

From Arithmetic to Zeta-Functions

Jürgen Sander • Jörn Steuding • Rasa Steuding
Editors

From Arithmetic to Zeta-Functions

Number Theory in Memory
of Wolfgang Schwarz

 Springer

Editors

Jürgen Sander
Institut für Mathematik und Angewandte
Informatik
Universität Hildesheim
Hildesheim, Germany

Jörn Steuding
Institut für Mathematik
Universität Würzburg
Würzburg, Germany

Rasa Steuding
Institut für Mathematik
Universität Würzburg
Würzburg, Germany

ISBN 978-3-319-28202-2 ISBN 978-3-319-28203-9 (eBook)
DOI 10.1007/978-3-319-28203-9

Library of Congress Control Number: 2016947422

Mathematics Subject Classification (2010): 11Axx, 11Bxx, 11Dxx, 11Jxx, 11Kxx, 11Lxx, 11Mxx, 11Nxx, 11Pxx, 01Axx

© Springer International Publishing Switzerland 2016

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made.

Cover image: Illustration is published with kind permission of © Nicola Oswald, 2015.

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG Switzerland



Published with kind permission of © Jürgen Wolfart, 2009

*Das schönste Denkmal, das ein Mensch
bekommen kann, steht in den Herzen seiner
Mitmenschen.¹*

Albert Schweitzer

¹The most beautiful memorial which can be erected for somebody is the imprint he leaves behind in the hearts of others.

Wenn er aber studiert, müssen ihm bedeutende ideale Ziele vorschweben

The headline above is a passage from a personal statement written by Wolfgang Schwarz shortly after finishing school, explaining his views and goals while looking forward to taking up studies at the University of Erlangen. Literally, it says that a student's motivation must be substantial and idealistic. At that time it was customary to write a proposal for enrollment at a German university.

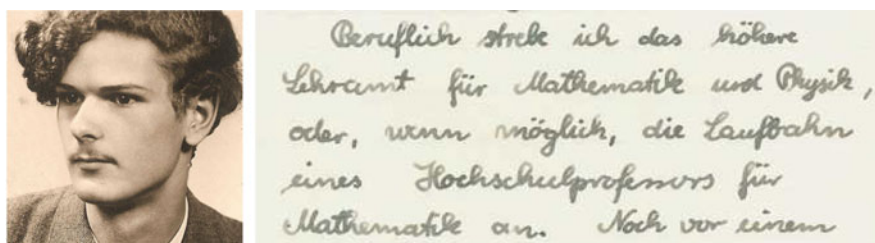


Fig. 1 *Left*: the 17-year-old Wolfgang Schwarz; *right*: excerpt from his essay. Published with kind permission of © Doris Schwarz, 2013

As a profession I aim at the position of a high school teacher of mathematics and physics, or, if possible, the career of a university professor of mathematics.²

This is another quotation from Wolfgang Schwarz's essay showing a rather self-confident young man with ambitious goals. And indeed, this promising young man lived up to his own expectations and became an outstanding mathematician. His field of research was number theory with an emphasis on analysis. Yet, his interests were as widespread as his knowledge, ranging from elementary to algebraic aspects of number theory, and from analysis to algebra. This broad spectrum is well reflected

²This is our translation of the German original (see Fig. 1). Please note that the German *höheres Lehramt* stands for teaching pupils from grade 6 to 13 at a gymnasium, and *Hochschule* at that time was equivalent to university.

in the variety of research articles and surveys in this memorial volume, some of them enriched with personal reminiscences.

Wolfgang Schwarz was born on the 21st of April 1934 in Selb (not far from Hof in northern Bavaria), which is well known for its famous porcelain manufactories. His father was a teacher at a primary school and, although he died untimely in 1945 during World War II, he left a deep impression on young Wolfgang, which is best manifested in his early curiosity about mathematics and natural sciences. Their mother raised Wolfgang and his brother Werner and encouraged their musical talents. In 1951 Schwarz obtained his university entrance certificate, called *Reifezeugnis* in German. He enrolled at the Friedrich-Alexander-Universität Erlangen to study mathematics and physics; among his teachers were Otto Haupt, Georg Nöbeling, Wilhelm Specht, and, most importantly, the eminent Theodor Schneider, who is famous for his elegant solution of the seventh Hilbert problem on the transcendence of numbers of the form α^β , where α is algebraic and β irrational.³ After passing the state examination for teachers in mathematics and physics in 1956 Wolfgang Schwarz continued his studies as an assistant of Schneider, first in Erlangen and later at the Albert-Ludwigs-Universität Freiburg (Fig. 2). His

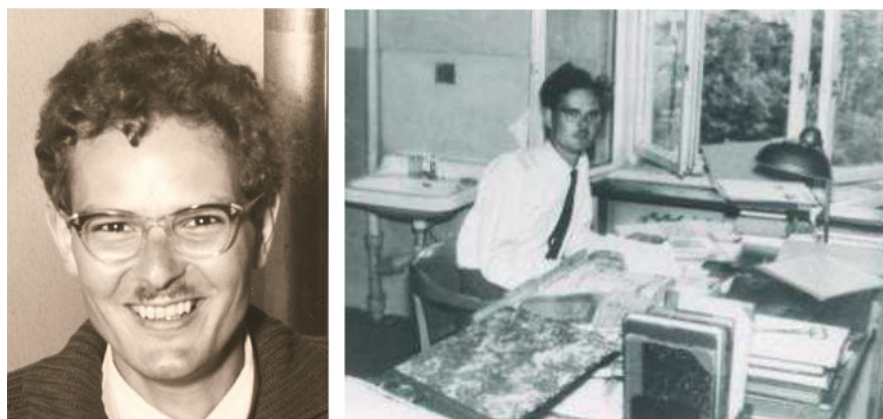


Fig. 2 Wolfgang Schwarz during his doctorate studies. Published with kind permission of © Doris Schwarz, 2013

doctorate thesis of 1959 dealt with questions about representations of integers as sums of powers of prime numbers. For his habilitation,⁴ accomplished in 1964 also at Freiburg, he studied so-called Tauberian theorems with applications to partitions.

The topics and methods used in both of his theses form a central part of twentieth-century analytic number theory. In this memorial volume, honoring the life and work of Wolfgang

³Independently, Alexander Gelfond provided another solution about the same time.

⁴Habilitation is a degree following the doctorate, at that time necessary to obtain the permission for independent teaching (*venia legendi*) and for appointment as a professor.

Schwarz, the reader will find several contributions closely related to Schwarz's work, e.g. those by Jörg Brüdern, Rainer Dietmann & Christian Elsholtz, and Rebecca Ulrike Jakob, all relying on the circle method, as well as Aleksandar Ivić's article on Tauberian theorems. Some of these new results use or extend previous ones obtained by Schwarz. In the background of János Pintz's article on recent progress in the theory of prime number distribution are sieve methods, which is another direction of research where Schwarz made significant contributions and published monographs at an early age.

In the late 1960s Wolfgang Schwarz was appointed professor at the Albert-Ludwigs-Universität Freiburg, and in 1969 he became Ordinarius at the Johann-Wolfgang-Goethe-Universität Frankfurt. In 1974, he and his family settled in the village of Ruppertshain near Königstein in the lovely Taunus mountain range north of Frankfurt. The Schwarz family quickly became acquainted with its new surroundings and made many friends.

Wolfgang Schwarz's mathematical ideas and inventions were creative and innovative. His broad knowledge together with his mastery of advanced techniques was and still is very impressive. He always had a desire for simplicity and elegance. This is well reflected in his numerous (more than one hundred) research articles (see the complete list of his publications subsequent to this preface). A few of them deal with elementary, diophantine and transcendental questions.

The articles of Régis de la Bretèche & Gérard Tenenbaum, Štefan Porubský, Andrzej Schinzel, Jan-Christoph Schlage-Puchta, Tarlok Shorey & Rob Tijdeman, and Jürgen Spilker are elementary in nature and lie at the core of number theory. Diophantine aspects or transcendence topics are the major theme of the contributions by Ioulia Baoulina & Pieter Moree, Valentin Blomer, Yann Bugeaud, Peter Bundschuh & Keijo Väänänen, Carsten Elsner, Manfred Madritsch & Robert Tichy, and Jörn Steuding; some of them are directly related to results of Schwarz.

One of Wolfgang Schwarz's major research interests was the study of arithmetical functions. Together with his colleague and friend Jürgen Spilker from Freiburg, Schwarz published the standard reference in this field, continuing important works by Wintner, Delange, Wirsing, Elliott, and Halász. Methods of probabilistic and functional-analytic flavor form an essential part of Schwarz & Spilker's *Arithmetical Functions* and provide a rather new perspective on this classical topic. In particular, the concept of Ramanujan expansions, studies of so-called related functions, and Gelfand's theory of maximal ideal spaces make reading this book a great pleasure.

In this memorial volume several examples of recent and past work on arithmetical functions can be found in the articles of Peter Elliott, Karl-Heinz Indlekofer, Aleksandar Ivić, Lutz Lucht, Eugenijus Manstavičius, and Friedemann Tüttas, some of them directly related to Wolfgang Schwarz's work: the contribution by Aleksandar Ivić includes aspects of Schwarz's study of the value-distribution of certain arithmetical functions; the surveys of Lutz Lucht and Karl-Heinz Indlekofer discuss, among other topics, questions about inversion of multiplicative functions, resp. integration theory for arithmetical functions.

Wolfgang Schwarz has always been interested in establishing international contacts. He pursued this objective as one of the main organizers of the number theory meetings at Oberwolfach for two decades from 1974 until 1994 (together with



Fig. 3 Wolfgang Schwarz and his first doctoral student Karl-Heinz Indlekofer discussing the divisor function in 1993. Published with kind permission of © Karl-Heinz Indlekofer, 1993

Hans-Egon Richert and Eduard Wirsing). The success and worldwide reputation of these workshops in the Black Forest emphasize the mathematical significance of both Wolfgang Schwarz and the group of German number theorists at this time. Another notable activity of Schwarz was the exchange program with the group of probabilists and number theorists around Jonas Kubilius at Vilnius University (even before Lithuania's independence in 1990).

In particular these Lithuanian contacts matched Schwarz's research interests on probabilistic number theory and applications to arithmetical functions, well reflected in the article by Eugenijus Manstavičius. However, questions around the Riemann zeta-function and its relatives should be mentioned here as well. To illustrate this line of research one may list the contributions of Ramūnas Garunkštis & Justas Kalpokas, and Antanas Laurinčikas. The article by Helmut Maier & Michael Rassias falls into this category, too. Another related paper by Nicola Oswald & Jörn Steuding traces this direction of investigation in the research area of the famous mathematician Adolf Hurwitz who was born in Hildesheim.⁵ This article on the history of number theory reflects a further research interest of Wolfgang Schwarz.

After Wolfgang Schwarz's retirement in 2002, the history of mathematics and, in particular, the development of number theory, which he had always considered to play an important role in mathematical thinking and education, became the central topic of his mathematical activities. His extraordinary knowledge of mathematical and historical details often led to relevant and helpful remarks at conferences, more than once opening up a new point of view on a discussed unsolved problem or

⁵The town that hosted the 2014 *ELAZ* conference where the first steps for this memorial volume were initiated.



Fig. 4 Wolfgang Schwarz playing the piano. Published with kind permission of © Doris Schwarz, 2013

a presented result. Wolfgang Schwarz's other passion was classical music, which took an even larger role in his life as he was getting older. He played the piano to an almost professional level, and he composed his own pieces of music, which regularly have been performed by members of his family and also by students of the Hochschule für Musik und Darstellende Kunst Frankfurt (for example, to celebrate his 75th birthday).



Fig. 5 Eva Schwarz and Valentin Blomer playing one of Wolfgang Schwarz's pieces at the memorial colloquium during the 2014 *ELAZ* conference. Published with kind permission of © Isa Lange, 2014

On Friday, the 19th of July 2013, Wolfgang Schwarz passed away at the age of 79. During the biannual conference *Elementare und Analytische Zahlentheorie* ($\mathcal{E}L\mathcal{A}Z$) hosted by the University of Hildesheim from July 26th to August 1st 2014, a special colloquium was held on Wednesday July 30th, dedicated to the memory of Wolfgang Schwarz. This was attended by his wife Doris and their daughter Eva, the conference delegates and several external guests. The idea to write a memorial volume to honor Schwarz's life and work was born at about the same time. The series of $\mathcal{E}L\mathcal{A}Z$ conferences had been initiated by Lutz Lucht in 2000 in order to provide a regular platform for young German number theorists to present their research and to meet with experts. Since then these workshops have continued to grow into an international conference with eighty participants from all over the world, invited morning lectures and parallel sessions in the afternoons. The program of the Memorial Colloquium at the Hildesheim $\mathcal{E}L\mathcal{A}Z$ together with a group photograph can be found on the final pages of this volume.

Besides Aleksandar Ivić's opening lecture as well as the Wednesday colloquium talks given by Lutz Lucht, Karl-Heinz Indlekofer, and Jörn Steuding honoring Schwarz's scientific work, this volume contains additional contributions presented at the $\mathcal{E}L\mathcal{A}Z$ conference, some of them with personal reminiscences, for example, the one authored by Štefan Porubský.⁶ The photograph on the previous page shows Valentin Blomer and Schwarz's daughter Eva playing one of Wolfgang Schwarz's own compositions during the memorial colloquium.

The third $\mathcal{E}L\mathcal{A}Z$ conference held in Mainz in 2004 was organized by Wolfgang Schwarz and this is just one of the many examples showing his dedication to support young researchers. Another one is his longstanding commitment to the *Cusanuswerk* at the University of Frankfurt as a student mentor, a position he was committed to for more than 30 years. It is noteworthy that this Catholic institution is named after the fifteenth-century universal scientist and mathematician Nicholas of Cusa. Over the years Schwarz and his Cusanuswerk students met twice a year at his home for an evening of culture and good food prepared by Doris.

One of his students from the Cusanuswerk, Anne Henke, is now professor of mathematics and has contributed an article on her field of research, namely representation theory. This work as well as the articles by Samuel Patterson on variations of Gaussian sums, Jürgen Sander & Torsten Sander on the interaction between number theory and (algebraic) graph theory, Eduard Wirsing on a classical function-theoretical result of Hardy, and Jürgen Wolfart & Benjamin Mühlbauer on dessins d'enfants would most likely have met with Schwarz's enjoyment in the linkage of different mathematical views.

⁶The 'Obituary for Wolfgang Schwarz' by K.-H. Indlekofer, L.G. Lucht, and J. Steuding, published in *Jahresber. Dtsch. Math.-Ver.* **116**, No. 3, 153–169 (2014) provides further information.

Yet, Wolfgang Schwarz was not only a researcher and teacher of mathematics, he also was well aware of the importance of activities beyond the blackboard. He engaged in university administration and was a member of the presidium of the *Deutsche Mathematiker Vereinigung*⁷ (DMV) for several years, being its president in 1986/1987. In 1992/1993 he was chairman of the *Konferenz der Mathematischen Fachbereiche*⁸ (KMathF).

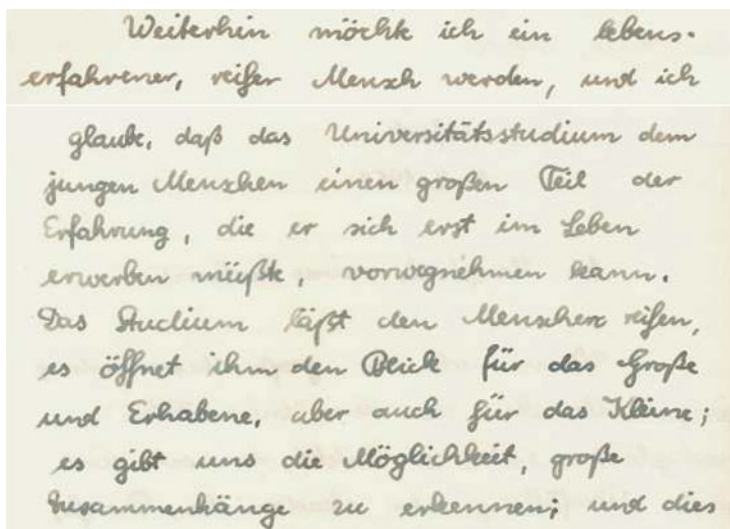


Fig. 6 Another quotation from Schwarz's essay of 1951. Published with kind permission of © Doris Schwarz, 2013

We conclude with a final quotation from Wolfgang Schwarz's essay for university entry:

Furthermore I wish to become an experienced and mature man. I believe that by studying at a university, a young person is able to anticipate much of the experience he would otherwise have to accumulate later in life. By studying, humans gain maturity, it enables one to view the great and the sublime, but also the small; it provides us with the opportunity to see the big picture [...].

In our opinion, these words do not only show a young man's perspective towards his future life, but they also reflect the lifelong motto of a great character who had so much empathy and was so supportive of his colleagues and friends. The mathematical community has lost an excellent and renowned number theorist and

⁷Association of German Mathematicians.

⁸Literally the 'Conference of the mathematical faculties and departments' in Germany; there seems to be no English name.

a true friend. We will miss him while doing mathematics, making music, when playing chess or hiking, or simply in daily life.

Hildesheim
Würzburg
November 2015

Jürgen Sander
Jörn and Rasa Steuding

Post Scriptum: We are grateful to Doris Schwarz for providing us with biographical details, her husband's essay quoted above and several photographs, and to his daughter Karin Schwarz for proofreading. Further photographs in this volume have been contributed by friends and colleagues. We admire the artistic talent of Nicola Oswald, who created the watercoloured portrait of Wolfgang Schwarz on the front cover. Moreover, we thank Lutz Lucht for additional information as well as Karl Dilcher and Jan-Hendrik de Wiljes for technical support. Last but not least, we wish to express our gratitude to Mario Aigner, Sonja Gasser and the whole team at Springer Basel and Heidelberg for making our idea into a book.

Universität Hildesheim
2003

Department of Algebra and Number Theory
Institute for Mathematics & Applied Computer Science
Conference Organizers:
Jürgen Sander
Martin Kreh
Jan-Hendrik de Wilies
Contact: elaz@ima.uni-hildesheim.de

ELAZ conference at the University of Hildesheim, Germany, July 28th – August 1st, 2014
(Elementary and Analytic Number Theory)

Invited speakers

Valeentin Blomer (Georg-August-Universität Göttingen)
Kathrin Brühmann (Universität zu Köln)
Jörg Brüdern (Georg-August-Universität Göttingen)
Karl Dilcher (Dalhousie University)
Christian Esholtz (Technische Universität Graz)
Karl-Heinz Indlekofer (Universität Paderborn)
Aleksandar Ivić (Univerzitet u Beogradu)
Simon Kristensen (Aarhus Universität)
Lutz Lucht (Technische Universität Clausthal)
Helmut Maier (Universität Ulm)
Pieter Moree (Max-Planck-Institut für Mathematik)
Nicola Oswald (Julius-Maximilians-Universität Würzburg)
Samuel Patterson (Georg-August-Universität Göttingen)
János Pintz (Magyar Tudományok Akadémiája)
Andrzej Schinzel (Polska Akademia Nauk)
Jean-Christophe Scholge-Puchta (Universität Rostock)
Jörn Steuding (Julius-Maximilians-Universität Würzburg)
Robert F. Tichy (Technische Universität Graz)
Robert Tijdeman (Universität Leiden)

Wednesday, 30th of July will be dedicated to **Wolfgang Schwarz**. Friends and colleagues will honour his life and scientific work in a series of talks.

Adolf Hurwitz

Conference Website: www.uni-hildesheim.de/ifa-4/institute/ima/abteilungen/algebra-und-zahlentheorie/elaz-2014

The poster of the ELAZ conference in Hildesheim. Published with kind permission of © Jürgen Sander, 2014

Wolfgang Schwarz's Publications

1. Zur Darstellung von Zahlen durch Summen von Primzahlpotenzen. I. Darstellung hinreichend grosser Zahlen. *J. Reine Angew. Math.* **205** (1960/1961), 21–47.
2. Zur Darstellung von Zahlen durch Summen von Primzahlpotenzen. II. *J. Reine Angew. Math.* **206** (1961), 78–112.
3. Weitere, mit einer Methode von Erdős-Prachar erzielte Ergebnisse. *Math. Nachr.* **23** (1961), 327–348.
4. Über die Summe $\sum_{n \leq x} \varphi(f(n))$ und verwandte Probleme. *Monatsh. Math.* **66** (1962), 43–54.
5. Irrationale Potenzreihen. *Arch. Math.* **13** (1962), 228–240.
6. Zur Darstellung einer Zahl als Summe einer k -ten Potenz einer potenzfreien Zahl und einer g -ten potenzfreien Zahl. *Arch. Math.* **14** (1963), 105–115.
7. Eine Bemerkung über fast-ganzzwertige Funktionen. *Monatsh. Math.* **67** (1963), 129–136.
8. Über die Lösbarkeit gewisser Ungleichungen durch Primzahlen. *J. Reine Angew. Math.* **212** (1963), 150–157.
9. Weitere, mit einer Methode von Erdős-Prachar erzielte Ergebnisse. II. *Monatsh. Math.* **68** (1964), 75–80.
10. *Einige Anwendungen Tauberscher Sätze in der Zahlentheorie*. Habilitationsschrift, Universität zu Freiburg i. Br., 1964.
11. Einige Anwendungen Tauberscher Sätze in der Zahlentheorie. A. *J. Reine Angew. Math.* **219** (1965), 67–96.
12. Einige Anwendungen Tauberscher Sätze in der Zahlentheorie. B. *J. Reine Angew. Math.* **219** (1965), 157–179.
13. Über die Anzahl Abelscher Gruppen gegebener Ordnung. I. *Math. Z.* **92** (1966), 314–320.
14. Einige Bemerkungen über periodische zahlentheoretische Funktionen. *Math. Nachr.* **31** (1966), 125–136.
15. Irrationale Potenzreihen. II. *Arch. Math.* **17** (1966), 435–437.
16. Über die Anzahl Abelscher Gruppen gegebener Ordnung. II. *J. Reine Angew. Math.* **228** (1967), 133–138.

17. Einige Anwendungen Tauberscher Sätze in der Zahlentheorie. C. Mahler's Partitionsproblem. *J. Reine Angew. Math.* **228** (1967), 182–188.
18. Remarks on the irrationality and transcendence of certain series. *Math. Scand.* **20** (1967), 269–274.
19. Eine Bemerkung über die Anzahl der Zahlen pr^2 unterhalb x . *Colloq. Math.* **19** (1968), 117–120.
20. Schwache asymptotische Eigenschaften schnell wachsender multiplikativer zahlentheoretischer Funktionen. *Monatsh. Math.* **72** (1968), 355–367.
21. Schwache asymptotische Eigenschaften von Partitionen. *J. Reine Angew. Math.* **232** (1968), 1–16.
22. Der Primzahlsatz. *Überblicke Math.* **1** (1968), 35–61.
23. Asymptotische Formeln für Partitionen. *J. Reine Angew. Math.* **234** (1969), 174–178.
24. *Einführung in Ergebnisse und Methoden der Primzahltheorie*. BI Hochschultaschenbuch, Mannheim 1969.
25. Bemerkungen zu einem Satz der Herren Turán und Clunie über das Verhalten von Potenzreihen auf dem Rande des Konvergenzkreises. *Publ. Math. Debrecen* **16** (1969), 67–73.
26. Eine Bemerkung zu einer asymptotischen Formel von Herrn Rényi. *Arch. Math.* **21** (1970), 157–166.
27. Schwache asymptotische Eigenschaften von Partitionen. In: *Number Theory* (Colloq., János Bolyai Math. Soc., Debrecen, 1968) North-Holland, Amsterdam 1970, 191–196.
28. Über Teiler von n der Gestalt $p - 1$. In: *Zahlentheorie* (Tagung, Math. Forschungsinst., Oberwolfach, 1970), *Ber. Math. Forschungsinst.*, Oberwolfach **5**, Bibliographisches Inst., Mannheim, 1971, 147–158.
29. with Spilker, J.: Eine Anwendung des Approximationsatzes von Weierstrass-Stone auf Ramanujan-Summen. *Nieuw Arch. Wisk.* **19** (1971), 198–209.
30. Bemerkungen zu einem Satz der Herren Turán und Clunie über das Verhalten von Potenzreihen auf dem Rande des Konvergenzkreises. II. *Publ. Math. Debrecen* **18** (1971), 129–137.
31. Berichtigung zu der Arbeit "Weitere, mit einer Methode von Erdős-Prachar erzielte Ergebnisse". *Math. Nachr.* **52** (1972), 385.
32. A remark on multiplicative functions. *Bull. London Math. Soc.* **4** (1972), 136–140.
33. with Indlekofer, K.-H.: Über B-Zwillinge. *Arch. Math.* **23** (1972), 251–256.
34. Über B-Zwillinge. II. *Arch. Math.* **23** (1972), 408–409.
35. Ramanujan-Entwicklungen stark multiplikativer zahlentheoretischer Funktionen. *Acta Arith.* **22** (1973), 329–338.
36. with Wirsing, E.: The maximal number of non-isomorphic abelian groups of order n . *Arch. Math.* **24** (1973), 59–62.
37. Ramanujan-Entwicklungen stark multiplikativer Funktionen. *J. Reine Angew. Math.* **262/263** (1973), 66–73.
38. Eine weitere Bemerkung über multiplikative Funktionen. *Colloq. Math.* **28** (1973), 81–89.

39. with Wallisser, R.: Über gewisse nichtfortsetzbare Potenzreihen. *Monatsh. Math.* **77** (1973), 63–66
40. with Wallisser, R.: Über gewisse nichtfortsetzbare Potenzreihen. II. *Monatsh. Math.* **77** (1973), 251–266.
41. Über Potenzreihen, die irrationale Funktionen darstellen. I. *Überblicke Mathematik* **6** (1973), 179–196.
42. Die Ramanujan-Entwicklung reellwertiger multiplikativer Funktionen vom Betrage kleiner oder gleich Eins. *J. Reine Angew. Math.* **271** (1974), 171–176.
43. with Spilker, J.: Über zahlentheoretische Funktionen, die $f(x + y) = f(f(x) + f(y))$ erfüllen. *Mitt. Math. Sem. Giessen* **111** (1974), 80–86.
44. Über Potenzreihen, die irrationale Funktionen darstellen. II. *Überblicke Mathematik* **7** (1974), 7–32.
45. *Einführung in Siebmethoden der analytischen Zahlentheorie*. Bibliographisches Institut, Mannheim-Vienna-Zürich, 1974.
46. Über die Ramanujan-Entwicklung multiplikativer Funktionen. *Acta Arith.* **27** (1975), 269–279.
47. Eine Bemerkung über Restklassencharaktere. *Math.-Phys. Semesterber.* **22** (1975), 185–192.
48. Zum zahlentheoretischen Werk von Enrico Bombieri. *Überblicke Mathematik* **8** (1975), 150–151.
49. *Einführung in die Zahlentheorie*. Wissenschaftliche Buchgesellschaft, Darmstadt, 1975.
50. with Lucht, L.G.: Über die Lösungsanzahl der Gleichung $f(n) = \kappa \cdot n$ für gewisse Klassen multiplikativer Funktionen. *Monatsh. Math.* **81** (1976), 213–216.
51. Aus der Theorie der zahlentheoretischen Funktionen. *Jber. Deutsch. Math.-Verein.* **78** (1976/77), 147–167.
52. with Spilker, J.: Mean values and Ramanujan expansions of almost even arithmetical functions. In: *Topics in Number Theory* (Proc. Colloq., Debrecen, 1974), Colloq. Math. Soc. Janos Bolyai, Vol. 13, North-Holland, Amsterdam, 1976, 315–357.
53. Liouville-Zahlen und der Satz von Baire. *Math.-Phys. Semesterber.* **24** (1977), 84–87.
54. with Zassenhaus, H.: In Memoriam Bohuslav Diviš. *J. Number Theory* **9** (1977), v–vii.
55. Vanishing sums of roots of unity. *Elem. Math.* **33** (1978), 141–143.
56. Survey on analogues of the Waring problem: Diophantine inequalities. In: *Conference on Additive Number Theory* (Bordeaux, 1977), Univ. Bordeaux I, Talence, 1978, 121–147.
57. Periodic, multiplicative number-theoretical functions. *Monatsh. Math.* **87** (1979), 65–67.
58. with Kopetzky, H.G.: Two conjectures of B. R. Santos concerning totitives. *Math. Comp.* **33** (1979), 841–844.

59. Some applications of Elliott's mean-value theorem. *J. Reine Angew. Math.* **307/308** (1979), 418–423.
60. with Spilker, J.: Wiener-Lévy-Sätze für absolut konvergente Reihen. *Arch. Math.* **32** (1979), 267–275.
61. with Ivić, A.: Remarks on some number-theoretical functional equations. *Aequationes Math.* **20** (1980), 80–89.
62. with Duttlinger, J.: Über die Verteilung der pythagoräischen Dreiecke. *Colloq. Math.* **43** (1980), 365–372.
63. with Kopetzky, H.G., Wirsing, E.: Nonnegative sums of roots of unity. *Arch. Math.* **34** (1980), 114–121.
64. Fourier-Ramanujan-Entwicklungen zahlen-theoretischer Funktionen und Anwendungen. In: *Festschrift of the 75th Jubilee of the Scientific Society at the Johann Wolfgang Goethe-Universität, Frankfurt/Main*, Steiner, Wiesbaden 1981, 399–415.
65. with Knopfmacher, J.: Binomial expected values of arithmetical functions. *J. Reine Angew. Math.* **328** (1981), 84–98.
66. with Spilker, J.: Remarks on Elliott's theorem on mean-values of multiplicative functions. In: *Recent Progress in Analytic Number Theory*, Durham 1979, 1981, 325–339.
67. with Spilker, J.: Eine Bemerkung zur Charakterisierung der fast-periodischen multiplikativen zahlentheoretischen Funktionen mit von Null verschiedenem Mittelwert. *Analysis* **3** (1983), 205–216.
68. with Heppner, E.: Benachbarte multiplikative Funktionen. *Studies in Pure Mathematics*, 323–336, Birkhäuser, Basel, 1983.
69. with Waterhouse, W.C.: The asymptotic density of supersingular Fermat varieties. *Arch. Math.* **43** (1984), 142–144.
70. with Knopfmacher, J.: Binomial expected values of arithmetical functions. II. In: *Topics in Classical Number Theory*, Vol. I, II (Budapest, 1981), 863–905, Colloq. Math. Soc. János Bolyai, 34, North-Holland, Amsterdam, 1984.
71. with Herzog, J.: Über eine spezielle Partitionenfunktion, die mit der Anzahl der abelschen Gruppen der Ordnung n zusammenhängt. *Analysis* **5** (1985), 153–161.
72. Über einige Probleme aus der Theorie der Primzahlen. *Sitzungsber. Wiss. Ges. Johann Wolfgang Goethe-Univ. Frankfurt am Main* **21** (1985), 61–118.
73. Another evaluation of an Erdős-Turán constant. *Analysis* **5** (1985), 343–345.
74. Remarks on the theorem of Elliott and Daboussi, and applications. In: *Elementary and Analytic Theory of Numbers* (Warsaw, 1982), 463–498, Banach Center Publ., **17**, PWN, Warsaw, 1985.
75. *Beispiele und BASIC-Programme zum Mathematikunterricht*. Bibl. Inst. Mannheim 1985.
76. with Spilker, J.: A variant of proof of Daboussi's theorem on the characterization of multiplicative functions with nonvoid Fourier-Bohr-spectrum. *Analysis* **6** (1986), 237–249.
77. A correction to: "Remarks on Elliott's theorem on mean-values of multiplicative functions" (Durham, 1979) and Some remarks on almost-even

- number-theoretical functions. In: *Analytic and Elementary Number Theory* (Marseille, 1983), 139–158, Publ. Math. Orsay, 86-1, Univ. Paris XI, Orsay, 1986.
78. with Porubský, Š.: Sums of additive functions on arithmetical semigroups. *Colloq. Math.* **53** (1987), 137–146.
79. Almost-even number-theoretical functions. In: *Probability Theory and Mathematical Statistics*, Vol. II (Vilnius, 1985), 581–587, VNU Sci. Press, Utrecht, 1987.
80. Bildungsinhalte des Mathematik-Studiums. In: Die Bildungswerte des Studiums Mathematisch-Naturwissenschaftlicher Fächer, WRK, *Dokumente z. Hochschulreform* 64/1988, 1987, 37–67.
81. Ramanujan expansions of arithmetical functions. In: *Ramanujan Revisited* (Urbana-Champaign, Ill., 1987), 187–214, Academic Press, Boston, MA, 1988.
82. with Hildebrand, A., Spilker, J.: Still another proof of Parseval's equation for almost-even arithmetical functions. *Aequationes Math.* **35** (1988), 132–139.
83. with Kappe, L.-Ch., Schlickewei, H.P.: Theodor Schneider zum Gedächtnis. *Jahresber. Deutsch. Math.-Verein.* **92** (1990), 111–129.
84. Geschichte der analytischen Zahlentheorie seit 1890. In: *Ein Jahrhundert Mathematik 1890–1990*, 741–780, Dokumente Gesch. Math., 6, Vieweg, Braunschweig, 1990.
85. with Maxsein, T., Smith, P.: An example for Gelfand's theory of commutative Banach algebras. *Math. Slovaca* **41** (1991), 299–310.
86. with Maxsein, T.: A new proof of a theorem concerning functions in different spaces of almost-periodic arithmetical functions. *Proceedings of the Amalfi Conference on Analytic Number Theory* (Maiori, 1989), 315–323, Univ. Salerno, Salerno, 1992.
87. Emeritierung von Friedrich Hirzebruch. *Mitt. Dt. Math. Ver.* 1993-2, 32–33.
88. with Spilker, J.: *Arithmetical Functions: An Introduction to Elementary and Analytic Properties of Arithmetic Functions and to Some of their Almost-Periodic Properties*. Cambridge University Press, 1994.
89. Some remarks on the history of the prime number theorem from 1896 to 1960. In: *Development of Mathematics 1900–1950*, ed. J.-P. Pier, Birkhäuser 1994 (Symposium Luxembourg 1992), 565–615.
90. Some aspects of the development of probabilistic number theory. In: *Probability Theory and Mathematical Statistics* (Vilnius, 1993), 661–701, TEV, Vilnius, 1994.
91. with Tschiersch, R.: *Studienführer Mathematik*. Karl Bock Verlag, Bad Honnef, 1994.
92. Die Mathematik gleicht einem Baum. *Forschung & Lehre* 10/1994, 434–437, Bonn.
93. Evaluation of mean-values of products of shifted arithmetical functions. *Note Mat.* **15** (1995), 235–250 (1997).

94. with Spilker, J.: A remark on some special arithmetical functions. In: *New Trends in Probability and Statistics* **4** (Palanga, 1996), 221–245, VSP, Utrecht, 1997.
95. On the number-theoretical work of Edmund Hlawka. In: *Proc. Conf. Analytic and Elementary Number Theory*, Vienna, July 18–20, 1996, 1–30, Universität Wien and Universität für Bodenkultur, Wien, 1997.
96. Wirsing's theorems; Delange's theorem; Elliott-Daboussi theorem; Halász' mean value theorem. Contributions to *Encyclopaedia of Mathematics*, Supplement I, Kluwer Academic Publishers, 1997.
97. with Burde, G.: Wolfgang Franz zum Gedächtnis. *Jahresber. Deutsch. Math.-Verein.* **100** (1998), 284–292.
98. Evaluation of mean-values of products of shifted arithmetical functions. II. In: *Number theory in progress*, Vol. 2 (Zakopane-Kościelisko, 1997), 1081–1098, de Gruyter, Berlin, 1999.
99. Nachruf auf Wolfgang Franz. *Sitzungsber. Wiss. Gesellschaft Frankfurt XXXVI* (1999), 379–383.
100. Some Results on Arithmetical Functions. In: *Prob. Theory and Math. Stat.*, Proc. 7th Internat. Vilnius Conference, August 12–18, 1998, Utrecht 1999, 653–664.
101. Almost-even functions as solutions of a linear functional equation. *Math. Slovaca* **50** (2000), 525–529.
102. Uniform-fast-gerade Funktionen mit vorgegebenen Werten. *Arch. Math.* **77** (2001), 1–4.
103. John Knopfmacher, analytic number theory, and the theory of arithmetical functions. *Quaest. Math.* **24** (2001), 273–290.
104. Zur Entwicklung einiger Gebiete der Zahlentheorie, insbesondere im 19. Jahrhundert. *Sitzungsber. Wiss. Ges. Univ. Frankfurt XXXIX* (2001), 75–123.
105. with Choi, E.-H.: Mean-values of products of shifted arithmetical functions. In: *Analytic and Probabilistic Methods in Number Theory* (Palanga, 2001), 32–41, TEV, Vilnius, 2002.
106. with Laurinčikas, A., Steuding, J.: Value distribution of general Dirichlet series. III. In: *Analytic and Probabilistic Methods in Number Theory* (Palanga, 2001), 137–156, TEV, Vilnius, 2002.
107. with Burde, G., Wolfart, J.: Ein Mathematiker mit universalem Anspruch. Über Max Dehn und sein Wirken am Mathematischen Seminar. *Forschung Frankfurt* 4/2002, 85–89.
108. with Laurinčikas, A., Steuding, J.: The universality of general Dirichlet series. *Analysis* **23** (2003), 13–26.
109. with Volkmann, B.: Hans Rohrbach zum Gedächtnis 27.2.1903–19.12.1993. *Jahresber. Deutsch. Math.-Verein.* **105** (2003), 89–99.
110. with Spilker, J.: Uniform-fast-gerade Funktionen mit vorgegebenen Werten. II. *Arch. Math.* **81** (2003), 534–537.

111. with Schlage-Puchta, J.-C., Spilker, J.: Uniformly-almost-even functions with prescribed values. III. *Ann. Univ. Sci. Budapest. Sect. Comput.* **22** (2003), 317–330.
112. with Schlage-Puchta, J.-C., Spilker, J.: Uniformly-almost-even functions with prescribed values. IV. Application of Gelfand theory. *Analysis* **24** (2004), 63–70.
113. with G. Burde, G., Wolfart, J.: Max Dehn und das Mathematische Seminar der Universität Frankfurt. *Mathematik im Fluss der Zeit. Algorismus* 44 (2004), 462–483.
114. *Aus der Geschichte der Frankfurter Mathematik. Festschrift zu den 100. Geburtstagen von Ruth Moufang, Gottfried Köthe, Wolfgang Franz.* Edited by Wolfgang Schwarz. *Schriften des Universitätsarchivs Frankfurt am Main*, 1, 2005.
115. with Wolfart, J.: Drei Säulen des Frankfurter Mathematischen Seminars. *Forschung Frankfurt* 2/2005, 55–59.
116. *Elementare und Analytische Zahlentheorie.* Proceedings of the 3rd Conference (ELAZ) held in Mainz, May 24–28, 2004. Edited by Wolfgang Schwarz and Jörn Steuding. *Schriften Wiss. Ges. Johann Wolfgang Goethe-Universität Frankfurt am Main*, Franz Steiner Verlag Stuttgart, Stuttgart, 2006.
117. with Lucht, L.G.: Zum Gedenken an Thomas Maxsein. In: *Elementare und Analytische Zahlentheorie*, 346–348, *Schriften Wiss. Ges. Johann Wolfgang Goethe Univ. Frankfurt am Main*, Franz Steiner Verlag Stuttgart, Stuttgart, 2006.
118. with Steuding, R., Steuding, J.: Universality for Euler products, and related arithmetical functions. In: *Analytic and Probabilistic Methods in Number Theory* (Palanga 2006), 163–189, TEV, Vilnius, 2007.
119. Some highlights from the history of probabilistic number theory. In: *Probability and Number Theory*, Kanazawa 2005, 367–419, *Adv. Stud. Pure Math.* **49**, Math. Soc. Japan, Tokyo, 2007.

Impressions

Mathematicians in general are sociable people. They meet at conferences and discuss their latest results and much more at conference dinners.



Published with kind permission of © Stefan Porubsky, 2004

The picture above shows Wolfgang Schwarz hosting a number theoretical meeting in Frankfurt in summer 2004, surrounded by (from left to right) Aleksandar Ivić, Antanas Laurinčikas, Jörn Steuding, Štefan Porubský, and Jürgen Wolfart.

Below a scene from a private round at Schwarz's home in the Taunus village Ruppertshain with guests. From left to right: Aleksandar Ivić, Andrzej Schinzel, Štefan Porubský, János Pintz, Wolfgang Schwarz, and Antanas Laurinčikas.



Published with kind permission of © Stefan Porubsky, 2004



Published with kind permission of © Stefan Porubsky, 1989

The photograph above shows Wolfgang Schwarz discussing mathematics with Karl Dilcher. In the next one, Rasa Steuding, Karl-Heinz Indlekofer, Kohji Matsumoto,

Wolfgang Schwarz, and Eugenijus Manstavicius (from left to right) having dinner in a Japanese restaurant.



Published with kind permission of © Jörn Steuding, 2005

The following photograph shows Wolfgang Schwarz during the 2012 *ELAZ* Conference at Schloß Schney talking with Lutz Lucht and his wife.



Published with kind permission of © Stefan Porubsky, 2012

Professor Schwarz discussing mathematics with his colleagues Profs. Albrecht Pfister (left) and Jürgen Spilker (right).



Published with kind permission of © Jürgen Wolfart, 2009

Below another example, Wolfgang Schwarz in discussion with Eugenijus Manstavicius; in the background Jürgen Sander (on the left) and Eduard Wirsing (in the middle).



Published with kind permission of © Stefan Porubsky, 2012

But sometimes mathematicians are silently watching someone else's talk. The next photograph shows Günter Pickert, Eduard Wirsing, Wolfgang Schwarz, and

Karl-Heinz Indlekofer (from left to right).



Published with kind permission of © Jürgen Wolfart, 2009

Some mathematicians are genuine globetrotters. Wolfgang Schwarz was one of those, and he was a passionate alpinist and mountaineer (see the photographs in Lutz Lucht's contribution).



Published with kind permission of © Peter Elliott, 1975

The last - rather romantic - photograph shows him at the Glaswaldsee in the Black Forest, while the photograph below, showing Wolfgang and Doris Schwarz, was taken during a rainy day at the Curonian Spit in Lithuania.



Published with kind permission of © Jörn Steuding, 2011

The Schwarz family has always been very hospitable. Below a photo showing the beginning of an excursion of a larger group of tourists in the Taunus range 2004.



From left to right: Wolfgang Schwarz, Jörn Steuding, Andrzej Schinzel, Doris Schwarz, János Pintz, Aleksandar Ivić, and Antanas Laurinčikas. Published with kind permission of © Stefan Porubsky, 2004

At conferences mathematicians give and listen to talks.



Published with kind permission of © Jürgen Wolfart, 2009

Eduard Wirsing (on the left) and Wolfgang Schwarz enjoying a humoristic moment at the *ELAZ* Conference 2012 in Schloß Schney.

Sometimes there are moments for celebration. At the *ELAZ* Conference 2004



Published with kind permission of © Stefan Porubsky, 2004

in Mainz Wolfgang Schwarz was celebrating his 70th birthday. As a representative of the Lithuanian delegation Eugenijus Manstavičius is handing over a hand-made Lithuanian sash.

Group photographs are another favourite amusement of mathematicians during their meetings.



Published with kind permission of © Peter Elliott, 1984

These pictures show Wolfgang Schwarz, Peter Elliott, Dieter Wolke, Ernst Heppner, Karl-Heinz Indlekofer, and Imre Katai (above from left to right) as well as Gérald Tenenbaum, once again Schwarz, Richard Warlimont, and Lutz Lucht (below from left to right) at a number theory conference at the Mathematisches Forschungsinstitut Oberwolfach in 1984.



Published with kind permission of © Peter Elliott, 1984



Published with kind permission of © Stefan Porubsky, 2012

Wolfgang Schwarz answering questions after his talk at the *ElAZ* Conference 2012 in Schloß Schney. In the background chairman Jürgen Sander.

At such occasions one can sometimes meet other mathematician's families. Wolfgang Schwarz and part of the Porubský family having coffee and ice cream.



Published with kind permission of © Stefan Porubsky, 1991

Contents

Wenn er aber studiert, müssen ihm bedeutende ideale Ziele vorschweben	vii
Wolfgang Schwarz's Publications	xvii
Impressions	xxv
Forbidden Integer Ratios of Consecutive Power Sums	1
Ioulia N. Baoulina and Pieter Moree	
A Note on the Negative Pell Equation	31
Valentin Blomer	
Localisation Conditionnelle de Diviseurs	41
Régis de la Bretèche and Gérald Tenenbaum	
A Ternary Problem in Additive Prime Number Theory	57
Jörg Brüdern	
An Improvement of Liouville's Inequality	83
Yann Bugeaud	
Guided by Schwarz' Functions: A Walk Through the Garden of Mahler's Transcendence Method	91
Peter Bundschuh and Keijo Väänänen	
Sums of Two Squares and a Power	103
Rainer Dietmann and Christian Elsholtz	
Multiplicative Functions and the Sign of Maass Form Fourier Coefficients	109
Peter D.T.A. Elliott	
On Error Sum Functions for Approximations with Arithmetic Conditions	121
Carsten Elsner	

Sum of the Lerch Zeta-Function over Nontrivial Zeros of the Dirichlet L-Function	141
Ramūnas Garunkštis and Justas Kalpokas	
Schur–Weyl Dualities Old and New	155
Anne Henke	
Arithmetic Functions: A Pivotal Topic in the Scientific Work of Wolfgang Schwarz	179
Karl-Heinz Indlekofer	
On Some Selected Works of Wolfgang Schwarz	201
Aleksandar Ivić	
Sums of Two Squares of Sums of Two Squares	217
Rebecca Ulrike Jakob	
The Joint Discrete Universality of Periodic Zeta-Functions	231
Antanas Laurinčikas	
Remembering Wolfgang Schwarz, His Life and Work	247
Lutz G. Lucht	
Dynamical Systems and Uniform Distribution of Sequences	263
Manfred G. Madritsch and Robert F. Tichy	
Asymptotics and Equidistribution of Cotangent Sums Associated with the Estermann and Riemann Zeta Functions	277
Helmut Maier and Michael Th. Rassias	
A Turán-Kubilius Inequality on Mappings of a Finite Set	295
Eugenijus Manstavičius	
Aspects of Zeta-Function Theory in the Mathematical Works of Adolf Hurwitz	309
Nicola Oswald and Jörn Steuding	
Selberg Sums: A New Perspective	353
Samuel J. Patterson	
Polignac Numbers, Conjectures of Erdős on Gaps Between Primes, Arithmetic Progressions in Primes, and the Bounded Gap Conjecture	367
János Pintz	
Idempotents and Congruence $ax \equiv b \pmod{n}$	385
Štefan Porubský	
Recent Developments on the Edge Between Number Theory and Graph Theory	405
Jürgen Sander and Torsten Sander	

The Leading Coefficients of Stern Polynomials 427
 Andrzej Schinzel

The Non-existence of Universal Carmichael Numbers 435
 Jan-Christoph Schlage-Puchta

Arithmetic Properties of Blocks of Consecutive Integers 455
 Tarlok N. Shorey and Rob Tijdeman

The GCD of the Shifted Fibonacci Sequence 473
 Jürgen Spilker

On Liouville Numbers: Yet Another Application of Functional Analysis to Number Theory 485
 Jörn Steuding

Natural Boundaries of Power Series with Multiplicative Coefficients in Algebraic Number Fields 509
 Friedemann Tüttas

A Minimal Proof of a Result of Hardy 523
 Eduard Wirsing

Regular Dessins with Abelian Automorphism Groups 527
 Jürgen Wolfart and Benjamin Mühlbauer

Impressions from the *ELAZ* Conference 2014 in Hildesheim 535