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22 September 1975

A1466 A6524 A6526

Dear Neil:

One troub@1/ with waiting so long to answer your letters is that I have that much more to write when I finally get down to the job.

About 3 AM many month ago, I was wakeful for no particular reason, and to while away the time I dreamed up a sequence to represent a decimal fraction by means of reciprocals of integers. So I calculated the values for pi and found that they were your Seq. 1935. (The reciprocals of the terms add up to the decimal part of pi.) I added one more term to what you had, and estimated the next one. The corresponding sequences for pi-1, e and e-1 are not in your book, so here they are:

π-3: 8, 61, 5020, 128541455, 162924332716605980, 2.878...x10³⁴ (Seq 1935)

new A652K π^{-1} : 4, 15, 609, 845029, 1010073215739, 1.300...x10²⁴

e-2: 2, 5, 55, 9999, 3620211523, 25838201785967533906, 3.408...x10³⁹ (525

 e^{-1} ; 3, 29, 15786, 513429610, 339840390654894740, 3.835...x10³⁵.

Recently I called Fred Gruenberger's attention to the paper on iterated square roots, $\sqrt{a_1 + \sqrt{a_2 + \dots}}$, in AMM 42 (1935), about page 419. He was excited and mystified and asked me to make some calculations. I pointed out that any number can be represented by an infinite number of different sequences a1 a2 ... under the radicals. Some of those generating π , 2, and the integers 1 to 10 are included with this letter.

Elinor Potter asked me to mention to you that Seq. 625 also gives the number of different buzzer calls that can be made with dots and dashes:

total 2

total 6

using 3 dots and/or dashes total 14 etc.

The analogous sequence 23k does not appear in your book. I wouldn't like to have to respond to buzzer calls using short and long dashes and dots.

I've been busy accumulating additional constants, with perhaps 12000 on hand now. This should increase by a couple thousand as the result of a new undertaking: I'm converting most of the sequences in Sloane to decimal numbers on the assuption that the sequences represent the partial quotients of continued fractions. Who is to say that Seq. 42, for example, may not have significance as a continued fraction? A sample is enclosed. It is tedious to enter the numbers, but the Wang does everything else, including comparing two independent entries of the Seid

1.37125 08223 84353 78386 56318 48092 36851 54778 67375

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Continued fraction - N.J.A. Sloane Seq. 42

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ous sequence TT for not appear in your back. I can be a very better an iong best very to the control of the con

sequence, and deciding how many decimals to print. This naturally will reduce errors substantially (and already has). Maybe I can get my older daughter, Anne, to do some of it. It will be interesting to compare these with my current file. Something unexpected may turn up. Do you have any suggestions, pro or con, about this project? I have one: Perhaps you could have your computer put out a similar calculation. It wouldn't have to be as bulky as your book because only the sequence number is needed to identify the constant. The other references could be omitted. I am omitting all sequences that contain a zero. Most of the sequences are good for 45D. When and if I publish a new table of constants, I hope to have them to 30D. Most of my file is calculated to 45D, with an occasional rounding error almost a certainty.

Since some of the sequences have a spurious 1 in front, a tabulation to say, 10 or 15D of $(N-1)^{-1}$ would get around this problem, where N is the number obtained directly from the sequence. Similarly, some sequences really begin 1, 1, ... and these can be recovered by calculating $1+N^{-1}$. Finally, we can just forget about those that are actually 1, 1, 1, ..., etc. All this information probably would go on a single line.

In Martin Gardner's book "New Mathematical Diversions from Scientific American" 1966, page 246, there are nine sequences listed. Only the first two of those appear in your book. I had planned to print out the others for you, but the Wang gave out after I started this letter.

Shanks and La1 in Math Comp $\underline{26}$ 271 (1972) give the sequence 1, 2, 4, 10, 24, 60, 156, 410, 1092.

In your letter of 26 January 1974 you questioned the accuracy of the continued fraction expansions that I sent you in my letter of 13 November 1973. You are right that they are in error, except for the 5th root of 6, and I have no idea why that is OK. I'm very sorry. In making these calculations I made a customary modification in my basic mutiple precision program, apparently failed to check it out, with the result that the terms are equivalent to about a 35D value instead of 44D or better. In any event, I've calculated the results anew, and added some others. My procedure is to get the partial quotients for numbers slightly smaller and slightly larger than the correct value, and retain only those partial quotients that agree. Some of these you have in your Supplement I, and they check as far as I carried them.

Have you brought out any further supplements? Have any errors in sequences been found?

I had some other sequences for you that were made early this year, but I've misplaced them. (I do this with nearly everything I own). When and if they are found I'll send them along.

Lois gave up her computer science studies at UC and got married last month. I can't remember whether you met her fiance Bob Elling.

In July Lois, Jim and I spent 2 weeks in Florida and some nearby states. We went mainly to see the blast-off of the Apollo rocket which later joined up with the Russian Soyuz. We got some good pictures, even from 11 miles away.

Jim is studying fish culture and enjoying it.

Sincerely,

Herman

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request of and deciding how many decimals to rise. Into materally will reduce errors substantially (and hiradly test). While I can get not also daughter. And, the does not seld of the little of the configuration of the sequence and the constant. The other references could be and the constant. The other references could be entired. I as on the constant of the sequence and of the sequences are conficulting all sequences that contains a configuration of the sequences are good for only. When and if I publish a new table of the sequences are good for only. When and if I publish a new table of constants. I hope to have then to 100, when and if I publish a new table of constants. I hope to have them to 100, when and if I publish a new table of constants. I hope to counding error allost a certificial.

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heis tave up her computer actionce studies at UC and got married last conti. I can't removaler wheeler you met her flate Bob Ellica.

In July Lois, Jim and I speak 2 weeks in 'Lorian and Jones author states. 'A work, probly to age the blast of the talk trolle restrict with the Wassian Soyus. We set come good alchers, even

in the dying fish culture and snjoying it

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