The Rankin Lectures 2008

Different numbers have different personalities. The number 5 is quirky and intriguing, thanks in large part to its relation with the golden ratio, the 'most irrational' of irrational numbers. The plane cannot be tiled with regular pentagons, but there exist quasiperiodic planar patterns with pentagonal symmetry of a statistical nature, first discovered by Islamic artists in the 1600s, later rediscovered by the mathematician Roger Penrose in the 1970s, and found in nature in 1984.

The Greek fascination with the golden ratio is probably tied to the dodecahedron. Much later, the symmetry group of the dodecahedron was found to give rise to a 4-dimensional regular polytope, the 120-cell, which in turn gives rise to the Poincaré homology sphere and the root system of the exceptional Lie group E_8. So, a wealth of exceptional objects arise from the quirky nature of 5-fold symmetry.

For a general audience

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