Scan

A7380
etc

Yanco & Bagchi
Apparently unpublished

Scan
add to A7380 &
many more

[YaBa94]

A7380
A11973
A45
A931
A1687
A7380
A7381
A160[illegible]8
A7387
A7388
A7389

K-TH ORDER MAXIMAL INDEPENDENT SETS IN PATH AND CYCLE GRAPHS

Richard Yanco
Department of Mathematics
Amherst College
Amherst, MA 01002-5000

Ansuman Bagchi
Department of Mathematical Sciences
Worcester Polytechnic Institute
Worcester, MA 01609

Abstract

A maximal independent set of a graph $\mathcal{G}$ is an independent set that is not contained properly in any other independent set of $\mathcal{G}$. In this paper we generalize maximality, apply the generalization to path and cycle graphs, and obtain closed-form equations for the number of k-th order maximal independent sets, with a given cardinality, in path and cycle graphs.

1 Introduction

Consider a simple graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $|\mathcal{V}| = n$. Let $\mathcal{S}$ be a set of vertices, $\mathcal{S} \subseteq \mathcal{V}$, where $|\mathcal{S}| = m$. We think of vertices in $\mathcal{S}$ as "colored," and those not in $\mathcal{S}$ as "uncolored." Thus $\mathcal{S}$ describes a two-coloring of $\mathcal{V}$.

$\mathcal{S}$ is an *independent set* of vertices iff no two vertices in $\mathcal{S}$ share an edge in $\mathcal{E}$; i.e., iff no two colored vertices are adjacent.

$\mathcal{S}$ is a *maximal independent set (m.i.s.)* of vertices iff no vertex in $\mathcal{V} - \mathcal{S}$ can be added to $\mathcal{S}$ without violating independence.

$\mathcal{S}$ is a *maximum independent set* of vertices iff it is a m.i.s. of greatest possible cardinality.

The *Hamming distance* between two subsets $\mathcal{S}$ and $\mathcal{T}$ is defined as

$$d_H(\mathcal{S}, \mathcal{T}) = |\mathcal{S} - \mathcal{T}| + |\mathcal{T} - \mathcal{S}|.$$

Any subset $\mathcal{T}$ for which $d_H(\mathcal{S}, \mathcal{T}) \leq k$ is called a *k-neighbor* of $\mathcal{S}$. All k-neighbors comprise the *k-neighborhood* of $\mathcal{S}$. Note that to transform $\mathcal{S}$ into any $\mathcal{T}$ at Hamming distance k, we must reverse the coloring of exactly k vertices. We define such a reversal as a *flip* of k vertices.

Combining results of Hammer and Rudeanu [1] and Bagchi and Williams [2], we define a *k-th order m.i.s.* as an independent set of larger cardinality than any independent set in its k-neighborhood. (See generally [3].) A first-order m.i.s. is simply an m.i.s., as previously defined: removing one vertex decreases cardinality, while adding one vertex violates independence. Note that a maximum independent set is a k-th order m.i.s., for all k.

2 A General Result

Theorem 2.1 *For all $k \geq 1$, every $(2k-1)$-th order m.i.s. is also a $(2k)$-th order m.i.s.*

Proof. Assume that the vertex set $\mathcal{S}$ is a $(2k-1)$-th order m.i.s. We wish to show that it is also a $2k$-th order m.i.s. We merely need to show that no vertex set $\mathcal{T}$, at Hamming distance $2k$ from $\mathcal{S}$, is an independent set of greater cardinality than $\mathcal{S}$. (If such a $\mathcal{T}$ existed at Hamming distance *less* than $2k$, we would violate our assumption.)

We must by definition flip exactly $2k$ vertices in $\mathcal{S}$ to create any $\mathcal{T}$ at Hamming distance $2k$. Since $|\mathcal{S}-\mathcal{T}|+|\mathcal{T}-\mathcal{S}| = 2k$, the number of vertices removed from $\mathcal{S}$ determines the number of vertices added to $\mathcal{S}$ (to form $\mathcal{T}$).

CASE 1: Greater than k vertices removed from $\mathcal{S}$. If $(k+a)$ vertices are removed from $\mathcal{S}$, $0 < a \leq k$, then $(k-a)$ vertices are added. This results in a set $\mathcal{T}$ of lesser cardinality than $\mathcal{S}$.

CASE 2: k or fewer vertices removed from $\mathcal{S}$. If $(k-a)$ vertices are removed from $\mathcal{S}$, $0 \leq a \leq k$, then $(k+a)$ vertices are added. This results in a net gain of $2a$ vertices. If $a = 0$, there is no net gain of vertices. If $a > 0$ and the resulting set is independent, there is a net gain of at least two vertices. But in that case we could choose to *not* add one of the gained vertices, resulting in a independent set $\mathcal{T}$ at Hamming distance $(2k-1)$ from $\mathcal{S}$ with at least one extra vertex, violating our assumption.

Thus every $(2k-1)$-order m.i.s. is also a $(2k)$-order m.i.s. □

Henceforth all orders k referred to in this paper are assumed odd.

For the rest of the paper, we will focus on path graphs of n vertices, P_n, and cycle graphs of n vertices, C_n.

3 Path and Cycle Graphs

3.1 Preliminaries

We define a *breakpoint* in a path or cycle graph to be any edge between two consecutive uncolored vertices. In a path graph, we imagine two additional edges beyond the end vertices, each of which is a breakpoint iff its corresponding end vertex is uncolored.

For convenience, we refer to the *distance between two breakpoints* as the number of vertices between the breakpoints. In a cycle graph, the distance between two breakpoints is the smaller number of vertices between them, and the distance between a breakpoint and itself may be regarded as infinite.

We say a set $\mathcal{S}$ *contains* a breakpoint iff $\mathcal{S}$ contains two adjacent uncolored vertices.

Proposition 3.1 *Given a m.i.s. $\mathcal{S}$ with two or more breakpoints, we may flip a consecutive string of vertices, bounded by any two breakpoints, to obtain another m.i.s. $\mathcal{T}$, where $|\mathcal{T}| = |\mathcal{S}| + 1$.*

Proof. We must show that to increase the cardinality of a m.i.s. $\mathcal{S}$ by one while maintaining independence, we may flip a set of vertices (1) which are consecutive, (2) which are bounded by two breakpoints, and (3) which, when flipped, will result in a net gain of one vertex.

(1) Suppose we increased the cardinality of a m.i.s. by flipping two or more disjoint sets of consecutive vertices, $\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_n$. Because we have increased cardinality by at least one, flipping $\mathcal{F}_k$ by itself, for some k, must have increased cardinality by at least one. Since we only *want* to gain one vertex, flipping one set of consecutive vertices is sufficient. Call it $\mathcal{F}$.

(2) Obviously $\mathcal{F}$ can contain no breakpoint: once flipped, the two consecutive uncolored vertices would become two colored vertices and violate independence. So $\mathcal{F}$ must contain alternating colored and uncolored vertices. Note that $\mathcal{F}$ must begin and end with an uncolored vertex, else flipping $\mathcal{F}$ in $\mathcal{S}$ results in no net gain of vertices. (The number of colored and uncolored vertices can differ by at most one; there must be an additional uncolored vertex if we are to gain a colored vertex after the flip.)

The end vertices of $\mathcal{F}$ are uncolored. If an uncolored end vertex of $\mathcal{F}$ is at the end of the path itself, it is by definition adjacent to a breakpoint. If the uncolored end vertex is in the middle of the path, the *adjacent* vertex which is not in $\mathcal{F}$ must be uncolored, else the flip would violate independence. Again, the uncolored vertex is adjacent to a breakpoint.

(3) Because we are flipping a string of vertices of alternating color, beginning and ending with an uncolored vertex, we are clearly flipping an odd number of vertices, $(2k+1)$, with $(k+1)$ uncolored and k colored. After the flip, we have $(k+1)$ *colored* vertices and k uncolored, a net gain of one. □

Proposition 3.2 *For any m.i.s. $\mathcal{S}$ in a path or cycle graph, let k_0 be the smallest distance between any two breakpoints (if two breakpoints do not exist, let k_0 be infinite). Then $\mathcal{S}$ is a k-th order m.i.s. iff $k_0 \geq k+2$.*

Proof. ($\Rightarrow$) Let $\mathcal{S}$ be a k-th order m.i.s. Because two consecutive uncolored vertices at the end of a path, or three consecutive uncolored vertices in the middle of a path or cycle clearly violate maximality, we are assured that $k_0 > 1$. Specifically, if $k = 1$, there are at least three vertices (uncolored, colored, and uncolored, respectively) between any two breakpoints. For $k > 1$, there is an alternating series of colored and uncolored vertices, beginning and ending with an uncolored vertex, between any two breakpoints (i.e., k_0 is odd).

Suppose $k_0 < k+2$, i.e., $k_0 \leq k$. (Remember that k and k_0 are both odd.) By Proposition 3.1 we can flip all k_0 vertices between the two closest breakpoints, to form an independent set with one additional vertex. This violates our assumption of k-th order maximality. Thus $k_0 \geq k+2$.

($\Leftarrow$) Let $k_0 \geq k+2$. We need to find a string of k or fewer vertices which begins and ends with a breakpoint. But by the defintion of k_0 such a string does not exist, so $\mathcal{S}$ is a k-th order maximal independent set.

Thus $\mathcal{S}$ is a k-th order m.i.s. iff $k_0 \geq k+2$. □

3.2 Path Graphs

We denote the set of k-th order m.i.s.'s of P_n as $XP_{k,n}$. We denote the set of k-th order m.i.s.'s of P_n *which contain m colored vertices* as $XP_{k,n,m}$.

To *add an uncolored vertex* to a graph, we add a vertex to the vertex set $\mathcal{V}$ but not to the subset $\mathcal{S}$. To *add a colored vertex* to a graph, we add a vertex to both $\mathcal{V}$ and $\mathcal{S}$. (Removing colored and uncolored vertices is done in the opposite manner.)

Lemma 3.3 *For $n \geq 2m-1$, each element of $XP_{k,n,m}$ contains exactly $n-2m+1$ breakpoints.*

Proof. We assume that there is at least one element in $XP_{k,n,m}$, else the lemma is irrelevant. We remove uncolored vertices from the path until we are left with the lone element of $XP_{k,2m-1,m}$, which begins and ends with a colored vertex, and alternates colorings within. (It has no breakpoints.)

To that coloring, we must add $n-(2m-1) = n-2m+1$ uncolored vertices to obtain any element of $XP_{k,n,m}$. Note that each uncolored vertex we add to the graph creates a breakpoint. □

Lemma 3.4 $|XP_{k,n,m}| = \left|XP_{1,n-(k-1)(n-2m),m-\frac{1}{2}(k-1)(n-2m)}\right|$.

Proof. First note that every element of both sets contains the same number of breakpoints, since $[n-(k-1)(n-2m)] - 2[m-\frac{1}{2}(k-1)(n-2m)] + 1 = n-(k-1)(n-2m) - 2m + (k-1)(n-2m) + 1 = n-2m+1$.

There are, then, $n-2m+1$ breakpoints in every element of either set, and $n-2m$ pairs of consecutive breakpoints. Between every such pair, to satisfy k-th order maximality there must be at least $k+2$ vertices; to satisfy first-order maximality, there must be at least 3 vertices.

There is a one-to-one correspondence between elements of $|XP_{k,n,m}|$ and $\left|XP_{1,n-(k-1)(n-2m),m-\frac{1}{2}(k-1)(n-2m)}\right|$:

(i) to each element of the former, we can add $(k+2)-3 = k-1$ alternating vertices between each pair of breakpoints to create a coloring satisfying k-th order maximality – a total of $(k-1)(n-2m)$ vertices are added, half of them colored;

(ii) to each element of the latter, we can remove $k-1$ alternating vertices from between each pair of breakpoints to create a coloring satisfying first-order maximality – a total of $(k-1)(n-2m)$ vertices are removed, half of them colored. □

Lemma 3.5 $|XP_{1,n,m}| = \binom{m+1}{n-2m+1}$.

Proof. As in Lemma 3.3, consider the lone element of $XP_{k,2m-1,m}$. To this we must add $n-2m+1$ breakpoints. There are $(m+1)$ places to add a vertex: between each of the $(m-1)$ pairs of colored vertices, or at either end. □

Theorem 3.6 $|XP_{k,n,m}| = \binom{km-(\frac{k-1}{2})n+1}{n-2m+1}$.

Proof. Follows immediately from Lemmas 3.4 and 3.5. □

Corollary 3.6.1 *The number of vertices m in a k-th order m.i.s. of a path graph P_n satisfies the inequality*

$$\frac{n(k+1)}{2k+4} \leq m \leq \frac{n+1}{2}$$

or, equivalently,

$$2m-1 \leq n \leq \frac{m(2k+4)}{k+1}.$$

Proof. By Theorem 3.6, $|XP_{k,n,m}| > 0$ iff $km-(\frac{k-1}{2})n+1 \geq n-2m+1 \geq 0$. □

Corollary 3.6.2 *If $n > k+2$ and $m > \frac{k+1}{2}$,*

$$\begin{array}{llcccc} \text{(a)} & |XP_{k,n,m}| & = & |XP_{k,n-2,m-1}| & + & |XP_{k,n-k-2,m-(\frac{k+1}{2})}|; \\ \text{(b)} & |XP_{k,n}| & = & |XP_{k,n-2}| & + & |XP_{k,n-k-2}|. \end{array}$$

Proof. **(a)** Follows immediately from Theorem 3.6.
(b) Follows immediately from (a). □

Observation 3.6.3 *Given any k, a, and b, there exist n and m such that*

$$|XP_{k,n,m}| = \binom{a}{b}.$$

Proof. Simply let $n = k(b-1)+2(a-1)$ and $m = (a-1)+\frac{(k-1)(b-1)}{2}$. (Note that $(k-1)$ is even, so $\frac{(k-1)(b-1)}{2}$ is an integer.) Then

$$\begin{aligned} a &= km - \left(\frac{k-1}{2}\right)n + 1 \\ b &= n - 2m + 1. \end{aligned}$$

□

3.3 Cycle Graphs

We denote the set of k-th order m.i.s.'s of C_n as $XC_{k,n}$. We denote the set of k-th order m.i.s.'s of C_n *which contain* m *colored vertices* as $XC_{k,n,m}$.

Theorem 3.7 **(a)** *If* $n = 2m$, $|XC_{k,n,m}| = 2$.
(b) *If* $n \neq 2m$, $|XC_{k,n,m}| = \frac{n}{n-2m}\binom{km-(\frac{k-1}{2})n-1}{n-2m-1}$.

Proof. **(a)** We can color every other vertex, and can flip the entire cycle to get a second valid coloring. Thus, for any k, there are two m.i.s.'s.

(b) Each member of $XC_{k,n,m}$ is a m.i.s. Between any two successive colored vertices, there must be at least one uncolored vertex but not more than two. Hence the gaps between colored vertices may be regarded as "singles" and "doubles."

The number of uncolored vertices in the m.i.s. is $n - m$; since each single contains one uncolored vertex, and each double two, we have $s + 2d = n - m$. Because the cycle has m gaps between colored vertices, and each gap is a single or a double, we have $s + d = m$.

This gives $s = 3m - n$ and $d = n - 2m$. (Note that each element of $XC_{k,n,m}$ has the same number of singles and doubles.) Since $d \geq 0$, $n \geq 2m$. By assumption, $n \neq 2m$, so $n > 2m$ and $d > 0$. Thus there is at least one double.

Label one vertex the "first" vertex in the cycle, and consider the set

$$\widetilde{XC}_{k,n,m} = \{XC_{k,n,m} | \text{the coloring begins with a double}\}.$$

By Proposition 3.2, we know that there must be at least $k + 2$ vertices between any two breakpoints; hence there are at least k vertices, beginning and ending with a colored vertex, between any pair of successive doubles. Since k is odd, there are at least $(k - 1)/2 = r$ singles between each pair of successive doubles. This implies $s \geq rd$: we must use rd singles to satisfy k-th order maximality.

We have complete freedom in placing the extra $s - rd$ singles. Since we are partitioning the singles into d groups, we have $\binom{(s-rd)+d-1}{d-1} = |\widetilde{XC}_{k,n,m}|$ possible partitions.

We wish to count *all* elements in $XC_{k,n,m}$. A naïve guess might be that $|XC_{k,n,m}| = n|\widetilde{XC}_{k,n,m}|$, because the "first" vertex is arbitrary. But if any element in $\widetilde{XC}_{k,n,m}$ had rotational symmetry, this would overcount.

Choose an arbitrary element in $\widetilde{XC}_{k,n,m}$. Suppose that a rotation of n/λ vertices, for some positive divisor λ of n, produces the same element. Let λ_0 be the largest such λ. (If the element does not have rotational symmetry, $\lambda_0 = 1$.)

The chosen element produces n/λ_0 distinct elements in $XC_{k,n,m}$. But notice that d/λ_0 of those elements are in $\widetilde{XC}_{k,n,m}$, and each of them would produce the same n/λ_0 elements in $XC_{k,n,m}$.

In this fashion, we can partition $XC_{k,n,m}$ into subsets containing n/λ_0 elements, and $\widetilde{XC}_{k,n,m}$ into subsets containing d/λ_0 elements; these subsets are in 1-1 correspondence. Thus

$$|XC_{k,n,m}| = \frac{n}{d}\left|\widetilde{XC}_{k,n,m}\right| = \frac{n}{d}\binom{(s-rd)+d-1}{d-1}.$$

Since $s = 3m - n$, $d = n - 2m$, and $r = (k-1)/2$, we have

$$|XC_{k,n,m}| = \frac{n}{n-2m}\binom{km - (\frac{k-1}{2})n - 1}{n-2m-1}.$$

□

Corollary 3.7.1 *The number of vertices m in a k-th order m.i.s. of a cycle graph C_n satisfies the inequality*

$$\frac{n(k+1)}{2k+4} \leq m \leq \frac{n}{2}$$

or, equivalently,

$$2m \leq n \leq \frac{m(2k+4)}{k+1}.$$

Proof. By Theorem 3.7(b), if $n \neq 2m$, $|XC_{k,n,m}| > 0$ iff $km - (\frac{k-1}{2})n - 1 \geq n - 2m - 1 \geq 0$, giving $\frac{n(k+1)}{2k+4} \leq m \leq \frac{n-1}{2}$. By Theorem 3.7(a), $n = 2m$ also implies $|XC_{k,n,m}| > 0$. Hence $|XC_{k,n,m}| > 0$ iff $\frac{n(k+1)}{2k+4} \leq m \leq \frac{n}{2}$. □

The formulae from parts (a) and (b) of Theorem 3.7 can be combined as follows:

Corollary 3.7.2 $|XC_{k,n,m}| = \frac{(n)[km-(\frac{k-1}{2})n-1]!}{(n-2m)![(k+2)m-(\frac{k+1}{2})n]!}$. □

Corollary 3.7.3 *If $n > 2k+2$ and $m > k$,*

(a) $|XC_{k,n,m}| = |XC_{k,n-2,m-1}| + |XC_{k,n-k-2,m-(\frac{k+1}{2})}|$;
(b) $|XC_{k,n}| = |XC_{k,n-2}| + |XC_{k,n-k-2}|$.

Proof. (a) Follows immediately from Corollary 3.7.2.
(b) Follows immediately from (a).

□

3.4 A Path and Cycle Graph Relation

Theorem 3.8 *For* $n > 11$, $|XC_{1,n}| = |XP_{1,n}| + |XP_{1,n-11}|$.

Proof. By induction. Repeated application of Theorems 3.6 and 3.7 implies

$$|XC_{1,12}| - |XP_{1,12}| = 29 - 28 = 1 = |XP_{1,1}|$$
$$|XC_{1,13}| - |XP_{1,13}| = 39 - 37 = 2 = |XP_{1,2}|$$
$$|XC_{1,14}| - |XP_{1,14}| = 51 - 49 = 2 = |XP_{1,3}|$$

Assume the relation holds for all $n' < n$. By Corollaries 3.6.2 and 3.7.3,

$$\begin{aligned}
|XC_{1,n}| - |XP_{1,n}| &= |XC_{1,n-2}| - |XP_{1,n-2}| + |XC_{1,n-3}| - |XP_{1,n-3}| \\
&= |XP_{1,n-13}| + |XP_{1,n-14}| \\
&= |XP_{1,n-11}|.
\end{aligned}$$

□

It is not known whether Theorem 3.8 generalizes to higher k.

4 Generalized Fibonacci Sequences

Green [4] considered the family of sequences

$$T_n^{a,b} = \sum_{\substack{ax+by=n \\ x\geq 0, y\geq 0}} \binom{x+y}{y},$$

where a and b are relatively prime. Note, for instance, that $T_n^{1,1} = 2^n$ and $T_n^{2,1} = F_n$.

Lemma 4.1 *All integer solutions of* $2x + (k+2)y = n + k + 2$ *are given by*

$$\begin{aligned}
x &= -\left(\frac{k+1}{2}\right) n + (k+2)m \\
y &= n - 2m + 1
\end{aligned}$$

Proof. Substituting, we see that x and y solve $2x + (k+2)y = n + k + 2$, for all $m \in \mathbf{Z}$. We want to show that x and y give *all* such solutions.

Let x, y be *any* solutions of $2x + (k+2)y = n + k + 2$; and set $m = 0$, above, to define

$$\begin{aligned}
x_0 &= -\left(\frac{k+1}{2}\right) n \\
y_0 &= n + 1
\end{aligned}$$

We thus have $2x + (k+2)y = n + k + 2 = 2x_0 + (k+2)y_0$. This implies that $(k+2)(y - y_0) = -2(x - x_0)$. Since k is odd, 2 and $(k+2)$ are relatively prime. Therefore $x - x_0 = m(k+2)$ and $y - y_0 = -2m$, for some $m \in \mathbf{Z}$. Hence $x = x_0 + (k+2)m = -(\frac{k+1}{2})n + (k+2)m$ and $y = y_0 - 2m = n - 2m + 1$. $\square$

Theorem 4.2 $|XP_{k,n}| = T^{2,k+2}_{n+k+2}$.

Proof. Follows from Lemma 4.1 and Theorem 3.7. $\square$

Corollary 4.2.1 $|XP_{-1,n}| = T^{2,1}_{n+1} = F_{n+1}$. $\square$

Note that Corollary 4.2.1 is not surprising. Proposition 3.2 implicitly assumed $k \geq 1$, but if $k = -1$ then that same proposition implies $k_0 \geq 1$. That is, for path and cycle graphs, $k = -1$ corresponds to the number of independent sets, *disregarding maximality*. It is well known (and easy to prove) that the number of independent sets in a path graph follows the Fibonacci sequence.

5 Generalized Lucas and Perrin Sequences

The number of independent sets in C_n is known to follow the Lucas sequence

$$L(1) = 1,\ L(2) = 3,\ L(n) = L(n-2) + L(n-1).$$

Because our formula for $|XC_{-1,n}|$ also gives the number of independent sets in C_n, we note the interesting identity

$$L(n) = \sum_{0 \leq m \leq \frac{n+1}{2}} \frac{n(n-m-1)!}{(n-2m)!m!}.$$

$\square$

For prime n, all terms in this sum are divisible by n, except that for $m = 0$. Hence the known identity $L(p) \equiv 1 \pmod p$ follows immediately.

Perrin [5] defined the sequence

$$P(1) = 0,\ P(2) = 2,\ P(3) = 3,\ P(n) = P(n-3) + P(n-2)$$

and noted that n divides $P(n)$ for all prime n. Perrin's sequence corresponds exactly to the sequence $|XC_{1,n}|$. We thus gain a second interesting identity:

$$P(n) = \sum_{\frac{n}{3} \leq m \leq \frac{n+1}{2}} \frac{n(m-n-1)!}{(n-2m)!(3m-n)!}.$$

$\square$

It is clear from Theorem 3.7 that for *all* $k \geq 1$ and all prime p, p divides $|XC_{k,p}|$. This suggests a natural generalization of the Perrin sequence. Hence we define $P_k(n) = |XC_{k,n}|$, where k is odd. Note that $L(n) = P_{-1}(n)$ for all n.

An obvious question is whether there exist *composite* n such that $n|P_k(n)$. The first such n for $P_1(n)$ is $271441 = 521^2$, discovered by Shanks and Adams [6] in 1982. The next three are 904631, 16532714, and 24658561[7].

Experimentation reveals that for $k \geq 3$, such composite n are quite common; but nearly all have a prime factor $\leq k$. Hence we define a *k-pseudoprime* to be a composite n such that $n|P_k(n)$ and such that all prime factors of n are greater than k.

Further experimentation suggests that k-pseudoprimes are scarce: over 1.37 million n, k pairs have been checked, revealing one 7-pseudoprime ($3481 = 59^2$); one 13-pseudoprime ($18769 = 137^2$); two 15-pseudoprimes ($17161 = 131^2$ and $22801 = 151^2$) and one 19-pseudoprime ($32041 = 179^2$). No pseudoprimes have been located for $k \in \{3, 5, 9, 11, 17\}$.

6 Acknowledgement

This work was supported in part by the National Science Foundation under grant number DMS-9200512. The authors thank David Cox of Amherst College for many useful conversations.

References

[1] P.L. Hammer and S. Rudeanu, *Boolean Methods in Operations Research and Related Areas*, Springer-Verlag, New York (1968) 209-228.

[2] A. Bagchi and A.C. Williams, k-order local optima and recognition problems for certain classes of pseudo-Boolean functions. Technical Report, RUTCOR, Rutgers University (1990).

[3] R.J. Yanco and A. Bagchi, k-th order maxima and independent sets. *Graph Theory Notes of New York*, **XXVI** (1994) 32-37.

[4] T.M. Green, Recurrent sequences and Pascal's triangle, *Math. Mag.* **41** (1968) 13-21.

[5] R. Perrin, Item 1484, *L'Intermédiare des Math.* **6** (1899) 76-77.

[6] W. Adams and D. Shanks, Strong primality tests that are not sufficient, *Math. Comp.* **39** (1982) 255-300.

[7] K. Brown, Perrin's triangle, Usenet posting on `sci.math`, March 1994.

A11973
A45
A931
A1687
A7380
A7381
A1608
A7387
A7388
A7389

$|XP_{-1,n,m}|$

Δ A11973

A-I

Path Length	Number of Colored Vertices												$\lvert XP_{-1,n}\rvert$
	0	1	2	3	4	5	6	7	8	9	10	11	
1	1	1	-	-	-	-	-	-	-	-	-	-	2
2	1	2	-	-	-	-	-	-	-	-	-	-	3
3	1	3	1	-	-	-	-	-	-	-	-	-	5
4	1	4	3	-	-	-	-	-	-	-	-	-	8
5	1	5	6	1	-	-	-	-	-	-	-	-	13
6	1	6	10	4	-	-	-	-	-	-	-	-	21
7	1	7	15	10	1	-	-	-	-	-	-	-	(e.g., 34 = 1+7+15+10+1)
8	1	8	21	20	5	-	-	-	-	-	-	-	55
9	1	9	28	35	15	1	-	-	-	-	-	-	89
10	1	10	36	56	35	6	-	-	-	-	-	-	144
11	1	11	45	84	70	21	1	-	-	-	-	-	233
12	1	12	55	120	126	56	7	-	-	-	-	-	377
13	1	13	66	165	210	126	28	1	-	-	-	-	610
14	1	14	78	220	330	252	84	8	-	-	-	-	987
15	1	15	91	286	495	462	210	36	1	-	-	-	1597
16	1	16	105	364	715	792	462	120	9	-	-	-	2584
17	1	17	120	455	1001	1287	792	330	45	1	-	-	4181
18	1	18	136	560	1365	2002	1287	792	165	10	-	-	6765
19	1	19	153	680	1820	3003	2002	1716	495	55	1	-	10946
20	1	20	171	816	2380	4368	3003	3432	1287	220	11	-	17711
21	1	21	190	969	3060	6188	5005	6435	3003	715	66	1	28657
22	1	22	210	1140	3876	8568	8008	11440	6435	2002	286	12	46368

↑ A45

A0931
Padovan seq
A-II

$\left|XP_{1,n,m}\right|$

Path Length	Number of Colored Vertices 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	$\left\|XP_{1,n}\right\|$
1	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1
2	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2
3	1	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2
4	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3
5	-	3	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4
6	-	1	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5
7	-	-	6	1	-	-	-	-	-	-	-	-	-	-	-	-	-	7
8	-	-	4	5	-	-	-	-	-	-	-	-	-	-	-	-	-	9
9	-	-	1	10	1	-	-	-	-	-	-	-	-	-	-	-	-	12
10	-	-	-	10	6	-	-	-	-	-	-	-	-	-	-	-	-	16
11	-	-	-	5	15	1	-	-	-	-	-	-	-	-	-	-	-	21
12	-	-	-	1	20	7	-	-	-	-	-	-	-	-	-	-	-	28
13	-	-	-	-	15	21	1	-	-	-	-	-	-	-	-	-	-	37
14	-	-	-	-	6	35	8	-	-	-	-	-	-	-	-	-	-	49
15	-	-	-	-	1	35	28	1	-	-	-	-	-	-	-	-	-	65
16	-	-	-	-	-	21	56	9	-	-	-	-	-	-	-	-	-	86
17	-	-	-	-	-	7	70	36	1	-	-	-	-	-	-	-	-	114
18	-	-	-	-	-	1	56	84	10	-	-	-	-	-	-	-	-	151
19	-	-	-	-	-	-	28	126	45	1	-	-	-	-	-	-	-	200
20	-	-	-	-	-	-	8	126	120	11	-	-	-	-	-	-	-	265
21	-	-	-	-	-	-	1	84	210	55	1	-	-	-	-	-	-	351
22	-	-	-	-	-	-	-	36	252	165	12	-	-	-	-	-	-	465
23	-	-	-	-	-	-	-	9	210	330	66	1	-	-	-	-	-	616
24	-	-	-	-	-	-	-	-	120	462	220	13	-	-	-	-	-	816
25	-	-	-	-	-	-	-	-	45	462	495	78	1	-	-	-	-	1081
26	-	-	-	-	-	-	-	-	10	330	792	286	14	-	-	-	-	1432
27	-	-	-	-	-	-	-	-	1	165	924	715	91	1	-	-	-	1897
28	-	-	-	-	-	-	-	-	-	55	792	1287	364	15	-	-	-	2513
29	-	-	-	-	-	-	-	-	-	11	495	1716	1001	105	1	-	-	3329
30	-	-	-	-	-	-	-	-	-	1	220	1716	2002	455	16	-	-	4410
31	-	-	-	-	-	-	-	-	-	-	66	1287	3003	1365	120	1	-	5842
32	-	-	-	-	-	-	-	-	-	-	12	715	3432	3003	560	17	-	7739
33	-	-	-	-	-	-	-	-	-	-	1	286	3003	5005	1820	136	1	10252
34	-	-	-	-	-	-	-	-	-	-	-	78	2002	6435	4368	680	18	13581
35	-	-	-	-	-	-	-	-	-	-	-	13	1001	6435	8008	2380	153	17991
36	-	-	-	-	-	-	-	-	-	-	-	1	364	5005	11440	6188	816	23833

A1687

$|XP_{3,n,m}|$

A-III

Path Length	Number of Colored Vertices																	$\lvert XP_{3,n}\rvert$
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
1	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1
2	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2
3	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1
4	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3
5	-	1	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2
6	-	-	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4
7	-	-	3	1	-	-	-	-	-	-	-	-	-	-	-	-	-	4
8	-	-	-	5	-	-	-	-	-	-	-	-	-	-	-	-	-	5
9	-	-	-	6	1	-	-	-	-	-	-	-	-	-	-	-	-	7
10	-	-	-	1	6	-	-	-	-	-	-	-	-	-	-	-	-	7
11	-	-	-	-	10	1	-	-	-	-	-	-	-	-	-	-	-	11
12	-	-	-	-	4	7	-	-	-	-	-	-	-	-	-	-	-	11
13	-	-	-	-	-	15	1	-	-	-	-	-	-	-	-	-	-	16
14	-	-	-	-	-	10	8	-	-	-	-	-	-	-	-	-	-	18
15	-	-	-	-	-	1	21	1	-	-	-	-	-	-	-	-	-	23
16	-	-	-	-	-	-	20	9	-	-	-	-	-	-	-	-	-	29
17	-	-	-	-	-	-	5	28	1	-	-	-	-	-	-	-	-	34
18	-	-	-	-	-	-	-	35	10	-	-	-	-	-	-	-	-	45
19	-	-	-	-	-	-	-	15	36	1	-	-	-	-	-	-	-	52
20	-	-	-	-	-	-	-	1	56	11	-	-	-	-	-	-	-	68
21	-	-	-	-	-	-	-	-	35	45	1	-	-	-	-	-	-	81
22	-	-	-	-	-	-	-	-	6	84	12	-	-	-	-	-	-	102
23	-	-	-	-	-	-	-	-	-	70	55	1	-	-	-	-	-	126
24	-	-	-	-	-	-	-	-	-	21	120	13	-	-	-	-	-	154
25	-	-	-	-	-	-	-	-	-	1	126	66	1	-	-	-	-	194
26	-	-	-	-	-	-	-	-	-	-	56	165	14	-	-	-	-	235
27	-	-	-	-	-	-	-	-	-	-	7	210	78	1	-	-	-	296
28	-	-	-	-	-	-	-	-	-	-	-	126	220	15	-	-	-	361
29	-	-	-	-	-	-	-	-	-	-	-	28	330	91	1	-	-	450
30	-	-	-	-	-	-	-	-	-	-	-	1	252	286	16	-	-	555
31	-	-	-	-	-	-	-	-	-	-	-	-	84	495	105	1	-	685
32	-	-	-	-	-	-	-	-	-	-	-	-	8	462	364	17	-	851
33	-	-	-	-	-	-	-	-	-	-	-	-	-	210	715	120	1	1046
34	-	-	-	-	-	-	-	-	-	-	-	-	-	36	792	455	18	1301
35	-	-	-	-	-	-	-	-	-	-	-	-	-	1	462	1001	136	1601
36	-	-	-	-	-	-	-	-	-	-	-	-	-	-	120	1287	560	1986

A7380

$|XP_{5,n,m}|$

A - IV

Path Length	Number of Colored Vertices 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	$\|XP_{5,n}\|$
1	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1
2	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2
3	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1
4	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3
5	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1
6	-	-	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4
7	-	-	1	1	-	-	-	-	-	-	-	-	-	-	-	-	-	2
8	-	-	-	5	-	-	-	-	-	-	-	-	-	-	-	-	-	5
9	-	-	-	3	1	-	-	-	-	-	-	-	-	-	-	-	-	4
10	-	-	-	-	6	-	-	-	-	-	-	-	-	-	-	-	-	6
11	-	-	-	-	6	1	-	-	-	-	-	-	-	-	-	-	-	7
12	-	-	-	-	-	7	-	-	-	-	-	-	-	-	-	-	-	7
13	-	-	-	-	-	10	1	-	-	-	-	-	-	-	-	-	-	11
14	-	-	-	-	-	1	8	-	-	-	-	-	-	-	-	-	-	9
15	-	-	-	-	-	-	15	1	-	-	-	-	-	-	-	-	-	16
16	-	-	-	-	-	-	4	9	-	-	-	-	-	-	-	-	-	13
17	-	-	-	-	-	-	-	21	1	-	-	-	-	-	-	-	-	22
18	-	-	-	-	-	-	-	10	10	-	-	-	-	-	-	-	-	20
19	-	-	-	-	-	-	-	-	28	1	-	-	-	-	-	-	-	29
20	-	-	-	-	-	-	-	-	20	11	-	-	-	-	-	-	-	31
21	-	-	-	-	-	-	-	-	1	36	1	-	-	-	-	-	-	38
22	-	-	-	-	-	-	-	-	-	35	12	-	-	-	-	-	-	47
23	-	-	-	-	-	-	-	-	-	5	45	1	-	-	-	-	-	51
24	-	-	-	-	-	-	-	-	-	-	56	13	-	-	-	-	-	69
25	-	-	-	-	-	-	-	-	-	-	15	55	1	-	-	-	-	71
26	-	-	-	-	-	-	-	-	-	-	-	84	14	-	-	-	-	98
27	-	-	-	-	-	-	-	-	-	-	-	35	66	1	-	-	-	102
28	-	-	-	-	-	-	-	-	-	-	-	1	120	15	-	-	-	136
29	-	-	-	-	-	-	-	-	-	-	-	-	70	78	1	-	-	149
30	-	-	-	-	-	-	-	-	-	-	-	-	6	165	16	-	-	187
31	-	-	-	-	-	-	-	-	-	-	-	-	-	126	91	1	-	218
32	-	-	-	-	-	-	-	-	-	-	-	-	-	21	220	17	-	258
33	-	-	-	-	-	-	-	-	-	-	-	-	-	-	210	105	1	316
34	-	-	-	-	-	-	-	-	-	-	-	-	-	-	56	286	18	360
35	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	330	120	452
36	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	126	364	509

A7381

$|XP_{7,n,m}|$

A-V

Path Length	Number of Colored Vertices																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	$\|XP_{7,n}\|$
1	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1
2	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2
3	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1
4	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3
5	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1
6	-	-	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4
7	-	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	-	1
8	-	-	-	5	-	-	-	-	-	-	-	-	-	-	-	-	-	5
9	-	-	-	1	1	-	-	-	-	-	-	-	-	-	-	-	-	2
10	-	-	-	-	6	-	-	-	-	-	-	-	-	-	-	-	-	6
11	-	-	-	-	3	1	-	-	-	-	-	-	-	-	-	-	-	4
12	-	-	-	-	-	7	-	-	-	-	-	-	-	-	-	-	-	7
13	-	-	-	-	-	6	1	-	-	-	-	-	-	-	-	-	-	7
14	-	-	-	-	-	-	8	-	-	-	-	-	-	-	-	-	-	8
15	-	-	-	-	-	-	10	1	-	-	-	-	-	-	-	-	-	11
16	-	-	-	-	-	-	-	9	-	-	-	-	-	-	-	-	-	9
17	-	-	-	-	-	-	-	15	1	-	-	-	-	-	-	-	-	16
18	-	-	-	-	-	-	-	1	10	-	-	-	-	-	-	-	-	11
19	-	-	-	-	-	-	-	-	21	1	-	-	-	-	-	-	-	22
20	-	-	-	-	-	-	-	-	4	11	-	-	-	-	-	-	-	15
21	-	-	-	-	-	-	-	-	-	28	1	-	-	-	-	-	-	29
22	-	-	-	-	-	-	-	-	-	10	12	-	-	-	-	-	-	22
23	-	-	-	-	-	-	-	-	-	-	36	1	-	-	-	-	-	37
24	-	-	-	-	-	-	-	-	-	-	20	13	-	-	-	-	-	33
25	-	-	-	-	-	-	-	-	-	-	-	45	1	-	-	-	-	46
26	-	-	-	-	-	-	-	-	-	-	-	35	14	-	-	-	-	49
27	-	-	-	-	-	-	-	-	-	-	-	1	55	1	-	-	-	57
28	-	-	-	-	-	-	-	-	-	-	-	-	56	15	-	-	-	71
29	-	-	-	-	-	-	-	-	-	-	-	-	5	66	1	-	-	72
30	-	-	-	-	-	-	-	-	-	-	-	-	-	84	16	-	-	100
31	-	-	-	-	-	-	-	-	-	-	-	-	-	15	78	1	-	94
32	-	-	-	-	-	-	-	-	-	-	-	-	-	-	120	17	-	137
33	-	-	-	-	-	-	-	-	-	-	-	-	-	-	35	91	1	127
34	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	165	18	183
35	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	70	105	176
36	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	220	240

C-I

$|XC_{-1,n,m}|$

Cycle Length	Number of Colored Vertices: 0	1	2	3	4	5	6	7	8	9	10	11	$\lvert XC_{-1,n}\rvert$
1	1	-	-	-	-	-	-	-	-	-	-	-	1
2	1	2	-	-	-	-	-	-	-	-	-	-	3
3	1	3	-	-	-	-	-	-	-	-	-	-	4
4	1	4	2	-	-	-	-	-	-	-	-	-	7
5	1	5	5	-	-	-	-	-	-	-	-	-	11
6	1	6	9	2	-	-	-	-	-	-	-	-	18
7	1	7	14	7	-	-	-	-	-	-	-	-	29
8	1	8	20	16	2	-	-	-	-	-	-	-	47
9	1	9	27	30	9	-	-	-	-	-	-	-	76
10	1	10	35	50	25	2	-	-	-	-	-	-	123
11	1	11	44	77	55	11	-	-	-	-	-	-	199
12	1	12	54	112	105	36	2	-	-	-	-	-	322
13	1	13	65	156	182	91	13	-	-	-	-	-	521
14	1	14	77	210	294	196	49	2	-	-	-	-	843
15	1	15	90	275	450	378	140	15	-	-	-	-	1364
16	1	16	104	352	660	672	336	64	2	-	-	-	2207
17	1	17	119	442	935	1122	714	204	17	-	-	-	3571
18	1	18	135	546	1287	1782	1386	540	81	2	-	-	5778
19	1	19	152	665	1729	2717	2508	1254	285	19	-	-	9349
20	1	20	170	800	2275	4004	4290	2640	825	100	2	-	15127
21	1	21	189	952	2940	5733	7007	5148	2079	385	21	-	24476
22	1	22	209	1122	3740	8008	11011	9438	4719	1210	121	2	39603
23	1	23	230	1311	4692	10948	16744	16445	9867	3289	506	23	64079
24	1	24	252	1520	5814	14688	24752	27456	19305	8008	1716	144	103682

AIG08

$|XC_{1,n,m}|$

C-II

Cycle Length	Number of Colored Vertices																	$\|XC_{1,n}\|$
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0
2	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2
3	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3
4	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2
5	-	5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5
6	-	3	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5
7	-	-	7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	7
8	-	-	8	2	-	-	-	-	-	-	-	-	-	-	-	-	-	10
9	-	-	3	9	-	-	-	-	-	-	-	-	-	-	-	-	-	12
10	-	-	-	15	2	-	-	-	-	-	-	-	-	-	-	-	-	17
11	-	-	-	11	11	-	-	-	-	-	-	-	-	-	-	-	-	22
12	-	-	-	3	24	2	-	-	-	-	-	-	-	-	-	-	-	29
13	-	-	-	-	26	13	-	-	-	-	-	-	-	-	-	-	-	39
14	-	-	-	-	14	35	2	-	-	-	-	-	-	-	-	-	-	51
15	-	-	-	-	3	50	15	-	-	-	-	-	-	-	-	-	-	68
16	-	-	-	-	-	40	48	2	-	-	-	-	-	-	-	-	-	90
17	-	-	-	-	-	17	85	17	-	-	-	-	-	-	-	-	-	119
18	-	-	-	-	-	3	90	63	2	-	-	-	-	-	-	-	-	158
19	-	-	-	-	-	-	57	133	19	-	-	-	-	-	-	-	-	209
20	-	-	-	-	-	-	20	175	80	2	-	-	-	-	-	-	-	277
21	-	-	-	-	-	-	3	147	196	21	-	-	-	-	-	-	-	367
22	-	-	-	-	-	-	-	77	308	99	2	-	-	-	-	-	-	486
23	-	-	-	-	-	-	-	23	322	276	23	-	-	-	-	-	-	644
24	-	-	-	-	-	-	-	3	224	504	120	2	-	-	-	-	-	853
25	-	-	-	-	-	-	-	-	100	630	375	25	-	-	-	-	-	1130
26	-	-	-	-	-	-	-	-	26	546	780	143	2	-	-	-	-	1497
27	-	-	-	-	-	-	-	-	3	324	1134	495	27	-	-	-	-	1983
28	-	-	-	-	-	-	-	-	-	126	1176	1155	168	2	-	-	-	2627
29	-	-	-	-	-	-	-	-	-	29	870	1914	638	29	-	-	-	3480
30	-	-	-	-	-	-	-	-	-	3	450	2310	1650	195	2	-	-	4610
31	-	-	-	-	-	-	-	-	-	-	155	2046	3069	806	31	-	-	6107
32	-	-	-	-	-	-	-	-	-	-	32	1320	4224	2288	224	2	-	8090
33	-	-	-	-	-	-	-	-	-	-	3	605	4356	4719	1001	33	-	10717
34	-	-	-	-	-	-	-	-	-	-	-	187	3366	7293	3094	255	2	14197
35	-	-	-	-	-	-	-	-	-	-	-	35	1925	8580	7007	1225	35	18807
36	-	-	-	-	-	-	-	-	-	-	-	3	792	7722	12012	4095	288	24914

A7387

$\left|XC_{3,n,m}\right|$

C-III

Cycle Length	Number of Colored Vertices 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	$\left\|XC_{3,n}\right\|$
1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0
2	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2
3	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3
4	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2
5	-	5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5
6	-	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2
7	-	-	7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	7
8	-	-	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	2
9	-	-	-	9	-	-	-	-	-	-	-	-	-	-	-	-	-	9
10	-	-	-	5	2	-	-	-	-	-	-	-	-	-	-	-	-	7
11	-	-	-	-	11	-	-	-	-	-	-	-	-	-	-	-	-	11
12	-	-	-	-	12	2	-	-	-	-	-	-	-	-	-	-	-	14
13	-	-	-	-	-	13	-	-	-	-	-	-	-	-	-	-	-	13
14	-	-	-	-	-	21	2	-	-	-	-	-	-	-	-	-	-	23
15	-	-	-	-	-	5	15	-	-	-	-	-	-	-	-	-	-	20
16	-	-	-	-	-	-	32	2	-	-	-	-	-	-	-	-	-	34
17	-	-	-	-	-	-	17	17	-	-	-	-	-	-	-	-	-	34
18	-	-	-	-	-	-	-	45	2	-	-	-	-	-	-	-	-	47
19	-	-	-	-	-	-	-	38	19	-	-	-	-	-	-	-	-	57
20	-	-	-	-	-	-	-	5	60	2	-	-	-	-	-	-	-	67
21	-	-	-	-	-	-	-	-	70	21	-	-	-	-	-	-	-	91
22	-	-	-	-	-	-	-	-	22	77	2	-	-	-	-	-	-	101
23	-	-	-	-	-	-	-	-	-	115	23	-	-	-	-	-	-	138
24	-	-	-	-	-	-	-	-	-	60	96	2	-	-	-	-	-	158
25	-	-	-	-	-	-	-	-	-	5	175	25	-	-	-	-	-	205
26	-	-	-	-	-	-	-	-	-	-	130	117	2	-	-	-	-	247
27	-	-	-	-	-	-	-	-	-	-	27	252	27	-	-	-	-	306
28	-	-	-	-	-	-	-	-	-	-	-	245	140	2	-	-	-	387
29	-	-	-	-	-	-	-	-	-	-	-	87	348	29	-	-	-	464
30	-	-	-	-	-	-	-	-	-	-	-	5	420	165	2	-	-	592
31	-	-	-	-	-	-	-	-	-	-	-	-	217	465	31	-	-	713
32	-	-	-	-	-	-	-	-	-	-	-	-	32	672	192	2	-	898
33	-	-	-	-	-	-	-	-	-	-	-	-	-	462	605	33	-	1100
34	-	-	-	-	-	-	-	-	-	-	-	-	-	119	1020	221	2	1362
35	-	-	-	-	-	-	-	-	-	-	-	-	-	5	882	770	35	1692
36	-	-	-	-	-	-	-	-	-	-	-	-	-	-	336	1485	252	2075

A7388

$\left|XC_{5,n,m}\right|$

C-IV

Cycle Length	Number of Colored Vertices																	$\|XC_{5,n}\|$
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0
2	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2
3	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3
4	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2
5	-	5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5
6	-	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2
7	-	-	7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	7
8	-	-	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	2
9	-	-	-	9	-	-	-	-	-	-	-	-	-	-	-	-	-	9
10	-	-	-	-	2	-	-	-	-	-	-	-	-	-	-	-	-	2
11	-	-	-	-	11	-	-	-	-	-	-	-	-	-	-	-	-	11
12	-	-	-	-	-	2	-	-	-	-	-	-	-	-	-	-	-	2
13	-	-	-	-	-	13	-	-	-	-	-	-	-	-	-	-	-	13
14	-	-	-	-	-	7	2	-	-	-	-	-	-	-	-	-	-	9
15	-	-	-	-	-	-	15	-	-	-	-	-	-	-	-	-	-	15
16	-	-	-	-	-	-	16	2	-	-	-	-	-	-	-	-	-	18
17	-	-	-	-	-	-	-	17	-	-	-	-	-	-	-	-	-	17
18	-	-	-	-	-	-	-	27	2	-	-	-	-	-	-	-	-	29
19	-	-	-	-	-	-	-	-	19	-	-	-	-	-	-	-	-	19
20	-	-	-	-	-	-	-	-	40	2	-	-	-	-	-	-	-	42
21	-	-	-	-	-	-	-	-	7	21	-	-	-	-	-	-	-	28
22	-	-	-	-	-	-	-	-	-	55	2	-	-	-	-	-	-	57
23	-	-	-	-	-	-	-	-	-	23	23	-	-	-	-	-	-	46
24	-	-	-	-	-	-	-	-	-	-	72	2	-	-	-	-	-	74
25	-	-	-	-	-	-	-	-	-	-	50	25	-	-	-	-	-	75
26	-	-	-	-	-	-	-	-	-	-	-	91	2	-	-	-	-	93
27	-	-	-	-	-	-	-	-	-	-	-	90	27	-	-	-	-	117
28	-	-	-	-	-	-	-	-	-	-	-	7	112	2	-	-	-	121
29	-	-	-	-	-	-	-	-	-	-	-	-	145	29	-	-	-	174
30	-	-	-	-	-	-	-	-	-	-	-	-	30	135	2	-	-	167
31	-	-	-	-	-	-	-	-	-	-	-	-	-	217	31	-	-	248
32	-	-	-	-	-	-	-	-	-	-	-	-	-	80	160	2	-	242
33	-	-	-	-	-	-	-	-	-	-	-	-	-	-	308	33	-	341
34	-	-	-	-	-	-	-	-	-	-	-	-	-	-	170	187	2	359
35	-	-	-	-	-	-	-	-	-	-	-	-	-	-	7	420	35	462
36	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	315	216	533

A7399

$|XC_{7,n,m}|$

C-V

Cycle Length	Number of Colored Vertices																	$\lvert XC_{7,n}\rvert$
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0
2	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2
3	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3
4	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2
5	-	5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5
6	-	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2
7	-	-	7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	7
8	-	-	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	2
9	-	-	-	9	-	-	-	-	-	-	-	-	-	-	-	-	-	9
10	-	-	-	-	2	-	-	-	-	-	-	-	-	-	-	-	-	2
11	-	-	-	-	11	-	-	-	-	-	-	-	-	-	-	-	-	11
12	-	-	-	-	-	2	-	-	-	-	-	-	-	-	-	-	-	2
13	-	-	-	-	-	13	-	-	-	-	-	-	-	-	-	-	-	13
14	-	-	-	-	-	-	2	-	-	-	-	-	-	-	-	-	-	2
15	-	-	-	-	-	-	15	-	-	-	-	-	-	-	-	-	-	15
16	-	-	-	-	-	-	-	2	-	-	-	-	-	-	-	-	-	2
17	-	-	-	-	-	-	-	17	-	-	-	-	-	-	-	-	-	17
18	-	-	-	-	-	-	-	9	2	-	-	-	-	-	-	-	-	11
19	-	-	-	-	-	-	-	-	19	-	-	-	-	-	-	-	-	19
20	-	-	-	-	-	-	-	-	20	2	-	-	-	-	-	-	-	22
21	-	-	-	-	-	-	-	-	-	21	-	-	-	-	-	-	-	21
22	-	-	-	-	-	-	-	-	-	33	2	-	-	-	-	-	-	35
23	-	-	-	-	-	-	-	-	-	-	23	-	-	-	-	-	-	23
24	-	-	-	-	-	-	-	-	-	-	48	2	-	-	-	-	-	50
25	-	-	-	-	-	-	-	-	-	-	-	25	-	-	-	-	-	25
26	-	-	-	-	-	-	-	-	-	-	-	65	2	-	-	-	-	67
27	-	-	-	-	-	-	-	-	-	-	-	9	27	-	-	-	-	36
28	-	-	-	-	-	-	-	-	-	-	-	-	84	2	-	-	-	86
29	-	-	-	-	-	-	-	-	-	-	-	-	29	29	-	-	-	58
30	-	-	-	-	-	-	-	-	-	-	-	-	-	105	2	-	-	107
31	-	-	-	-	-	-	-	-	-	-	-	-	-	62	31	-	-	93
32	-	-	-	-	-	-	-	-	-	-	-	-	-	-	128	2	-	130
33	-	-	-	-	-	-	-	-	-	-	-	-	-	-	110	33	-	143
34	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	153	2	155
35	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	175	35	210
36	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	9	180	191