analysis differ little from those computed through a simpler membrane theory analysis. Therefore, the necessary engineering accuracy could be obtained by membrane solutions for dead load.

## 2. Numerical bending solutions for wind load.

The variations of the internal forces,  $N_\phi$ ,  $N_\theta$ ,  $N_{\phi\theta}$ ,  $M_\phi$ , and  $M_\theta$ , along the meridian of the shell are shown in Figs. 7 through 11, and the circumferential distribution of maximum wind load force are given in Figs. 12 through 14. All the data for these plots are calculated based upon the nondimensional numerical wind load stress resultants in Reference (8) which were obtained by summing Fourier series harmonics from  $n = 0$  to  $n = 20$ . For design purposes, the quantities of the forces at some points are taken from Figs. 7 through 11. By using these values and combining them with those in Table 1, the total forces are tabulated in Table 2.

TABLE 1. Comparisons between Long Hand Membrane Solutions and  
Computer Bending Solutions for Dead Load

	Vertical Meridional		Long hand		Computer bending		
	distance	angle	membrane solutions		solutions		
	Y (ft)	$\phi$ (deg.)	$N_{\phi}$ (lb/in)	$N_{\theta}$ (lb/in)	$N_{\phi}$ (lb/in)	$N_{\theta}$ (lb/in)	$M_{\phi}$ (lb-in/in)
Base of shell	-270	72.344	-1345.33	-301.61	-1345.1	-228.66	129.91
	-260	72.583	-1317.33	-295.30	-1323.3	-274.02	-22.16
	-240	73.114	-1260.41	-282.83	-1261.1	-282.95	0.49
	-220	73.731	-1201.57	-270.40	-1206.4	-271.87	-0.24
	-200	74.447	-1140.90	-258.00	-1140.7	-257.87	-0.33
	-180	75.281	-1077.60	-245.35	-1075.3	-244.80	-0.30
	-160	76.250	-1010.24	-232.08	-1009.7	-231.71	0.01
	-140	77.373	- 941.03	-218.07	- 937.8	-217.30	-0.30
	-120	78.663	- 868.49	-202.93	- 866.3	-202.31	-0.26
	-100	80.140	- 787.79	-185.00	- 789.1	-185.18	-0.35
	- 80	81.808	- 703.24	-164.69	- 701.7	-164.10	-0.13
	- 60	83.659	- 613.90	-141.26	- 613.5	-140.97	-0.20
	- 40	85.671	- 519.24	-114.33	- 518.6	-113.88	-0.03
	- 20	87.804	- 420.04	- 84.08	- 420.5	- 84.01	-0.22
Throat of shell	0	90.000	- 317.42	- 51.38	- 317.2	- 51.01	0.16
	20	92.196	- 212.37	- 17.15	- 211.5	- 16.52	-0.04
	40	94.329	- 106.04	17.25	- 106.2	17.58	0.46
Top of shell	60	96.341	0.00	50.59	0.0	50.98	0.85

1 Kg. = 2.2046 lb.

1 in. = 2.54 cm.

TABLE 2. Total Design Forces.

Y (ft)	Dead load				Wind load				Maximum effect				
	$N_{\phi}$ (lb/in)	$N_{\theta}$ (lb/in)	$M_{\phi}$ (lb-in/in)	$M_{\theta}$ (lb-in/in)	$N_{\phi}$ (lb/in)	$N_{\theta}$ (lb/in)	$M_{\phi}$ (lb-in/in)	$M_{\theta}$ (lb-in/in)	$N_{\phi}$ (lb/in)	$N_{\theta}$ (lb/in)	$M_{\phi}$ (lb-in/in)	$M_{\theta}$ (lb-in/in)	$M_{\theta}^*$ (lb-in/in)
-270	-1345.1	-228.6	129.91	3763.2	705.6	0	2418.1	477.0	-1058.4	129.9 <sup>+</sup>	0		
-240	-1261.1	-282.9	0.49	4100.0	-895.3	-156.3	2838.9	-1178.2	-931.3	-155.8	47.5		
-200	-1140.7	-257.8	-0.33	4190.0	-718.8	-81.3	3049.3	-976.7	-875.0	-81.6	75.0		
-160	-1009.7	-231.7	0.01	4185.0	-473.4	-84.4	3175.3	-705.1	-831.3	-84.4	56.3		
-120	-866.3	-202.3	-0.26	3818.7	-243.8	-106.3	2952.4	-446.1	-950.0	-106.6	86.3		
-80	-701.7	-164.1	-0.13	2975.0	-125.0	-96.9	2273.2	-289.1	-1175.0	-97.0	100.0		
-40	-518.7	-113.9	-0.03	1862.5	-203.1	31.4	1343.8	-317.0	-1193.8	31.4	6.3		
0	-317.2	-51.0	0.16	1106.3	-343.8	-43.8	789.1	-394.8	-828.1	-43.6	71.3		
40	-317.3	17.6	0.46	456.3	-485.9	75.0	350.1	-468.3	-243.8	75.5	-81.8		
60	0	50.9	0.85	0	-635.0	0	0	-575.0	0	0.8	0		

\*  $N_{\theta\phi}$  and  $M_{\theta}$  are zero for dead load condition — These values are from the wind load condition.

$N_{\theta\phi}$  are at  $\theta = 45^{\circ}$ .

1 Kg. = 2.2046 lb.

1 in. = 2.54 cm.

+  $M_{\phi_{max}}$  = 1226.9 lb-in/in at 5' from base of shell.

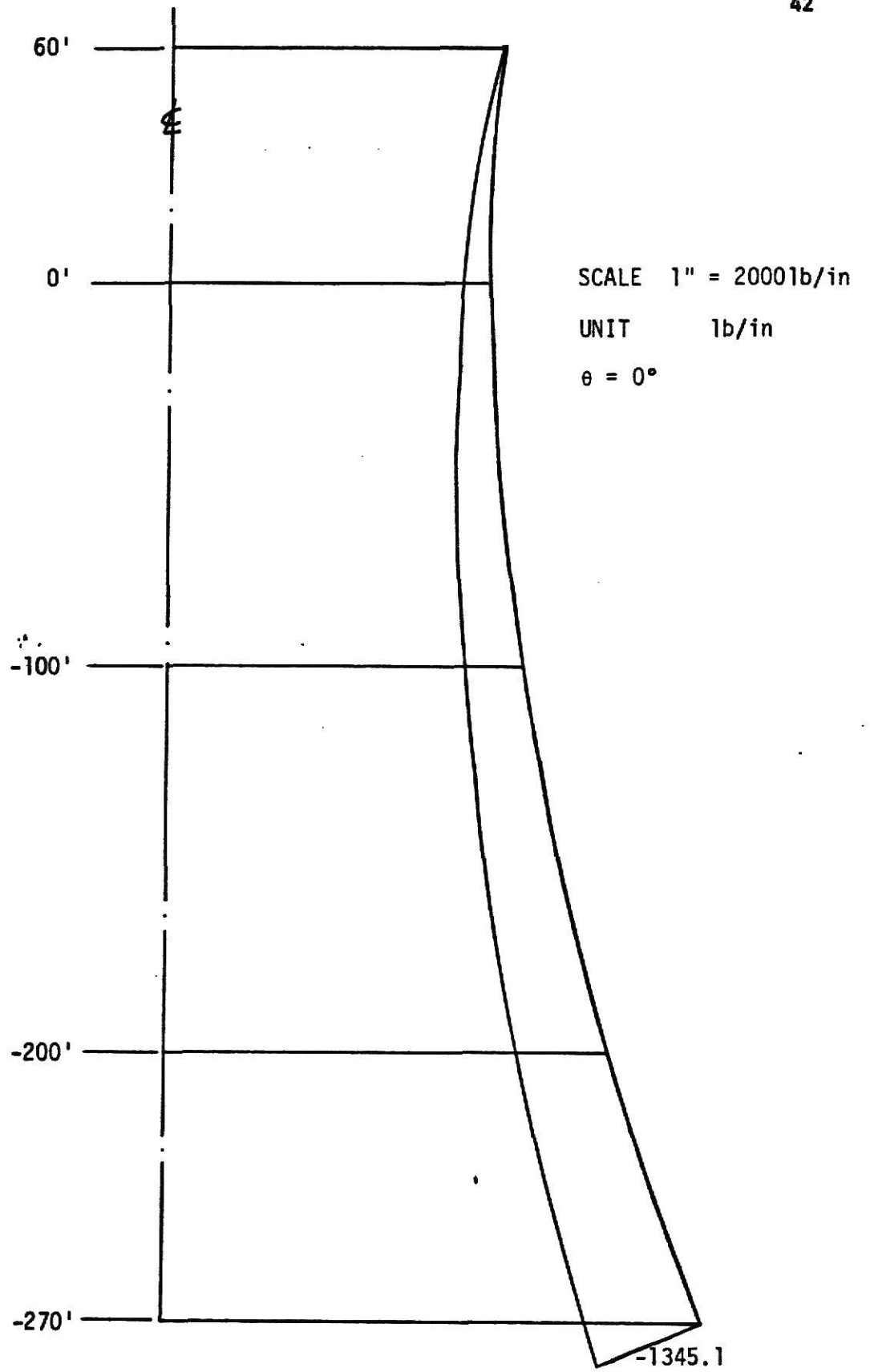


FIG. 4. Variation of  $N_\phi$  Forces due to Dead Load

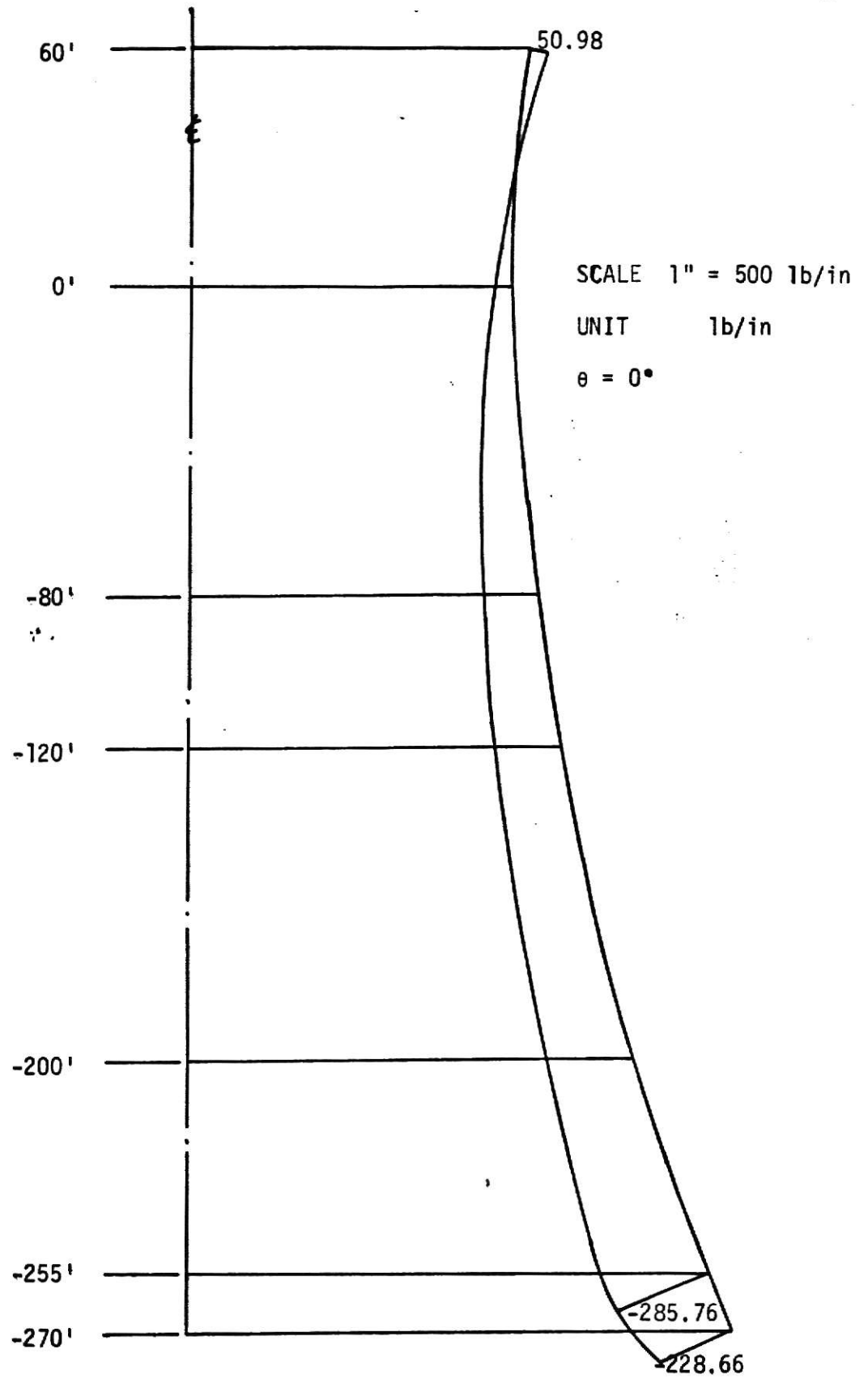


FIG. 5. Variation of  $N_\theta$  Forces due to Dead Load



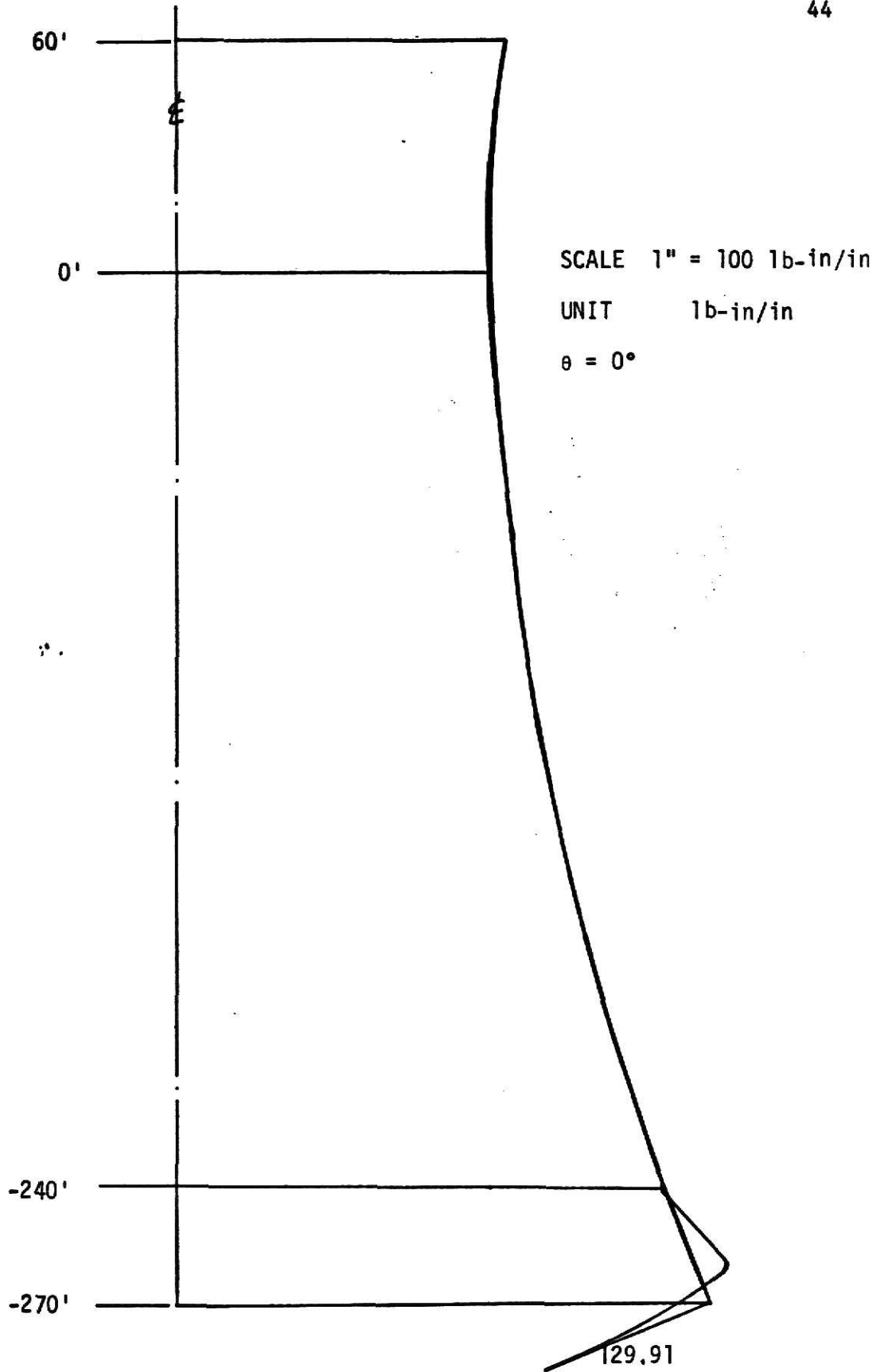


FIG. 6. Variation of  $M_\phi$  Moments due to Dead Load

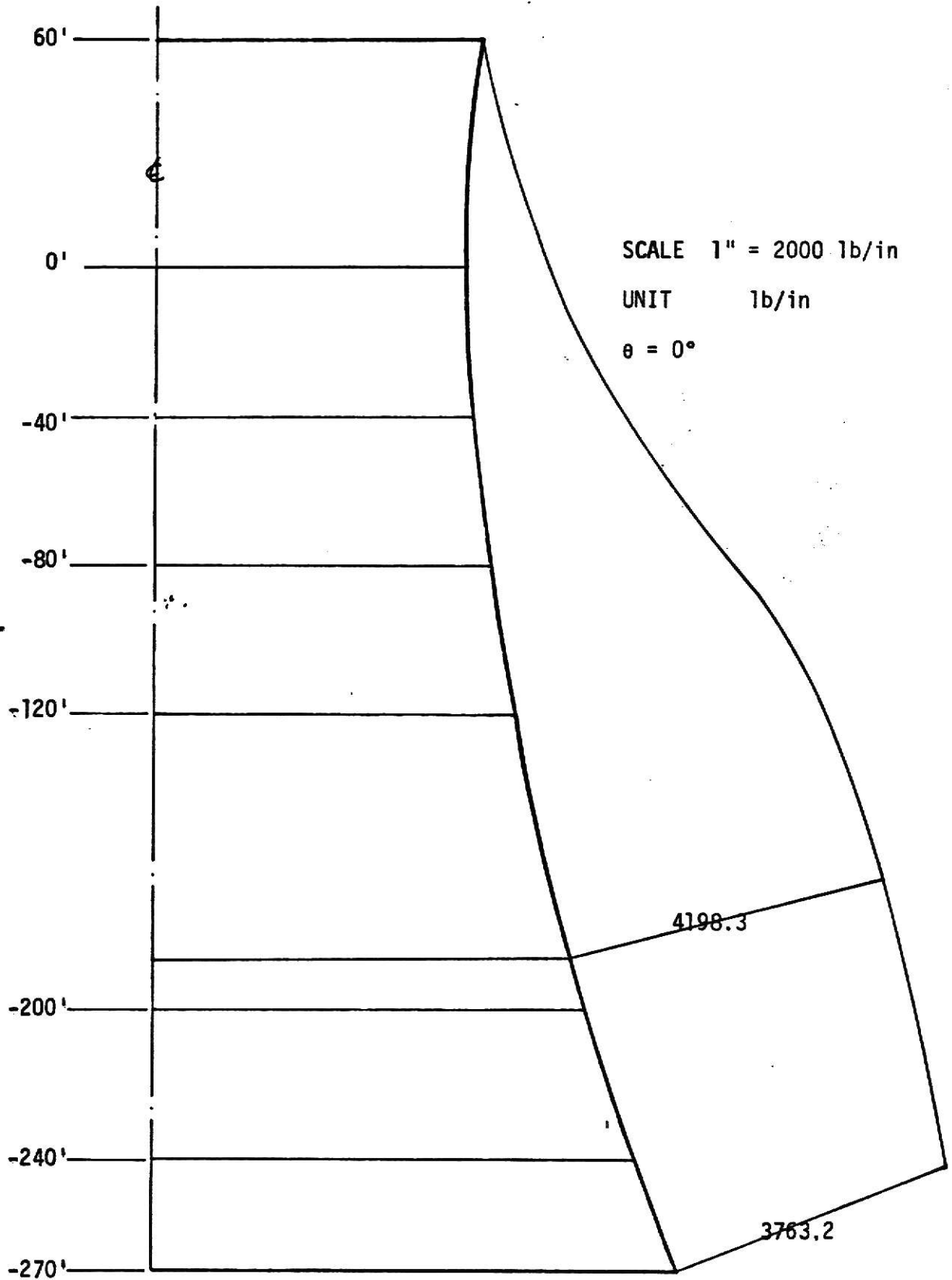


FIG. 7, Variation of  $N_\phi$  Forces due to Wind Load

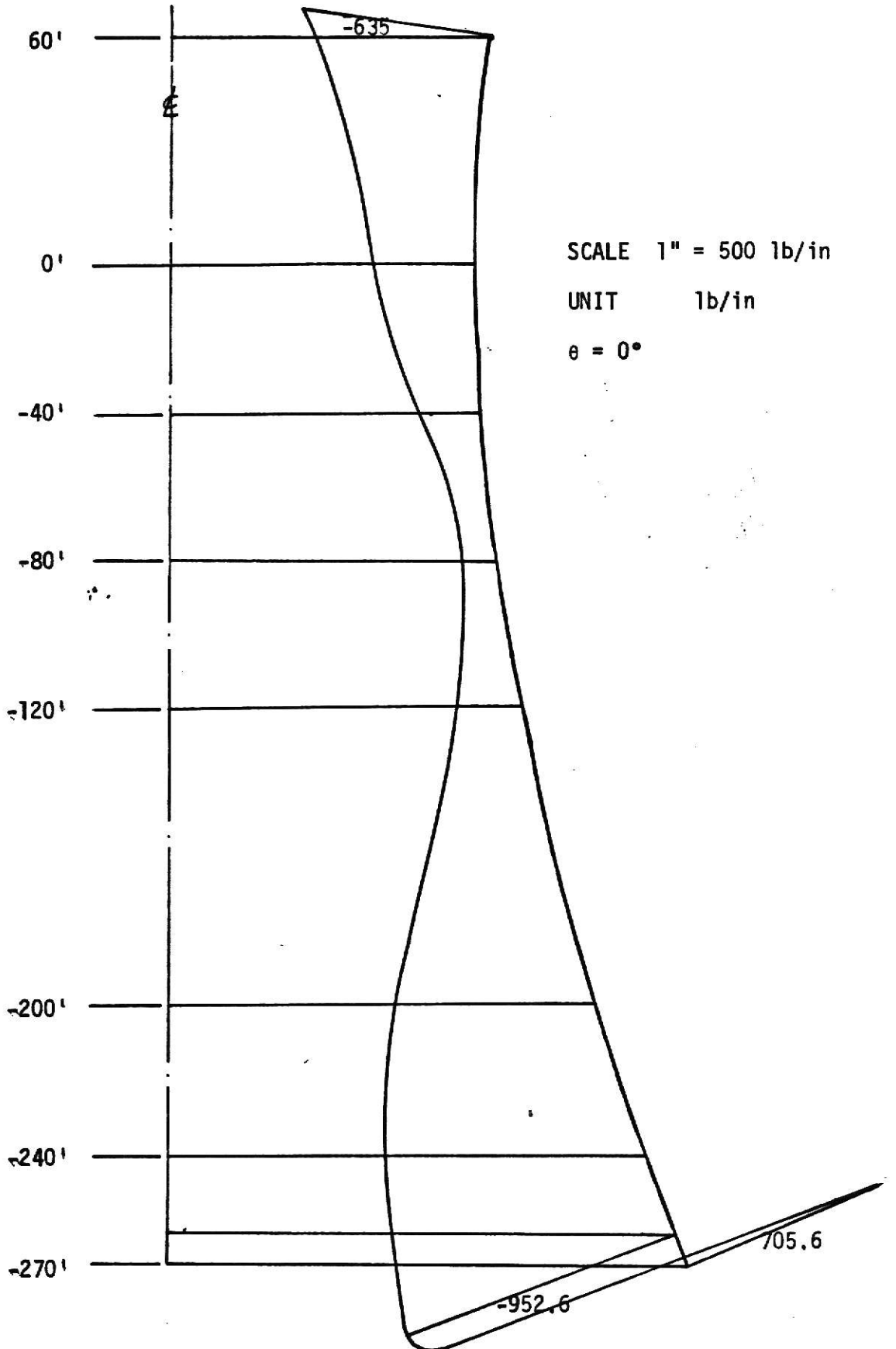


FIG. 8. Variation of  $N_\theta$  Forces due to Wind Load

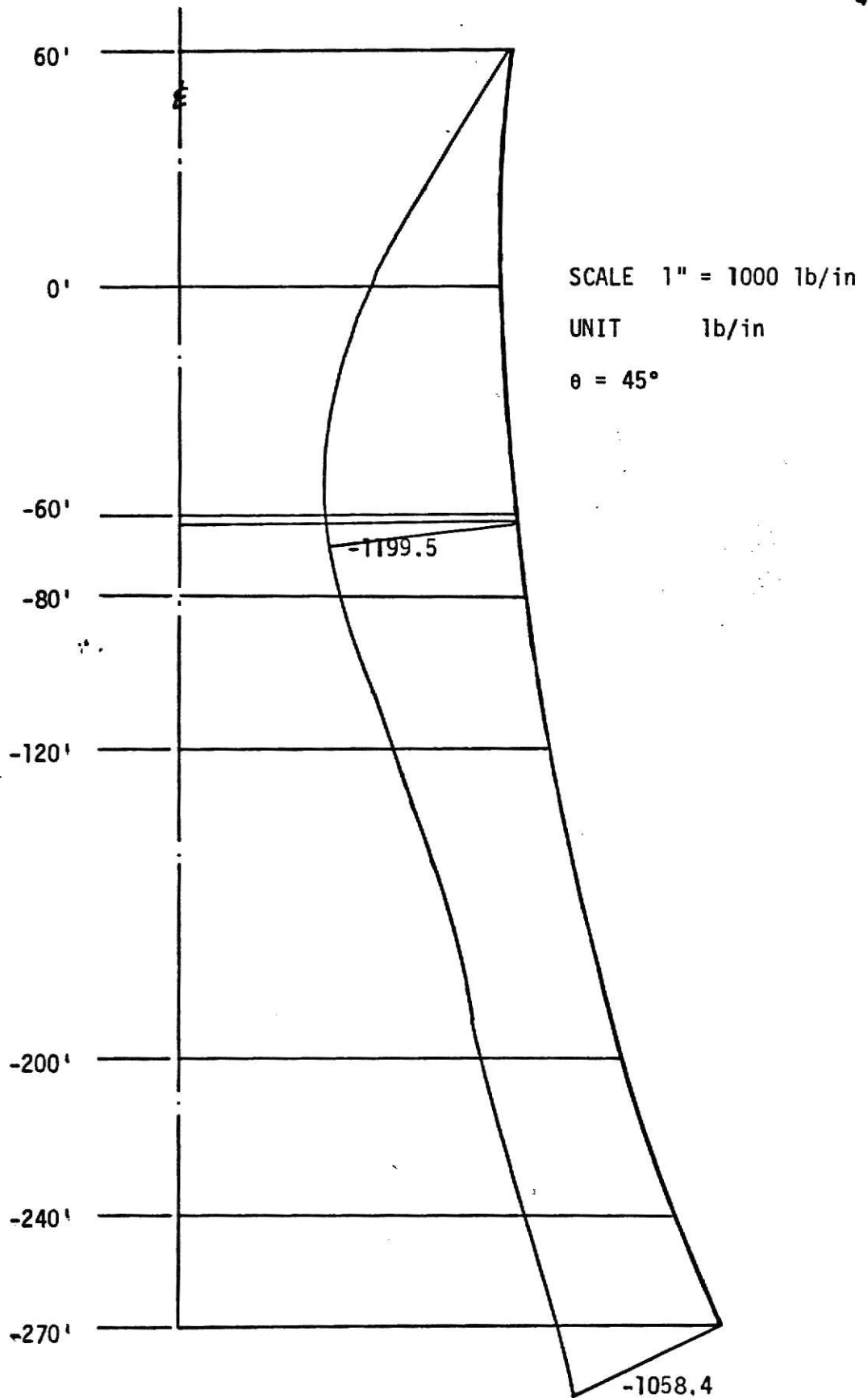


FIG. 9. Variation of  $N_{\theta\phi}$  Forces due to Wind Load

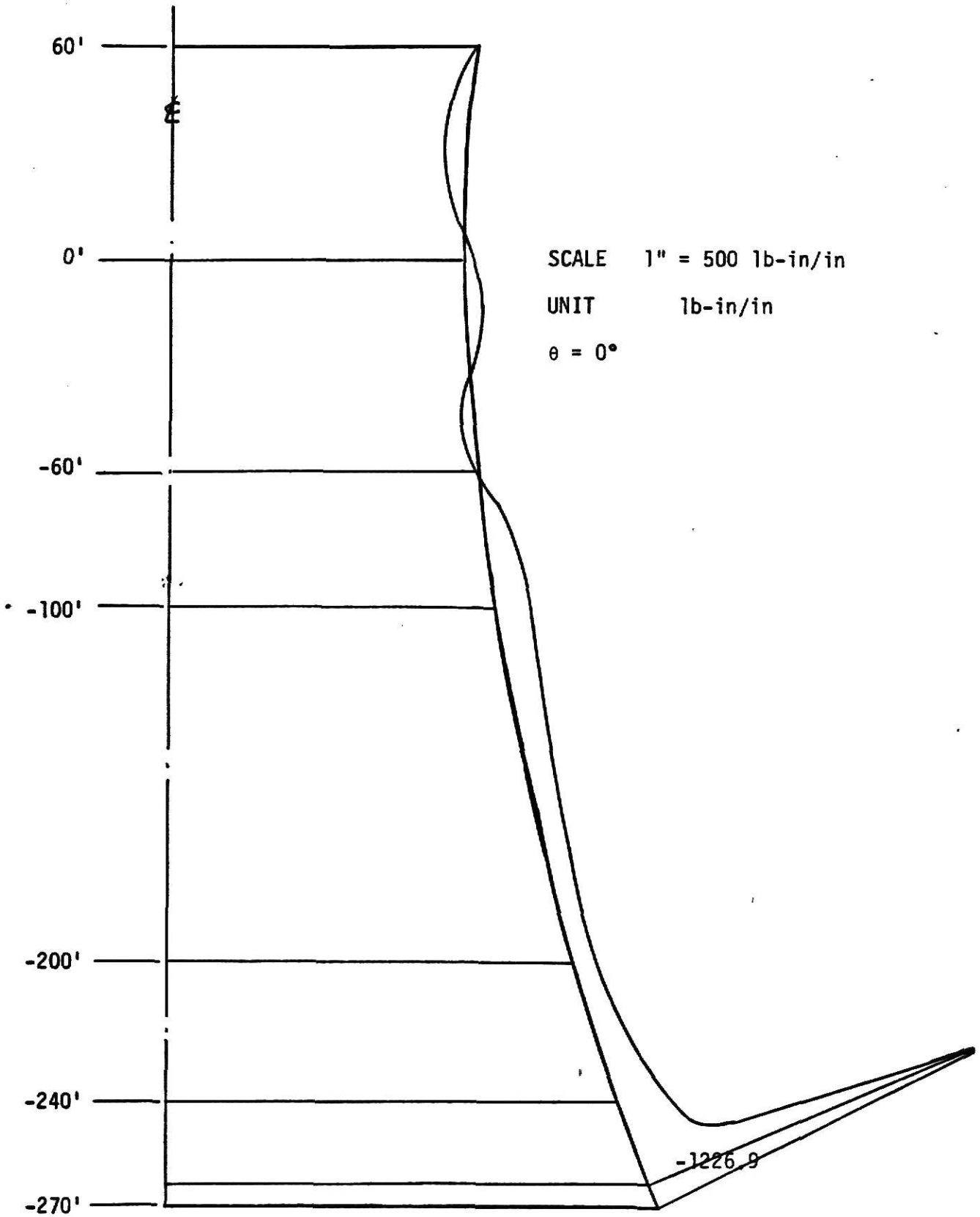
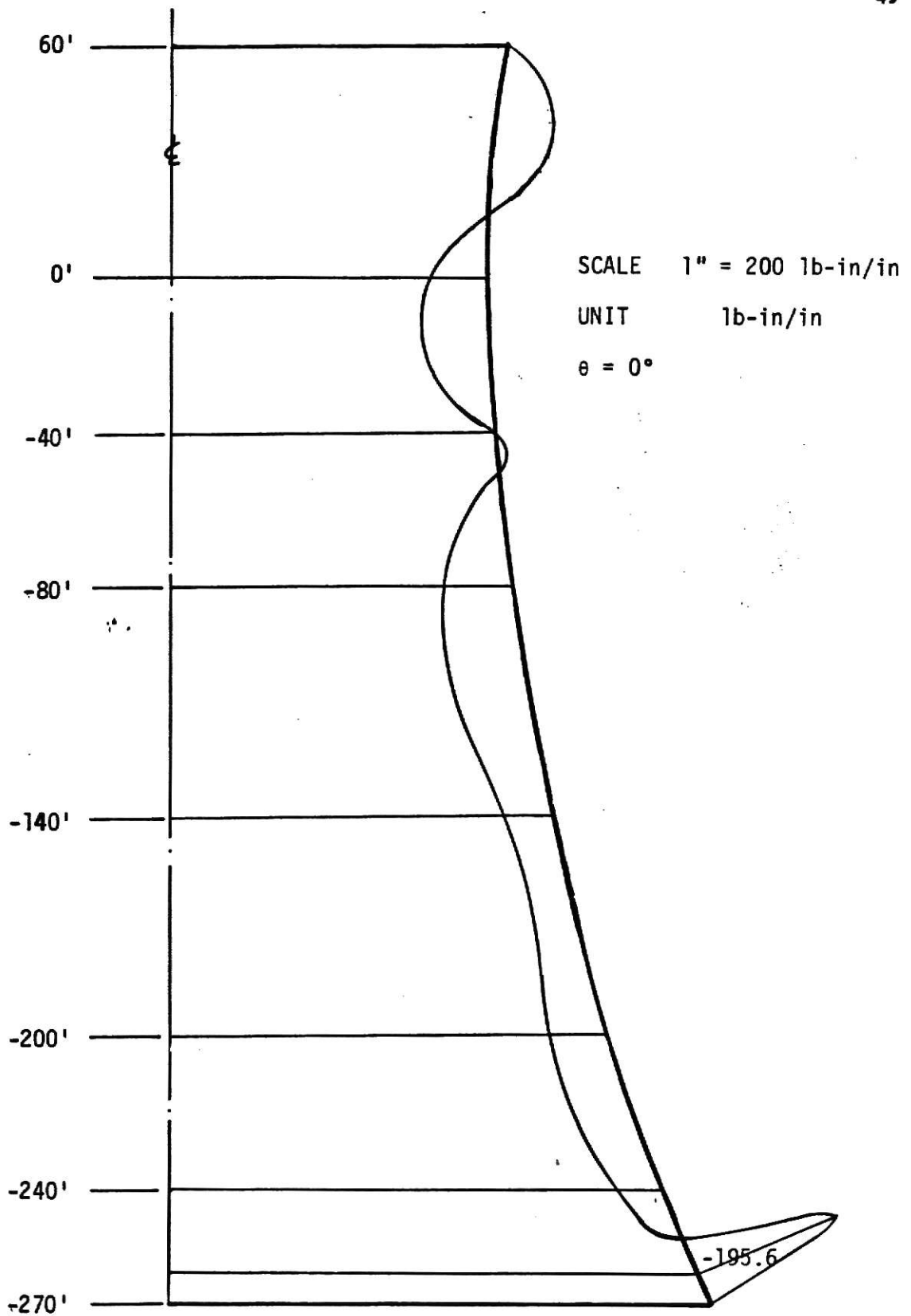


FIG. 10. Variation of  $M_\phi$  Moments due to Wind Load



FIG, 11. Variation of  $M_\theta$  Moments due to Wind Load

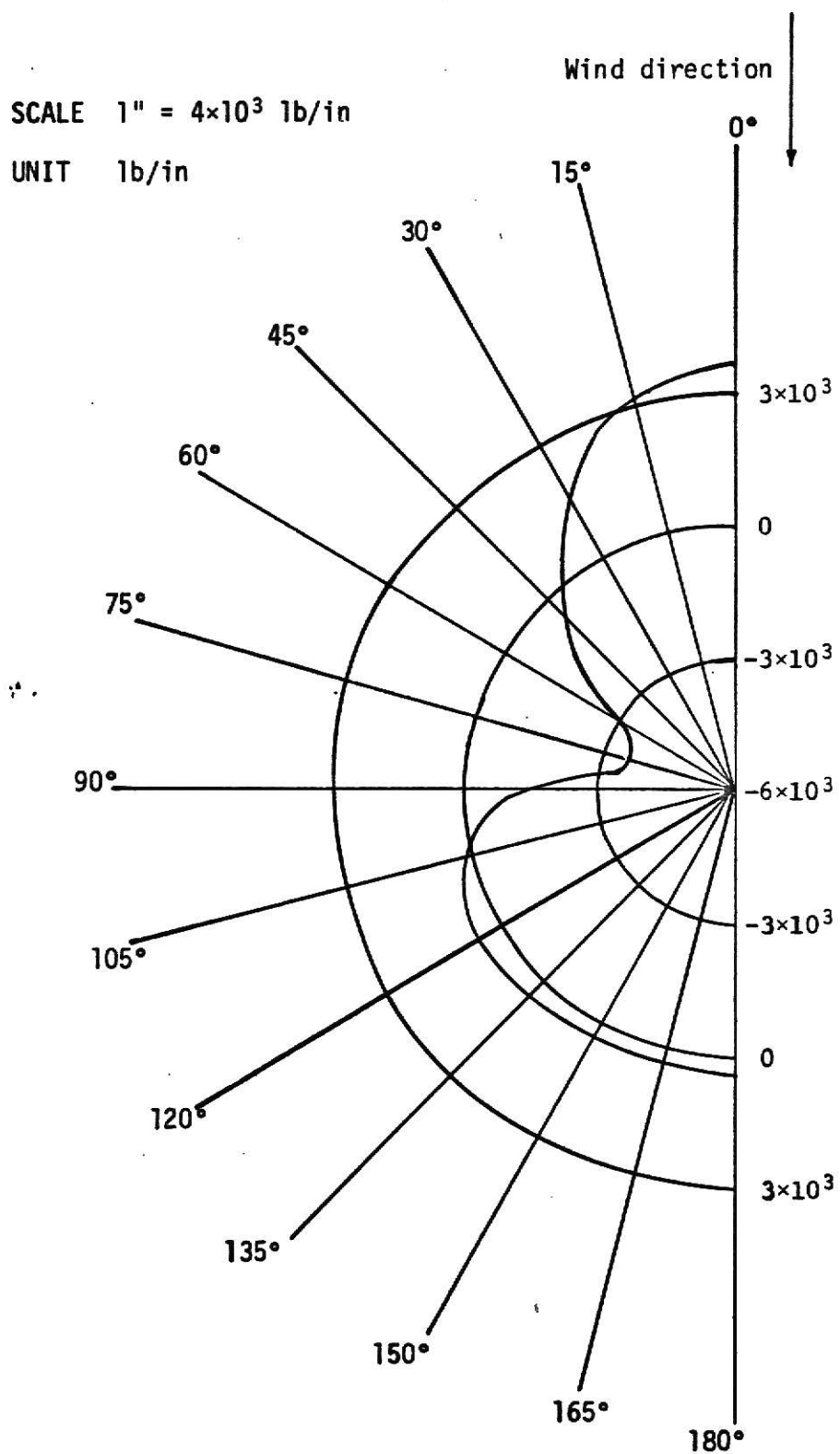


FIG. 12. Circumferential Distribution of  $N_\phi$  Forces at Base due to Wind Load

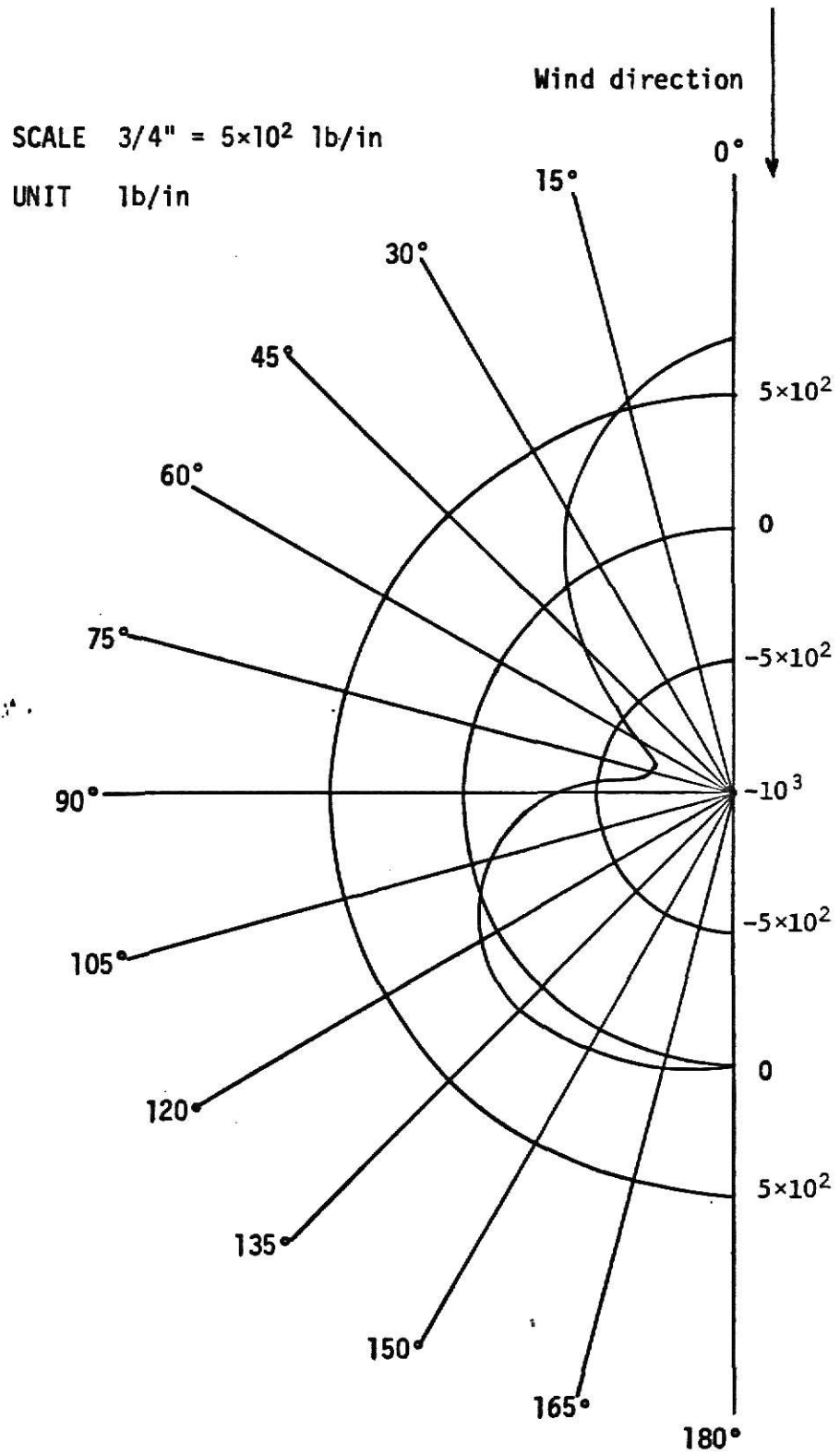


FIG. 13. Circumferential Distribution of  $N_{\theta}$  Forces at Base due to Wind Load



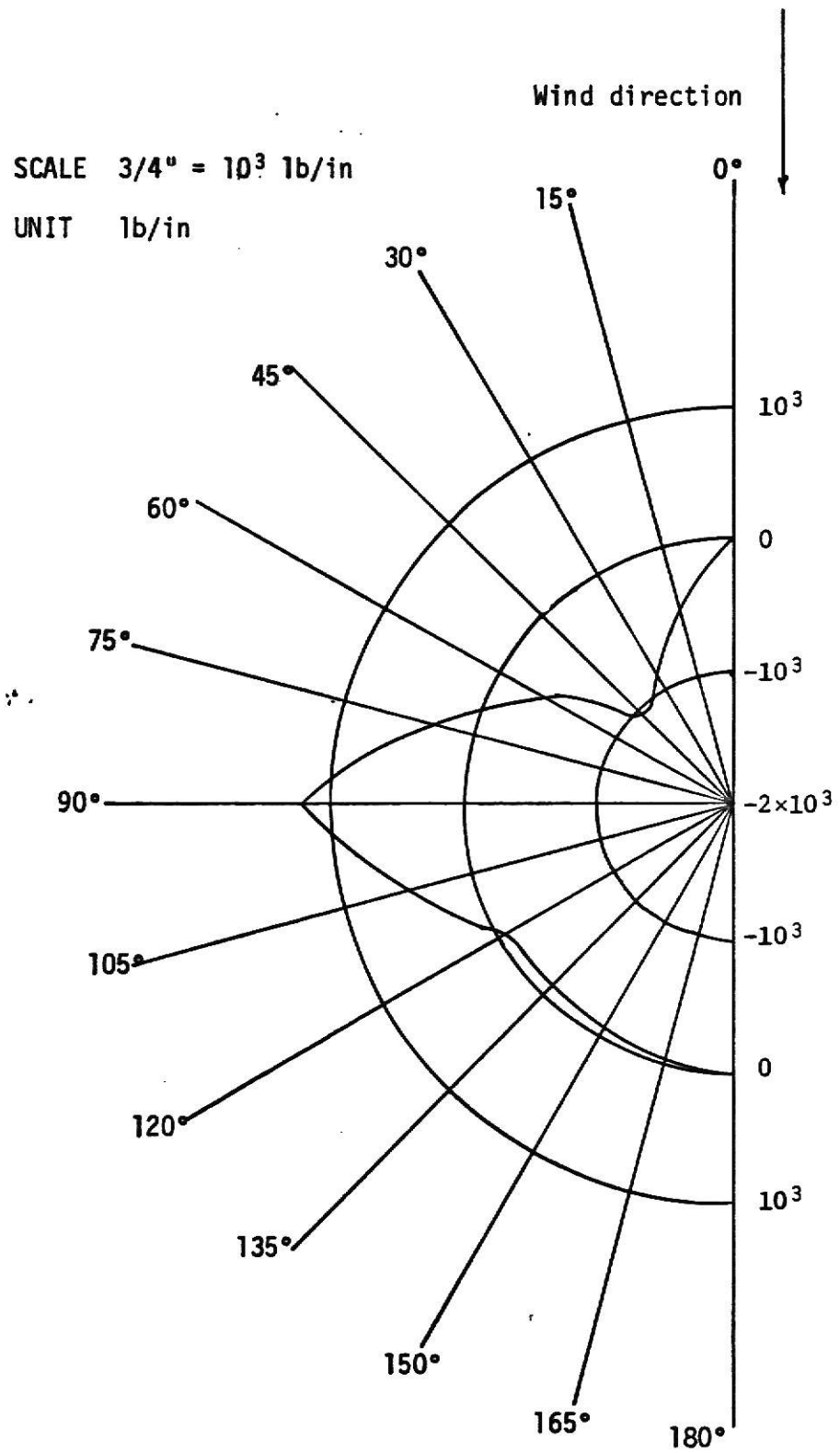


FIG. 14. Circumferential Distribution of  $N_{\theta\phi}$  Forces at Base due to Wind Load

### 3. Design example

#### (1). Reinforcement ( using elastic method.) (11, 16, 17)

$$f_s = 24000 \text{ psi}, \quad f_c = 1800 \text{ psi}, \quad f_y = 60000 \text{ psi}, \quad f'_c = 4000 \text{ psi}.$$

The shell reinforcing in each direction should not be less than 0.35% of the cross sectional area of concrete (11).

$$A_s = 0.0035h \text{ in}^2/\text{in},$$

$$\text{for } h = 5 \text{ in}, \quad A_s = 0.21 \text{ in}^2/\text{ft}.$$

Also, the maximum spacing of bars in each layer should not exceed twice the shell thickness, or no more than 18 in. In this design, it is 10 in (11).

#### a. For meridional steel

The steel cross-sectional area can be calculated by the following formula.

$$A_s = \frac{N_\phi}{f_s},$$

in this, the bar number and the spacing are chosen and given in the Table 3. based on the maximum values of  $N_\phi$  given in Table 2.

#### b. For circumferential steel

The force  $N_\theta$  is in tension at the base only and is not big enough for governing the steel design. Use #4 with spacing 10" bar as circumferential steel for the whole tower.

c. Steel quantities for the in-plane shear forces

As can be observed from Fig. 14, the maximum shear force  $N_{\phi\theta}$  is at  $\theta = 42^\circ$  from the wind direction, but from Figs. 12 and 13  $N_\phi$  and  $N_\theta$  are found almost equal to zero at the same locations. In order to determine the steel necessary to resist the tensile forces, the principal forces obtained from the maximum in-plane shear forces must be evaluated. This can be solved by using the governing equations that

$$T_p = \pm \sqrt{N_{\phi\theta}^2}$$

at  $\theta = 45^\circ$ ,

in which

$T_p$  = the principal forces whose values are the same as those of  $N_{\phi\theta}$  (absolute value). The plane on which the first principal forces act is given by  $\delta = 45^\circ$  measured in a clockwise direction from the force on which  $N_\phi$  acts. With these, suppose the reinforcement is put in the direction  $45^\circ$  from the meridian. The steel cross sectional area  $A_{sd}$  for the principal tensile forces can be calculated by the following equation,

$$A_{sd} = \frac{|N_{\phi\theta}|}{f_s}$$

For this, bar number and spacing are given in Table 4.

d. Meridional reinforcement for  $N_\phi$ ,  $M_\phi$

Figs. 7 and 10 indicate the maximum effect for  $N_\phi$ ,  $M_\phi$  being

in the place 5.5 ft from the bottom on which  $N_\phi = 2500$  lb/in,  $M_\phi = -1226.9$  lb-in/in. Observing Fig. 15a and 15b, the meridional reinforcement  $A_{st}$  of the maximum effect for  $N_\phi$ ,  $M_\phi$  is calculated as follows

$$\text{The deviation } e = \frac{M_\phi}{N_\phi} = 0.49 \text{ in,}$$

$$A_{st} = \frac{N_\phi(e + jd)}{f_s jd} = \frac{2500(0.49 + 0.875 \times 2.5)}{24000 \times 0.875 \times 2.5}$$

$$= 0.1275 \text{ in}^2/\text{in} = 1.53 \text{ in}^2/\text{ft.}$$

In this result, the spacing of the first and second region near the base shown in Table 3 should be changed to 3" C. C.

e. Reinforcement for  $M_\theta$

The maximum  $M_\theta$  near the middle is equal to 106 lb-in/in. The reinforcement

$$A_{st} = \frac{M_\theta}{f_s jd} = \frac{106 \times 12}{24000 \times 0.875 \times 3}$$

$$= 0.02 \text{ in}^2/\text{ft.}$$

The circumferential steel cross-sectional area is much more than this value.

(2). Steel placement

Figure 17 shows the steel placement of vertical plane.

(3). The adequacy of the design with respect to ACI 318-71 requirements (16)

a. The maximum steel area/per foot should be less than

$$\frac{7.2hf'_c}{f_y} = \frac{7.2 \times 5 \times 4000}{60000} = 2.4 \text{ in}^2,$$

or

$$\frac{29000h}{f_y} = \frac{29000 \times 5}{60000} = 2.42 \text{ in}^2. \quad \text{O.K.}$$

The deviation of the diagonal reinforcement does not exceed

10 degree. O.K.

b. The maximum spacing

Because  $4\phi\sqrt{f'_c} = 227.68 \text{ psi} < \text{the computed tensile stresses}$   
 due to design load, the maximum spacing allowed could be three  
 times the thickness,  $3h = 15''$ . (capacity-reduction factor  $\phi=0.9$ )

c. The ratio of the minimum reinforcement per foot to the concrete area is equal to 0.0014.

For #4 @ 10",  $A_s/\text{ft} = 0.24 \text{ in}^2/\text{ft}$ , and

$$\frac{\text{steel area}}{\text{concrete area}} = \frac{0.24}{12 \times 5} = 0.004 > 0.0014. \quad \text{O.K.}$$

d. The minimum reinforcement ratio in the tensile zone at any portion shall not be less than 0.0035.

For #4 @ 10",  $A_s/\text{ft} = 0.24 \text{ in}^2/\text{ft}$ ,

$$\frac{0.24}{12 \times 5} = 0.004 > 0.0035. \quad \text{O.K.}$$

TABLE 3. Bar Number and Spacing for Meridional Reinforcement corresponding to  $N_{\phi}$  at  $\theta = 0^{\circ}$ .

Region	Y (ft)	Corresponding $N_{\phi}$ (lb/in)	$A_s$ ( $\text{in}^2/\text{ft}$ )	Bar No.	Spacing (in)c.c
1	-270 - -260	2418.1	1.209	6	4
2	-260 - -220	2838.9	1.419	6	3½
3	-220 - -180	3049.3	1.525	6	3
4	-180 - -140	3175.3	1.588	6	3
5	-140 - -100	2952.4	1.476	6	3½
6	-100 - -60	2273.2	1.137	6	4½
7	-60 - -20	1343.8	0.672	6	7½
8	-20 - 20	789.1	0.396	6	10
9	20 - 60	350.1	0.175	4	10

TABLE 4. Bar Number and Spacing for Diagonal Reinforcement corresponding to  $N_{\theta\phi}$  at  $\theta = 45^{\circ}$ .

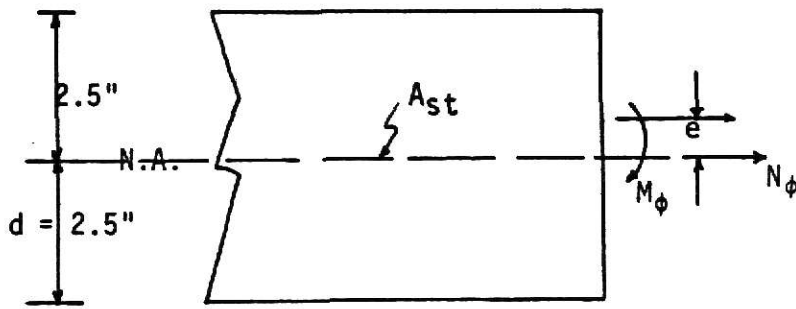
Region	Y (ft)	Corresponding $N_{\theta\phi}$ (lb/in)	$A_{sd}$ ( $\text{in}^2/\text{ft}$ )	Bar No.	Spacing (in)c.c
1	-270 - -260	-1058.4	0.529	4	4
2	-260 - -220	- 931.3	0.466	4	5
3	-220 - -180	- 875.0	0.438	4	5
4	-180 - -140	- 831.3	0.416	4	5
5	-140 - -100	- 950.0	0.475	4	4½
6	-100 - -60	-1175.0	0.588	4	4
7	-60 - -20	-1193.8	0.597	4	3½
8	-20 - 20	- 828.1	0.414	4	5½
9	20 - 60	- 243.8	0.122	4	10

1 Kg. = 2.2046 lb

1 in. = 2.54 cm

TABLE 5. Circumferential Wind Pressure and Fourier Coefficients

Angle from windward meridian, $\theta$	Wind pressure Coefficient	Fourier harmonic n	Fourier Coefficient $P_{zn}$
0°	1.0	0	-0.3923
15°	0.8	1	0.2602
30°	0.2	2	0.6024
45°	-0.5	3	0.5046
60°	-1.2	4	0.1064
75°	-1.3	5	-0.0948
90°	-0.9	6	-0.0186
105°	-0.4	7	0.0468
120°	-0.4		
135°	-0.4		
150°	-0.4		
165°	-0.4		
180°	-0.4		



Calculation of Meridional Reinforcement for  $M_\phi$ ,  $N_\phi$

FIG. 15a

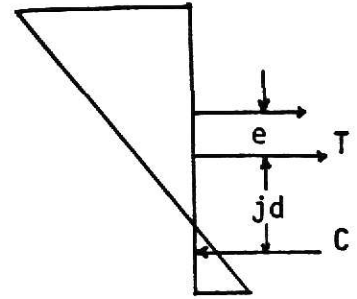


FIG. 15b

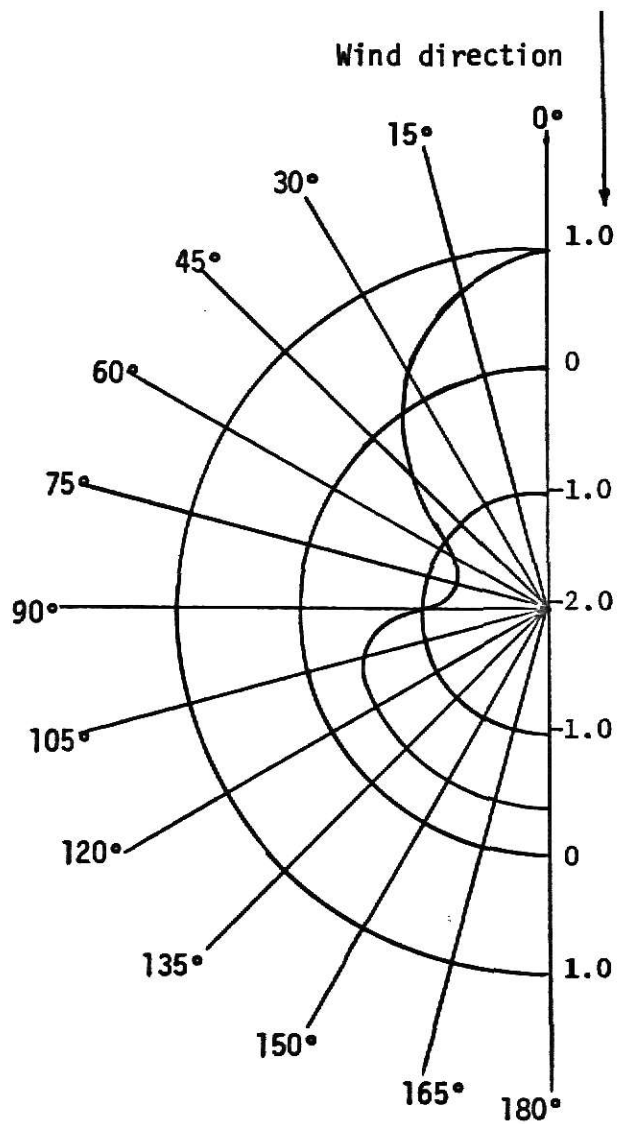


FIG. 16. Circumferential Distribution of Wind Pressure  $P_{zn}$



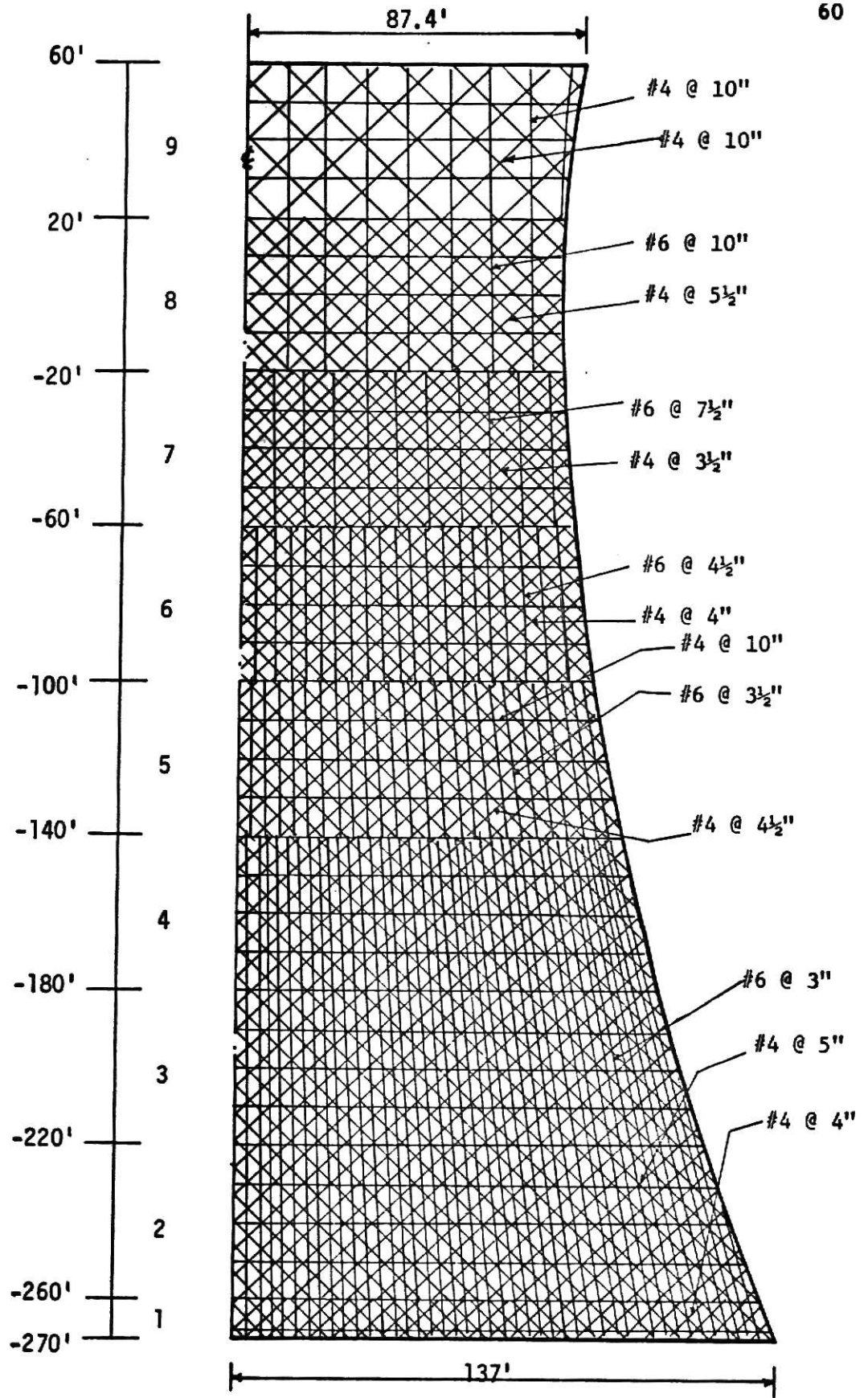


FIG. 17. Steel Placement of Vertical Plane

## VI. DISCUSSION AND CONCLUSIONS

1. It has been shown that the maximum values of the in-plane force resultants are strongly dependent on the circumferential distribution of wind loading (7). The values in Table 5 and Fig. 16 appear to be the best available at the present time (11).
2. The dynamic analysis of the seismic response is beyond the scope of this Report. It is known the bending forces due to seismic loads are significant.
3. For the thinner shells, the possibility of net tension under wind load in one or both directions requires careful design consideration.
4. The analysis and prediction of the buckling capacity of cooling tower shells with realistic geometry and boundary conditions are not presented in this report.
5. The bending solutions for wind load obtained from a computer program (14) were obtained, and were based upon only the first four Fourier harmonics. The results were much too inaccurate for design purposes.
6. From a design standpoint, it is of interest to observe that although the bending forces are not large, the corresponding hoop forces due to wind load near the base of the shell are significantly different from those observed in the membrane analysis.
7. For this typical shell, the bending forces are small compared to the direct forces.
8. Since the wind direction is arbitrary, only the maximum force resultants are of design interest

9. It is of interest to note that at the points  $\theta = 42^\circ$ , where  $N_{\theta\phi}$  is maximum, the values of  $N_\phi$  and  $N_\theta$  are relatively small. Also the maximum positive  $N_\phi$  and  $N_\theta$  occur at  $\theta = 0^\circ$  where  $N_{\theta\phi} = 0$ . For design purpose, the maximum and minimum principal forces should be checked at these locations.

## ACKNOWLEDGEMENTS

The writer wishes to express his sincere appreciation to his advisor, Dr. Stuart E. Swartz, for devoting a great deal of time to instructing and guiding the writer during the investigation.

Also, he is grateful to Dr. Robert R. Snell, Head of the Department of Civil Engineering, Dr. Peter B. Cooper, Dr. Kuo-Kuang, Hu, and Dr. Hugh S. Walker for serving on the advisory committee and reviewing the manuscript.

Besides, he is very grateful to his wife for helping him in correction and typing days and nights.

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## APPENDIX.- NOTATION

The following symbols are used in this report:

- $A, A', A_1,$   
 $A_2, A_3, A_4$  = constants of integration;  
 $\tilde{A}, \tilde{A}'$  = complex constants of integration;  
 $A_j(\phi), A_j'(\phi)$  = variables defined by Eqs. (4.17a) and (4.30d);  
 $A_b$  = the base area of the tower measured at pond sill;  
 $A_s, A_{sd}, A_{st}$  = the reinforcement steel cross-sectional area;  
 $a$  = throat radius;  
 $B_j(\phi), B_j'(\phi)$  = variables defined by Eqs. (4.17b) and (4.30e);  
 $b$  = shell parameter defined by Eq. (4.1b);  
 $C$  = an efficiency factor as the performance coefficient  
for tower size selection;  
 $C_c$  = an empirical coefficient taken to be 0.052;  
 $C_j, C_j',$   
 $D_j, D_j'$  = constants of integration in segment  $j$ ;  
 $D$  = Duty Coefficient for size selection;  
 $E$  = Young's modulus;  
 $f_c$  = compressive working strength of concrete,  $0.45 f_c'$ ;  
 $f_c'$  = compressive strength of concrete;  
 $f_s$  = working strength of steel,  $0.4 f_y$ ;  
 $f_y$  = ultimate strength of steel;  
 $G(\phi)$  = variable defined by Eq. (4.15c);  
 $G_1(), G_{2n}()$  = operators defined by Eqs. (4.38e) and (4.41c);

- $g$  = gravity load per unit area of the surface;  
 $H$  = the height of the tower measured above sill level;  
 $H'(\phi)$  = variable defined by Eq. (4.15b);  
 $h$  = shell thickness;  
 $\Delta h$  = the change in total heat of the air passing through  
the tower;  
 $i, j, m$  = segment designation;  
 $l$  = arc length;  
 $M_\phi, M_\theta$  = meridional, and circumferential bending moment  
resultants, respectively;  
 $M_{\phi\theta}, M_{\theta\phi}$  = twisting moment resultants;  
 $\dot{m}_\phi$  = nondimensional meridional bending moment resultant;  
 $m_{\phi n}$  =  $m_\phi$  for nth harmonic;  
 $m_\theta$  = nondimensional circumferential bending moment resultant;  
 $m_{\theta n}$  =  $m_\theta$  for nth harmonic;  
 $m_{\phi\theta}, m_{\theta\phi}$  = nondimensional twisting moment resultants;  
 $m_{\theta\phi n}$  =  $m_{\theta\phi}$  for nth harmonic;  
 $N_\phi, N_\theta$  = meridional, and circumferential stress resultants,  
respectively;  
 $N_{\phi\theta}, N_{\theta\phi}$  = shearing stress resultant;  
 $N(\phi)$  = variable defined by Eq. (4.15a);  
 $\tilde{N} = \tilde{N}_\phi + \tilde{N}_\theta$ ;  
 $\tilde{N}_n = \tilde{N}_{\phi n} + \tilde{N}_{\theta n}$ ;  
 $\tilde{N}_{nh}$  = homogeneous part of  $\tilde{N}_n$ ;  
 $\tilde{N}_{np}$  = particular part of  $\tilde{N}_n$ ;  
 $\tilde{N}_\phi, \tilde{N}_\theta, \tilde{N}_{\theta\phi}$  = complex stress resultants defined by Eqs. (4.37);



$$N_{\phi n} = N_{\phi} / \cos n\theta;$$

$$N_{\theta n} = N_{\theta} / \cos n\theta;$$

$$N_{\theta\phi n} = N_{\theta\phi} / \sin n\theta;$$

$n$  = harmonic number;

$n_{\phi}$  = nondimensional meridional stress resultant;

$n_{\phi n}$  =  $n_{\phi}$  for  $n$ th harmonic;

$$n'_{\phi n}(\phi) = n_{\phi n} / \cos n\theta;$$

$$n^*_{\phi n}(\phi) = n'_{\phi n} \text{ obtained in membrane analysis;}$$

$n_{\theta}$  = nondimensional circumferential stress resultant;

$n_{\theta n}$  =  $n_{\theta}$  for  $n$ th harmonic;

$$n'_{\theta n}(\phi) = n_{\theta n} / \cos n\theta;$$

$$n^*_{\theta n}(\phi) = n'_{\theta n} \text{ obtained in membrane analysis;}$$

$n_{\phi\theta}$ ,  $n_{\theta\phi}$  = nondimensional shearing stress resultant;

$n_{\theta\phi n}$  =  $n_{\theta\phi}$  for  $n$ th harmonic;

$$n'_{\theta\phi n}(\phi) = n_{\theta\phi n} / \sin n\theta;$$

$$n^*_{\theta\phi n}(\phi) = n'_{\theta\phi n} \text{ obtained in membrane analysis;}$$

$P_{\phi}$ ,  $P_{\theta}$ ,  $P_z$  = meridional, circumferential, and normal load component, respectively;

$P$  = constant reference load intensity per unit area of middle surface;

$p_{\phi}$  = nondimensional meridional load component;

$p_{\phi n}$  =  $p_{\phi}$  for  $n$ th harmonic;

$p_{\theta}$  = nondimensional circumferential load component;

$p_{\theta n}$  =  $p_{\theta}$  for  $n$ th harmonic;

$p_z$  = nondimensional normal load component;

$p_{zn}$  =  $p_z$  for  $n$ th harmonic;

- $Q_\phi$  = meridional transverse shear;  
 $Q_\theta$  = circumferential transverse shear;  
 $q_c$  = critical buckling pressure along the windward meridian.  
 $q_\phi$  = nondimensional meridional transverse shear;  
 $q_{\phi n}$  =  $q_\phi$  for nth harmonic;  
 $q_\theta$  = nondimensional circumferential transverse shear;  
 $q_{\theta n}$  =  $q_\theta$  for nth harmonic;  
 $R_o$  = horizontal radius;  
 $R_\phi, R_\theta$  = meridional, and circumferential radius of curvature,  
 respectively;  
 $r_\phi, r_\theta$  = nondimensional meridional, and circumferential radius  
 of curvature, respectively;  
 $S$  = vertical distance from throat to base of shell;  
 $s$  = base radius;  
 $T$  = vertical distance from throat to top of shell;  
 $T_p$  = Principal force;  
 $\Delta T$  = the change of temperature of the water passing through  
 the tower;  
 $t$  = top radius;  
 $\Delta t$  = the change between the dry bulb air temperature and  
 aspirated wet bulb air temperature;  
 $U$  = meridional displacements;  
 $u$  = nondimensional meridional displacement;  
 $u_n$  =  $u$  for nth harmonic;  
 $u_n'(\phi) = u_n / \cos n\theta$ ;

$V$  = circumferential displacements;

$v$  = nondimensional circumferential displacements;

$v_n$  =  $v$  for  $n$ th harmonic;

$v_n'(\phi) = v_n / \sin n\theta$ ;

$W$  = normal displacement;

$W_d$  = total vertical load above the level  $\phi$ ;

$W_L$  = water load;

$w$  = nondimensional normal displacement;

$w_n$  =  $w$  for  $n$ th harmonic;

$w_n'(\phi) = w_n / \cos n\theta$ ;

$w_t$  = deviation in thickness;

$Y_j, Y_j''$  = variables defined by Eqs. (4.22c) and (4.22d);

$y$  = vertical coordinate;

$Z_j', Z_j''$  = variables defined by Eqs. (4.33c) and (4.33d);

$z$  = normal coordinate;

$\alpha$  = variable defined by last equation in Eq. (4.48);

$\alpha_c$  = an empirical coefficient taken to be 2.3;

$\alpha_n$  = Fourier coefficient for harmonic  $n$ ;

$\bar{\alpha} = b^2/a^2$ ;

$\beta$  = variable defined by Eq. (4.45);

$\beta_j$  = variable defined by Eq. (4.15f);

$\beta_j'$  = variable defined by Eq. (4.30c);

$\bar{\beta}$  = variable defined by Eq. (4.49b);

$\psi$  = variable defined by first equation in Eq. (4.9);

$\psi'(\phi) = \psi / \cos n\theta$ ;

- $\psi'_j = \psi'(\phi)$  in segment j;  
 $\psi'_{jh}$  = homogeneous part of  $\psi'_j$ ;  
 $\psi'_{jp}$  = particular part of  $\psi'_j$ ;  
 $\tilde{\psi} = \tilde{\psi}_n(\phi) \cos n\theta$ ;  
 $\tilde{\psi}_n$  = variable defined by Eq. (4.40c);  
 $\phi$  = meridional coordinate;  
 $\phi_{ij}$  = angle locating junction of segments i and j;  
 $\phi_{jm}$  = angle locating junction of segments j and m;  
 $\phi_s$  = angle locating base of shell;  
 $\phi_t$  = angle locating top of shell;  
 $\epsilon_\phi, \epsilon_\theta$  = strains;  
 $\eta$  = variable defined by second equation in Eq. (4.9);  
 $\tilde{\eta} = \tilde{\eta}_n(\phi) \sin n\theta$ ;  
 $\tilde{\eta}_n$  = variable defined by Eq. (4.40d);  
 $\omega$  = shear strain;  
 $\mu$  = Poisson's ratio;  
 $\nu$  = variable defined by Eq. (4.37d);  
 $\rho(\phi)$  = variable defined by Eq. (4.15d);  
 $\rho_j(\phi)$  = average value of  $\rho$  in segment j;  
 $\gamma = v/r_\theta \sin\phi$ ;  
 $\gamma'(\phi) = \gamma/\sin n\theta$ ;  
 $\gamma'_j(\phi) = \gamma'$  in segment j;  
 $\gamma'_{jh}$  = homogeneous part of  $\gamma'_j$ ;  
 $\gamma'_{jp}$  = particular part of  $\gamma'_j$ ;  
 $\theta$  = circumferential coordinate;  
 $\xi$  = auxiliary variable defined by Eq. (4.6c);

$\delta_1 - \delta_{11}$  = variables defined by Eqs. (4.48);

$\chi_1, \chi_2, \chi_3$  = variables defined by Eqs. (4.13a) through (4.13c);

$\chi_4$  = variable defined by Eq. (4.28b);

$\zeta = u/\sin\phi$ ;

$\Delta(\phi)$  = variable defined by Eq. (4.15e);

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**AN ABSTRACT OF A MASTER'S REPORT**

**submitted in partial fulfillment of the**

**requirements for the degree**

**MASTER OF SCIENCE**

**Department of Civil Engineering**

**KANSAS STATE UNIVERSITY**

**Manhattan, Kansas**

**1976**

## ABSTRACT

This report deals with the analysis and design of hyperbolic cooling towers. The purpose of the study may be summarized as follows:

1. Formulae are given to enable the size of cooling towers to be determined for a given cooling duty.
2. The membrane solutions and bending solutions for the stress resultants and displacements in hyperbolic cooling towers subjected to dead load and wind load are presented.
3. Numerical examples are presented to provide a comparison between the results of the long hand membrane solutions and computer bending solutions.
4. Design tables and force variations by numerical bending solutions under dead load and wind load are given to facilitate the design of this structure.
5. Design considerations based upon the ACI-ASCE report (11) and ACI 318-71 (16) are expressed for the design example.