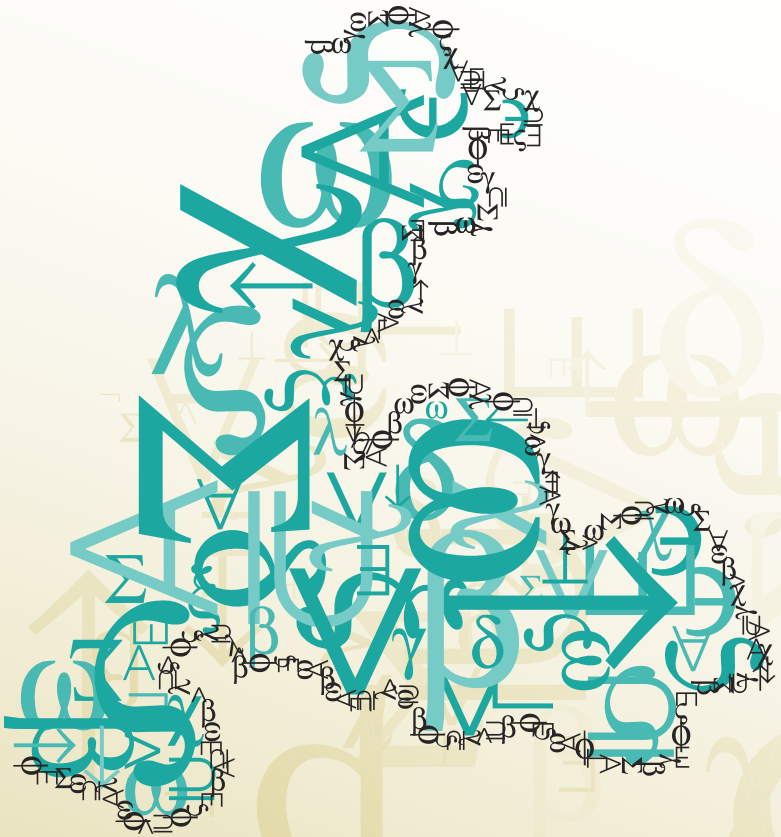
Logic at Notre Dame



Logic at Notre Dame

The University of Notre Dame's Mathematics and Philosophy Departments have a long and dedicated involvement with formal logic. In this small pamphlet, we offer two vignettes from that history, an essay by John Dawson on Kurt Gödel's time at Notre Dame and an essay by Mic Detlefsen on Thoralf Skolem's time here. Gödel visited the Mathematics department, as Dawson details, in the winter of 1939, teaching two seminars and working on an outline of his proof of the consistency of GCH. Skolem was a regular visitor to the Department, teaching seminars and conducting research in set theory on eight separate occasions from 1957 through 1962.

The history of logic at Notre Dame also includes Innocentius Bochenski's year-long teaching position in the Philosophy Department in 1955–56. Following Bochenski's departure, the department was joined by Boleslaw Sobocinski, who took up a regular faculty position and founded the *Notre Dame Journal of Formal Logic*.

We are fortunate to be able to continue a robust tradition of teaching and research in logic. Our logic group includes faculty members, post-docs, and graduate students from both the Mathematics and Philosophy departments, offers PhDs through both the regular Mathematics and Philosophy programs, and sponsors a joint PhD in Logic and Foundations of Mathematics.

Kurt Gödel at Notre Dame

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During the 1930s the Mathematics department at the University of Notre Dame was host to a number of distinguished European scholars. Among them were Emil Artin and Karl Menger—and also, for a single semester in the spring of 1939, Kurt Gödel, already celebrated for his incompleteness results published eight years before, and soon to become still more renowned for his consistency results in set theory, obtained in the summer of 1937 and first publicly announced in a brief note the following year in the *Proceedings of the National Academy of Sciences*.

Unlike many of the scholars who came to America in the wake of Hitler's conquests, Gödel was not a political refugee. He was not Jewish and was deemed to be "a thoroughly apolitical person" by one of the Nazi functionaries who vetted one of his applications for a leave of absence (a judgment which, however, also implied that he lacked "any inner commitment to National Socialism").[1] Nevertheless, despite his mathematical fame, at the time of his visit to Notre Dame Gödel's academic and financial position was precarious. He had never held a position beyond that of *Privatdozent* (unsalaried lecturer, whose income from teaching came solely from students' tuition), and much of the family wealth on which he had earlier depended for support had been lost during the Austrian economic collapse in 1931. In addition, he had suffered several episodes of depression during the years 1934–37 that had prevented him from teaching on a regular basis; indeed, after receiving his *Dozentur* in March of 1933 he had taught only three courses at the University of Vienna prior to his coming to Notre Dame. Worse still, his authorization to teach (*Lehrbefugnis*) had officially lapsed in April of 1938, a month

[1] *Dawson 1997*, p. 146.

after the Austrian *Anschluß*.[2] And by the summer of 1938 most of those with whom he had studied or had contact through the Vienna Circle had dispersed or died, leaving him isolated: his dissertation advisor, Hans Hahn, had suffered a fatal heart attack in the summer of 1934 following emergency cancer surgery; Moritz Schlick, the leader of the Vienna Circle, had been assassinated in June of 1936, after which the Circle itself soon ceased to function; and Menger, in whose mathematical colloquium Gödel had participated actively ever since 1929, had left in 1937, seizing the offer of a professorship at Notre Dame as a means of escaping the growing specter of Nazism.

No wonder, then, that Gödel was receptive when Menger broached the idea that he too should come to Notre Dame. Notre Dame's president at that time, John Cardinal O'Hara, was determined to strengthen the University's academic reputation by recruiting eminent European scholars to its faculty.[3] It was presumably he who had extended the offer of a faculty position to Menger, and when Menger became chairman of the mathematics department, just a year after his appointment, he in turn "resolved to help…develop advanced studies of mathematics" there.[4] Gödel was not a Catholic, as were many of those whom O'Hara had recruited, and from his correspondence with his mother we know that he was disdainful of some Catholic doctrines (influenced in part, no doubt, by his experience of the Catholic clerico-fascist state that had developed in Austria in the years preceding the *Anschluß*.) Nor, for their part, did the logicians then at Notre Dame have much interest in Gödel's work: According to Menger, "at [all] the Catholic universities in the United States in the mid-1930s, logic was completely dominated by the writings of Jacques Maritain and the philosophical school of Laval University," which was "quite opposed to mathematical ('merely formal') logic."[5] But Menger said nothing about that in his correspondence with Gödel.[6] His initial invitation, dated 22 May 1937, merely asked whether Gödel would consider coming to a "liberal Catholic University east of Chicago" that had "a first-class chemistry department" and desired "to upgrade its mathematics and physics departments." And in a telegram of 12 September 1937 he stressed that in the winter of 1938 (when he had originally hoped Gödel would be able to come to Notre Dame) "Artin will be here and a few good students, all looking forward to you. Quiet opportunities for work, cheaper living costs than in the East. Teaching duties at most three hours per week and a seminar every second week."

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[2] A year later, during his stay at Notre Dame, the position of *Dozent* would be abolished altogether.
[3] See *Strich* 1981 for a more detailed account of O'Hara's achievements in that regard.
[4] *Menger 1994*, p. 215. [5] *Ibid*., p. 216. [6] Reproduced in *Gödel 2003*, pp. 83–133.

In the meantime, though, Gödel had written Menger that while he was "in principle" agreeable to the idea of coming to Notre Dame (and that "becom[ing] acquainted with the operation of a Catholic American university would…be very interesting" to him), "the summer semester of 1938 would be the earliest" that he could consider.[7] For Phillip Frank had tentatively invited him to spend the fall of 1937 in Prague (a proposal that later failed to materialize), and in the winter semester of 1938 he expected to return to Princeton, where he had spent the 1933–34 academic year and had come again briefly in the fall of 1935. He emphasized, however, that due to the health problems he had experienced during that latter visit (one of the episodes of depression mentioned earlier), he did not wish to be obligated to stay more than one semester.

Interestingly, Gödel did not tell Menger then that just three weeks earlier he had at last succeeded in proving the consistency of the generalized continuum hypothesis with the axioms of Zermelo-Fraenkel set theory. (He did so only in his later letter to him of 15 December.) But he did tell von Neumann about that during a visit the latter made to Vienna later that summer—a revelation that so excited von Neumann that he then urged Gödel to come to Princeton to lecture on his discoveries there.

A rather protracted correspondence subsequently ensued among Menger, O'Hara, von Neumann, and Oswald Veblen, involving numerous delays and crossings of letters in the mail. Eventually, however, all the parties became aware of each other's efforts, and a solution acceptable to all was worked out: In a letter to Gödel of 24 December 1937, Veblen proposed that Gödel spend the fall semester of 1938 at Notre Dame and the winter term of 1939 at the Institute for Advanced Study; Gödel, however, preferred to reverse that order, and that was the arrangement that was finally agreed upon.

Further negotiations followed concerning the courses that Gödel would teach at Notre Dame. Menger suggested that he offer "a very elementary introduction to logic" three hours per week, plus "an advanced discussion either of the continuum problem or of the *Entscheidungs-problem*" for two hours once a week.[8] Gödel, however, expressed concern that he was "not very well-suited to giving an elementary course of lectures, on account of insufficient knowledge of English, insufficient experience at elementary lectures and insufficient time for preparation." He proposed instead that he "give a three-hour course of lectures on the axiomatization of set theory," including his own results on the continuum hypothesis and the axiom of choice.[9] In the end, a compromise was reached: Gödel

[7] Gödel to Menger, 3 July 1937. [8] Menger to Gödel, 20 May 1938. [9] Gödel to Menger, 25 June 1938.

would lecture on the advanced material for three hours per week, while he and Menger together would offer a joint seminar on introductory logic. Menger also hoped that Gödel would help him to turn the colloquium he had organized at Notre Dame into one like that in Vienna, but in that he was frustrated.[10]

Gödel arrived in South Bend in January of 1939, where he lodged at the Morningside Inn from 27 January until 31 May.[11] According to Menger's later recollection, he "appeared to be in fairly good health" but "not particularly happy"[12]—hardly a surprising circumstance, given both the political events then taking place in Austria and the fact that Gödel had married in September of 1938, just weeks before leaving for America, but had left his wife Adele behind in Vienna. Regrettably, Gödel himself left no extant account of his semester at Notre Dame; his letters to Adele have not been preserved, and his surviving correspondence with his mother and brother begins in March 1940, after his emigration to the United States. But it is clear that he spent a very busy and productive semester there, writing out lectures notes for his two courses (which together fill thirteen manuscript notebooks in his *Nachlaß*[13]) and preparing an outline of his consistency proof for the generalized continuum hypothesis for publication in the *Proceedings of the National Academy of Sciences*.

It seems remarkable, given that Gödel had lectured on his consistency proofs just the semester before in Princeton (his notes on which themselves occupy seven manuscript notebooks in his *Nachlaß*), that he would take the trouble to write out much of that same material again. One might suppose that he did so because the audience at Notre Dame was less sophisticated than that in Princeton. Nevertheless, he stated at the outset of the Notre Dame lectures that "this course of lectures is not intended as an introduction to set theory, as one might suspect from the title. I shall discuss only very special problems, particularly questions concerning the axiom of choice and Cantor's conjecture, about the power of the continuum, and its generalizations. As an introduction I shall give a brief survey of the theory of

[10] *Menger 1994*, pp. 220–21. One reason (as reported in *Strich 1981*, p. 26) was that whenever Menger was present at meetings of the colloquium, "Gödel would scarcely broach the matter at hand before Karl would dart in and take over." [11] As is confirmed by receipts in Gödel's *Nachlaß* (box 13b, folder 26) and by the addresses on letters sent to him by Notre Dame officials (box 13a, folder 6). Yet Thomas Strich, a 1934 Notre Dame graduate who was a graduate student at the time of Gödel's visit, later recalled Gödel "brooding in the Lyons Hall [dormitory] annex, right above my own nest" (*Strich 1981*, p. 26), and Menger (*1994*, p. 221) likewise claimed that Gödel had "lived on campus," at least "for a large part of the semester." Perhaps Gödel used the room on campus as an office during the day. [12] *Menger 1994*, p. 221. On the other hand, in an undated letter to Veblen written sometime during Gödel's residence at Notre Dame, Menger mentioned that Gödel had become "ill with a bad flu which kept him in bed for more than a week, in his room for another, and very weak for a third." (Veblen papers, Library of Congress) [13] Boxes 7c and 8a, folders 52–56 and 58–65.

transfinite cardinals and ordinals without giving the proofs for the theorems."

Despite his demurral about his lack of experience in giving elementary lectures, Gödel had also lectured once before on basic topics in logic—in a course at the University of Vienna in the summer of 1935. But his notes for that course[14] are fragmentary and in Gabelsberger shorthand, evidently prepared just for his own use. In contrast, his notes for the Notre Dame lectures are detailed and written in English, in a leisurely and quite readable style.

Gödel's Notre Dame continuum lectures were never published, and likely never will be, because of their overlap with his many other treatments of that subject.[15] On the other hand, Menger reported that Gödel's introductory lectures on logic were intended to appear as a booklet in the series *Notre Dame Mathematical Lectures*, the same series in which Artin's well-known text on Galois theory was published. That series was planned to serve as the Mathematics Department's contribution to the Notre Dame centenary celebration in 1942. But wartime exigencies, in particular, the establishment by the U.S. Navy of a large training center for naval officers at Notre Dame, diverted the energies of Notre Dame faculty, especially the mathematicians, away from other work. Consequently, no further volumes in the series appeared, and the centenary celebration never took place.[16]

Gödel's Notre Dame lectures on logic were also considered for publication in his *Collected Works*. In the end, however, they were not selected for inclusion therein.[17] Recently, however, Pierre Cassou-Nogués has studied those lectures in detail and published three brief excerpts from them.[18] He notes (p. 69) that the style of the lectures "contrasts sharply with the tone of [Gödel's] later philosophical papers," and "gives…insight into his views on logic prior to his philosophical investigations." The lectures began with a presentation of the propositional calculus, including a proof of the completeness and independence of the axioms chosen. After a brief discussion of Gentzen's sequent calculus, Gödel went on to the predicate calculus, stating (but not proving) both its completeness and the undecidability of the decision problem for validity. That was followed by a discussion of the calculus

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[14] Gödel *Nachlaß*, box 7b, folder 31. [15] Especially his 1940 monograph *The consistency of the axiom of choice and of the generalized continuum hypothesis with the axioms of set theory*, based on notes by George W. Brown on the Princeton lectures. However, the treatment in the Notre Dame lectures, like that in Gödel's 1939 *Proceedings* paper, defines the constructible sets inductively in terms of definability with ordinal parameters, not, as in the monograph, in terms of closure under eight fundamental operations on classes (a more efficient, but much less perspicuous approach). [16] *Menger 1994*, pp. 225–26. [17] See *Dawson and Dawson 2005* for a discussion of the circumstances that caused those and other items to be omitted. [18] *Cassou-Nogués 2009*.

of classes and relations and of Russell's type theory. The course ended with an analysis of the classical antinomies, including the Liar paradox.[19]

Of those who attended Gödel's courses (both offered as electives for graduate students only) little record is preserved. According to Menger, about twenty were present at the beginning of the logic course, half of whom were "mathematicians who attended the lectures to their conclusion," whereas the other half "consisted of older philosophers and logicians, occasionally joined by one or another member of the physics department." Among the former group, the name of one (Frederick P. Jenks) is known, because one of his homework assignments is preserved in Gödel's *Nachlaß*.[20] Of the latter, Menger recalled that "Professor Yves Simon, a student and friend of Maritain,...made a special effort to take advantage of Gödel's presence." Artin also attended every other week, on days that he came from Indiana University to conduct a course on algebra.[21]

It was during Gödel's residence at Notre Dame that Hitler invaded Czechoslovakia, heightening Gödel's concern about his mother, who had moved back to her villa in Brno late in 1937, and his wife, with whom he longed to be reunited. Against all advice to the contrary, therefore, he returned to Vienna at the end of the semester, where he soon found himself with no means of support, was declared fit for Nazi military service, and was once attacked on the street by a gang of young ruffians. Only through the assistance of colleagues at the Institute for Advanced Study (especially von Neumann and Institute director Frank Aydelotte) did he manage, a few months later, to surmount all the bureaucratic difficulties and secure permission to return once again (this time with Adele as well) to the United States.[22] Yet for all that, on 30 August 1939, just days before Hitler's invasion of Poland, Gödel wrote Menger merely to say that he had "had a lot of running about to do" since his return to Vienna, and to ask how the examinations for the logic course had turned out.[23]

Although Menger said that he was "immensely relieved" when Gödel arrived safely in America once more, their correspondence dropped off sharply after that, and Menger confessed that he no longer felt the same warmth toward Gödel that he once did.[24] The main reason appears to have been Menger's feeling that Gödel had failed to display concern for the plight of other intellectuals whose situations were far more desperate than his own.

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[19] For some further details about that course, see *Dawson 1997*, pp. 135–36. [20] Box 13a, folder 5.
[21] All quotations in this paragraph are from *Menger 1994*, p. 220. [22] For a detailed account of those negotiations and his emigration via the trans-Siberian railway, see chapter VII of *Dawson 1997*.
[23] A letter that Menger thought might have set "a record for unconcern on the threshold of world-shaking events." (*Menger 1994*, p. 225) [24] *Ibid*.

After his emigration Gödel rarely ventured far from Princeton, and he never taught a credit course again. From time to time Menger visited Princeton and spoke with him, and on one of those occasions Gödel revealed that he was unaware that Artin was then also living in Princeton.[25] Nor does Gödel seem to have had any contact there with Jacques Maritain, who, by a strange quirk of fate, was invited to Princeton in 1949 and ended up living in a house on the very same street as Gödel (Linden Lane). There Maritain established a Center for Thomist Studies, and on his departure from Princeton in 1960, left the home to Notre Dame, which later converted it to a center for visiting Catholic scholars.[26]

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The papers of Karl Menger are held in the manuscript collection of the Perkins Memorial Library at Duke University. Those of Oswald Veblen are in the manuscripts division of the Library of Congress. Gödel's *Nachlaß* is owned by the Institute for Advanced Study, Princeton, and is on indefinite loan to the manuscripts division of the Firestone Library at Princeton University.

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[25] *Menger 1994*, p. 226. [26] *Johnson 1983*

Skolem At Notre Dame

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During the latter years of his life (1887–1963), Skolem was several times a visiting faculty member in the Mathematics Department at Notre Dame. Specifically he held the following appointments:

1. 1957/58 academic year. Visiting Professor, Mathematics
2. 1958 Summer Session. Visiting Distinguished Lecturer in NSF Mathematics Teacher Training Institute, sponsored by the Department of Mathematics
3. 1959 Spring semester, Visiting Professor, Mathematics
4. 1959 Summer Session, Summer Session Faculty, Mathematics
5. 1960 Spring semester, Visiting Professor, Mathematics
6. 1960 Summer Session. Summer Session Faculty, Mathematics
7. 1961 Summer Session. Summer Session Faculty, Mathematics
8. 1962 Summer Session. Summer Session Faculty. Mathematics

He both taught and conducted research during these visits. The graduate seminars he taught included the following:

1. A seminar on *Abstract Set Theory* during the fall semester of 1957[1]
2. A seminar on *Foundations of Set Theory*, spring semester 1958
3. *Algebra Topics*, spring semester 1959
4. *Advanced Topics in Number Theory*, spring semester 1959
5. *Combinatorial Problems*, spring semester 1960, summer session 1960, summer session 1961 and summer session 1962[2, 3]

[1] The notes from these lectures are collected in [Sko62], available online at the *Project Euclid* website.

Among the scholarly projects he worked on were his monograph on set theory [Sko62] and the following articles, published in the *Notre Dame Journal of Formal Logic*.

i. "Investigations on a comprehension axiom without negation in the defining propositional functions," volume 1, numbers 1–2 (1960): 13–22
ii. "Proof of some theorems on recursively enumerable sets," volume 3, number 2 (1962): 65–74
iii. Addendum to: "Proof of some theorems on recursively enumerable sets," volume 4, number 1 (1963): 44–47
iv. "Studies on the axiom of comprehension," volume 4, number 3 (1963): 162–170

Skolem's work was marked by long-standing philosophical interests in the foundations of mathematics. It was therefore only natural that he should also have interacted with the Philosophy Department during his visits to Notre Dame. Among other things, he participated in the newly founded Philosophy Department Colloquium in the spring of 1960, commenting on a paper by Professor Kenneth Sayre, still an active member of the department. The paper Skolem commented on was entitled "Gasking on Arithmetical Incorrigibility" and was subsequently published as [Say62].

Skolem sketched his philosophical views in his influential early essay [Sko23], where he argued that to adequately found arithmetic (and other areas of mathematics) requires that we recast arithmetical reasoning in a way that does not depend on unbounded quantification over infinite domains.

> *If we consider the general theorems of arithmetic to be functional assertions and take the recursive mode of thought as a basis, then that science can be founded in a rigorous way without use of Russell and Whitehead's notions "always" and "sometimes"*....[I]t will often be advantageous to introduce apparent variables; but we shall require that these variables range over only finite domains, and by means of recursive definitions

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[2] Skolem gave this course different descriptions, one of which was "This course develops a number of basic ideas and techniques commonly used in attacking combinatorial problems, specifically; mappings, partitions, schubfach principle, the input-output formula, recursive relations, formal power series, and generating functions. The value of these ideas and techniques is illustrated by their use in solving a large variety of problems. The course concludes with the study of several famous problems: Graphs, Königsberg Bridge Problem, Four Color Problem, Philip Hall's Theorem and its application to the Marriage Problems. Five periods a week; three credits." This is taken from the *Bulletin of Information of Notre Dame for the Summer Session*, 1961 (June 19–Aug 3). Notre Dame Archives. [3] The summer seminars were part of a regularly recurring teachers' training seminar known as *The Notre Dame Mathematics Teacher Training Program*. This program was begun by Professor Arnold Ross in the summer of 1947. Other visitors during Skolem's years included Richard Brauer, Sarvadaman Chowla, Max Dehn, Paul Erdös, Solomon Lefschetz, Kurt Mahler and Harry Vandiver.

> we shall then always be able to avoid the use of such variables. [Sko23], 304

Skolem himself described these views as "consistently finitist" and as "built upon Kronecker's principle that a mathematical determination is a genuine determination if and only if it leads to its goal in a *finite* number of steps" (*op. cit.*, 333).[4] He retained these views to the end of his life.

But though Skolem's views were finitist in their skepticism toward the use of infinite domains of quantification, they were not in all respects the same as those of Hilbert, the foundational thinker with whose name the term 'finitist' has most often been associated. Specifically, Skolem demurred from "strict" or "extreme" formalist views ([Sko53], 28) which conceived of proofs as ultimately based on "perceptions of a sensual character" (*loc. cit.*). He attributed such views both to Hilbert and to his fellow Norwegian mathematician Axel Thue.[5]

> ...the difference between the procedure set forth here and that of the Hilbert school is that I recommend to set up formal systems...after having first seen by finitary reasoning their consistency or in other words their conservative character, whereas in Hilbert's system, we first ignore the question of interpretation, i.e. first set up a formalism without interpretation and then try afterwards to prove something called consistency. [Sko53], 33[6]

Skolem was especially critical of uses of nondenumerable infinities in mathematics. He thus praised [Lor55] for its encouragement of a skepticism concerning the absolutely nondenumerable, an attitude Skolem described as "thoroughly justified" ([Sko57], 290). He also rejected attempts (specifically Dedekind's) to found arithmetic on logical laws, arguing that the logicists' "logical intuitions are rather uncertain, whereas the arithmetical ones known as recursive or inductive reasoning are quite clear and completely safe" ([Sko55], 377). He thus maintained that "it is better and more justified not to set forth arithmetic as a part of logic, but to develop it independently" (*loc. cit.*). He even suggested that it may be possible to found *set theory* on arithmetic.[7]

Overall, Skolem sought balance and sobriety in his foundational views and encouraged others to do the same. This was nowhere more apparent than in his

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[4] This is my translation. It differs from that in [VH67], 333 chiefly by using 'determination' rather than 'definition' as a translation of the German 'Bestimmung'. 'Bestimmung' is a more general idea that seems to apply to both definitions and arguments. [5] I cannot comment on the plausibility of attributing this view to Thue. In my view, though, Hilbert's view of proof is not accurately described as being a matter of perceptual judgment. [6] Of Thue he wrote, "Thue often said…that mathematical proofs should in the last instance be of the form: 'Now I did *A* and by doing that I noticed *B*'. Of course this means that our proofs shall be perceptions of a sensual character. The strict formalist mathematics turns out to be a sort of sensualist mathematics." ([Sko53], 28) [7] "It may even be possible to…base set theory on arithmetic." ([Sko55], 377)

Cambridge lecture of 1950 where he summed up his foundational attitude as follows:

> ...the fear that mathematics will be crippled by the restriction to the use of only free variables is exaggerated....it may look different to mathematicians accustomed to analysis...and those only working in the theory of numbers, but there are certainly many more ways of treating mathematics than we know today.
>
> I am no fanatic, and it is not my intention to condemn the nonfinitistic ideas and methods. But I should like to emphasize that the finitistic development as far as it may be carried out has a very great advantage with regard to clearness and security. Further there may be good reason to conjecture that it can be carried out very far, if one would make serious attempts in that direction. [Sko50], 526–27[8]

Acknowledgement: I'm grateful to the staff of the University of Notre Dame Archives for their help in locating pertinent archival materials concerning Skolem.

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[8] For two recent, interesting discussions of Skolem's foundational ideas, see [Wan96] and [Coh05].

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