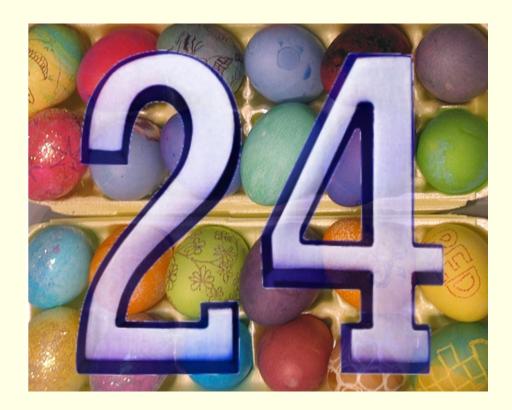# The Rankin Lectures 2008

The numbers 12 and 24 play a central role in mathematics thanks to a series of 'coincidences' that is just beginning to be understood. One of the first hints of this fact was Euler's bizarre 'proof' that

$$1 + 2 + 3 + 4 + \cdots = -1/12$$

which he obtained before Abel declared that 'divergent series are the invention of the devil'.

Euler's formula can now be understood rigorously in terms of the Riemann zeta function, and in physics it explains why bosonic strings work best in $26 = 24 + 2$ dimensions. The fact that

$$1^2 + 2^2 + 3^2 + \cdots + 24^2$$

is a perfect square then sets up a curious link between string theory, the Leech lattice (the densest known way of packing spheres in 24 dimensions) and a group called the Monster. A better-known but closely related fact is the period-12 phenomenon in the theory of 'modular forms'. We shall do our best to demystify some of these deep mysteries.

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